Shell-based Peridynamics for the Modelling of Isotropic and Composite Structures

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Contents

Preface

With great humility, I present this Peridynamics monograph, which is a culmination of my research based on my doctoral thesis. This work aims to provide valuable insights and references for researchers and academics in the field of Peridynamics.

I am deeply grateful for the opportunity to undertake this endeavour. First and foremost, I would like to express my sincere appreciation to my doctoral supervisor Professor Naceur Hakim. His unwavering guidance and support have been instrumental in shaping my academic pursuits. His wisdom and expertise have had a profound impact on my growth in the field of Peridynamics. I would also like to extend my heartfelt gratitude to my family and friends. Their steadfast support and encouragement have consistently been a source of strength throughout my journey. Throughout the process of writing this monograph, I have been fortunate to receive assistance and collaboration from many colleagues. I would like to express my gratitude to those who have generously shared their knowledge and insights, enriching the content of this work. Lastly, I would like to express my profound appreciation to the Cambridge Scholars Publishing, financial support from Entrepreneurship and Innovation Support Program for Chongqing Overseas Returnees (cx2023003), and financial support from National Natural Science Foundation of China (62303079). I hope that this monograph will serve as a valuable resource and contribute to the further development of Peridynamics.

In closing, I humbly present this Peridynamics monograph as a testament to the collective efforts and support from my supervisor, family, and friends. May it inspire and contribute to the ongoing advancements in the field of Peridynamics.

Ruqing BAI

CHAPTER I

Fundamental Peridynamics

1.1 Introduction

Nowadays, numerical simulation is commonly established as a cost-effective alternative for understanding the complex behavior of structures in a wide range of applications, including civil [1], mechanical [2], aeronautical [3], and aerospace engineering [4]. Generally, the deformation of a structure is often described using ordinary/partial differential equations (ODEs/PDEs) and cannot be easily solved by analytical methods [5]. In the process of numerical simulation, the domain of the physical problem is divided into a collection of sub-domains, each represented by sets of equations that can be expressed by ODEs/PDEs. Then, all sets of equations are systematically recombined into the final algebraic system of nonlinear equations to mimic the complex physical phenomenon.

Among all numerical techniques, the Finite Element (FE) method is well established and represents the most widespread numerical method for simulating multi-physics engineering problems [6]. In the FE technique, the physical domain is subdivided into a ”finite number of elements,” and nodal state variables are generally used to map an approximate solution inside the element by means of ”shape functions.” The elements are then assembled to generate an algebraic system of equations [7], which is solved using various numerical solution strategies. This method is now well established and has led to the development of numerous commercial FE software such as ABAQUS, ANSYS, LS-DYNA, etc.

In the FE method, the predefined mesh can present some shortcomings and often restricts the use of FE in certain situations. Firstly, it is common knowledge that creating a ”good” mesh is not only time-consuming but also cannot always be generated automatically. Constructing a ”good” mesh for complex geometries is typically much more time-consuming than the FE simulation itself [8]. Secondly, the shape of the element also affects the accuracy of the FE solution, and poor element shapes often cause divergence in nonlinear analysis, leading to inaccurate results. Moreover, standard FE is not suitable for simulating discontinuous problems because spatial derivatives are undefined in the presence of dislocations, microcracks, etc. Although many researchers have developed solutions, such as cohesive zone elements (CZE) [9] and extended finite element method (XFEM) [10], to handle cracks, there remains a major challenge within the FE framework, which assumes that a body remains continuous when it deforms.

The difficulties encountered in FE can be overcome by using local methods such as molecular dynamics (MD) [11] or the atomic lattice model (ALM) [12]. However, MD and ALM have focused on providing a fundamental understanding of the basic physical processes of dynamic fracture, rather than being predictive [13]. Additionally, both methods require significant computational resources, and ALM is insufficient to model fracture processes in real-life structures.

The nonlocal continuum theory was introduced to account for long-range effects [14, 15, 16]. In nonlocal continuum theory, a material point interacts only with other material points within a fixed radius region, denoted as δ, and the interaction is assumed to vanish beyond δ. When δ becomes infinitely small, nonlocal theory converges to classical continuum mechanics (CCM) [17], or conversely, it approaches MD. Therefore, nonlocal theory establishes a connection between CCM and MD. The relationship between the local model, nonlocal model, and molecular dynamics is illustrated in Figs. 1.1.1 (a), (b), and (c). According to the reference [18], nonlocal theory is a relatively effective alternative for solving complex nonlinear problems, as it is computationally less demanding than CCM or MD when considering long-range effects.

