Boundary Parametric Control of Oscillations in Heat and Fluid Flows

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To my dear wife Olga
for her love and support

Parametric control of oscillations in heat and fluid flows creates new possibilities for the intensive development of new technologies, devices and facilities that are highly efficient and profitable. Such technologies are based on the use of parametric oscillations in continua, which act as a working body, or use of phenomena, which would be impossible without effective suppression of various instabilities. Some periodic or quasi-periodic influences of regular or random character are widespread in technological installations and processes. These include oscillations of a field (electric, magnetic, acoustic, temperature, vibration, etc.) or those conditions under which acyclic changes of certain system parameters cause oscillations of other parameters causing, in turn, parametric oscillations of the system.

Boundary parametric excitation and suppression of oscillations in continuous media promises an emergence of new physical principles for creation of processes or considerable improvement of indicators of the existing and projected installations. Methods based on use of strong resonance effects that will allow the development of new energy and resource-saving technologies important for intensive development of the economy are especially promising.

The book presents results from new technologies developed based on newly discovered and studied phenomena: film flow dispersion and granulation with cooling rates up to 10^4 K/s for the production of new super materials, protection of metallurgical aggregate machines using artificially controlled garnissage, etc.

The monograph is an author's translation of the previous Russian edition, revised and supplemented (© Kazachkov I.V., 2016).

Referees:
Professors of the Royal Institute of Technology, Energy Technology Dept:
Björn Laumert and Andrew Martin

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BASIC SYMBOLS

a - Film thickness, *m;* ε - dimensionless film thickness
a_j - Heat diffusivity coefficient for *j*–th medium, m^2/s
$\vec{B}$ - Magnetic induction vector, $T=Vb/m^2=kg \bullet c^{-2} \bullet A^{-1}$ - tesla is the SI derived unit of Magnetic flux density. One tesla is equal to one weber per square meter; named in 1960 in honour of Nikola Tesla
b - Characteristic radius of the particles in a granular layer; also, a radius of the vertical jet spreading on the horizontal plate, *m*
b_1 - Characteristic radius of the pores in a granular layer, *m*
c_p, c_v - specific isobar and isochoric heat capacity, *J/kg•K*
$\vec{D}$- Electrical induction vector (also called electrical displacement) the sum of two vectors of different nature: electric field intensity E (the main characteristic of the field) and the polarization P (determines the electrical state of the substance in this field, C/m^2
d - Diameter of the particles (granules), *m*
$\vec{E}$ - The electric field intensity, *V/m*
e - Specific energy density, J/m^3
$\vec{f}$ - Volumetric force density, *N/m*
g - Acceleration due to gravity, m/s^2
$G_{k,m}$ - Parameter of feedback control for wave numbers *k,m*, $W/(m^2 \bullet K)$
$\vec{H}$ - Vector of the magnetic field strength, *A/m*
h - Height of the granular layer, *m*
I - Electric current, *A*
$i = \sqrt{-1}$ - Imaginary unit,
J_n, Y_n - The Bessel functions of the first and second kind of *n*-th order
I_n, K_n - The modified Bessel functions of the first and second kind
P - Stress tensor, p - pressure, N/m^2
$\vec{F}_{\mu k}$ - Vector of surface forces, N/m^2
Q - Heat flux, W/m^2
q_v - Specific energy due to volumetric heat sources, W/m^3

q_n - Specific energy flux through boundary surface, *W/m²*

r_0 - Nozzle radius or radius of the cylindrical channel, *m*

$R = 8.3143 \pm 0.0012 \dfrac{J}{K \cdot mol}$ - The universal gas constant,

S - Area, *m*

s - Entropy, *J/K*

s_{12} - Specific interfacial surface of the heterogeneous continua, *1/m*

S_{ij} - Interfacial surface separating phases with indices *i* and *j*, m^2

t - Time, *c*

T - Temperature, *K;* θ - dimensionless temperature

U - Voltage or electric potential, *V*

u,*v*,*w*- The velocity vector $\vec{v}$ components, *m/s*

V - Volume, m^3

α_j - Volumetric content of *j*-th phase in heterogeneous system

β_p, β_T - The baric and thermal expansion coefficient, m^2/N, *1/K*

γ_e - Specific electrical conductivity, *1/(Ω·m)* (*Ohm·m*)

δ - Thickness of the skin-layer, *m*

κ - Heat conductivity coefficient, *W/*(*m·K*)

λ_{21} - Specific heat of transition from phase 2 into phase 1, *J/kg*

λ_{jc} - Heat transfer coefficient from *j*-th medium to surrounding, *W*/(m^2·*K*)

μ - Dynamic viscosity coefficient, *Pa·s*

μ_e - Dielectric permittivity of medium (farads per meter), *F/m*

μ_m - Magnetic permeability of medium (henries per meter), *H/m*

ν - Kinematic viscosity coefficient, m^2/s

ν_m - Magnetic viscosity coefficient, m^2/s

ξ - Structure parameter of granular medium

ρ- Density, kg/m^3

ρ_e - Volumetric density of charges, C/m^3

σ - Surface tension coefficient, *N/m*

χ- Amplitude of interface perturbation, *m;* ζ - dimensionless χ

ω - Frequency, *1/s=Hz*

Indices and mathematical symbols

in, ex **-** Internal, external
0 - Parameter of the unperturbed medium
m - Parameters of the electromagnetic field
n, τ - Projections of function on normal and tangent surface direction
re, im - Real and imaginary parts of complex value
c - Surrounding medium
*- Parameters of Eigen oscillations (bottom index) or parameters belonging to melting boundary; also, critical values (top index)
ij - Ratio of the values with indices *i* and *j*
s,l - Parameters of solid and liquid phases, correspondingly
∇, Δ - Differential operators: gradient and Laplace; also, difference
div - Divergent
x, $\cdot$ - Vector and scalar product of vectors, respectively

