The Framework of Modern Optics

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CONTENTS

ABBREVIATIONS

is equal to - =,
is identical with - ≡,
is approximately equal to; is roughly equal to - ≈,
is proportional to - ∝,
if and only if - ⇔,
implies - ⇒,
is element of - ∈,
is not an element of - ∉,
contains or includes - ∋,
The set of all natural numbers – $\mathcal{N}$,
The set of all real numbers – $\mathcal{R}$,
Phase space – $\mathcal{M}$,
Ultrashort pulse – USP,
Dispersion delay line – DDL,
Wigner distribution function – WDF,
Full width at half maximum – FWHM,
Black body radiation – BBR,
Liquid crystal – LC,
Liquid crystal element – LCE,
Dynamic scattering mode – DSM,
Chaotic phase screen – CPS,
Point spread function – PSF,
Optical transfer function – OTF,
Lens – *L*,
Objective – O,
Eyepiece (or ocular lens) – EYE,
Collector lens –C,
Focusing screen – FS,
Front focal length – FFL,
Back focal length – BFL,
Optical path length – OPL,
Optical path difference –OPD,
Graded-Index Lenses - GRIN lens,

compare with - cf. (short for the Latin: “confer/conferatur”),
that is - i.e., (short for the Latin: “id est”),
… and other similar things - etc. (short for the Latin: “et cetera”).

Radiometry terms

Flux - F [W].
Exitance at a specified point (x,y) on the emitting surface - $M(x,y)$ [W/m^2].
Irradiance at a specified point (x,y) on the surface irradiated - $E(x,y)$ [W/m^2].
Iuminous intensity in a specified direction (θ,φ) - $J(\theta,\varphi)$ [W/sr].
Phase radiance at point (x,y) in direction (p,q) - $B_n(x,y;p,q)$ [W/(sr·m^2)].
Phase volume - volρ [sr·m^2].

PART 0

INTRODUCTION

There is the well-known joke "light is the darkest place in physics" [1]. Note that the natural source of light is also something dark, ideally, the so-called “black body”, which absorbs all incident electromagnetic radiation, regardless of frequency or angle of incidence. The radiation emitted by the black body in thermal equilibrium with its environment is the black body radiation (BBR). In the case of high temperatures (the Sun, the filament of an incandescent lamp, etc.), a significant portion of BBR electromagnetic spectrum is light, i.e., visible to the human eye. (Typically, the human eye can detect wavelengths of BBR in the wavelength range from 380 to 700 nanometers.)

By deriving a formula for the observed spectrum of BBR, in 1900, Max Planck postulated that an electrically charged oscillator in a cavity could only change its energy in discrete amounts, *quanta*, E [2]:

$$E = h\nu, \qquad (1)$$

where ν is the frequency of its associated electromagnetic wave, h is the Planck constant. The word *quanta* (singular *quantum,* Latin for *how much*) was used before 1900 to mean particles or amounts of different quantities. As quantization of energy is more suitable for particles than waves, Albert Einstein [3] proposed to call the virtual particle of light with the energy (1) a *light quantum* (German: *ein Lichtquant*). Later, an American physical chemist, Gilbert Newton Louis [4], proposed another, more common now, name for this virtual particle, the “photon” (from Ancient Greek φῶς, φωτός (phôs, phōtós), “light”).

Wave-particle duality states that every entity may be described as either a particle or a wave [5]. In optics, wave-particle duality is interpreted as the existence of light effects that cannot be explained on the basis of wave theory. For example, the Nobel Prize in Physics 1921 was awarded to Einstein for his explaining in 1905 the external photoelectric effect in terms of photons [3]. In 1915, Einstein predicted a new previously unknown effect, the effect of stimulated emission, where a photon interacts with an excited molecule or atom and causes the emission of a second photon having the same frequency, phase, polarization and direction [6]. In 1960, an American engineer and physicist, Theodore Harold Maiman [7], practically realized this effect by constructing a "light amplification by stimulated emission of radiation" (laser).

According to the “wave-particle” duality, the science of light is comfortably divided into optics (wave theory of light, studying the transformation of light in optical devices of various types) [8] and photonics (quantum theory of light, studying the interaction of light with matter) [9]. The word “photonics” sounds similar to the word “electronics”.