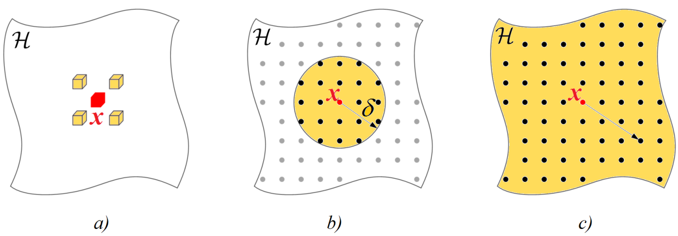

Figure 1.1.1. Relationship between: (a) local model, (b) nonlocal model, and (c) molecular dynamics.

1.2 Nonlocal methods

Due to the nature of nonlocal theory, various types of nonlocal theories have been introduced that use the displacement field instead of the derivatives of the state variable field [16, 19, 20]. However, this was initially developed only for a one-dimensional medium [16, 20]. A three-dimensional nonlocal model was derived by approximating a discrete periodic lattice structure as continuous media [19]. More recently, the Peridynamic (PD) method was introduced to solve discontinuous problems without the need for spatial derivative equations [21]. Compared to previous nonlocal theories [16, 19, 20], Peridynamics is more versatile as it can handle one-, two-, and three-dimensional media. In contrast to traditional nonlocal theory [19], Peridynamics provides a nonlinear material response with respect to displacements. Furthermore, in Peridynamics, discontinuous problems are treated as part of the solution rather than as part of the problem.

Due to the flexibility of nonlocal theory (meshfree/meshless) and the non-use of classical mesh, meshfree methods have become one of the hottest topics in computational mechanics. An increasing number of researchers are devoting their investigations to the development of meshfree methods, and various meshfree methods have been proposed and used to solve ODEs and PDEs. Meanwhile, meshfree methods are being applied to an increasingly wide range of practical engineering applications. Compared to the form of PDEs used in the computation process, meshfree methods can be classified into three distinct groups:

1.2.1 Methods based on strong formulation

To estimate the strong-form of PDEs using meshfree methods, PDEs are usually discretized at points using various forms of collocation, such as Peridynamics (PD) [21, 22], the smoothed particle hydrodynamics method (SPH) [23, 24], the finite points method (FPM) [25, 26], and the meshfree collocation method (MCM) [27, 28]. PDEs in their strong-form are discretized straightforwardly without the need for variational formulation, hence eliminating the requirement for numerical integration. The resulting discretized equations are simple and fast to implement, making the methods truly meshfree. However, these methods can often be unstable and less accurate, especially in cases of non-uniform nodal distribution or irregular computation domains.

1.2.2 Methods based on weak formulation

In weak-form-based methods, the PDEs of a problem are first converted into integral equations, thus the field variables require only half the order of continuity compared to those using the strong formulation [29]. Integral operations can regularize the solution, making meshless methods based on the weak formulation more stable and accurate. Although very accurate in solving numerous different engineering problems, this type of meshfree method is not considered ”truly” meshfree, as it still requires background cells (FE mesh) for the integral operation of the weak forms [30].

This family of meshfree methods has been under active investigation by researchers since the early 1990s. Typically, it includes methods such as the diffuse element method (DEM) [31] generated by moving least squares (MLS) [32], the element free galerkin (EFG) method [33], the radial point interpolation method (RPIM) [34], and the reproducing kernel particle method (RKPM) [35], which improve the SPH approximation to satisfy consistency requirements using correction functions.

To overcome the drawback of necessitating an integration background mesh, local weak-form methods using the local Petrov-Galerkin approach were proposed by Atluri [36], known as the meshless local petrov-galerkin (MLPG) method. Other typical local weak-form methods include the method of finite spheres (MFS) [37], developed using the MLPG principle, and the hp-cloud method [38]. It is important to note that when meshfree local weak-form methods employ a delta function as the weight function, they become meshfree strong-form methods.