Basic dimensionless criteria

Eu, Re, We, Fr - Hydrodynamic Euler, Reynolds, Weber, Froude criteria
Oh, Ga, Pr, Be, Bo - Kinematic Ohnesorge, Galileo, Prandtl, Batchelor, Bond numbers, respectively
Nu, Pe, Bi - Heat transfer criteria: Nusselt, Peclet and Biot numbers
$Bi_{k,m}$ - Modified Biot number
Gr, Da, Ra- Grashof, Darcy and Rayleigh numbers
Ja, K - Phase transition criteria: Jacobi and Kutateladze numbers
Al, Ha - Electromagnetic Alfven and Hartman criteria
Eu_m, Re_m, We_m - Magnetic Euler, Reynolds and Weber numbers
Fo - Fourier number (dimensionless time for temperature change)
Eu_g - Vibration Euler number
Ca - Capillary number
Ar - Archimedes number
Sh - Strukhal number

INTRODUCTION

Parametric oscillations in continua, especially at the boundaries separating different media (solid, liquid, gaseous), have attracted the attention of many researchers because of their originality and widespread applications in nature, technological processes and technical systems. However, significant development of the theory of parametric oscillations only began in the 19th century with the fundamental studies of Faraday, Savart, Magnus, Plateau, Rayleigh, Chernov, and other renowned scientists. Later, the theory was significantly developed by N. Bohr, K. Weber, W. Ohnesorge, A. Haenlein, W. Tollmien, S. Chandrasekhar, C.C. Lin, G. Schlichting, T. Benjamin, L.D. Landau, P.L. Kapitza, J.B. Zeldovich, M.A. Lavrentiev, S.S. Kutateladze and others.

Significant contributions to the development of various areas of the theory of Eigen and parametric oscillations in continua and their use in engineering and technology were made by J. Lighthill, J. Whitham, J. Joseph, G.A. Ostroumov, V.E. Nakoryakov, R.I. Nigmatullin, A.G. Butkovsky, Yu.I. Samoylenko, V.M. Entov, A.P. Sukhorukov, L.A. Ostrovsky, A.F. Kolesnichenko, A.I. Nakorchevskii, and others.

The **current relevance of the problem** arises from the intense development of new technologies, creation of devices and mechanisms that are highly efficient and profitable based on the use of parametrically controlled oscillations in continuum media. In addition, stability of systems that would be impossible to achieve in the absence of various means for the effective suppression of oscillations, is in focus. Periodic or quasi-periodic influences of regular or random nature are widespread in technological installations and processes [48, 89, 99, 200, 299, 435, 469]. These include fluctuations of a field (electric, magnetic, acoustic, temperature, vibration, etc.) or those conditions under which acyclic changes of certain parameters in a system causes oscillations of other parameters provoking, in turn, parametric oscillations of the whole system or some part of it.

Parametric excitation and suppression of oscillations in continua is promising in terms of the emergence of new physical principles for the creation of processes, or in terms of the considerable improvement of the effectiveness of existing and projected installations, together with an increase in their possible uses. Methods based on the use of strong (e.g., resonance) effects that will allow the development of new energy and resource-saving technologies are particularly promising.

Many parametric instability phenomena are observed in fluid flows complicated by phase transformations and chemical reactions [2, 35, 62, 88, 194, 195, 277], in magneto-hydrodynamics (MHD) and plasma physics [45, 68, 79, 91, 222, 226-228, 389], thermal hydraulics of granular and underground systems [66, 104, 198, 216, 345], biological fields [115, 256, 497] and others [75, 95, 201, 279, 299]. Parametric oscillations in continua are of great theoretical interest and have continuously expanding practical applications resulting in intensive development of the underlying theory. Parametric excitation of oscillations and suppression of random or regular oscillations (system stabilisation, process stabilisation), are equally important.

The ability to improve technological processes based on the use of parametric oscillations in continua was first revealed more than a century ago: for example, in 1879, D.K. Chernov [393] was the first to note the positive influence of mechanical oscillations on the quality of a crystallizing ingot. However, up until now, recommendations for the choice of parameters for external influences have often been contradictory or completely absent. For control of the crystallisation of the metal structure, the recommended range of oscillations is 1-250 Hz [89]. It is also known that low-frequency mechanical, electromagnetic and thermal oscillations or a combination thereof often crush the structure of metal making it finer by a factor of 2-10 times [318, 356]. Nevertheless, a pulsing thermal field can result in the alignment of the larger-sized crystals.

The temporal modulation of parameters in continua influences the stability of the physical-mechanical and chemical processes and can result in the stabilization of a certain kind of instabilities [66, 75, 155, 226-228, 238, 435] or, by contrast, to excitement of instabilities [3, 61, 68, 79, 89, 111, 120, 129, 160-163, 281-283, 466]. In general, the processes of parametric excitation and suppression of oscillations in continuum media are interconnected, especially when significant non-linearity of the processes and a presence of complicating factors exists: heterogeneity of the physical media properties, relaxation, etc.

