

Introductory Property Valuation Mathematics

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By

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Thankfully dedicated to my parents:
Philomena Ighiwiyi Onwuanyi (1932-1996),
who stood for doing the right thing and did things right

&

Ichie Amawulu Nwalioba Osii Onwuanyi,
KCOB, FPSN (1924-2009),
who was a patriot and a champion of good causes.

*Written to explain concepts, simplify learning, lessen difficulties
and facilitate understanding*

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PREFACE

Valuation is a core subject in estate management education in Nigerian universities and polytechnics. Being a subject in professional education, it is first encountered by students at the tertiary level. The subject is introduced through preliminary studies which are founded upon mathematical precepts. These are described as the mathematics of valuation. However, valuation is not totally about mathematics. This reading comes from the nature of valuation as a professional function of the valuer, a process undertaken by investigation, as well as an outcome of objective and subjective factors. It is true that there is mathematics in valuation, but there is more to the subject. This suggests that mathematics merely facilitates valuation. In effect, the logic of mathematics introduces an objectivity which enhances the validity of outcomes produced by the partly subjective process of valuation.

Valuation does not require the valuer to be a mathematician, but to possess and demonstrate a level of mathematical capacity. To expatiate further, the logic in the aphorism: *"a bird in hand is worth two in the bush"* has a mathematical side, but its true meaning is commonsensical, not mathematical in nature. Unquestionably, two birds, if available, would be more valuable than one, but common-sense suggests that there is actuality, and therefore more value, in definite possession than in the anticipation of possession. Numerically, two is larger than one, but in this case, one is commonsensically superior because the other option is supposed, and therefore, uncertain. The value in having one real bird, rather than none, is better explained by common sense than mathematics. Therefore, an understanding of the aphorism is facilitated by mathematics, but its significance does not lie in mathematics. Again, that a certain sum held today can be invested to produce a higher value in future or that today's worth of a known contractual, future financial inflow would be relatively lower are instances of economic logic which are provable only by mathematical logic. By this role, mathematical logic facilitates an understanding of these economic concepts. This is the case with valuation.

Thus, mathematical precepts provide the foundation of valuation studies. A good grounding in the introductory stages is essential for the student of valuation. Areas of doubt should be cleared up as quickly as possible and not allowed to linger as this may create fundamental gaps in knowledge

which may lead to lasting challenges. A good, timely and enduring grasp of introductory valuation mathematics is essential for a good performance in the subject of valuation at undergraduate level, for a professional career in valuation and advanced studies in the subject. Such a background also prepares the student for an understanding of the inadequacies of the traditional investment valuation method and the rationale for improved methods as have evolved, or are evolving, in response. The concept of value in general, and property value in particular, an understanding of the property industry and market, the valuer's functions and methods, the purposes, basis and methods of valuation, the valuation formulae (and tables) and their application in the assessment of value are the starting points of valuation studies.

For many reasons, a book on property valuation cannot avoid making references to the science of economics, which also, may be described in this regard as the science of pricing, given the role of the price mechanism in all economic activity. For one, property valuation relates to land which happens to be a fundamental issue in economics. Furthermore, the concept of rent which underlies valuation, is founded in the rent theory of economics. Again, interest as the cost of capital and the opportunity cost of holding money, guides investors' decisions and offers a basis for the comparative assessment of investment profitability. In addition, price, value and cost are all market concepts which apply to valuation and define economic activity by respectively enabling, justifying, rewarding, and ultimately, sustaining it. Yet again, valuations are interpretations of the property market which is a forum for economic activity. Thus, valuation derives from the broader realm of economics, adopting the latter's theory of value, and also, the annuity and depreciation concepts of accounting science, another discipline of economics.

Valuation mathematics is not a new topic in any way as it has been treated by many authors over the years. The same applies to the ancient principle of compound interest from which valuation mathematics derives. Therefore, this book is one more attempt to travel a familiar route, hoping that dark areas will become better lighted, bends more easily negotiated and uneven surfaces smoothened.