1.2.3 Methods based on weak-strong formulation

This family of methods was first developed by GR Liu and Gu [39]. In this approach, both the strong-form and the local weak-form are used to discretize the same set of PDEs, but they are applied to different groups of points, each carrying different types of equations/conditions. The strong formulation is used for all internal nodes and nodes on the essential boundaries. The local weak form is used only for nodes near boundaries with derivative boundary conditions, which are difficult to handle by the collocation method. Fewer background cells are used for integration compared to the weak-form methods, making this method more stable and efficient.

1.3 Peridynamics and its applications

The main difference between Peridynamics and classical continuum mechanics is that the former is based on integral equations, rather than differential equations. In Peridynamics, displacements and internal forces are allowed to have discontinuities and other singularities without the need for special crack growth treatment.

Essentially, the solid is discretized into particles, where each particle, denoted as $\boldsymbol{x}$, interacts with its surrounding particles, denoted as $\boldsymbol{x'}$, within a finite distance, δ, which limits an influence zone called the "*Horizon*" $\mathcal{H}_x$ (see Fig. 1.3.1). Interactions within the Horizon occur through pairwise forces, $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{x'})$ (force per unit volume squared), between $\boldsymbol{x}$ and $\boldsymbol{x'}$, referred to as "*bonds*" representing the material cohesion and depending on the material behavior [40].

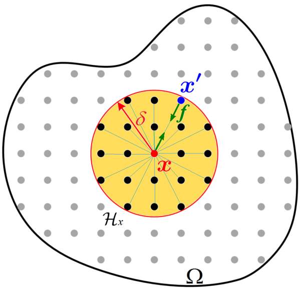

Figure 1.3.1. Principle of the Peridynamics.

Damage can be introduced into Peridynamics by allowing the "*bonds*" to break irreversibly. Breakage occurs when a "*bond*" is stretched in tension (or possibly compression) beyond a prescribed critical value, representing the maximum strength of the material.

Peridynamics has three different categories [41]: bond-based Peridynamics (BPD), state-based Peridynamics (SPD), and non-ordinary state-based Peridynamics (NSPD). As an example of standard Bond-Based Peridynamics, the particle j with volume dV_j exerts on particle i with volume dV_i a force $\boldsymbol{f}(\boldsymbol{x}_i, \boldsymbol{x}_j)dV_idV_j$, called a "*pairwise bond*" force density, as shown in Fig. 1.3.2.

In BPD, $\boldsymbol{f}(\boldsymbol{x}_i, \boldsymbol{x}_j)$ is based on the relative position vector $\boldsymbol{s} = \boldsymbol{x}_j - \boldsymbol{x}_i$ between the two particles i and j in the deformed configuration C (Fig. 1.3.2). It is defined as a force per unit volume squared and is given by:

$$\boldsymbol{f}(\boldsymbol{x}_i, \boldsymbol{x}_j) = f(\boldsymbol{x}_i, \boldsymbol{x}_j)\, \boldsymbol{t} \qquad \text{with} \qquad \boldsymbol{t} = \frac{\boldsymbol{s}}{|\boldsymbol{s}|}. \tag{1.3.1}$$

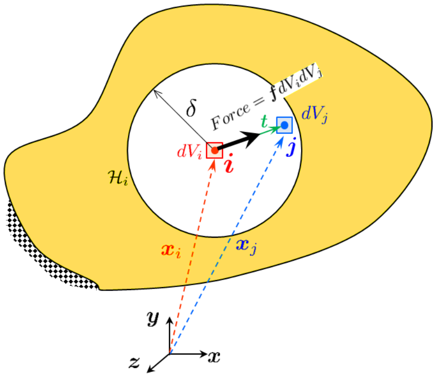

Figure 1.3.2. Principle of Bond-Based Peridynamics for continua.

It is also assumed that the bond force density $f(\boldsymbol{x}_i, \boldsymbol{x}_j)$ derives from an energy density known as "*micropotential*" (energy per unit volume squared). Peridynamics provides the ability to link different length scales and can be viewed as the continuum version of MD. This method addresses multiphysics and multiscale failure prediction within a common framework. Moreover, Peridynamics is a truly particle-based method that uses only the particle distribution as a computational basis and is based on a strong formulation of the governing equations, which are directly discretized. In recent years, Peridynamics has garnered extensive interest from researchers in solving a wide range of problems involving various types of structures.