Intensive development of the technology for generation of parametric oscillations in continua is associated with their successful application in many areas of the economy: structural control of MHD flows [78, 91, 315], intensification of chemical and technological processes [190, 192, 321], localization of heating [219, 254, 331-333]. The considerable spread of vibration and acoustic methods resulted from an increase in technological processes [3, 26, 35, 48, 56, 89, 200, 280, 299, 508, 521], that also have not been studied in detail [321, 326, 378, 457].

In respect of their impact on continua (technological processes), parametric oscillations can be classified as follows: optimizing, intensifying, transforming and stabilizing. The first type of parametric oscillations improves only the process or parts of it (acoustic granulation and centrifugation, etc. [299]), the second type increases the process' speed (MHD granulation [201, 202], acoustic dissolution). The third type helps to achieve an essentially new regime or process, which would be impossible in the absence of parametric action (vibration dispersal of liquids, crushing of the structure of an ingot, spouting of drops from a surface of a vibrating layer of liquid. The fourth type makes it possible to carry out processes in continuum media in a stable regime (decrease in hydraulic resistance to moving bodies in a liquid by stabilization of the laminar mode of flow [199, 432, 563], ensuring chemical reactions with unstable parts can be run, etc.). Clearly, a strict separation of these four types of parametric impact on the processes in continua does not exist.

The ***effects*** arising from parametric oscillations in continua can be subdivided into ***first order*** (small-amplitude oscillations defined by frequency, intensity and speed of propagation in a medium) and ***second order*** (powerful perturbations of the medium causing the non-linear phenomena, violation of continuity, etc.).

Despite the large number of papers on separate classes of parametrically excited (suppressed) oscillations in continua, research relating to the problem of parametric excitation and suppression of oscillations at the interfaces of continua is still at an initial stage. Therefore, the objective of this monograph is to research the specified problem by considering three task classes:

- film flows under the prevailing contribution of the inertia forces,
- phase transition boundaries between liquid and solid states in channel-type flow,
- and granular gas-saturated media,

which have not been studied in enough detail, by application of different types of parametric action: electromagnetic and thermal fields, vibrations, etc. The parametric oscillations considered are mainly of the third (transforming) and fourth (stabilizing) type (and sometimes the second type).

The focus is revealing the opportunities for forecasting the reaction of continua to various external influences, especially resonant influences. The latter can form a basis for creating essentially new, highly effective power and resource-saving technologies, e.g., for creating powders and granules by means of film-MHD and vibration-type granulation machines [81, 100, 202]. Parametric oscillations having both first and second-order effects on continua interfaces are considered.

For the studied phenomena, the essential influence of the regularities of parameter spatial change, dependent on which parametric oscillations are generally represented by various physical fields in the form of partial differential equations (PDEs), is characteristic. Therefore, according to the *Law of Requisite Variety of W.R. Ashby* [406], the external influences also have to be fields distributed in space and time and described by the PDE. External action can be transferred to each point of the continua (volumetric control) or to its boundary (boundary control). The most effective volumetric impact on the processes in continua is rarely realized. An alternating electromagnetic field penetrates into a conducting medium only up to a skin-layer thickness; an acoustic wave also fades in a medium, etc.

Boundary control in continua, the theory of which was developed by A.G. Butkovsky with colleagues [66, 67], T.K. Sirazetdinov [354, 355], Yu.P. Ladikov and his team [227, 228], etc., is relatively easily realized. Yu.I. Samoylenko proposed linear layered and fibrous artificial media possessing high resolution (reaction of the medium to external action can be as close as possible to the delta function) for electromagnetic control of processes in continua [339]. The theory of such systems was developed in [226, 340]. Where the nature of the operating influences is concerned, the problems of boundary control are subdivided into three classes [227] (control of mass, momentum and energy flows), in each of which there are three types of control: with feedback, by programming, and by determination of the dynamic properties of an object based on the boundary values of its parameters.

This research is mainly devoted to the problems of the second (disintegration of film flows by parametric oscillations) and third (stabilization of the phase transition boundaries of thin films of a solid phase) classes with the first and second types of influences. In parametric suppression of oscillations, at the phase transition boundaries, the energy exchange is crucial. However, the main role is played by momentum exchange in case of electromagnetic or vibration initiation of surface oscillations in film flows.

Open-loop and closed-loop control systems are applied to excitation and suppression of the parametric oscillations in continua (with feedback):

- the first type of control system is when the return influence of the continua on the actuation device is negligible (for example, electromagnetic influence on the medium if magnetic Reynolds numbers are small),

- the second control system type is for non-linear media and stabilization of fast-proceeding processes, etc.

Thus, in both cases, at the first stage of research, it is necessary to reveal boundary impact regularities acting on the technological process, while in the second stage it is necessary to define the structure and parameters of the device providing the required impact on continua.

Modern continuum mechanics merges in many areas of physics and applies to various technical and biological systems with a broad application of mathematical modeling and computers. The following problems have been considered:

- programmed control with the open-loops relating to the improvement of the structure of ingots [318, 356],
- vibration mixing and intensification of heat transfer [89, 200],
- stabilization of technological processes [75, 97, 227],
- parametric excitation of the waves on liquid surfaces [163, 214, 521],
- stabilization of the flows in the boundary layers using suction and blow-in [192, 563],
- and many others [27, 96, 130, 279, 421, 440, 461].

Many new tasks have been solved in recent decades [25, 42, 129, 215, 236, 255, 326, 334, 335, 359, 437, 515].