AIM OF THE BOOK

The aim of this text is to introduce the subject of valuation mathematics in a simplified fashion which would enable beginners to gain an enduring grounding. This is important for the following reasons. Firstly, it comes as a new subject to students at the level of tertiary or professional education. Secondly, valuation mainly lays the foundation for training as a valuer. Thirdly, it is key to understanding advanced valuation which is taught in later years of study. Fourthly, a weak background in valuation and a failure to grasp elementary principles at the appropriate stage may lower the student's overall performance. It is self-evident that the quality of the future professional is determined mainly by the quality of his preparation. This, in turn, is substantially dependent on the level of comprehension (or learning and understanding) attained particularly in the foundational years. The book approaches its objective in stages. It starts by explaining the basis of property value which is the concern of all property valuations. Furthermore, detailed explanations are given of the compound interest formulae from which the valuation formulae derive. Important issues required for a thorough understanding of the formulae are explained. These include identification of the main valuation formulae and their variants, the various ways in which the formulae can be classified, their similarities and dissimilarities and inter-relationships. Worked examples are used to explain inter-relationships of the formulae. An innovation is that inter-relationships are demonstrated by calculations made in a forth and back manner. Instances of the application of the formulae in property valuation are also given. The book covers an essential aspect of introductory studies in property valuation for estate management students in Nigerian universities and polytechnics and for people elsewhere who need to undertake academic or professional studies in valuation or seek an understanding of its mathematics. Valuations are expressed in units of currency. The unit of currency used in this book is the Nigerian *Naira*. It is represented by the symbol (₦). Each unit of the Naira comprises 100 kobo which is represented by *k*.

—N. Onwuanyi
Chukwuemeka Odumegwu Ojukwu University
November, 2023

INTRODUCTION

Property, which also is described as landed property, real property or real estate, consists of land and improvements of land. These are assets of value which can be bought and sold. Transactions in these assets are founded upon information and knowledge. Buyers would need to know the amount of money to pay whilst sellers would need to know what price to ask or expect from prospective buyers. Both decisions require valuations, the advice given by a valuer to his client, whether buyer or seller. Valuations involve calculations. There are many methods of valuation, including the investment method which is founded on compound interest theory. This book specifically looks at the basis of the calculations which are necessary for carrying out investment valuations of property. The fundamental principle in the investment valuation method is that a relationship exists between the income produced by a property investment and its capital value. Thus, it is a method which involves ascertainment of the net annual income from a property investment, and its conversion thereafter, to a figure which represents its capital value, for which amount it may be exchanged in the market. The guidance provided by valuations is important so that property is not exchanged at a price in excess of its value or sold at a price which is far below it. This accords with the rationality of economic activity and market efficiency which require that the factors of production, one of which is landed property, be appropriately rewarded. Accurate valuations contribute to market efficiency.

A property valuation can be perceived in four main ways. First, it is an estimate of value, usually market value, which the International Valuation Standards Council (IVSC), in performing its role of developing and setting technical and ethical standards for valuations, describes as:

“The estimated amount for which an asset or liability should exchange on the valuation date between a willing buyer and a willing seller in an arm’s length transaction, after proper marketing and where the parties had each acted knowledgeably, prudently and without compulsion”. (IVSC, 2023: 20:14)

A second definition, which highlights also the value typically produced in a property valuation, is given by the Royal Institution of Chartered Surveyors, as follows:

“The open market value is the best price obtainable in a transaction completed on the valuation date based upon the following assumptions:

(i) a willing seller (a hypothetical owner who is neither eager or reluctant i.e. not forced but not at a price which suits only him/her).

(ii) prior to the sale, a reasonable period to market the property and complete all the legal formalities was available.

(iii) during this period, the state of the market was the same as at the date of valuation.

(iv) any bid from a special purchaser is excluded.

(v) all parties acted knowledgeably, prudently and without compulsion”

—Royal Institution of Chartered Surveyors (2019)

Secondly, property valuation refers to the detailed report, prepared by a certified valuer on the value, usually market value, of a property.

Thirdly, it is a process because it has a beginning and an end. Since valuations are done upon the request of clients, the process starts when the client issues instructions to the valuer and logically ends upon the delivery of a report.

Fourthly, it is a quantitative and analytical activity. It involves the gathering of information, the taking of measurements and the execution of calculations. This is due to the financial nature of property investment. The compound interest principle, from which the valuation formulae are derived, is also used by other financial analysts who practice in the fields of accountancy, banking and business management, *inter alia*. Just as financial analysis requires information, so does property valuation.

According to the Royal Institution of Chartered Surveyors, valuation information usually comes from five sources. These are the client, property inspection, property analysis, market analysis and the public. With these sources of information and the application of valuation techniques and skills, the professional valuer is able to deliver an informed opinion on the value of different types of property for a variety of purposes.

