

# Electron Beams in a Plasma



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By

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## PREFACE

In this book the author summarizes and generalizes the results of scientific research and applications of a very diverse field of low temperature plasma physics - the interaction of electron beams with plasma and, as a consequence of this interaction, a beam plasma discharge. Of course, this study is incomplete - the author has his own many years of experience as an experimenter in some parts of this field, and therefore a very limited overview of the theory of the processes under study is given here. Probably the volume of the individual sections is not proportionate to the volume and to importance of the results achieved in the various fields of research and application. Nevertheless the author hopes that this review will be useful as introduction to this field for undergraduate and graduate students specializing in plasma physics, as well as for other specialists in various fields of physics and technology.

The author expresses deep gratitude to Prof. E.M. Oks and Prof. P.S. Strelkov, who took an active part in the preparation of this book, I.A. Sorokin and R.A. Selivanov for assistance in technical design, L. Alekseeva for very carefully editing of the text.

# 1. INTRODUCTION

Intensive researches in processes of collective interaction of an electron beam with plasma begins with a phenomenon discovered independently in theoretical works of A.I. Akhiezer and Y. B. Fainberg and D. Bom and E.P. Gross in 1949. They revealed that when the electron beam moves through the plasma, instability develops, which manifests itself in increased fluctuations of velocity and density of plasma and beam electrons in the frequency range, close to the Langmuir frequency of the plasma. The first experimental confirmation of this effect is described in (Kharchenko 1960), where a transformation of the velocity distribution of the electron beam passing through an independently created plasma was discovered: initially almost monochromatic beam scatters, and its distribution function over velocities takes the form of a plateau. This work stimulated a large stream of theoretical and experimental works on this topic. A linear theory of the interaction of the beam and plasma in two qualitatively different current cases: monochromatic (“cold”):  $\Delta V < V_0$  and “hot”: ( $\Delta V \sim V_0$ ) beam. A quasi-linear theory describing the transformation of the distribution function of plasma with a beam in the case of a hot beam was created. Later a nonlinear theory of the interaction of cold beam with plasma was built. At the same time, experimental studies of the interaction of “cold” beam and plasma, as well as a system where the scattering of the beam over longitudinal velocities occurs due to propagation in a magnetic field is inhomogeneous along its length.

Quite quickly after start of this work, it was found that at a sufficiently high electron density of the beam dense plasma is generated with a density, significantly exceeding (by 2-3 orders of magnitude) the density of the primary plasma formed due to ionization of the gas at collisions of beam electrons with gas molecules. L.D. Smullin and W.D. Getty first described this effect and explained it: ionization is carried out by electrons heated in high-frequency fields which generated during the development of beam instability (Smullin and Getty 1962; Getty and Smullin 1963). They gave to

this effect the name “beam plasma discharge” (BPD). In these and subsequent works thresholds of ignition of the BPD in a high magnetic field have been experimentally studied, and were determined spectra of oscillations and types of instabilities responsible for the formation and maintenance of BPD, energy relationships in BPD.

Some years later, it was found that BPD can develop without external magnetic field (Popovich et al. 1973).

Proposals for applications of this effect appeared rather quickly: for heating plasma in thermonuclear installations (Alexeff et al. 1963), for creating microwave generators and amplifiers (Bernashevsky et al. 1965), in plasma chemistry (Ivanov et al., 1973; Ivanov and Soboleva 1978), in active geophysical experiments (Cambou et al. 1975; 1978). The formation of BPD at injection of the electron beam from the rocket into the lower ionosphere was discovered.

First works on the theory of BPD appeared in 1976 (Lebedev et al. 1976; Galeev et al. 1977). They were stimulated, on the one hand, by starting research and development of plasma microwave electronic devices, on the other hand, work on the application of electron beams in active geophysical experiments.

Work on thermonuclear research was quickly stopped: it was revealed that electrons can be heated in BPD, but not ions. Plasma processing technology also has not been developed: although due to strong disequilibrium ( $T_e \gg T_i$ ) it is possible to carry out reactions that are not implemented in equilibrium chemistry, and obtain new materials, but due to the low density of the reagents productivity of such processes were too low for practical applications. Further direction of plasma chemistry found development in technologies of synthesis and processing of thin films for micro- and nanoelectronics and modification of the surface of dielectrics and metals.

Work on plasma electronics was stimulated by high gain coefficient of a signal compared to classical devices such as traveling wave tubes (TWT). The highest electronic efficiency expected in accordance with the quasilinear theory (Vedenov 1963; Shapiro and Shevchenko 1968) and the



absence of metallic slowing structures seemed promising for creating plasma amplifiers and generators of millimeter band.

However, experiments (Berezin 1965; Shustin 1969) showed that the electron efficiency was not so high: it was revealed that a quasilinear theory is not applicable in conditions of real experiments. The actual efficiency turned out to be significantly lower than the electronic efficiency: the conversion of excited plasma waves into electromagnetic waves of waveguide structures occurs with significant losses. The hope of implementing millimeter-wave plasma devices also did not materialize: it turned out to be impossible to create a plasma of the required density in a narrow channel, while maintaining a significant part of the electron beam with the initial speed.

A feature of BPD is the generation of intense broadband oscillations of density of electrons and corresponding radio emission. In the 60s, plasma noise signal generators were developed for use in electronic warfare devices. However, they could not compete with the vacuum noise generators created in the same years based on circuits with traveling wave tubes (TWT): their efficiency was significantly lower, they were poorly controlled, although the possibilities for tuning the frequency range were higher than vacuum noise oscillators. Further, plasma microwave devices were implemented in the form of hybrid devices using metal slow-wave structures filled with plasma (Zavjalov et al. 1994). Subsequently, this direction led to the birth of relativistic plasma electronics (Barker and Schamiloglu 2001; Kuzelev et al. 2018).