- ***One-dimensional structures:*** The one-dimensional dynamic response of an infinite rod was investigated by Weckner *et al.* [42] to study the effects of long-range forces. The authors reported that the significant dispersion characteristic occurring in an infinite rod can be easily captured by Peridynamics. This nonlinear dispersion can be reduced to the local dispersive relation by considering specific boundary conditions when the "*Horizon*" tends to zero [43, 44]. The 1D Peridynamics model has been used to solve heat transfer problems, including those involving cracks [45, 46]. For instance, Peridynamics has been investigated for modeling 1D heat conduction problems in a wide range of materials [47, 48, 49, 50], including brittle structures made of glass and rock [51, 52, 53].

- ***Beams and plates:*** State-Based Peridynamics were used to analyze the deformation of structural beams using the Euler-Bernoulli model [54, 55], and the Kirchhoff-Love plate model was developed by Grady *et al.* [56] based on Non-Ordinary State-Based Peridynamics. Peridynamics dispersions were studied and compared with classical theory, and the Peridynamics results closely matched the physical response of structures [57, 58, 59]. For failure prediction, a beam with an offset notch was implemented and simulated to capture crack initiation and growth [60, 61]. Modal analysis of beams, including those with cracks, has been conducted by Freimanis *et al.* [62]. Kilic *et al.* [63] studied crack growth in glass plates.

- ***Composite structures:*** Predicting the failure of composite structures remains a challenge. However, using Peridynamics, damage is simulated in a much more realistic manner compared to the classical continuum method. Madenci and Oterkus [41] developed the Peridynamics laminate theory, which includes transverse normal and shear deformations. Among the Peridynamics composite models, the deformation of composite laminates subjected to uniaxial [64] and biaxial loading [65] was predicted. Colavito *et al.* [66] and Xu *et al.* [67] investigated the delamination process of composite plates under low-velocity impact. Oterkus and Madenci [68] employed a similar concept to analyze laminates with quasi-isotropic lay-ups. Hu *et al.* [69] developed a Peridynamics composite model that accounts for the variation of bond micromodulus based on the angle between the bond direction and fiber orientation. As an extension of this model, Hu *et al.* [70] developed an energy-based approach to remove bonds located between adjacent plies, simulating delamination under different mode mixity conditions. Moreover, fiber-reinforced composite plates with defects, such as central cracks, central holes, and pre-existing cracks, were presented to predict the correct failure mechanisms of matrix cracking, fiber breakage, and delamination without resorting to any special treatments [41, 71, 72, 73]. More details on the various applications of Peridynamics in composite structures can be found in the cited source [74].

- ***Multi-scale modeling:*** The emergence of advanced manufacturing technologies, such as 3D printing [75, 76, 77, 78, 79], has facilitated the design of microstructured materials. However, microscale defects, including micro-cracks and voids, continue to pose challenges in fully modeling these materials. Multiscale methodologies have emerged as a promising solution to address these challenges. Within the peridynamic framework, several approaches have been proposed. Silling [21] introduced the coarsening approach to represent microstructural behavior using fewer degrees of freedom, initially for 1D problems. This approach was extended to 2D structures by Galadima *et al.* [80]. Although limited to static conditions, the coarsening approach shows promise. Another noteworthy approach is model order reduction using static condensation, as proposed by Galadima et al. This methodology reduces the number of degrees of freedom, similar to the coarsening approach, but allows for the analysis of both static and dynamic problems. Homogenization techniques within the peridynamic framework have also been developed. Madenci *et al.* [81] introduced the peridynamic unit cell homogenization approach, enabling the determination of thermoelastic properties in heterogeneous microstructures with defects.

- ***Fracture analysis:*** The application of Peridynamic theory has been instrumental in addressing numerous fracture-related challenges. Kilic and Madenci [63] explored dynamic crack propagation in quenched glass plates. Foster *et al.* [82] introduced an energy-based failure criterion, causing irreversible Peridynamic bond breakage when the stored strain energy density surpasses a critical value within the bond. Within the Peridynamic PD framework, Dipasquale *et al.* [83] delved into various failure criteria, examining both critical stretch-based and critical energy density-based criteria in continuum and discrete forms. Additionally, Hu *et al.* [84] derived a nonlocal J-integral through a bond-based formulation, utilizing the central difference algorithm to obtain derivatives of physical quantities. Extending this approach, Panchadhara and Gordon [85] employed nonlocal stress intensity factors to assess crack initiation and propagation. In recent studies, Imachi *et al.* [86, 87, 88] scrutinized displacement fields and the collapsed stress tensor within the Peridynamic framework. Derivatives of physical quantities were approximated using the moving least squares (MLS) [89] method to compute dynamic stress intensity factors (DSIFs). These DSIFs were subsequently applied in modeling crack propagation and arrest.