Structure of the Book

Just as value is a market concept, so also is valuation. Every valuation should look to the market since it is essentially an interpretation of the market. The first considerations are, therefore, the market concepts of price and value, property value and property rights. This is followed by a consideration of the features of property which make it desirable for ownership and exchange. Then there is a rationalisation of property markets in respect of which distinctions are made between the Western world, the

old Eastern block and Africa. The book considers the role of the valuer as the producer of valuations and the skills and methods employed in discharging that function. After this, the valuation formulae are identified and categorised into the six main formulae and their variants. Further, classifications are made and explanations given of the main formulae and their functions. Particularly, the meaning and function of the Year's Purchase multiplier, a valuation formula used in the investment method, is explained in detail with appropriate calculations. This is followed by demonstrations of relationships of the formulae, both direct and indirect, by worked examples. The book concludes with a brief mention of new valuation formulae.

The contents are designed to cover aspects of the *Introduction to Valuation* module for estate management, quantity surveying and urban and regional planning courses in Nigerian universities and polytechnics. The content requires students to acquire knowledge of the basis of value, property value, the property market, the valuer's role, the valuer's methods and skills, valuation methods, the valuation process and the mathematics of valuation, *inter alia*. Standard introductory texts on property valuation generally extend beyond valuation mathematics in content, but the focus of this book is valuation mathematics. The contents are essential to the implementation of the Core Curriculum-Minimum Academic Standards (CC-MAS) for the B.Sc. estate management course in Nigeria as introduced in 2022 by the National Universities Commission.

CHAPTER 1

BASIS OF VALUE

Undeniably, profit is the motive for economic activity. However, a profit cannot be produced without the creation and delivery of value through activity of an economic nature. This underscores the importance of value to all economic actors. It also demonstrates why obtaining value is the rational goal of all economic activity. Producers create value, sellers offer value whilst buyers seek value. All of these groups need to attain value for their sustenance. In value, there is a commonality of motive and expectations between makers and takers. Both parties find in the marketplace the right platform for their interactions. Thus, value connects producers with consumers. Goods produced and services offered enable customers either to create further value or achieve a state of satisfaction on which they place a value. The key consideration is value, whether one is involved in producing a good, selling it, or purchasing or using it. A maker who does not create value will not have takers because takers have the expectation of receiving value. Economic actors may undertake production primarily to meet their own needs at the level of subsistence, but the wider goal in modern times and economies is to produce more and exchange the extra output so that the income earned can be used by producers to meet other needs. Sale of extra production output can take place only in the markets, so the producer's or seller's profit is determined by the market, not by his own whims, needs or preferences. Usually, what features as price in the marketplace consists of production cost plus a markup. However, since markets can rise and fall, price will tend to be dynamic. Whatever the market decides, that is, the exchange amount agreed between a seller and a buyer at any point in time, is the appropriate value of the good offered for sale. Thus, value is a factor of the market, just as price is. Nonetheless, the producer of a good will always continue to produce and supply until that level where marginal cost equals marginal revenue. That is, the revenue obtained from the sale of one additional unit of output should be equal to the cost of producing it. Beyond this level, there would be no incentive for the continuation of production since it means that revenue received or receivable is lower than the production cost incurred or estimated. In sum, price leads to value, whilst value emanates from price. Price defines the seller's expectation, value

defines the buyer's response. Aspects of price and value are compared and explained in the table below.

Table 1: Comparing price and value

	Price	Value
Rationale	expresses the expectations of sellers	expresses the expectations of buyers
Features	is asked by sellers	is sought by buyers
	offers access to a useful item	delivers satisfaction to user of the item
	encapsulates the seller's motive	encapsulates the buyer's motive
	characterises every offered market item	exists in every purchased market item
Essence	is compensation for economic activity involving goods production & services provision	is the satisfaction, advantage or benefit enjoyed from use of purchased goods and services
Composition	is based upon cost input plus mark-up or profit	is based on perceived utility, expected satisfaction and affordability
	is <i>one-way</i> in effect because it is established from the producer's or seller's perspective	is <i>two-way</i> in effect because it is determined from both producer/seller and buyer/user perspectives or interactions
Size	is usually the same for all buyers	is individually assessed by each buyer
Behaviour	may rise or fall in accordance with expectations	may vary amongst users

