

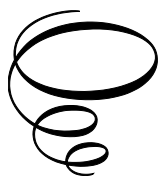
Fundamentals of Reactor Physics

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By

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and María Laura Moreira

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Für meine Tochter, kleine Heuschrecke genannt ...

An einem Tag im Winter, 2023.-

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Chapter 1

The Atom

“Daher ist die Aufgabe nicht sowohl zu sehen, was noch Keiner gesehen hat, als, bei Dem, was jeder sieht, zu denken, was noch Keiner gedacht hat.”

Arthur Schopenhauer

This chapter explains the basic building blocks of all matter in our universe, the atoms of the chemical elements, and focuses on the concepts and physics involved to better understand the neutron-induced fission processes that take place in nuclear reactors. It begins with a brief explanation of the structure of the cloud of electrons that surrounds the atomic nucleus and continues with the structure of the nucleus itself. The importance of knowing the interactions of the subatomic particles involved in the structure of the nucleus, known as nucleons, is directly related to the analysis of the energy released in the physical processes that take place during the operation of nuclear reactors, *i.e.* nuclear fission. To this end, a practical calculation of the binding energy of the nucleons is explained, based on the masses of the constituent nucleons and the mass of the nucleus formed, followed by the basics of a rather more elaborate theory, the Weizsäcker formula, where its physical concepts are explained. A very useful approximation for estimating the

binding energy of nucleons in reactor physics calculations is also presented.

1.1 The chemical elements

1.1.1 Introduction

All the constituents of the universe are made up of one or a mixture of different substances, the basic building blocks of which are called *molecules*. Molecules are the smallest part of a substance that retains the same chemical properties as the substance from which it is made. All existing molecules in the universe are made up of a relatively small number of basic building blocks called *elements*. Many of these elements were already known in ancient times, such as iron, copper, gold, silver and lead, although at that time the concepts of element and substance were ignored and the elements themselves (such as those mentioned above) were confused with alloys, mixtures and also with the product of chemical reactions (such as fire). Many years later, this knowledge was the source of alchemy.

From the 8th century onwards, alchemists managed to advance in the knowledge of materials by discovering new elements, substances and compounds, but without a scientific basis. The first scientific discovery of an element took place in 1669 when Hennig Brand¹ discovered phosphorus. In the 17th century, with the development of chemistry, the properties of the substances discovered by the alchemists were studied, but now using the scientific method. In this stage of chemistry the concepts of substance, mixture, chemical reactions, etc. began to be better understood.

¹Hennig Brand (*Hamburg, Germany, c. 1630 - †*ibid.*, c. 1692 or c. 1710) German merchant, glassblower, pharmacist and amateur alchemist, best remembered for his discovery of phosphorus around 1669.

1.1.2 Atom and molecule concepts

Many years later, when chemistry was already established as a science, a much clearer idea of the structure of substances (molecules and atoms) was developed. On 6 March 1869, the Russian chemist Mendeleev² published the periodic table of the elements in his work “Principles of Chemistry” under the title “The Correlation of the Properties and Atomic Weights of the Elements”³. In this work, the 63 elements known at that time were arranged in increasing order of atomic weight into seven groups with similar chemical properties. In 1871, Mendeleev was able to determine the properties of the previously undiscovered elements gallium, scandium and germanium.

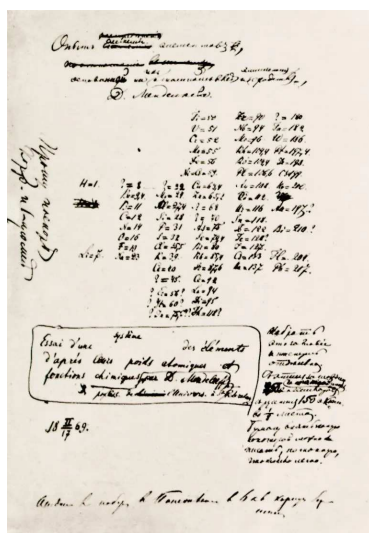


Figure 1.1: Mendeleev's 1869 handwritten draft of the periodic table.

²Dmĭtriy Ivánovich Mendeléyev (*Tobolsk, Russia, 27.Jan.1834 - †Saint Petersburg, Russia, 20.Jan.1907) Russian chemist, famous for having discovered the underlying pattern in what is now known as the periodic table of the elements.

³Mendeleev D. 1869. Sootnoshenie svoistv s atomnym vesom elementov. *Zh. Russ. Khim. Obshch.*, **1**(2/3): 60-77 and *Z. Chem.*, **1**(5):405-406.

Ueber die Beziehungen der Eigenschaften zu den Atomgewichten der Elemente. Von D. Mendelejeff. — Ordnet man Elemente nach zunehmenden Atomgewichten in verticale Reihen so, dass die Horizontalreihen analoge Elemente enthalten, wieder nach zunehmendem Atomgewicht geordnet, so erhält man folgende Zusammenstellung, aus der sich einige allgemeinere Folgerungen ableiten lassen.