The ability to create plasma with an electron beam quickly gave rise to the proposal to use ribbon electron beams to create quickly switchable plasma reflectors in antenna technology (Fernsler et al. 1998). This work was carried out at the Naval Research Laboratory in the USA in the 90s, but, apparently, did not find wide application. However, this work stimulated the creation of emitters of large-width ribbon beams. The same laboratory later began to develop work on the application of beam excited plasma with ribbon electron beams (energy 1-2 keV, current density 1-20 mA/cm<sup>2</sup>) in plasma processing technologies for microelectronics (Meger et al. 2001)

The most widespread use of beam plasma in microelectronics was achieved in work at the LAPPS facility created at the Naval Research Laboratory in the USA (Walton et al. 2015). The history of the use of beam plasma in plasma technologies for micro- and nanoelectronics is described in more detail in that paper.

In the works of E.M. Oks and his collaborators at the Institute of High-Current Electronics of the Russian Academy of Sciences and the Tomsk University of Control Systems and Radioelectronics (TUSUR) another area of application of beam plasma: the synthesis and modification of materials, primarily dielectrics was developed (see reviews Yushkov et al. 2022; Klimov et al. 2017) and the works cited there). In this case, sources of electron beams of various power with a plasma cathode, operating in the forevacuum energy range (Oks 2006; Klimov et al. 2020), and plasma sources created on their basis for technological installations are used (Klimov et al. 2017; Burdovitsin and Oks 2021).

Since 2003 study and applications of BPD in a weak magnetic field in micro- and nanoelectronics technologies (Shustin et al., 2009; 2014) began.

Both research on relativistic microwave electronics and technological applications of a beam plasma and BPD currently continue to develop and find practical applications.

## 2. ELECTRON BEAM EXCITED PLASMA

Following to papers (Fernsler et al.1998; Meger et al. 2001), we will give a description of beam plasma physics.

When an electron beam with an energy in the range  $10^2$ - $10^4$  eV propagates through a gas with a pressure  $<10^3$  Pa, the main processes leading to the loss of its energy are dissociation (in a molecular gas), ionization and excitation of gas molecules.

The continuity equation determines the balance of production and loss of charged particles in a stationary plasma:

$$\beta n^2 - D_a \nabla^2 n = \frac{J_b}{e} N \sigma(W_b). \quad (2.1)$$

Here  $\beta$  is the electron-ion recombination coefficient,  $D_a$  - the ambipolar diffusion coefficient,  $j_b$  - the beam current density,  $N$  - the gas density,  $\sigma(W_b)$  - the effective beam ionization cross section.

For  $W_b \sim 1$  keV  $\sigma \sim 10^{-16}$  cm<sup>2</sup> in oxygen, argon and similar gases. The dissociation coefficient is very small in atomic gases, and in molecular gases it is of order  $10^{-7}$  cm<sup>3</sup>/s. The ambipolar diffusion coefficient depends on the magnetic field; at magnetic field induction  $\sim 100$  Gs (LAPPS installation conditions) can be approximated by the value  $10/p$  cm<sup>2</sup>/s, where  $p$  is the gas pressure in Pa.

Using (2.1), it can be shown that recombination limits the ion flux arriving at a surface distant from the beam by the value

$$F_{im}(\Delta x) = -D_a \nabla n \leq 35 \frac{D_a^2}{\beta \Delta x^3}. \quad (2.2)$$

Here  $\Delta x$  is the distance from the edge of the beam to this surface. Such flows come from both sides of the ribbon beam.

Dissociative recombination converts each molecular ion into 2 free radicals, the number of radicals accordingly increases as

$$F_r = \frac{I_b N}{e} (\sigma_e + \sigma_d) - 2F_{im}(\Delta x) \quad (2.3)$$

Here  $\sigma_d$  is the dissociation cross section,  $I_b$  is the beam current.

Since in atomic gases  $\beta = 0$ , there is no flow of radicals in them, and the flow of ions from the beam is constant:

$$F_{ia} = I_b N \sigma_e / 2e \quad (2.4)$$

According to (2.2) – (2.4), the flows of ions and radicals can be adjusted independently by varying the gas composition and distance  $\Delta x$ . As an example, we point out that a beam 1 cm wide with a current density  $j_b = 10 \text{ mA/cm}^2$  creates the flows of ions and radicals of more than  $10^{16} \text{ cm}^{-2}/\text{s}$  with an energy of  $<6 \text{ eV}$  at a pressure of 5 Pa.

When the main mechanism of charged particles loss is recombination, the plasma density on the axis is determined as

$$n_0 = \frac{I_b N \sigma_e}{e D_a} \Delta x \quad (2.5)$$

For example, at  $\Delta x > 20 \text{ cm}$  in argon with the parameters defined above, the density reaches  $10^{13} \text{ cm}^{-3}$ .

The energy loss of an electron beam when passing through a gas is defined as

$$dW_b/dz \sim (ZN \ln \varepsilon_b) / W_b, \quad (2.6)$$

Here  $W_b$  is the beam electron energy,  $z$  is the length of the propagation region,  $Z$  and  $N$  are the atomic number and density of the gas.

For commonly used gases (Ar, N<sub>2</sub>, O<sub>2</sub>), a beam with an electron energy of 2 keV while traveling 1 m at a gas pressure of 0.5 Pa, loses approximately 15% of its energy.

When electrons collide with gas molecules, the beam is also scattered at angles. To minimize beam scattering, a magnetic field parallel to the beam is used. In practice, a magnetic field of 100-200 Gs is sufficient to maintain  $r_b=1$  cm at a distance of 1 m.

When a beam propagates in a plasma, the beam instability may occur, however, if the beam scattering by speed satisfies the condition  $\Delta v_b \geq (\pi \omega_b^2 / \omega_p)^{1/2}$ , where  $v_b$  is the collision frequency,  $\omega_p = (e^2 n_p / m \epsilon_0)^{1/2}$  is the electron plasma frequency, beam instability is strongly suppressed. This condition is violated at a significant beam current density ( $>100$  mA/cm<sup>2</sup>), leading to formation of BPD.

Experimental studies of electron beam excited plasma (EBEP) were carried out at the LAPPS facility that will be described in Chapter 6. The experiments showed, that in a plasma dominated by diffusion loss (argon) mean ion energy is about 3 eV, and in a plasma dominated by electron-ion recombination in the gas phase (O<sub>2</sub>, Cl<sub>2</sub>) this value doesn't exceed 1.5 eV.