The following observations are also relevant. Price precedes value. Price is payment whilst value is benefit. Value is a response to price, and therefore, the justification of price. The journey to value is made through price - from offer or ask prices; to bid prices; to optimal prices and then traded prices which are final, and therefore, true reflections of value. Where the subjective valuations or estimates (respectively offers and bid prices) of seller and buyer coincide, an optimal price is reached based upon which a transaction can take place. This becomes the traded price. Thus, value is determined by the interactions between parties in the market. Where a prospective purchaser discovers that the price of an item exceeds his own subjective estimate or valuation, the rational decision would be not to purchase it. Since value is established between buyer and seller at a given point in time, changes in desires, expectations and subjective estimates over time will lead to changes in price and value. These changes are responsible for market trends. The market may at any time take a rising or falling trend or may achieve stability, depending upon prospects as interpreted by participating actors through their estimates of value in response to price. Value is, therefore, not a constant, but remains dynamic as determined by market conditions.

Investment choices and decisions are made at macroeconomic and microeconomic levels in an economy. At all times, the price mechanism provides a guide for all producers of goods, suppliers of services and participants in exchange. Price is the guide not only because it reflects production cost, but also, paves the way to value as determined by buyer and seller interactions. As mentioned, producers in the modern economy are motivated by the desire to create and sell extra output so as to meet other needs, and thereby, grow their businesses. The necessary decisions which have to be taken usually feature price. Price plays the important role of signaling producers as to what items are in demand and those which are not. From price levels, consumers also gain knowledge which is displayed in their buying habits. Price levels also incentivise producers to create either more output or less output at any particular point in time. This is described by Adam Smith as an invisible hand which regulates the economy. It does this by sending prices up when there is a scarcity and down when there is a surplus. It is also able to increase or decrease supply indirectly through the instrumentality of price.

The concept of value is usually discussed in conjunction with the associated concepts of price and cost. The three are market factors. It was mentioned earlier that price precedes value, but it is also true that cost precedes price in so far as production is concerned. Cost is fundamental to all economic

activity. Every created item has a cost just as any created item which has a value must have a cost which is usually incorporated in its price. This is rational because production involves the assembly of costly factors. Price reflects cost, else the producer would go unrewarded, and may therefore, be forced out of business. Producer-sellers fix prices which reflect *actual* production cost and a *provision* for sufficient profit. The price at which an item is eventually sold in the market is the price at which the seller's minimum acceptable offer intersects with the buyer's maximum affordable price. In this way, value is established as a factor of the market through price also as a factor of the market. The transaction prices imply value because they are the outcomes of market interactions. At this point the buyer obtains value for his money whilst the seller obtains recompense. The subjective valuations of an item may vary amongst people because estimates of actual utility and satisfaction levels differ. Yet, value will always be the outcome of negotiations between parties who come to the market with different subjective estimates or subjective valuations of an item. Cost, price and value are the critical concepts which determine whether or not economic activity and exchange can take place. The first two rationalise production whilst the third determines the capacity of the producer to sustain production and remain in business.

Coming to landed property, it is rational that owners would choose to sell or grant a lease only on favourable terms, chief of which is price, the principal guide to economic activity. So also do the producers of new buildings consider price. Similarly, the owners of existing buildings would refer to price before taking decisions on whether to retain, sell or lease out their property. In sum, price (as a guide to profit or reward), incentivises producers, sellers and buyers. If the price were not right, economic activity would be impossible to rationalise.

CHAPTER 2

PROPERTY VALUE

Property value may be expressed either in annual or capital terms. In annual terms, it is the yearly entitlement of the landlord or the yearly obligation of the tenant. Property value in capital terms is the summation of the series of entitlements of the landlord, (or owner of other property interest such as the leasehold or sub-leasehold), spread over the life of the investment. By paying the purchase price, a new investor becomes entitled to this stream of income, and thereby, obtains value. Similarly, by selling the investment, the existing owner offers and delivers value to the new investor. Annual values produce capital values through the normal capitalisation of the former. However, capitalised values can also be used to establish annual equivalent values through a process of decapitalisation. The ascertainment of annual value is a necessary step towards estimating capital value. This is why the value of a property investment is defined as consisting of the sum of the present values of current and anticipated future income flows. Such flows are typically annual in nature and occur all through the duration of the investment.