In general, the problem of parametric excitation and suppression of the oscillations in continua can be formulated as follows. After a schematization of the physical phenomenon, allocation of its most essential and minor parameters and creation of the physical and mathematical models with a subsequent optimum choice among them (in a certain sense), it is then necessary to investigate the regularities of system perturbations in space-time. This leads to establishment of the correlations of type $F_n(A_j,\omega_j,\vec{k}_j;\vec{A}_j^*,\omega_*,\vec{k}_*)=0$, where $A_j,\omega_j,\vec{k}_j$ are the amplitude, frequency and wave vector of j-th perturbation, respectively. $\vec{A}_j^*$ is a vector of the perturbed parameters of the system (process) of dimension J. Parameters marked with an asterisk belong to the Eigen fluctuations of a system, F_n are the sought functions of parameters (for example, differential), $n=\overline{1,N}$, $N\geq J$ (if $N<J$ the task becomes uncertain). The behaviour of continua is defined by parameters of the Eigen fluctuations and external influences.

Thus, the problem of excitation (suppression) of the parametric oscillations can be reduced to a determination of the physically realized parameters $A_j, \omega_j, \vec{k}_j$ providing the necessary mode, or the set type of a vector function $\vec{A}_j^*(x, y, z, t)$. In particular, in the event of parametric oscillations at the interfaces separating different media, it is often possible to reduce a task to the solution of the equation for boundary oscillations (a crystallization surface, a film flow surface, etc.) and solution of the equation for the external exciting force (field). The problems of excitation-suppression of the oscillations of the set type demand control of all other perturbations because the energy pumping in a certain wavy mode in the non-linear systems can lead to its transfer into other modes, which may result in a highly complicated regime.

The purpose of this monograph is development of a set of the physical, mathematical and numerical models of the parametric excitation and suppression of oscillations at the interfaces separating continuous media:

- for carrying out the computational, physical and natural experiments,
- for revealing the new phenomena and parametric effects, and for their use in improvement of the existing processes,
- and for creation of the prospectively highly efficient and profitable technological processes.

The scientific novelty of this work is a development of the theory and applications of the parametric excitation and suppression of oscillations at the boundaries of continua on the samples of three classes: flat and radial spreading film flows of viscous incompressible liquids, both conducting and non-conducting; liquid to solid-state phase-transition surfaces; and heterogeneous granular media. The external influences considered are the alternating electromagnetic, vibration, acoustic and thermal fields. Not only linear, but also non-linear parametric oscillations are investigated and the results of theoretical research are confirmed and supplemented with the corresponding experimental data.

General and specific peculiarities of the parametric oscillations and the new parametric effects are uncovered. The first chapter contains a general statement and description of the problems studied, while the various parametric oscillations in continua are considered based on a common methodological foundation. The assessment of the status of the problems, analysis of their features, prospects for further development and the main difficulties of different characters are given.

The second and the third chapters are devoted to a class of problems relating to the parametric excitation and suppression of oscillations of a surface of flat and radial film flows of conducting and non-conducting fluids, both free surface as well as surfaces restricted by some plates. For the non-conducting viscous incompressible fluid, the investigations are done subject to vibrations and acoustic fields in the form of progressive or standing waves, etc.

Chapter 4 contains the results of investigations on the parametrically excited and suppressed oscillations at the liquid to solid state phase-transition boundaries (generally in relation to stabilization of a surface of a thin solid layer, which may be melted in some localised regions, by use of the electromagnetic fields and automatic heat flux control systems).

In chapter 5, the parametric oscillations at the boundaries separating phases of the heterogeneous system in a space that is prone to the temperature perturbations are considered taking into account the non-linearity of the processes and physical properties of media and the heterogeneity.

Experimental confirmation of the consistent patterns determined theoretically and identification of the new parametrical effects on physical models and experimental facilities are presented in the sixth, final chapter of the monograph.

This *new scientific field* represents the theoretical research on physical and mathematical models, as well as the experimental installations concerning the various parametric oscillations at the interfaces of continua, including revealing of the new effects and actions used in essential technological applications. The mathematical models are kept simple but adequately reflecting the most essential separate features if not all the studied process. Complex non-linear models are realized using the computational methods. Models, generally rather simple, are suitable for computational experiments and possess high enough accuracy even based on the finite-difference numerical schemes with relatively small number of the grid nodes (some of them can be implemented on personal computers).

The *validity and reliability* of the results obtained follow from the validity and reliability of the initial theoretical basis of parametric oscillations in continuum media and the models applied. Also, from strict substantiated statements of the tasks solved and the methods for their solution. Finally, from the experimental proof of the results and then comparison of the results with the known limit cases.

The *practical value* of the work consists in application of the developed mathematical models of parametric control to the interfaces of continuum media in order to calculate actual physical situations and select the technological solutions providing a significant increase in quantitative and/or qualitative levels of the performance.

Data on the new technological processes based on the discovered new phenomena are provided in the monograph, and some patents on the invented methods and devices for the granulation of materials together with the outcome of their implementation in industry are described. In particular, description of the film granulator for Zn, Al, Mg, etc. and their alloys, which has currently no analogues in the world, and which was developed and introduced by the author with the colleagues, is presented.

All three classes of the tasks considered are important for the film granulator:

- parametric control of the liquid metal film disintegration (dispersing mode),
- stabilization of the phase transition boundaries (protection of the channel walls against destruction and, at the same time, protection of the liquid metal flow in a channel against contamination with particles of other materials),
- and thermal hydraulics of the granular medium (selection of the optimal cooling regime for granules and achievement of the highest possible cooling rate for the drops in their solidification).

The processes studied are described by the PDE arrays for the mass, momentum and energy conservation for both homogeneous and heterogeneous media. In addition, the electromagnetic field equations and some known thermodynamic state equations are applied.