Let us limit the field of our research to optics but use mathematical tools and methodology from quantum theory and electronics to describe it.

0.1. Mathematical tools

Since the beginning of the twentieth century, the mathematical tools of quantum theory have developed rapidly. Noted above, the convergence of scientific interests in the field of laser physics led to the penetration into optics of powerful mathematical tools of quantum theory:

1) Feynman’s path integral, asymptotic Fourier transforms, and the stationary phase approximation made it possible to interpret Fermat’s principle and the eikonal theory (Hamiltonian optics) from the point of view of wave optics [10-12].
2) The application of the Wigner distribution function (WDF) allowed us to explain the basic concepts and laws of radiometry from the point of view of the theory of partial coherence [13].
3) The WDF application significantly simplified the transition from the wave representation of optics (in the form of Fourier optics) to its geometric representation (in matrix form) [14].

After World War II, in addition to quantum physics, electronics, in particular radio engineering, rapidly developed. Since light and radio waves are electromagnetic radiation in adjacent ranges, the problems of optics and radio engineering are close. It was quite natural that the mathematical tools of electronics and radio engineering began to be used in optics. So radio optics [15, 16] (or Fourier optics [17]) and microwave optics [18-21] appeared.

0.2. The principle of selective ignorance

In addition to modern mathematical tools, we can also use a modern way of organizing the material. Note that geometry is not only "the art of correct reasoning on incorrect figures" [22] but, since the time of Euclid, geometry has also been studying the ways of constructing theories. In philosophy, there is a principle named Occam's razor, which states that "entities should not be multiplied beyond necessity" [23]. In geometry, a similar principle is called the “principle of selective ignorance” [24]. In practice, the principle is implemented in the form of two specialized methods: on the one hand, it is an “identification method" [25] and on the other hand, it is an “axiomatic method” [24].

By applying the identification method we look for similarities in the things under study and view them from the same point of view, ignoring their differences [25]. Francis Bacon wrote [26]: “Men’s labour should be turned to the investigation and observation of the resemblances and similarity of things… for these, it is which detect the unity of nature, and lay the foundation for the constitution of the sciences”. Investigation is a process whereby we relate novel features to things that we already know, and various forms of identification method representations (modelings, analogies and dualities) are means for this.

A modeling emphasizes correspondences in the physical process under study with a similar, but more simple, visual or intuitive clear physical process. For example, there is a strong similarity between all types of wave phenomena; therefore, if we study one type of wave, we can understand a lot about all types of waves. The simplest waves are waves propagating in mechanical media since it is easy to create conditions under which the observer can directly control the time-space parameters of the wave [27].

An analogy emphasizes correspondences in certain relations of two objects. “Analogy is a sort of similarity. Similar objects agree with each other in some respect, analogous objects agree in certain relations of their respective parts” [22].

A duality emphasizes one-to-one correspondences in the mathematical structures of two theories. One theory is dual to the other theory if the concepts and theorems from the first theory translate in a one-to-one fashion into concepts and theorems from the second theory. In this case, if the second theory is proved then the first theory is also proved (see Appendix). The mathematical duality, which is especially important for us, is a powerful method of theory construction: it allows us to relate the theory we are creating with the theory that we already know so that the created theory will be "logically arranged no worse than the known theory". Note that this mathematical duality strongly differs from the physical wave-particle duality discussed above.

There are many mathematical dualities in optics, but in this chapter, we need only three of them:

1) The **optical-mechanical duality** [28-30] makes it possible to construct ray and wave optics as Hamiltonian mechanics and Feynman's interpretation of quantum mechanics, respectively. In particular, in Hamiltonian mechanics, there is the **"Hamiltonian picture – Liouville picture" duality** [31], which allows us to convert phase transformations of ray optics into energy transformations.
2) The **time-space duality** is based on the similarity between the diffraction of paraxial monochromatic light beams and the dispersion of ultrashort pulses [32, 33].
3) **Bohr's principle of correspondence,** according to which a new, more general theory must contain the old theory as a special case [34].

By applying the axiomatic method to the construction of theory we select some properties of its objects that we are carefully listing, while others are ignored [24]. In the future, when constructing a theory, it is forbidden to appeal to anything other than the listed properties (axioms) and the laws of logic. Note that an increase in the number of axioms reduces the application field of the theory but increases its power.