The Concept of Rent

The investment attraction in land is that it is an economic asset which has the capacity to produce value in the form of income. The income derived from land is described as rent. Since property valuations usually rely on rental income as evidence of property value, it is important to consider the nature of rent. The concept of rent was established in Classical rent theory through farmland fertility analysis. This primarily established that the price of agricultural land is determined by its fertility. In addition, farmland productivity analysis established that land earns an income known as rent. Again, that the amount of rent earned by land (and the amount a tenant-farmer would be willing to pay as rent) is dependent on the surplus of sales revenue over production costs (including profit). The surplus is described as the pure earnings of land because land has no cost of production. Thus, the more productive the land, the more a tenant farmer would be willing to pay for its use. Rent theory was developed by the Classical economists

whose works are collectively described as the theories of surplus value. The meaning is that rent is a surplus value which is created by land when put to use for economic purposes. Whilst theorising on rent can be traced as far back as the 18th century with the significant input of James Anderson, it was David Ricardo's 19th century work which compellingly demonstrated the income-earning capacity of land by using the example of agricultural land cultivated by tenant-farmers. Ricardo's theory is usually described as the theory of economic rent. In the process, Ricardo described rent as:

"that portion of the produce of the earth which is paid to the landlord for the use of the original and indestructible powers of the soil"

—David Ricardo *"On the Principles of Political Economy and Taxation"*
(1817: chapter 2: pp.59)

The suggestion is that the tenant pays rent to the owner or landlord of agricultural land for use of the natural productive properties of the soil which the former exploits through its cultivation. Such farmers would rationally need to have an idea as to how much profit they could make on the land before deciding whether or not to take a tenancy. The knowledge would guide them as to how much rent they could afford to pay for the land. Crop production on such land would also require co-operating factors in the forms of labour, capital and enterprise. The possible outcomes could be positive or negative. It would be positive where there was a surplus of total revenue over total cost. On the other hand, the outcome would be negative if total cost was to exceed total revenue. Where the outcome is a surplus, it is described as economic rent or the earnings attributable to the land itself since all other co-operating factors have been rewarded. A surplus would be indicative of the viability of the farming activity proposed for that particular year. Logically, the process will have to be repeated the following year if changes have occurred in all or some of the other three co-operating factors: labour (workers' wages), capital (input, seed, fertilisers, etc.) and also enterprise (the level of profit or reward acceptable to the entrepreneur which may move in sympathy with either rising or falling total costs). A situation in which total cost exceeds total revenue means that the land will earn no rent that year if that particular crop were to be cultivated. This may be because the crop is no longer in high demand, leading to a low revenue expectation, or the cost of production is high relative to the revenue earning potential. It may also be due to both factors.

On the contrary, if the demand for the crop increases and revenue also increases relative to the cost of the co-operating factors, this would lead to greater earnings for the land and an increased capacity of the tenant-farmer

to pay rent to the landowner. Ricardo's theory was postulated as an explanation for the rising rents of agricultural land in Europe. This coincided with the rising price of *corn*, a term then used to refer to all grain, including wheat, maize, oats and barley. The common belief was that the high rent was caused by landowners' demand of high rents from tenant-farmers. Ricardo demonstrated that the rise in farmland rents was due to the rising price of corn in the market. In addition, he attributed the rising price of corn to its scarcity, not the demands of landowners. Two of his fundamental assertions were that:

"Corn is not high because a rent is paid, but a rent is paid because corn is high".

—David Ricardo "*On the Principles of Political Economy and Taxation*" (1817: chapter 2: pp.63) and that

"...the rent of land is the effect, and never the cause, of the high value of their produce".

—David Ricardo "*On the Principles of Political Economy and Taxation*" (1817: Chapter 3: pp.76)

These statements reflected the truth in James Anderson's earlier 18th century declaration on the high price of corn in Scotland, that:

"It is not the rent of the land that determines the price of its produce, but it is the price of its produce which determines the rent of the land".

—James Anderson "*An Enquiry into the Nature of the Corn Laws with a view to the New Corn Bill proposed for Scotland*" (1777)

Therefore, the sale price of corn determines the rent for farmland used for corn production, rather than the rent paid for the land itself determining the sale price of corn. Thus, rent arises from the profitability of production. This meant that farmland rent was produce price-determined, but not produce price-determining. This finding established the economic principle of derived demand. The principle states that the demand for a factor is derived from the demand for the products of the factor. Thus, the demand for landed property such as shops, offices, etc., is derived from the demand for the economic value which they produce in the use to which they are put. The value produced is the basis of the subjective valuations which people use to express their preferences and assessments of worth. The greater the demand of consumers for the goods or value produced on these premises, the greater the subjective valuations which prospective property buyers and sellers make of the premises. The same applies to people who wish to let such property.