The mathematical methods applied are the theory of integral transformations, methods for averaging the differential and integral-differential operators, reductive perturbation method for the non-linear PDE array, numerical methods for the non-linear PDEs (split method), physical modelling of the fields and of interacting media, etc.

Therefore, the concept is adopted here: investigate the problem of parametric excitation and suppression of the oscillations in three different systems (film flows, phase-transition boundaries and granular media) from united methodological positions, and reveal both features for each of them and the general regularities. The mathematical and physical modelling of the phenomena are applied to expand the areas of the studied phenomena. Their comparative analysis in common areas makes it possible to acquire data on adequacy of the constructed models and reliability of the technique applied. This monograph presents:

1. The theory of parametric excitation and suppression of the oscillations in film flows subject to the electromagnetic fields and vibration.

2. The theoretically revealed and experimentally studied new phenomena of the soliton-like and shock wave disintegration of the film flow into the drops of given sizes.
3. Theory of parametric control of the phase-transition boundaries from a liquid to a solid state on the channel walls.
4. Thermal hydraulic oscillations in granular saturated media: localization of dissipative processes and local abnormal heating due to the non-linear heat conductivity of vapour, etc.
5. Experimental facilities and experiments with the processes of the parametric control for the wavy film flow, and with the newly created devices and processes.

CHAPTER 1

STATE-OF-THE-ART PROBLEMS OF THE PARAMETRIC EXCITATION AND SUPPRESSION OF OSCILLATIONS IN CONTINUA

The extensive development of modern industry and equipment requires research of the parametrically excited (caused by the periodic exciting force) or simply parametric oscillations in continuum media (continua): dynamic stability of elastic systems, fluctuations of plates, covers, rods, oscillation of liquids in vibrating vessels, in pipelines, pumps, etc. Currently know-how concerning multiple methods of the parametric excitation and suppression of oscillations in continua, in particular, in the liquids exists: for example, physics of the superficial phenomena and thin films [20, 46, 61, 62, 77, 84, 85, 95-97, 103, 127-134, 206-209, 423].

The new directions in natural sciences [98, 213-215, 279-284, 365] have arisen in research into the non-linear processes of different physical-mechanical and chemical systems: thermo-hydrodynamic and magneto-hydrodynamic instability, catastrophes, bifurcations, self-oscillations, etc. In recent years, it has been demonstrated that an ideal means of energy transfer in the non-linear systems is via solitons [277,285-289,497], with a mathematical description for shallow water given by Korteveg-De-Vriz [492]. M.A. Lavrentyev presented a strict proof of their existence for a liquid of finite depth [223]. Steady individual waves are formed because of mutual compensation of the non-linearity and dispersion, which ideally combine the properties of a particle and a wave [115, 497].

A number of the new interesting phenomena were discovered in the studies of thermal waves [86, 140, 337]. An example is the non-linearity caused by a dependence of the physical properties of the medium on coordinates and time, in particular, the dependence of the heat conductivity coefficient on temperature, which goes to zero at a front of thermal wave. The presence of some sources and sinks in a medium lead to an inertial delay in the speed of perturbation spreading. The revealed mechanism of volumetric heat absorption [219] was suggested as one possible explanation for the thermal self-isolation of a ball lightning [253]. Mathematical models of the non-linear heat conductivity with adaptation to the actual cases make it possible to describe a wide class of the diffusion problems, distribution of thermal and electromagnetic fields, etc.

Problems of parametric excitation and suppression of the oscillations in continua, in particular at the boundaries, connect with research into media properties. Creation of adequate physical and mathematical models of continuum media and the processes in them, research into methods that have an effective impact on these processes, and control of the processes are the most important problems of modern continuum mechanics. Some features of the class of tasks noted in the introduction, the applied mathematical models of continua and the methods for excitation and suppression of oscillations are described and analysed here (the main focus of attention is boundary control in continua). In addition, a number of unsolved challenges, both existing and future are listed.

1.1. Physical-mathematical models of continua

Creation of physical and mathematical models of continua and subsequent research into the physical-mechanical and other processes is most frequently based on the hypotheses of continuity and continuous n-times differentiability of all functions describing parameters almost everywhere except for individual points, lines or surfaces, where the discontinuities are allowed. This model creation allows a phenomenological approach applying the methods of classical calculus and mathematical physics.

However, although this phenomenological approach made it possible to solve a set of problems in continuum mechanics, which became known as classical, and this approach is now one of the most frequently applied, it has to be stated that many processes in continua do not comply with this physical model. For example, the acceleration fields in turbulent flow cannot be described by a class of continuous or quasi-continuous functions, and the film flow compressed in a direction tangential to a free surface cease to be differentiable in this direction even if it remains continuous (saw-tooth surface with the teeth perpendicular to it).

The other examples include spraying and cavitation. Here it is impossible to individualize the volumetric domains because the continuum turns into a set of the free points. In heterogeneous media, the velocity and temperature fields, etc. become fractured and combination of two different fields in one continuum results in a polysemy of parameters that belong not to the individualized point of a medium but to a space point, where the individual points of the various parameters combine.

A statistical approach and diverse variation methods [31, 53, 125, 138, 308, 410] have been applied to the non-classical problems of continuum mechanics that do not satisfy the phenomenological hypotheses. Because of the mathematical complexity, a statistical (microscopic) approach is used most frequently to justify phenomenological (macroscopic) models of continua provided the model is not unique (discharged gases, plasma, etc.). Strictly speaking, there are no real continua in nature, although the continuity hypothesis describes them well at the macro level and the continua model allows using the powerful theory of continuous functions, differential and integral calculus.