[illegible]

1. Die nach d. Grösse des Atomgewichts geordneten Elemente zeigen eine stufenweise Abänderung in den Eigenschaften.
2. Chemisch-ähnliche Elemente haben entweder übereinstimmende Atomgewichte (Pt, Ir, Os), oder letztere nehmen gleichviel zu (K, Rb, Cs).
3. Das Anordnen nach den Atomgewichten entspricht der *Werthigkeit* der Elemente und bis zu einem gewissen Grade der Verschiedenheit im chemischen Verhalten, z. B. Li, Be, B, C, N, O, F.
4. Die in der Natur verbreitetsten Elemente haben *kleine* Atomgewichte.

Figure 1.2: Mendeleev's scientific article in *Zeitschrift für Chemie*, 1869.

A few years later his thesis was confirmed: the element germanium was discovered in 1871, gallium in 1875 and scandium in 1879. Figure 1.3 shows the periodic table of the elements of the IUPAC (International Union of Pure and Applied Chemistry).

IUPAC Periodic Table of the Elements

1 H Hydrogen +1 1.008																	2 He Helium +2 4.002 4.0026																																											
3 Li Lithium +1 6.941	4 Be Beryllium +2 9.012 9.0122	<div>Key:</div> <div> <div>Atomic Number</div> <div>Symbol</div> <div>Name</div> <div>Group</div> <div>Period</div> <div>Block</div> </div>														5 B Boron +3 10.811	6 C Carbon +4 12.011	7 N Nitrogen +3 14.007	8 O Oxygen +2 15.999	9 F Fluorine -1 18.998	10 Ne Neon 0 20.180																																							
11 Na Sodium +1 22.990	12 Mg Magnesium +2 24.305	13 Al Aluminum +3 26.982	14 Si Silicon +4 28.086	15 P Phosphorus +3 30.974	16 S Sulfur +4 32.06	17 Cl Chlorine -1 35.45	18 Ar Argon 0 39.948	19 K Potassium +1 39.098	20 Ca Calcium +2 40.078	21 Sc Scandium +3 44.956	22 Ti Titanium +4 47.88	23 V Vanadium +5 50.942	24 Cr Chromium +3 51.996	25 Mn Manganese +2 54.938	26 Fe Iron +2 55.845	27 Co Cobalt +3 58.933	28 Ni Nickel +2 58.693	29 Cu Copper +1 63.546	30 Zn Zinc +2 65.38	31 Ga Gallium +3 69.723	32 Ge Germanium +4 72.63	33 As Arsenic +3 74.922	34 Se Selenium +4 78.96	35 Br Bromine -1 79.904	36 Kr Krypton 0 83.798	37 Rb Rubidium +1 85.468	38 Sr Strontium +2 87.62	39 Y Yttrium +3 88.906	40 Zr Zirconium +4 91.224	41 Nb Niobium +5 92.906	42 Mo Molybdenum +6 95.94	43 Tc Technetium +7 98.906	44 Ru Ruthenium +4 101.07	45 Rh Rhodium +3 102.91	46 Pd Palladium +2 106.91	47 Ag Silver +1 107.87	48 Cd Cadmium +2 112.41	49 In Indium +3 114.82	50 Sn Tin +4 118.71	51 Sb Antimony +3 121.76	52 Te Tellurium +4 127.6	53 I Iodine -1 126.91	54 Xe Xenon 0 131.29	55 Cs Cesium +1 132.91	56 Ba Barium +2 137.33	57 La Lanthanum +3 138.91	58 Ce Cerium +3 140.12	59 Pr Praseodymium +3 140.91	60 Nd Neodymium +3 144.24	61 Pm Promethium +3 144.91	62 Sm Samarium +3 150.36	63 Eu Europium +3 151.96	64 Gd Gadolinium +3 157.25	65 Tb Terbium +3 158.93	66 Dy Dysprosium +3 162.50	67 Ho Holmium +3 164.93	68 Er Erbium +3 167.26	69 Tm Thulium +3 168.93	70 Yb Ytterbium +3 173.05	71 Lu Lutetium +3 174.967
39 Rb Rubidium +1 85.468	40 Sr Strontium +2 87.62	41 Y Yttrium +3 88.906	42 Zr Zirconium +4 91.224	43 Nb Niobium +5 92.906	44 Mo Molybdenum +6 95.94	45 Tc Technetium +7 98.906	46 Ru Ruthenium +4 101.07	47 Rh Rhodium +3 102.91	48 Pd Palladium +2 106.91	49 Ag Silver +1 107.87	50 Cd Cadmium +2 112.41	51 In Indium +3 114.82	52 Sn Tin +4 118.71	53 Sb Antimony +3 121.76	54 Te Tellurium +4 127.6	55 I Iodine -1 126.91	56 Xe Xenon 0 131.29	57 Cs Cesium +1 132.91	58 Ba Barium +2 137.33	59 La Lanthanum +3 138.91	60 Ce Cerium +3 140.12	61 Pr Praseodymium +3 140.91	62 Nd Neodymium +3 144.24	63 Pm Promethium +3 144.91	64 Sm Samarium +3 150.36	65 Eu Europium +3 151.96	66 Gd Gadolinium +3 157.25	67 Tb Terbium +3 158.93	68 Dy Dysprosium +3 162.50	69 Ho Holmium +3 164.93	70 Er Erbium +3 167.26	71 Tm Thulium +3 168.93	72 Yb Ytterbium +3 173.05	73 Lu Lutetium +3 174.967																										
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Figure 1.3: Periodic table of the elements of the IUPAC, 4 May 2022.