In a situation where cost exceeds revenue, the solution may be to switch to the cultivation of a different crop which has better revenue prospects relative to production cost. The farmer may also decide to revise the cost estimates to make production more efficient. A situation of efficiency would occur if less input could be applied to produce the same output level or more output produced with the same level of input. This can happen through the introduction of innovations and technology.

In keeping with the Ricardian focus on farming which is an economic or business activity, this section illustrates the theory of rent as a surplus or the theory of surplus value, by considering three examples in which land is used for business purposes. Table 2 presents four scenarios. They all concern a parcel of farmland on which cropping is done sequentially over a four-year period. The first scenario shows a surplus of revenue over costs. In the second, there is an increase in the surplus as revenue exceeds cost by a greater margin. The third describes a situation where there is a drop in the size of the surplus. In the fourth situation, cost exceeds revenue as a result of which there is no surplus. The first two scenarios describe the main issue in the theory. However the other two scenarios are included for further elucidation.

Table 2: Rent as a surplus: Land used for crop production

S/N	Scenarios				
		1 st Year	2 nd Year	3rd year	4 th Year
	Farm Input	₦	₦	₦	₦
1	Labour	300,000	360,000	400,000	478,500
2	Capital	700,000	650,000	710,000	748,500
3	Enterprise	250,000	350,000	275,000	357,000
	Total revenue	1,295,000	1,438,000	1,477,000	1,524,000
	Total cost	1,250,000	1,360,000	1,415,000	1,584,000
	Revenue less Cost	45,000	78,000	62,000	-60,000
	Outcome	A Surplus	An increase in the surplus	A fall in the surplus	No surplus

Where revenue deducted from cost produces a surplus, the farmer would be willing to pay rent to the landowner only up to the maximum of the surplus. This is the case in Scenarios 1&2. Where there is a rise in the surplus, the tenant would be willing to pay more to the landlord as the land is earning more rent (Scenario 2). A fall in the surplus reduces the earnings of the land (Scenario 3), and therefore, the tenant's ability to pay unlike in the two preceding years. In the event that the tenant's estimation of production costs less revenue fails to produce a surplus, he would not farm on the land that year because the rent would be unaffordable (Scenario 4). A way out is to farm another crop which produces more revenue relative to cost or revise the cost of input to ascertain if a surplus can be achieved so that he can continue in business. This may require reducing his profit expectation, labour costs and capital employed. The last two courses of action may not always be feasible because they will impact output, and thereby, revenue and profitability whilst the first may be unacceptable to the farmer.

The above example comes from agrarian society. An example can also be drawn from modern society to demonstrate how a surplus can be produced by bare land which is used to provide a service. An apt example should be a parcel of land which is located in the commercial section of a small, but busy town. Due to the site's location on a shopping street, and a scarcity of car parking spaces, the owner applies for, and is granted, permission for car parking purposes. Details of the arrangement are given in Table 3. The site can accommodate up to 30 vehicles. The tariff is fixed at ₦600-00 daily which is comparable with the rate charged by other car park operators in the town.

Table 3: Rent as a surplus: Land used for providing a service

	Per Day	Per Month	Per Annum	Total Revenue	Total Costs	Return to the Land
Business Revenue 30 spaces @₦600-00	₦18,000	₦540,000	₦6,480,000	₦6,480,000	₦6,084,000	₦396,000
Business Costs						
(i) Wages: 2 Attendants @ ₦5,000 each	₦10,000	₦300,000	₦3,600,000			
(ii) Sundries	₦3,000	₦90,000	₦1,080,000			
(iii) Entrepreneur's return	₦3,900-00	₦117,000	₦1,404,000			

Since this is an informal, day-to-day business, the revenues and costs are stated in daily, monthly and annual terms. Daily turnover is ₦18, 000 giving a monthly turnover of ₦540, 000. Business expenses in the form of wages for two attendants, municipal charges and sundries amounting to ₦1, 080, 000. The owner's reward as entrepreneur is ₦1, 404,000-00 (30% of business costs). The total cost of providing the service is ₦ 6,084,000. When total revenue is deducted from total costs (which includes profit), the balance is ₦ 396,000. This balance can be described as the pure earnings of the land, all other co-operating factors having been rewarded. Under the same assumptions in the example above, if another person rather than the owner of the land were the operator of the car parking service and was required to pay rent for the land, the balance produced would be the maximum amount which would be offered as rent.