Summarizing the results of the information presented above, we can draw the following conclusions.

When the pressure of the neutral gas through which the beam propagates is less than  $10^{-2}$  Pa, the density of the electron beam excited plasma (EBEP) does not exceed the density of the beam.

At low beam density ( $\sim 10$  mA/cm<sup>2</sup>) and at gas pressures above 1 Pa, EBEP density exceeds greatly the electron beam density. It's value strongly depends on the type of gas and geometry of the system. Main parameters of the described technological installations are: electron density  $\sim 10^{10}$  cm<sup>-3</sup>, electron temperature  $\sim 3$  eV for plasma of atomic gases, 1-1.5 eV for molecular gases.

At greater pressure and beam density of around 100 mA/cm<sup>2</sup> the beam plasma density can exceed essentially the beam density, and beam instability develops, leading, under certain system parameters, to the formation of a beam plasma discharge, which radically changes the properties of the plasma.

### 3. INTERACTION OF THE BEAM WITH PLASMA

#### 3.1. Linear effects

The simplest model of a plasma-electron beam system is collisionless cold ( $T_e=0$ ) plasma with charge density  $n_p$ , and a boundless beam of electrons with density  $n_b$  passing through it, moving at a constant velocity  $V_0$ . Such plasma is an isotropic continuous medium with dielectric constant  $\varepsilon_p=1-\omega_p^2/\omega^2$ . Thus, for  $\omega < \omega_p$   $\varepsilon_p < 0$ . For charge density waves of electron flow moving through a permeable infinite dielectric, it is easy to obtain the dispersion equation from the equations of motion, continuity and Poisson's equation:

$$(\omega - \Gamma V_b)^2 = \frac{\omega_b^2}{\varepsilon} \quad (3.1)$$

This equation makes it possible to find the propagation constant:

$$\Gamma = \frac{\omega}{V_0} \pm \frac{\omega_b}{V_0 \sqrt{\varepsilon}} \quad (3.2)$$

One can see that for a medium with  $\varepsilon < 0$  the propagation index  $\Gamma$  is complex, i.e. one of the charge density waves increases along the beam path length.

When substituting the value  $\varepsilon = \varepsilon_p$  into the equation (3.2), it is easily transformed to the well-known form (Ahiezer and Fainberg. 1951; Briggs 1964):

$$\frac{\omega_p^2}{\omega^2} + \frac{\omega_{pb}^2}{(\omega - \Gamma V_0)^2} = 1 \quad (3.3)$$

From the continuity equations and the Poisson equation it is clear that the electric field vector of the space charge  $e$  is shifted by  $\pi/2$  relative to the current vector  $I$  and therefore interaction is reactive in nature:

$$P = \frac{\omega}{2\pi} \int I E \partial t = 0 \quad (3.4)$$

The physical mechanism of this instability type is determined by the reversal of the sign of the Coulomb forces between bunches of beam electrons: at  $\omega < \omega_p$ . The charges induced by the beam in the plasma are phased in a way that the electrostatic forces of the space charge are directed inside the bunch of the beam electrons and compact it. From the numerous designations for this type of instability adopted in the literature, we will use the term “instability on beam space charge waves”, which most closely reflects its essence.

The equation (3.3) can be solved both in the form  $\Gamma(\omega)$  ( $\text{Im } \omega = 0, \Gamma = k + i\chi$ ), and in the form  $\omega(\Gamma)$  ( $\text{Im } \Gamma = 0, \omega = \omega_0 + i\gamma$ ). We will mainly analyze solutions in the first of these forms, which are more consistent with the conditions of experiments with stationary or quasi-stationary conditions of beam injection into the plasma and a fairly extended interaction region.

Upon reaching plasma resonance ( $\omega \rightarrow \omega_p$ ) spatial increment  $\chi = \text{Im } \Gamma \rightarrow \infty$ . The infinite growth of  $\Gamma$  naturally disappears with a more realistic formulation of the problem, taking into account the transverse limitation of the beam and plasma, their finite temperature and collisions with heavy particles. In particular, only taking into account the finiteness of the electron temperature leads to an expression for the maximum growth rate in the form (Fainberg 1962):

$$\chi_{\max} = \frac{\sqrt{3}}{2} \left( \frac{n_b}{2n_p} \right)^{1/3} \frac{\omega_p}{V_0} \left( \frac{V_0}{V_T} \right)^{2/3}. \quad (3.5)$$

Here  $V_T$  is the average thermal velocity of plasma electrons.

Since this instability is convective (Kalmykova and Kurilko 1988), the system can experience an increase of amplitudes of the electric field and beam density oscillations along the length of the interaction region, specified at the beginning of the system by thermal fluctuations, external influence (beam modulation) or feedback.

The conditions under which instability on the waves of a space charge can exist “in its pure form” are not met in experiments. In reality, in a spatially limited plasma, an electron beam interacts with various eigen modes of

waves in the plasma, similar to the interaction in electron-beam microwave devices.

As follows from the general concepts of the theory of coupled waves (Louisell 1960), the interaction of a beam eigen wave with a wave in the medium or structure through which it propagates can lead to increase of the waves only near their synchronism. Thus, to analyze possible regions of existence of beam instability in a plasma, it is necessary to find the types of waves with which the electron beam space charge waves can be synchronized. First of all let us point to the longitudinal Langmuir wave with the dispersion relation:

$$\omega^2 = \omega_p^2 + (kV_T)^2 \quad (3.6)$$

The Langmuir wave is synchronized with the space charge wave of the electron beam near the frequency

$$\omega \cong \omega_p \left(1 + \frac{1}{2} \frac{V_T^2}{V_0^2}\right)$$

If the plasma is in a constant magnetic field, the dispersion of electromagnetic waves in it is described by the Appleton-Hartree dispersion equation (Ginzburg and Rukhadze 1975):

$$Ak^4 + B \frac{\omega^2}{c^2} k^2 + C \frac{\omega^4}{c^4} = 0, \quad (3.7)$$

$$A = \varepsilon_{\perp} \sin^2 \theta + \varepsilon_{\parallel} \cos^2 \theta, B = -\varepsilon_{\perp} \varepsilon_{\parallel} (1 + \cos^2 \theta) - (\varepsilon_{\perp} - g^2) \sin^2 \theta, C = \varepsilon_{\parallel} (\varepsilon_{\perp}^2 - g^2), \varepsilon_{\perp}, \varepsilon_{\parallel},$$

$$\langle \varepsilon \rangle = \begin{vmatrix} \varepsilon_{\perp} & ig & 0 \\ -ig & \varepsilon_{\perp} & 0 \\ 0 & 0 & \varepsilon_{\parallel} \end{vmatrix};$$