In physics, the axiomatic method takes the form of the *first principles method* [35], in which the most fundamental principles of physics are used as axioms. The number of the fundamental principles is limited, and they are clear. Therefore, reducing some theories to the first principles makes it easier to understand their contents. The first principles method is "a way of thinking such that the laws of physics are becoming evident" [36] when "surprising facts" transform into "a matter of course" [37]. The application of this method implies the construction of a theory from a clean slate, including re-defining its fundamental concepts and re-deriving its basic relationships.

0.3. Classification of optical theories

Contrary to BBR, a laser can generate two new special types of radiation: highly coherent monochromatic waves and short pulses. Note that the durations of short pulses have a fundamental limit, namely the cycle period of the monochromatic wave from the center of their spectra. The short pulse whose duration approaches the fundamental limit is called an ultrashort pulse (USP) [32, 33]. During the propagation of USP through an optical medium, two specific features are noted: the duration of USP essentially depends on the dispersion of the medium, and the width of its spectrum essentially depends on the Kerr nonlinearity of the medium. Optics of USPs is a special branch of optics, ultrafast optics (or time optics).

The mathematical tools used to describe optics phenomena depend on the source of radiation. Therefore, it is natural to compare three types of sources with three branches of optics: optics of thermal radiation, optics of monochromatic waves (monochromatic optics, space optics), and optics of ultrashort pulses (ultrafast optics, time optics). The structure of the proposed book is shown in Fig. 0-1.

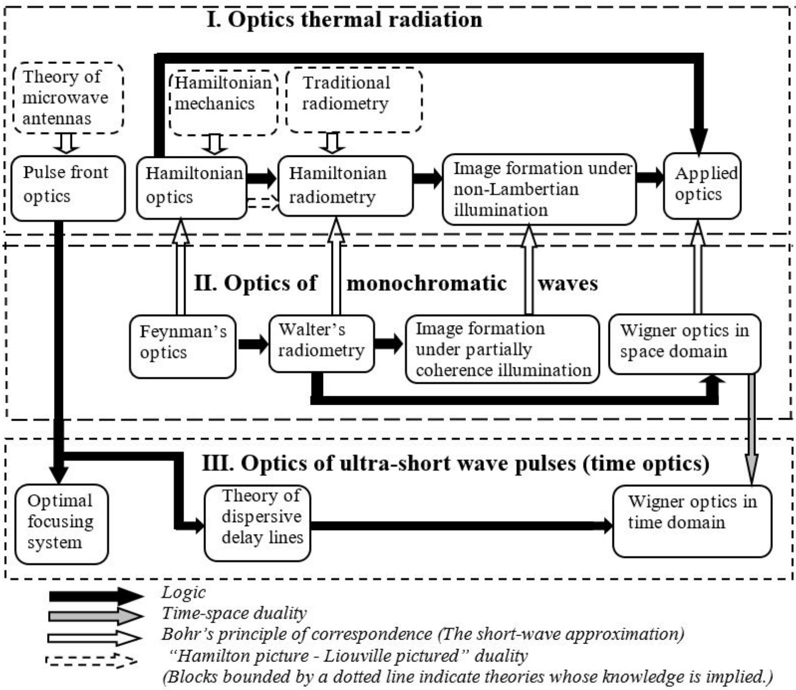

Fig. 0-1. The structure of the theoretical chapters of the book.

Part I: Optics of thermal radiation

The basic concept of thermal radiation is a light wave pulse which looks like the wave pulse the water. The modeling gives a visual version of such concept and also explains the appearance in optics of such characteristics and relationships as wave function, polarization, scalar approximation, retro-system, the principle of superposition, the reversibility principle, Huygens' principle, the principle of tautochronism, the time-space duality, the unfolding scheme - folding scheme duality, the reflector - refractor duality, and the pulse front - ray duality.

According to the "pulse front - ray" duality, optics of thermal radiation is comfortably divided into 1) **pulse front optics** based on a duality noted in the theory of microwave antennas, the tautochronism principle, the unfolding technique and 2) **Hamiltonian optics (**ray optics in Hamiltonian form) based on the principle of least time and Hamilton's optico-mechanical duality.