In the above two cases, land without any permanent structures or physical improvements was involved. Rent came from using the land in its natural state. In the first, production took place whilst in the second a service was provided. However, there is another instance, more common in modern society, where rent arises from land which has been improved by the erection of buildings or structures, that is, developed land.

Where land has been developed, the same principle applies as for undeveloped land which is given out for rent. A tenant pays rent because it works to his advantage. This is basically because it is affordable. Affordability means that it facilitates the production of acceptable value from the use of the premises. This occurs where revenue sufficiently exceeds cost in accordance with the tenant's desires and expectations. If the tenant using the property, land and building(s) in this case, finds that his business costs in the location are rising faster than revenue, it means he is becoming less competitive, losing rather than gaining, because what he pays out cannot sustain the business. The tendency would be for him to seek an affordable property or consider leaving that line of business. Table 4 explains the situations which may arise by using the example of a barber's business located in a rented shop. The yearly tenancy agreement has a clause which commits the tenant to paying a 10% increase in rent every year because it is the average annual rate of rental growth in the area. This is not unusual in an inflationary economy.

Upon entry into the property, the barber offered to pay a rent of ₦50, 000 which was accepted by the landlord on the understanding that this sum would be adjusted upwards by 10% each subsequent year, as earlier mentioned. The ₦50, 000 rent for the first year is successfully covered by

Table 4: Rent and business profitability: The example of a barber's shop

Revenue(₦)	Year 1 850,000	Year 2 915,500	Year 3 935,600	Year 4 780,060
<u>Cost(₦)</u>				
(i) Wages	360,000	382,000	390,000	372,000
(ii) Equipment & Consumables (clippers, razors, etc)	70,000	74,000	77,000	73,000
(iii) Electricity charges (including standby generator & fuel)	50,000	56,000	58,000	58,000
(iv) Sundries	15,000	16,500	17,500	18,200
(v) Municipal charges	60,000	65,000	68,500	58,800
(vi) Interest on capital employed (generator, furniture & equipment, etc)	<u>40,000</u>	<u>50,500</u>	<u>51,000</u>	<u>61,000</u>
	595,000	644,000	662,000	641,000
(vii) Entrepreneurial return (30 % on cost)	178,500	193,200	198,600	192,300
Total Cost	773,500	830,500	860,600	833,300
Revenue less Cost	76,500	85,000	75,000	53,240
Amount available for rent (attributable to land & improvements made by landlord)	76,500	85,000	75,000	53,240
Rent to be paid	50,000	55,000	60,500	66,550

the ₦76,500 surplus produced in that year. At the close of business in the second year, the ₦85,000 excess of revenue over cost means that the barber can afford to pay the new, increased rent of ₦55,000. The third year shows that the sum available for rent has dropped to ₦75,000 as a result of rising costs and a slower growth in revenue. However, the ₦75,000 available is still able to pay the upwardly reviewed rent of ₦60,500. In the fourth year, there is a fall in revenue as well as a rise in cost. This leads to a 29% drop in the sum available for rent compared with the preceding year. The available sum of ₦53,240 falls below the rent liability of ₦66,550 for that

year. Thus, the business is operating at a loss since it has become unprofitable for the barber to pay the contractual 10% increment and remain profitable.

In the present shop example, the surplus produced is described in the table as the amount available for the land and improvements of the land. Rather than all the value coming from land only as in economic rent, there are now two components of value: the land and the improvements. The reasons are as follows. Land, on its own, can be used to create value through rent. Putting up a building on land increases its usefulness by boosting its rent-producing capacity. This means that building upon land enhances its value. After putting up a building on land, it becomes impracticable to use the land purely as land. The land must thenceforth be used together with the building which now has inhered in it, merging both elements into one unit for the purpose of producing value or satisfaction. However, it needs be mentioned in passing that land and building retain their original status as two elements of cost which were brought together for the purpose of creating value. This is important where there is a need to find out the amount of capital tied up in an investment.