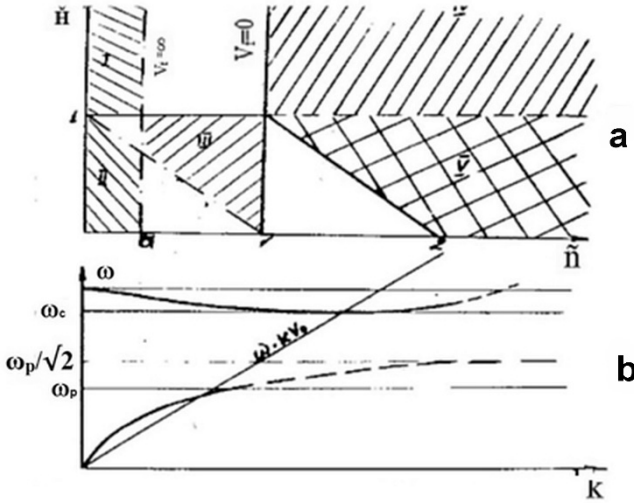
$g$  – components of tensor

$\varepsilon_{\perp} = 1 - \omega_p^2 / (\omega^2 - \omega_c)$ ,  $\varepsilon_{\parallel} = 1 - \omega_p^2 / \omega^2$ ,  $g = \omega_p^2 / (\omega^2 - \omega_c^2)$ ,  $\omega_c$  – an electron cyclotron frequency.



Near the critical angle  $\theta = \arctg(-\varepsilon_{\parallel} / \varepsilon_{\perp})^{1/2}$  one of the waves satisfying equation (1.7) is slow and almost longitudinal ( $E_{\parallel} \gg E_{\perp}$ ) and therefore can be carried out synchronously with the beam's airfield.

The slow electric waves of a plasma cylinder in a vacuum were analyzed in (Trivelpiece and Gould 1959; Jensen 1970; Kondratenko 1976). The results of the analysis can be briefly summarized as follows (see Fig. 3.1).



**Fig. 3.1.** Regions of existence of electric waves in a plasma waveguide of radius  $a$  in the space of parameters  $\tilde{n} = (\omega_p/\omega)^2$  and  $\tilde{H} = \omega_c/\omega$  (a) and dispersion curves for a plasma column in vacuum (b).

When  $\omega_p < \omega_c$  there are two regions of slow waves: a wave with normal dispersion (direct wave - region IV in Fig. 3.1) and  $\omega_c < \omega < (\omega_p^2 + \omega_c^2)^{1/2}$  - a wave with strong anomalous dispersion (backward wave - region III). Both of these waves, in contrast to slow waves of slowing structures, are characterized by a maximum field  $E_z$  on the axis of the plasma waveguide, i.e. they are voluminous. When  $\omega_c < \omega_p$  there is also a backward wave in the range  $\omega < \omega < (\omega_p^2 + \omega_c^2)^{1/2}$ , and a forward wave changes its structure at frequency  $\omega = \omega_c$  the wave is bulk, in range  $\omega_c < \omega < (\frac{\omega_p^2 + \omega_c^2}{2})^{1/2}$  the slow

wave becomes a surface wave (region V). A surface wave cannot, naturally, exist in a plasma that completely fills a metal waveguide. At the same time, in this configuration additional eigenmodes appear - fast waves of the metal waveguide, perturbed by the plasma filling with  $\varepsilon > 0$  (regions I and II).

The transverse inhomogeneity of the plasma density in the waveguide leads to deformation of the dispersion characteristics and changes in the structure of the field in it. In particular, the inhomogeneity can lead to very strong attenuation or disappearance of the reverse or surface modes of the waveguide (De Santis 1965).

Note that the dispersion equation of the plasma waveguide with an electron beam is similar to the general equation for the motion of a beam in a slow wave structure (Pierce 1951). This is a consequence of the unity of the physical interaction mechanism in such system and the mechanism of interaction in the travelling wave tubes (TWT) and the backward wave oscillators (BWO).

Due to the strong dispersion in the plasma waveguide, the group velocity of the wave is significantly lower than in the conventional slow-wave structures. The volumetric nature of waves in the plasma has following result: the coupling coefficient of the beam with the wave is  $R_c = \bar{E}^2 / 2kP > 1$ , while in the slow wave structures with surface wave it is always  $R_c \ll 1$ . Here  $\bar{E}$  is the electric field of the wave, averaged over the cross section of the interaction region,  $k$  is the wave number,  $P$  is the wave power. In interaction regions where  $\omega < \omega_p$ , the synchronous interaction is superimposed by instability of the waves of space charge. All these factors determine the high efficiency of interaction between the beam and waves in the plasma waveguide, leading to a rapid increase in wave intensity from initial disturbances and to the formation of dense space charge clumps in the beam. These properties of the beam-plasma system revealed in theoretical researches were demonstrated in first researches directed to creation plasma electron devices (Boyd et al. 1958; Bogdanov et al. 1960; Kislov and Bogdanov 1960).