Based on the “Hamiltonian picture – Liouville picture” duality, it is possible to construct Hamilton radiometry of non-Lambert sources. In turn, Hamiltonian radiometry is closely related to such branches of optics as an image formation under non-Lambertian illumination and applied optics (Fig. 0-1).

According to Bohr's principle of correspondence, Hamiltonian radiometry of non-Lambert sources will contain the methodology of traditional radiometry of Lambert sources.

In Part I, in addition to the chapters shown in Fig. 0-1, Chapter I.6 is devoted to the designing of such subsystems of photo and film devices as radiometers, a condenser of a projector and a viewfinder of a reflex camera.

Part II. Optics of monochromatic waves

A monochromatic wave looks like a periodic wave on the surface of the water. This modeling gives a visual version of the principle of interference and the Huygens-Fresnel principle. Note that the monochromatic wave is a special case of wave pulse. Therefore, the application field of monochromatic optics is less than that of thermal source optics, however, its mathematical tools, based on the interference principle (for example, the Huygens-Fresnel principle), are more powerful and efficient than those, based on the superposition principle (for example, Huygens’ principle).

The natural development of the Huygens-Fresnel principle is the *integral over all possible paths* used in Feynman’s interpretation of quantum mechanics. The optical-mechanical duality, supplemented by the double meaning of spatial frequencies [15], allows us to present Feynman’s interpretation of wave optics (Feynman’s optics). The basic concept and relations of Feynman’s optics in the short-wavelength approximation correspond to the basic concept and relations of Hamiltonian optics. The short-wavelength approximation is the other side of Bohr's principle of correspondence.

The introduction of WDF into optics made it possible, on the one hand, to explain the basic concepts and laws of radiometry from the point of view of the theory of partial coherence (Walter’s radiometry), to describe an image formation under coherence illumination and, on the other, to create Wigner optics in space domain which combining wave optics (in the form of Fourier optics) and ray optics (in the matrix form).

In Part II, in addition to the chapters shown in Fig. 0-1, Chapter II.5 is devoted to a model source of non-Lambertian radiometry, and Chapter II.6 is devoted to image quality.

Part III. Optics of ultrashort wave pulses

A special case of the reversibility principle is the “illuminating device - focusing system” duality. By combining this duality with the focusing properties of elliptical and parabolic mirrors and the “method of elementary reflectors” from illumination engineering, it is possible to create and calculate an optimal two-mirror focusing system.

Any USP can be considered as an infinite set of monochromatic waves of different angular frequencies for which the concept of a zero-distance pulse front transforms into the concept of “a zero-phase wave front”. Combining this concept with the fundamental symmetry of time and space, the theory of dispersion delay lines can be created by using the method of first principals.

The time-space duality radically simplifies to create Wigner optics in the time domain (time optics) on the base of Wigner optics in the space domain (space optics).

Appendix

Duality was discovered by J. Gergonne (1771–1859) in projective geometry [38]. In 1825, he noted that the “point” and “straight line” are dual objects, and the property “the line passes through the point” corresponds to the property “the point lies on the line”. By using the concept “is incident with”, the above statements can be rewritten in a more “symmetric” form: “the point is incident with the line” and “the line is incident with the point”, respectively. If one of the dual statements is true, then the other is also true. For example, the true statement “two points are incident with one and only one straight line” corresponds to the dual true statement “two straight lines are incident with one and only one point”.

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PART I

OPTICS OF THERMAL RADIATION

Optics of thermal radiation involves the illumination of objects of interest to us with “natural” light emitted by the sun, a fire or an incandescent bulb.

To clarify the basic concepts and relationships of optics of thermal radiation, we will use mechanical modeling [1]. Mechanical modeling illustrates the fundamental principles of the theory of wave pulses and gives a visual version of the physical situation that strips away its inessential aspects.

Mechanical waves can propagate in one-, two- or three-dimensional media. The mechanical model of wave pulse in one-dimensional space is the wave pulse on the spring (Fig. I-1) and in two-dimensional space—wavefronts on the surface of the water (Fig. I-8). A ripple tank can illustrate the moving of wave pulses on the surface of the water. In contrast to the wave in one-dimensional space, for which there is only one path from an emitter point to a detector point, for the wave in two-dimensional space, there are different possible paths from an emitter to a detector.