For the systems that do not comply with the continuity hypothesis of an occupied space, the fractal theory (objects of fractional dimension) and the developed theory of fractional differential calculus, (not just integer derivatives and integrals as in classical calculus) are more suitable. In this monograph, I generally use a phenomenological approach according to which creation of the mathematical models of continua is based on assumption that each point of the medium (physically infinitesimal volume) is characterized by a set of the defining parameters introduced based on the experimental data and on the theoretical investigations.

Nowadays, based on the phenomenological approach, such a set of the mathematical models of various classes of problems has been developed, taking into account their specific features, resulting in a need arising for their systematization and development on the basic principles of mathematical modelling of processes in continua. Therefore, when developing new complex systems of the mathematical and numerical models, it is necessary to proceed based on a modular principle in order to allow the greatest possible unification of the modelling process and facilitate the use of the mathematical and computer numerical models by various researchers in various tasks.

The equations of dynamics of continua

Adopting the phenomenological approach, the creation of mathematical models of continua is based on an assumption that each point of the medium (physically infinitesimal volume), whose physical-mechanical state is characterized by a set of the defining parameters, is introduced based on the experimental and theoretical data or on the statistically averaged functions (temperature, for example).

The general equations of the dynamics of continua with any structure (including the heterogeneous mix considered without phase interaction, which cannot be accounted when studying the movement of a heterogeneous system as a uniform complex continuous medium), may be represented in the form:

$$\partial\rho / \partial t = -div\left(\rho\vec{v}\right), \tag{1.1}$$

$$\rho\left(\partial\vec{v} / \partial t + \vec{v}\nabla\vec{v}\right) = divP + \rho\vec{F} - \sum_{j=1}^{N}\vec{v}\nabla\left(\rho_j\vec{v}_j\right), \tag{1.2}$$

$$\rho\left(\partial e / \partial t + \vec{v}\nabla e\right) = div\left(\vec{q} + P\vec{v}\right) + \rho\vec{F}\vec{v} + \sum_{j=1}^{N}\left[\rho_j\vec{F}_j\vec{v} - div\left(\rho_j e_j\vec{v}\right)\right], \tag{1.3}$$

with ρ- density of heterogeneous medium, $\vec{v}$ - velocity vector for heterogeneous medium, *t*- time, *P*- stress tensor, $\vec{F}$- volumetric force, $\rho_j, \vec{v}_j$- parameters of the medium's components (likewise for the other parameters with indices), *e*- specific density of energy, q - specific volumetric energy influx.

Equation (1.1) is the mass conservation equation, (1.2) - conservation of momentum equation, (1.3) – conservation of energy equation. For reversible processes, the uncompensated heat is equal to zero. With an exception of the internal energy and entropy, the other functions of state and the additional thermodynamic relations are used. In case of a heterogeneous medium when an exchange of mass, momentum and energy between the phases inside the volume or at the boundaries must be taken into account, the terms for the exchange of mass, momentum and energy between the phases of the heterogeneous mix must be explicitly specified in the equation array (1.1) - (1.3). This is the main problem in the mechanics of heterogeneous media because in most cases it is not clear how to define the corresponding magnitudes of exchange of the mass, momentum and energy between the phases of heterogeneous continuum media.

Each phase occupies some part of an elementary volume of the heterogeneous medium: the volumetric contents of *N* phases α_j satisfies the equations $\sum_{j=1}^{N}\alpha_j = 1$, $\rho = \sum_{j=1}^{N}\rho_j\alpha_j$ (the density of the medium is expressed based on the actual densities of the phases). At each point of the heterogeneous medium, *N* parameters are defined for the continuum (densities, velocities, temperatures, etc.). The set of continua for each phase filling the same volume is called the multi-speed continuum[1].

Using the Gauss-Ostrogradsky's formula in a region of continuous or quasi-continuous movement of a continuous medium, it is possible to pass from the integral balance equations to the differential equation array describing thermo-hydrodynamic processes in the heterogeneous medium taking into account the joint movement of phases and interfacial exchange of the mass, momentum and energy. The main obstacle to use of this system in the mathematical modelling of heterogeneous media arises from the need for a specification of the laws of phases' interactions that is extremely difficult[2].

The system of differential equations obtained from the conservation equations for each component of the heterogeneous mix through the summation over the whole mix may be used as a rather weak manifestation of interfacial interaction in the heterogeneous medium for the description of the processes happening in it. However, remarkably, the momentum and energy conservation equations depend on the relative movement of phases inside the heterogeneous mix. Here when subsequently considering the mass forces, essentially only the gravitational and electromagnetic forces are considered. An inflow of external energy (the so-called Joule heat) or energy of the vibratory action transmitted through the boundary interface is taken into account.

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[1] Summation of the equations of the mix for all phases gives the equations of heterogeneous medium taken as a uniform system, without taking internal structure into account. Such a model does not exhibit the features of interfacial interaction in a heterogeneous mix. In contrast, accounting for an interaction of the phases' macroscopic inclusions results in a need to account for the conditions of joint deformation and movement of phases, influences of the form and the number of inclusions, their distributions in a space, phase transformations, etc. If physical-mechanical processes in continua are rather precisely described by continuous or quasi-continuous functions of the coordinates and time, the system of integral conservation equations can be replaced by the corresponding differential equations. However, for the real continuous media prone to external influences, the classical methods may be unacceptable, and all that remains are numerical methods based on the use of integral correlations used for fractured fields and media if integration by Riemann is replaced by integration by Lebesgue.