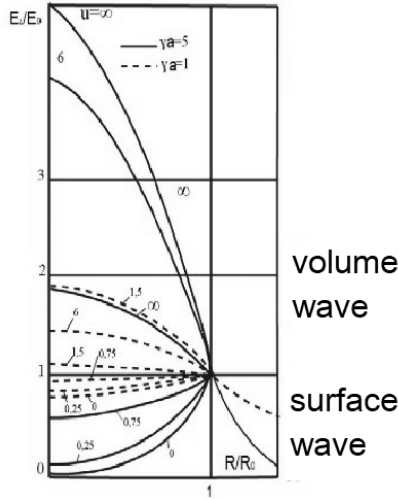
As instability develops in the area where the linear theory of interaction is valid, the wave power in the waveguide increases exponentially.

Accordingly, the average power of the electron beam over the oscillation period decreases. This is the fundamental difference between the interaction of the beam with synchronous waves and the interaction with induced charges in a collisionless boundless plasma described above.

As instability develops in the area where the linear theory of interaction is valid, the wave power in the waveguide increases exponentially. Accordingly, the average power of the electron beam over the oscillation period decreases. This is the fundamental difference between the interaction of a beam with synchronous waves and the interaction with induced charges in a collisionless infinite plasma described above.

Even with a uniform spectrum of initial fluctuations, the intense growth of oscillations in the region of linear interaction should lead to monochromatization of oscillations (Apel 1969):

$$\delta k = \frac{\Delta k (\ln 2)^{1/2}}{(2\gamma_{\max} \tau)^{1/2}} = [3(\ln 2)^{1/2} 2^{5/6}] \left(\frac{n_b}{n_p}\right)^{1/3} \frac{\omega_p}{V_0} (\gamma_m \tau)^{-1/2}. \quad (3.8)$$



**Fig. 3.2.** Distribution of the electric field of plasma waveguide waves along the radius (Kislov and Bogdanov 1960). The numbers on the curves are the ratio  $u = \omega/\omega_p$ ,  $\gamma$  is the wave propagation constant.

Experimentally, the transformation of the oscillation spectrum with the development of beam instability was observed both in the system beam - independently created plasma (Apel 1969) and in the BPD in a high magnetic field (Shustin et al. 1969). In both cases, at the linear stage, a decrease in the width of the oscillation spectrum was observed as their total intensity increased, and at the nonlinear stage, a broadening of the spectrum was observed, the reasons for which we will discuss below.

Equations (3.3) - (3.5) were obtained under the assumption that all beam electrons have the same initial (unperturbed) velocity. In reality, there is always a spread of velocities equivalent to a certain “kinetic” temperature. This spread is determined by the origin of the beam and can be of a truly thermal nature or significantly exceed the thermal spread. For example, the beam is scattered on velocities when it is generated in the plasma of a gas discharge or in the Earth’s magnetosphere. In the latter case, the spread of electron velocities  $V_{Tb}$  can be comparable to the beam velocity. Rigorous analysis (O’Neil and Malmberg 1968) shows that the instability described by equations (3.3-3.4) occurs only under the condition:

$$V_{Tb}/V_0 < (n_b/n_p)^{1/3}. \quad (3.9)$$

When the opposite condition is met kinetic instability develops. The elementary mechanism underlying it is the interaction of the beam with waves having phase velocities  $v_{ph} < V_0$ . For these waves, the number of particles with velocities exceeding the velocity of the wave (Cherenkov emitters) exceeds the number of particles moving slower than the wave and absorbing radiation energy. Thus, the mechanism of this instability is a reversal of the well-known collisionless Landau damping. Since the radiation is accompanied by the phasing of particles in the field of the excited wave, coherent interaction occurs, and the emitted wave grows exponentially in space.

The increment of kinetic instability is significantly less than the maximum increment of the hydrodynamic instability analyzed above:

$$\gamma_{\max} \cong \frac{\omega_p}{2} \frac{n_b}{n_p} \left( \frac{V_0}{V_{Tb}} \right)^2, \quad (3.10)$$

and the frequency and phase velocity differ little from the corresponding values for a plasma wave unperturbed by the beam.

As shown in this paper by numerical integration of the dispersion equation in general form, the hydrodynamic and kinetic instabilities smoothly transform into each other as the beam temperature changes. The topology of the dispersion curves of various unstable types of waves changes at a critical value of the parameter  $S_c$ :

$$S_c = \frac{V_{Tb} n_b}{V_0 n_p} \left( \frac{6V_T^2}{V_0^2} + 2 \frac{\lambda_{nm}^2 V_0^2}{\omega_p^2 \omega^2} \right)^{1/3} \approx 1.5 \quad (3.11)$$

where  $\lambda_{nm}$  is the transverse eigenvalue of the plasma waveguide, determined by its geometry.

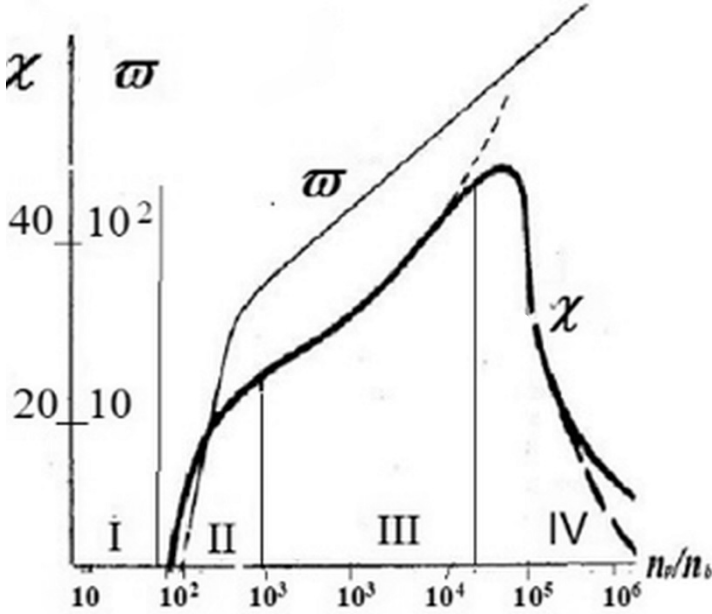
Let us determine the beam instability region in the space of the basic plasma parameters. For simplicity, we consider the case of one-dimensional interaction ( $\omega_c/\omega_p \gg 1$ ) on Langmuir plasma waves. Let the beam parameters be the velocity  $V$ , its density  $n_b = I_0/eV_0 S_b$  and the dimensions of the plasma waveguide are fixed. Figure 3.3 shows qualitatively the dependences of the maximum gain and the corresponding frequency on the plasma density.