1.4 Motivations of the book

The Peridynamics theory is a relatively new method, and therefore it requires further investigations. Hence, this book introduces some complements and improvements for the Peridynamics theory concerning isotropic and composite structures.

As it is known thin structure models accounting for transverse shear deformation may lead to the shear locking phenomenon [90] which affects the accuracy of the Peridynamics [57].

As it has been reported in FE formulation, shear locking in thin structures can be alleviated by using a reduced integration technique [91, 92, 93]. For the sake of stability and generality we propose in this work a mixed formulation (inspired from the one used in FE) which combines the Peridynamics with a mixed formulation [94, 95, 96, 97] to alleviate the shear locking problem which arises especially when thickness of beam or plates tends to zero. Meanwhile, the mixed formulation concept will be extended for multilayered structures and fiber-reinforced composites with very thin plies.

Although the Peridynamics has been already widely used for the study of isotropic and composite structures in statics and dynamics including cracks, however structures under low-velocity impact using the Peridynamics with different boundary conditions are currently lacking. Moreover, the composite lamina with pre-existing crack will be studied in this work, by including matrix cracking and fiber breakage under different angles of the fibers. Besides, the fiber-reinforced composites under quasi-static and dynamic loading conditions will be investigated, to have a complete overview of the performances of the Peridynamics.

1.5 Outline of the book

This book is organized into five Chapters:

- Chapter I presents a general introduction including a review of the Peridynamics.
- In Chapter II, the Peridynamic formulation for continuum media will be presented. Then basics of the Peridynamic equations of motion and damage definition will be detailed. Some numerical applications will be investigated.
- In Chapter III, the Peridynamics method for the modeling of beam structures is introduced. At first the continuum model of the Timo-

shenko beam will be recalled with the basic assumptions, kinematics and constitutive relations. Then a specific Peridynamics Timoshenko beam model will be addressed. Two techniques for the alleviation of the shear locking problem are introduced, namely the reduced integration method and the mixed formulation method. The resulting Peridynamic Timoshenko beam model does not suffer from shear locking phenomenon. Several numerical applications of beam structures are discussed, showing the effectiveness of the proposed Peridynamic Timoshenko beam model.

- In Chapter IV, the Peridynamics method for modeling plate structures is discussed. Initially, we revisit the continuum model of the Reissner-Mindlin plate, outlining its fundamental assumptions, kinematics, and constitutive relations. Subsequently, we delve into a specific Peridynamics Reissner-Mindlin plate model, addressing the shear locking problem through the introduction of two techniques: the reduced integration method and the mixed formulation method. Consequently, the resulting Peridynamic Reissner-Mindlin plate model is free from shear locking phenomena. We further discuss several numerical applications of plate structures, demonstrating the effectiveness of the proposed Peridynamic Reissner-Mindlin plate model.

- In Chapter V, the Peridynamic method will be extended for the modeling of fiber-reinforced thin composite structures. At first, the Peridynamic approach for the modeling of a lamina will be discussed and crack propagation prediction will be detailed. Then, the Peridynamic method will be extended to analyze composite laminate structures. Finally, some numerical applications will be presented and validated by comparison to reference solutions from the literature.

1.6 Conclusion

This Chapter provided an overview of the strengths and limitations of the classical FE method, along with the advancements proposed by scholars within the FE framework to address discontinuity issues. The nonlocal theory, bridging classical continuum mechanics and molecular dynamics, was introduced. Notably, the Peridynamics method, a popular nonlocal theory, has garnered attention for its effectiveness in tackling discontinuity problems. We summarized the latest applications of the Peridynamics method in one-dimensional structures, beams, plates, composite structures, multiscale modeling, and fracture analysis. Additionally, we outlined the motivation behind this book and provided an overview of the subsequent Chapters. This discussion serves as a foundation for readers to gain a comprehensive understanding of the Peridynamics method and its diverse applications.

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