[2] The law of deformation of heterogeneous medium does not only depend on the velocity fields, pressure, and temperature of phases; therefore, determination of the regularities of interfacial interaction even for special cases is a very complex challenge. Nevertheless, accounting for fields' ruptures at the boundary interfaces is essential for some practically important tasks (similar to those considered in chapter 5).

Therefore, the system of differential equations must also include the field equations:

$$div\vec{B} = 0\,,\ \partial\vec{B}/\partial t = -rot\vec{E}\,,\ \vec{j} = \partial\vec{D}/\partial t + rot\vec{H}; \quad (1.4)$$

$$\rho d\vec{v}/dt = divP + \rho\vec{g} + \rho_e\vec{E} + \vec{j}\times\vec{B}\,,\ \rho_e = div\vec{D}; \quad (1.5)$$

$$\rho de/dt = \vec{E}'\cdot\vec{j}' + \rho\vec{g}\vec{v} + \Phi + div\left(\vec{q}^{ex} - \vec{q}^{in}\right) - pdiv\vec{v}\,, \quad (1.6)$$

where $\vec{E}'\cdot\vec{j}'$ is the Joule heat, Φ is the dissipation function, $\vec{E}' = \vec{E} + \vec{v}\times\vec{B}$, $\vec{q}^{ex}, \vec{q}^{in}$ are external and internal heat fluxes, and $\vec{j}' = \vec{j} - \rho_e\vec{v}$ are vectors of the electric field and current density in the coordinates connected to the moving volume.

The stress tensor for a Newtonian fluid (this monograph primarily considers Newtonian fluids) is presented in the form of the sum of a spherical tensor and a deviator: $P_{ik} = -p\delta_{ik} + \tau_{ik}$, where the summation is done by a so-called "mute" index, and δ_{ik} is the Kronecker symbol. In the dissipative function $\Phi = \tau_{ik}v_{i,k}$, where *i,k* are the "mute" indices, $v_{i,k}$ is the derivative of the *i*-th velocity component for the *k*-th coordinate. The heat flow is calculated according to the Fourier's law: $\vec{q} = -\kappa\nabla T$, where κ is the coefficient of thermal conductivity. The Ohm's law for the electric current density is as follows:

$$\vec{j} = \rho_e\vec{v} + \gamma_e\left(\vec{E} + \vec{v}\times\vec{B}\right). \quad (1.7)$$

The system (1.1) - (1.3) can be transformed to a divergent form that is very important in the numerical simulations. In addition to the above, some other correlations are used to close the system of equations, as well as the empirical dependencies of physical characteristics of the media on the parameters of state (pressure, temperature), etc. For instance, κ is, generally speaking, a function of pressure and temperature, and for the linear materials without polarization, the constitutive relations connect the magnetic and electric induction vectors to the electric and magnetic field intensity vectors through the equations:

$$\vec{B} = \mu_m\vec{H}\,,\quad \vec{D} = \mu_e\vec{E}. \quad (1.8)$$

All continuum media considered here (homogeneous, as well as heterogeneous ones) are assumed as the double-parametric [350], so that their thermodynamic functions (*e*, *p*, *s*, etc.) are determined by the two parameters of state. This allows using the Gibbs correlation

$$de = Tds - pdV, \tag{1.9}$$

where the entropy for incompressible fluid is

$$s = c_V \ln\left(p / \rho^{c_{p/V}} \right) + const, \tag{1.10}$$

therefore, $de = c_V dT + const$, which for an ideal gas taking account of the Clausius-Clapeyron relation yields:

$$p = \rho RT, \quad c_p dT = c_V dT + d(p/\rho), \tag{1.11}$$

Here c_p is, generally, a function of temperature.

When considering heterogeneous media, it is assumed that the properties of each phase are defined based on a condition of filling the total volume with this phase. The temperature of phases is entered on the basis of assumption of local thermodynamic balance within a phase. Further, the non-stationary differential equations for the non-isothermal movement of a viscous incompressible fluid (1.1) - (1.3), (1.4) - (1.6) (conducting or non-conducting) in the Cartesian and cylindrical coordinate systems are used. Therefore, it is expedient to write them here for the rather general case with constants μ_m, ν_m and variables ρ, μ, κ. In each case equations (1.7) - (1.11) for an actual continuum model are used.

Cartesian coordinate system

Without limiting the generality, it is possible to assume that the axis *z* is directed against gravity. Then, assuming the magnetic force prevails over the electric one, and taking into account the above-mentioned factors, the system of differential equations (1.5), (1.6) is written as:

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) + \frac{\partial}{\partial x}\left(p + 0,5\mu_m \left|\vec{H}\right|^2\right) = \frac{\partial}{\partial x}(\mu\frac{\partial u}{\partial x}) +$$

$$= \frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y}) + \frac{\partial}{\partial z}(\mu\frac{\partial u}{\partial z}) + \mu_m\left(H_x\frac{\partial H_x}{\partial x} + H_y\frac{\partial H_x}{\partial y} + H_z\frac{\partial H_x}{\partial z}\right),$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)+\frac{\partial}{\partial y}\left(p+\frac{\mu_m}{2}\left|\vec{H}\right|^2\right)=\mu_m(H_x\frac{\partial H_y}{\partial x}+$$

$$+H_y\frac{\partial H_y}{\partial y}+H_z\frac{\partial H_y}{\partial z})+\frac{\partial}{\partial x}(\mu\frac{\partial v}{\partial x})+\frac{\partial}{\partial y}(\mu\frac{\partial v}{\partial y})+\frac{\partial}{\partial z}(\mu\frac{\partial v}{\partial z}),$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)+\frac{\partial}{\partial z}\left(p+\frac{\mu_m}{2}\left|\vec{H}\right|^2\right)=\mu_m(H_x\frac{\partial H_z}{\partial x}+$$

$$+H_y\frac{\partial H_z}{\partial y}+H_z\frac{\partial H_z}{\partial z})+\frac{\partial}{\partial x}(\mu\frac{\partial w}{\partial x})+\frac{\partial}{\partial y}(\mu\frac{\partial w}{\partial y})+\frac{\partial}{\partial z}(\mu\frac{\partial w}{\partial z}),$$