In section I there is no instability on Langmuir waves, since the dispersion curve of the wave in the waveguide does not intersect with the dispersion curve of the SCW beam. Wave coupling occurs when

$$\frac{\lambda_{01} V_0}{\omega_p a} \cong 1 \quad (3.12)$$

( $2.4 < \lambda_{01} < 3.65$  is the lowest intrinsic number of the plasma waveguide). This

relationship determines the critical plasma density at which instability begins.



**Fig. 3.3.** Normalized maximum gain  $\chi = \gamma V_0 / \omega_p$  and the corresponding frequency of the excited wave  $\varpi = \omega / \omega_b$  depending on the ratio of plasma and beam densities  $n_p/n_b$ . For the calculation, the values of the beam and plasma parameters were taken:  $V_0 = 2 \cdot 10^9$  cm/s,  $I_0 = 0.3$  A,  $b = a = 10$  cm,  $T_e = 2$  eV,  $\Delta V_{Tb}/V_0 = 0.1$ ,  $v_0/\omega_p = 10^{-4}$ ,  $V_0 = 2 \cdot 10^9$  cm/s,  $I_0 = 0.3$  A,  $b = a = 10$  cm,  $T_e = 2$  eV,  $\Delta V_{Tb}/V_0 = 0.1$ .

As the ratio of the plasma density to the beam density increases, the thermal spread of the beam velocities plays an increasingly significant role - the growth of instability increment is weakened, and when

$$\frac{n_p}{n_b} > \left( S_c \frac{V_0}{V_{Tb}} \right)^3 \frac{V_0^2}{6V_T^2} - \frac{2\lambda_{01}^2 V_0^2}{\omega_p^2 a^2} \quad (3.13)$$

hydrodynamic instability is replaced by kinetic instability, its enhancement coefficient of which already decreases with the plasma density (region IV).

If we take into account the collisions of plasma electrons with the heavy particles of a frequency  $\nu_e$ , then the decrease in the coefficient will go even more sharply (dotted line in Fig. 3.2), and at

$$\frac{n_p}{n_b} \geq \left( \frac{V_0}{V_T} \right)^2 \frac{\omega_p}{\nu_e}, \quad (3.14)$$

the system stabilizes (Singhaus 1964; Self et al. 1971). The stabilization mechanism is determined by the fact that a weak beam cannot compensate for wave attenuation due to collisional energy dissipation.

The threshold nature of beam instability was observed in many experimental studies devoted to the interaction of a beam with plasma at a high magnetic field (see for example (Nezlin et al. 1969; Bogdankevich et al. 1970; Leont'ev et al. 1970)). Determination of the dependence of threshold values of the beam current or primary plasma density on other parameters as magnetic field strength, beam speed, etc. is used in these works to identify the type of instability and test the linear theory.

The presence of an upper critical plasma density, limiting the region of existence of instability of this type, has been recorded in a number of works with both beam and independently generated plasma. In addition to the collision mechanism mentioned above, the nonlinear transfer of energy from excited oscillations into non-resonant types of waves (Tsytovich 1967) can play a significant role. The widening of the electron velocity distribution function observed in many researches is much greater than can be expected only when taking into account collisions of beam electrons with heavy particles, testifies in favor of the second mechanism.

Previously we analyzed the linear stage of beam instability in the case when only purely longitudinal perturbations of particle velocity and density could exist in the beam, i.e. in the case of a sufficiently strong magnetic field ( $\omega_c \gg \omega_b$ ). The case of beam instability in a plasma without a magnetic field, paradoxically, presents a significantly greater difficulty for theoretical

analysis. This is due to the fact that in the general case of a transversely limited beam and plasma, the electric field of the excited wave inevitably has both longitudinal and transverse components, which entails two-dimensional motion of the beam, as well as perturbation of its boundary ( $b=f(E)$ ). Therefore, a correct detailed analysis of instability in the system beam - unmagnetized plasma was carried out only in extreme cases of a transversely unbounded system ( $k_{\perp}b, k_{\perp}a \gg 1$ ) or a thin beam ( $k_{\perp}b < 1$ ), when the beam disturbances can be considered purely longitudinal. In the first case, the dispersion equation, and, consequently, the expressions for the frequency range and wave amplification coefficient coincide with the expressions for the case of infinite plasma in an infinite magnetic field. In the second case, taking into account the limited nature of the system leads to the same equations if the ratio of the beam and plasma densities is multiplied by the factor  $Q < 1$ , determined by averaging the effective field over the cross section of the system. The  $Q$  factor depends on the geometry of the system; for a transversely homogeneous beam in an infinite plasma it is estimated (Rogashkova 1980) as:

$$Q = 1 - 2I_1(k_e b) K_1(k_e b) \quad (3.15)$$

where  $I_1$  and  $K_1$  are Bessel functions.

An analysis of the role of beam limitation when excitation of oscillations by a beam in an isotropic or weakly magnetized ( $\omega_c \ll \omega_p$ ) plasma was also carried out in (Romanov and Filippov 1961; Azarova et al. 1983). The following conclusions follow from them: 1) In the case when the beam has a finite radius ( $k_{\perp}b \geq 1$ ), in addition to the excitation of the main, axially symmetric mode of oscillations, the excitation of higher types of waves, as well as a surface wave, can play a significant role. 2) At the kinetic stage of instability, the efficiency of oscillations excitation decreases significantly at  $k_{\perp}b \geq 3$ .