1D (one-dimensional) modeling

Consider an infinite homogeneous coil spring stretched from end to end. If we shake one end of the spring, then the neighboring elements of the spring displace, and the displacement repeats itself over and over. The single displacement running along the spring (Fig. I-1a and b) is called a **wave pulse** [1].

Suppose that the wave pulse propagates along the z-axis. The spring is mentally divided into elements of length Δz lying in the neighborhood of point z. The coordinate z does not depend on time t and labels the equilibrium position of each element Δz. The motion of adjacent elements of the spring is almost the same. Therefore the wave is conveniently described by a continuous "wave function" $\boldsymbol{\psi}(z;t)$, characterized by a displacement vector $\boldsymbol{\psi}$ with the components [2].

$$\boldsymbol{\psi}(z;t) = (\psi_x(z;t),\ \psi_y(z;t),\ \psi_z(z;t)). \quad (1)$$

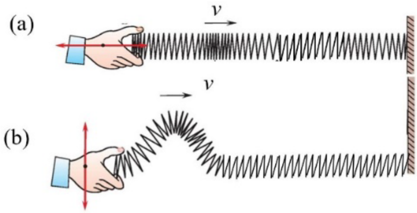

Fig. I-1. The wave pulses as a function of position, i.e., snapshots of: a longitudinal wave (a) and a transverse wave (b).

The direction of motion of the individual element of coils is the same as the direction of vibration of the source of the disturbance. In the stretched spring, two types of wave pulses can be observed:

Longitudinal waves, in which the displacement vectors $\psi(z;t)$ are parallel to the z-direction (Fig. I-1a)

$$\psi_{II}(z;t) = \psi_z(z;t) \quad . \qquad (2a)$$

Transverse waves, in which the displacement vectors $\psi(z;t)$ are perpendicular to the *z*-direction (Fig. I-1b)

$$\psi_{\perp}(z;t) = (\psi_x(z;t),\ \psi_y(z;t)). \qquad (2b)$$

Thus, any transverse wave pulse can be decomposed into two orthogonal components, e.g., in a vertical plane $\psi_{\perp}(z;t) = \psi_y(z;t)$, and the horizontal plane $\psi_{\perp}(z;t) = \psi_x(z;t)$. This feature of transverse waves is called **polarization**. The first is said to be vertically polarized, and the other is said to be horizontally polarized. (Polarization generally just means “orientation.” It comes from the Greek word polos, for the axis of a spinning globe). Note that the easiest way to control the polarization state of the transverse wave is by controlling the direction of the shaking (Fig. I-1b).

Polarization of the wave pulse

If there is no opportunity to control the wave pulse generation, then the problem is to somehow select the desired polarization state from the existing complicated superposition of different states. The polarization state is crucial for the same physical processes. For example, suppose a transverse wave pulse on a spring is passing through a slit in the screen. In that case, its magnitude depends on the orientation of the polarization plane with respect to this slit: a wave pulse will pass through the slit if its plane of polarization is parallel to the slit (Fig. I.1-2), but a wave pulse will not pass through the slit (Fig. I.1-3) if its plane of polarization is perpendicular to the slit. In this case, the first slit is called the polarizer, and the second slit is called the analyzer [2].

The slit in the screen can also be used to select the desired polarization state from the existing complicated superposition of different states, i.e., the slit transforms unpolarized transverse wave pulses into linearly polarized ones.

If, behind the first screen with the slit (polarizer), we place a second screen with the slit (analyzer), then the transverse wave pulse will pass through both slits if they are parallel (aligned) and will not pass if they are perpendicular (crossed).

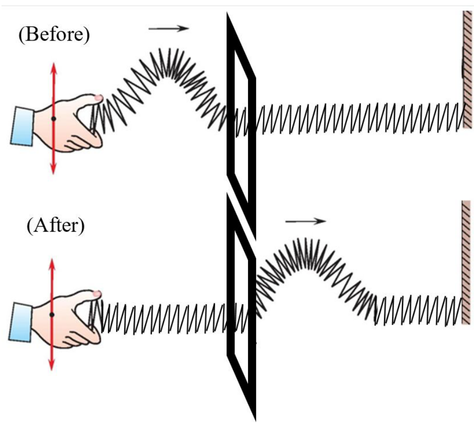

Fig. I-2. A vertically polarized wave pulse passes through the vertical slit.