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0, \qquad (1.12)$$

$$\rho c_v\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\right)+p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=$$

$$=\frac{\partial}{\partial x}(\kappa\frac{\partial T}{\partial x})+\frac{\partial}{\partial y}(\kappa\frac{\partial T}{\partial y})+\frac{\partial}{\partial z}(\kappa\frac{\partial T}{\partial z})+\frac{\partial q_x^{ex}}{\partial x}+\frac{\partial q_y^{ex}}{\partial y}+\frac{\partial q_z^{ex}}{\partial z}+$$

$$+\nu_m\left[\left(\frac{\partial H_z}{\partial y}-\frac{\partial H_y}{\partial z}\right)^2+\left(\frac{\partial H_x}{\partial z}-\frac{\partial H_z}{\partial x}\right)^2+\left(\frac{\partial H_y}{\partial x}-\frac{\partial H_x}{\partial y}\right)^2\right]+$$

$$+0,5\mu\left[\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)^2\right],$$

$$\frac{\partial H_x}{\partial t}=\nu_m\left(\frac{\partial^2 H_x}{\partial x^2}+\frac{\partial^2 H_x}{\partial y^2}+\frac{\partial^2 H_x}{\partial z^2}\right)+\frac{\partial}{\partial y}(uH_y-vH_x)+$$

$$+\frac{\partial}{\partial z}(uH_z-wH_x),\quad \frac{\partial H_y}{\partial t}=\frac{\partial}{\partial z}\left(vH_z-wH_y\right)+\frac{\partial}{\partial x}(vH_x-uH_y)+$$

$$+\nu_m\left(\frac{\partial^2 H_y}{\partial x^2}+\frac{\partial^2 H_y}{\partial y^2}+\frac{\partial^2 H_y}{\partial z^2}\right),\quad \frac{\partial H_z}{\partial t}=\frac{\partial}{\partial x}\left(wH_x-uH_z\right)+$$

$$+\frac{\partial}{\partial y}\left(wH_y - vH_z\right) + \nu_m\left(\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2}\right), \quad \frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0 \cdot$$

The potential part of the electromagnetic field (magnetic pressure) was separated in (1.12), therefore this form is called the symmetrical form.

Cylindrical coordinate system

In a cylindrical coordinate system (r, φ, z), the coordinate surfaces are cylinders $r = const$, semi-plates $\varphi = const$ and plates $z = const$, therefore differential equation array (1.12) transforms to:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial r} + \frac{\rho u}{r} + \frac{\partial(\rho v)}{r\partial \varphi} + \frac{\partial(\rho w)}{\partial z} = 0,$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v\partial u}{r\partial \varphi} + w\frac{\partial u}{\partial z} - \frac{v^2}{r}\right) + \frac{\partial}{\partial r}\left(p + \frac{\mu_m\left|\vec{H}\right|^2}{2}\right) =$$

$$= \mu_m\left(H_r\frac{\partial H_r}{\partial r} + \frac{H_\varphi \partial H_r}{r\partial \varphi} + \frac{H_z \partial H_r}{\partial z} - \frac{H_\varphi^2}{r}\right) + \frac{\partial}{r^2\partial \varphi}\left(\mu\frac{\partial u}{\partial \varphi}\right) +$$

$$+\frac{\partial}{\partial r}\left(\mu\frac{\partial u}{\partial r}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right) + \frac{\partial(\mu u)}{r\partial r} - \frac{\mu}{r^2}\left(2\frac{\partial v}{\partial \varphi} + u\right),$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{v\partial v}{r\partial \varphi} + \frac{w\partial v}{\partial z} + \frac{uv}{r}\right) + \frac{\partial}{r\partial \varphi}\left(p + \frac{\mu_m}{2}\left|\vec{H}\right|^2\right) =$$

$$= \mu_m\left(H_r\frac{\partial H_\varphi}{\partial r} + \frac{H_\varphi \partial H_\varphi}{r\partial \varphi} + H_z\frac{\partial H_\varphi}{\partial z} + \frac{H_r H_\varphi}{r}\right) + \frac{\partial}{\partial r}\left(\mu\frac{\partial v}{\partial r}\right) +$$

$$+\frac{\partial}{r^2\partial \varphi}\left(\mu\frac{\partial}{\partial \varphi}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial v}{\partial z}\right) + \frac{\partial(\mu v)}{r\partial r} + \frac{\mu}{r^2}\left(2\frac{\partial u}{\partial \varphi} - v\right),$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + \frac{v\partial w}{r\partial \varphi} + w\frac{\partial w}{\partial z}\right) + \frac{\partial}{\partial z}\left(p + \frac{\mu_m}{r^2}\left|\vec{H}\right|^2\right) + \rho g =$$