### 3.2. Nonlinear stage of instability.

To determine the conditions for the occurrence of a beam plasma discharge and its radio physical properties, it is not enough to know the interaction characteristics of at the linear stage of the instability. The intensity of the



electric fields determines the balances of energy and particles in the discharge. Spatio-temporal spectra of the oscillations can be obtained only by analyzing the nonlinear development of the interaction beam with plasma. Therefore, let us now consider the nonlinear stage of instability development. The nonlinearity of interaction manifests as the deviation of the law of increase in the amplitude of oscillations from exponential; deformation of the beam electron distribution function; change in the vibration spectrum (generation of harmonics, side and combination frequencies). The main parameters characterizing this stage of instability are the relative losses of beam energy, the intensity of the RF field and the kinetic temperature of the beam electrons and plasma.

The first works (Vedenov 1963) and (Shapiro, Shevchenko 1968), describing the nonlinear development of beam instability, related to the kinetic type of instability. They were based on approximation of weakly turbulent plasma, i.e. plasma, where the wave energy density is small compared to thermal energy, although it significantly exceeds the energy density of initial oscillations - thermodynamically equilibrium noise in a plasma without a beam:

$$N_p T / N_D < W < n T,$$

where  $N_D$  is the number of electrons in the sphere of Debye radius.

For these conditions, a quasilinear theory was developed, where the distribution function (EDF) of beam electrons is divided into rapidly oscillating and slowly varying parts. In this case, the behavior of the slowly changing part of the distribution function is described by the diffusion equation in phase space, and the rate of growth or decay of plasma oscillations is determined by the formulas of linear theory.

The main results of the quasilinear theory for the plasma beam system are reduced to the following. As a result of the development of oscillations from the level of thermal noise, the distribution function scattered in velocity space (see Fig. 3.4), eventually acquiring the form of a plateau ( $\partial f / \partial V = 0$ ) in the range from  $V_1 \approx V_T$  to  $V_2 = V_0 [1 + (n_b / 6 n_p)^{1/3}]$ . At the same time, the spatial spectrum of excited oscillations  $S(k)$  scattered, since at each given moment of time oscillations are intensively excited with a phase velocity close to the

velocity of the lower limit of the distribution function, where  $\partial f / \partial V = \max$ . The frequency spectrum of oscillations is determined by the dispersion dependence of the system  $\omega(k)$ . The characteristic time for the establishment of a plateau-like DF and of the stationary spectrum is:

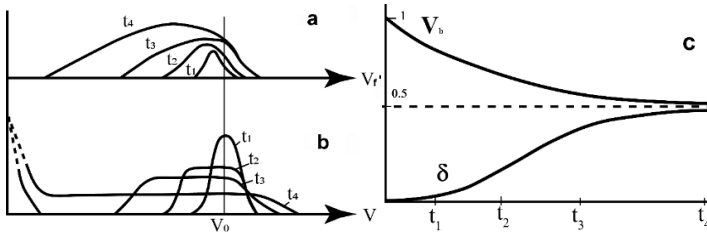
$$\tau_{\infty} = \frac{n_p}{n_b} \ln \frac{W_{\infty}}{W_0}, \quad (3.16)$$

where  $W_0$  is the energy density of initial oscillations, estimated for equilibrium thermal noise as  $W_0 = nT_e/N_D$ . The energy relationships during relaxation can be estimated based on the laws of the beam current conservation and the total power of the beam and wave:

$$n_{b\infty} \approx 2n_{b0}; \quad P_{\infty}/P_{b0} \approx 1/2.$$

Thus, neglecting the processes of dissipation and nonlinear wave transformation, half of the beam power is ultimately spent to exciting of Langmuir waves in the plasma.

The problem of three-dimensional relaxation of the beam-isotropic plasma system is much more difficult to analyze. Qualitative analysis and computer modeling (Romanov and Filippov 1961; Rogashkova 1980; Azarova et al. 1983) led to the following results.



**Fig. 3.4.** Quasilinear one-dimensional relaxation of a beam in plasma. Transformation of the oscillation spectrum  $S(v)$  (a), the form of the average electron velocity distribution function  $f(V)$  (b), the change in the average velocity of beam particles  $V_b$  and the relative loss of its energy  $\delta$  in time (c). Curves designated by indices  $t_1...t_4$  in Fig. (a, b), correspond to time points  $t_1...t_4$  on graph (c).

At the initial stage of interaction, due to the angular dependence of the kinetic instability increment  $\gamma \cos \theta = k_z/k_0$ , quasi-one-dimensional relaxation of the beam occurs with the formation of a plateau in longitudinal velocities and the excitation of waves in a narrow cone near the direction of beam motion.

The spectrum of excited waves shifts towards the low longitudinal phase velocities. Next, the beam electron distribution function (EDF) becomes isotropic.

As stated above, the methods of quasilinear theory are applicable only to kinetic beam instability, while in the most experiments the beam at the entrance to the system is practically monochromatic, i.e. the conditions for the hydrodynamic instability are met. Attempts to apply the quasi-linear theory in this case were based on the assumption that at the initial, hydrodynamic stage of instability, the beam is scattered in velocities and a wide spectrum of oscillations is excited, after which a transition to the kinetic instability occurs. Then the quasi-linear approach with a corresponding correction of the initial conditions may be applied. A more consistent analysis and computer modeling of the conditions for interaction a plasma and a beam with a small initial velocity spread, however, gave a qualitatively different picture. It was established (Biskamp and Welter 1972), that for the quasilinear theory to be valid, the following condition must be satisfied:

$$k_m[\Delta(\omega/k)] > \omega_{pb},$$

where  $k_m$  is a wave vector of a wave with the maximum linear increment  $\gamma_{max}$ ,  $\Delta(\omega/k)$  is the half-level width of the curve in the space of phase velocities.

Otherwise, as stated above, even at the linear stage the excited wave approaches to monochromatic one, and other oscillations with frequencies and wave vectors different from those corresponding to the maximum linear increment are suppressed. This wave grows until the depth of the potential well for beam electrons in the wave field don't exceed some critical energy.

After this, the beam electrons are captured by the wave, they oscillate in the potential well of the wave, and the corresponding oscillations of the wave amplitude occur. As a result, a new instability develops - instability on trapped particles, or satellite instability, leading to self-modulation of the wave in time and space, i.e. to generating of side frequencies (satellites) in the spectrum and to the formation of a complex, multi-hump structure of the EDF of beam electrons.