In the extant case, the land and the building on it share a common characteristic, which is, that they jointly, as one unit, make space available for the business to operate. However, availability of the shop space does not create value, only a potential arising from the obvious effort made to advance the usefulness of the land. Thus, this factor of production remains passive (just like uncultivated farmland) until the tenant or business operator puts it into use for production (like the tenant-farmer). As mentioned in Chapter 1, a profit can be produced only if value can be successfully created and delivered through an economic activity. The shop operator's economic activity can create and deliver value if his business revenue adequately exceeds cost, thereby justifying the payment of rent for the premises. The profitability of the business done in the space provided will be dependent on the desire for the services of the barber. The revenue-cost relationship will decide whether or not a surplus will be produced and the size of it. This will be reflected, in turn, on the value of the shop space, a situation which illustrates the principle of derived demand as earlier mentioned. So the profitability of the services of the barber determines the rent-producing ability of the shop space.

Since the surplus produced by tenanted farmland does not involve any contributions from the landowner, the value is directly attributable to the land. It becomes the entitlement of the landowner who, as the presumed investor of capital in the purchase of the land, receives an earned income.

Otherwise, the income may be described as an unearned entitlement of the landowner. In the case of the tenanted shop, the landowner contributes as producer of the shop space and facilitator of the opportunity for value to be created by the tenant. On his part, the tenant supplies the necessary input which, in combination with the landlord's efforts, enables the production of value from which it becomes possible for rent to be earned and paid. The rent produced is for the entire landed property (land and improvements). This is the rental value of the shop, the manifestation of its rent-producing capacity. As demonstrated in Table 4, this value will rise or fall according to changes in the barber's business. This will also be the case with any other economic activity undertaken on the premises. The meaning is that the tenant's ability to pay determines the financial sustainability of a business tenancy. Similarly, the profitability of a tenant's activities on the rented property determines his ability to pay. This is the same for farmland as described in rent theory. For every business tenant, the profitability of occupying a property is the most important consideration. This determines the rent-producing capacity of property. It is the basis of property value - the rental value and capital value of property.

The preceding examples explain the nature of rent and how people may rationalise its payment. From the three examples given above, the common thread is that rent is a value which is generated where economic activity which takes place on undeveloped or improved land produces a surplus of revenue over costs. This justifies the rent paid for the property which a business occupies. If the rent cannot be justified by the earnings of the business, then the business is unviable in that location. Despite the status of land as a primary factor of production, it remains in a passive state until put to work to create value, either in its bare state or an improved one. Creating value involves combining land with co-operating factors which can work together to achieve productive and profitable outcomes.

Types of Rent

All the preceding examples demonstrate that rent is economic in nature. In transactions involving property, as between landlords and tenants or lessors and lessees, rent is understood as a contractually agreed amount of money payable or paid on a periodic basis to the owner of property by its occupier in consideration of the benefit of occupation. Such rent may be described as 'rent' or 'lease rent'. However, there are certain other 'rents' which relate to property.

(i) Ground rent

This is a payment for undeveloped land under a ground lease. This kind of lease entitles the lessee to occupy and use the land, including putting up a building on it. However, at the end of the term, possession of the land and building reverts to the land owner.

(ii) Pepper corn rent

This is a symbolic or nominal rent. It usually has little economic value.

(iii) Rack rent

This is a rent which reflects the full annual value of a property. It is also known as the full rental value, that is, the rent which realistically reflects the income-producing capacity of a property at a particular point in time as determined by demand and supply factors.

(iv) Passing Rent

The rent which a tenant pays under a subsisting lease. It may be the rack rent (full rental value) or a negotiated amount. It is also known as the lease rent.

(v) Head Rent

This describes the rent paid under a lease where the amount in question is lower than the full rental value of the property, thereby creating a profit rent.

(v) Profit Rent

The difference between the rent paid under a lease and the full rental value of the property. The difference constitutes a profit to the lessee because it provides the opportunity to sublet the (subject to the lessor's consent). By this arrangement, a leasehold interest is created by the granting of a sublease to a third party, making the lessee a head tenant who pays a head rent to the lessor.

Outside the property sector, particularly in the business and finance world, 'rent' has been adopted as a word for all payments made for the use of hired assets such as cars, plants and other equipment. These all are income-producing assets just like land and property.

Why People Invest in Landed Property

Real estate is highly sought after because of its utility. This comes in many dimensions. One of these is its capacity to provide shelter which is a basic