Studies of the evolution of the beam and wave at this stage of interaction in many works were carried out using methods of qualitative theoretical analysis and computing experiment. Let us summarize their results here, focusing for definiteness on the picture of the spatial development of oscillations in a system with one-dimensional motion of an electron beam ( $\omega - k_{\parallel}V_0 < \omega_c$ ,  $\omega_b < \omega$ ,  $k_{\perp}V_{\perp}/\omega_c < 1$ ). Here we emphasize that in this model only the nonlinearity associated with the electron beam is taken into account, and the plasma is considered as a linear medium.

Figure 3.5 shows the spatial evolution of the oscillation amplitude at the fundamental frequency, the beam EDF, averaged over the oscillation period, and the relative beam energy losses (Lavrovskii et al.1972). The figure shows that the wave grows exponentially until it reaches an amplitude sufficient to capture electrons in the potential well (point  $\tilde{z} = 2.2$  on the Fig.3.5). At this point, a narrow bunch is formed containing almost all the electrons of the beam (“macroparticle”), and their velocity distribution represents an almost plateau-shaped function, which also contains significantly accelerated electrons, in contrast to the conclusions of the quasi-linear theory:  $V_{max} \approx (1 + 3 \frac{V_0 - V_f}{V_0})$ .

Further, the electrons of the bunch oscillate in the potential well of the wave, exchanging energy with it. An electron in a potential well of the monochromatic wave is an anharmonic oscillator - the period of its oscillations depends on the amplitude, i.e. on the energy of the electron in the reference frame associated with the wave, and on the depth of the well. Since the average energy of the electrons significantly exceeds the group velocity of the wave, each subsequent portion of the beam electrons experiences the influence of the field excited by the previous portion, as a

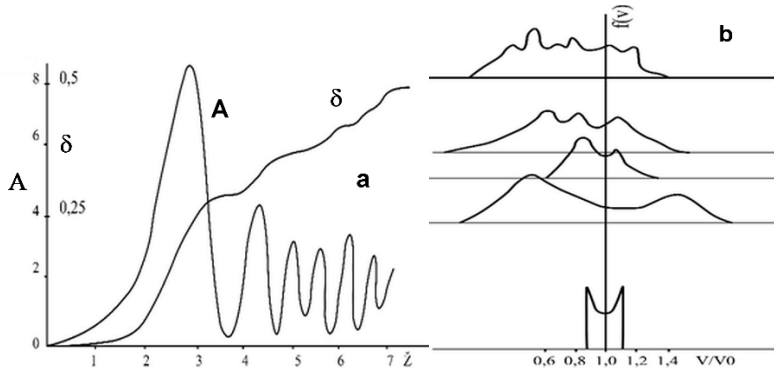
result, the oscillation frequency of the captured electrons and the ratio between their number and the number of passing electrons continuously changes. These factors lead to the fact that spatial oscillations turn out to be irregular, and the amplitude of the main wave decreases.

Experimental verification of the theoretical results described above was carried out on systems with a weak electron beam ( $I_0 < 10$  mA,  $U_0 = 100-400$  V) and, as a rule, with initial modulation at a fixed frequency (Roberson and Gentle 1971; Gentle and Lohr 1973; Kovalenko 1983). The conclusions of the theory, describing the spatial evolution of the system parameters (the amplitude of the main wave, its harmonics and satellites, electronic efficiency) and the picture of the transformation of the EDF are reliably confirmed.

However, already on an early stage of experimental research, effects were discovered that definitely did not fit into the ideas following from the theory. Firstly, numerous experimental studies have shown that the development of beam instability is typically characterized by the excitation of noise-like oscillations with a wide continuous spectrum of frequencies. This result is in apparent contradiction with the conclusions of the theory of coherent interaction, predicted that the wave packet get to monochromatic one, as the wave grows. Secondly, the smooth transformation of the EDF of the beam electrons from delta-shaped to plateau as instability develops, observed in the same experiments, contradicts the picture of nonlinear interaction described above, from which it follows that the velocity distribution function of beam electrons should have a complex shape of the curve with many humps, changing during beam relaxation.

Both in numerical and physical experiments, the effect of stochastization of generated oscillations is clearly observed, with the spectrum width reaching or even exceeding  $0,1\omega_0$ . It is the stochastization of oscillations that determines the intense heating of plasma electrons in the paraxial region. Let us recall that in physical experiments the generation of noise-like oscillations was observed regularly, and this was explained by interaction of the excited high-frequency waves with other types of excited oscillations. In (Bliokh et al. 1998; 2003) spatial development of convective instability was theoretically studied at an initially given wave frequency

(plasma amplifier model), and a significant effect was revealed: the development of beam instability at the nonlinear stage gives rise to the generation of ion-acoustic and magneto-acoustic oscillations leading to expansion spectrum of an initially monochromatic signal.



**Fig. 3.5.** Distribution along the length of the interaction region of the dimensionless amplitude of the wave field  $A$  with frequency  $\omega$ , relative losses of beam energy  $\delta$  and the type of DF of beam electrons in terms of velocities at various points in space.  $\tilde{z} = \chi z$  (Lavrovsky(a) et al. 1972).

It was experimentally shown in (Lavrovsky (b) et al. 1972; Lavrovsky 1974) that the interaction processes in BPD are nonstationary even under stationary conditions of beam injection into the discharge plasma. Therefore this study by methods suitable for stationary processes gives results distorting the physical picture of interaction. Non-stationarity is expressed in the fact that oscillations in the system, perceived by the inertial instruments as broadband stochastic oscillations, are chaotically alternating quasi-monochromatic or narrow-band wave trains corresponding to the excitation of the system at various eigen modes (Fig. 3.6). Accordingly, the beam velocity distribution function recorded during the existence of one eigen mode is a complex multi-hump structure, and the smooth plateau-like appearance of the EDF is the result of averaging of “instantaneous” EDF. The averaged characteristics of both high-frequency oscillations and the velocity distribution function of the beam electrons mask the true nature of the interaction. The expansion of the spectrum of oscillations and their