Introductory Physics for Engineers

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To the children of Gaza

“The duty of the man who investigates the writings of scientists, if learning the truth is his goal, is to make himself an enemy of all that he reads, and, applying his mind to the core and margins of its content, attack it from every side.”

Form The book of optic
Al-Hassan Ibn-El-Haytham
965-1040 AD

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PREFACE

The provided textbook focuses on basic concepts that covers topics of Newton mechanics, oscillation, electromagnetics, fluidic and thermodynamics. It elaborates the three laws of motions as well as the laws of conservation of energy and momentum. It then links the laws of motion to oscillating movement. The concept of oscillation is used to explain electrical oscillators and electromagnetic radiation. The book then specified two chapters to give brief introduction to fluid systems and thermodynamics. That includes the definition of fluid and the basics of fluid mechanics that include static systems as well as fluid dynamics. Fundamental concepts such as heat, entropy, heat transfer and thermal engines are covered.

While writing the book, the authors kept in mind that it is targeting first year undergraduate engineering students. Hence, we find it necessary to start the book with a brief mathematical review to cover some needed background such as vectors, vector operations, trigonometry, differentiation and integration. That is followed by introducing scalar and vector quantities such as displacement, velocity and acceleration. The chapter is made to provide a brief revision material for students who might not have covered all or some these topics in mathematics during their pre-university education.

Additionally, during the course of the book, the authors were careful to ensure that level and use of mathematics gradually increase. For each concept introduced, full mathematical derivations are provided for the governing equations. This might seem to mathematically challenge some student groups. However, all of the needed mathematical operations are briefly covered in the first chapter.

—The authors

CHAPTER ONE

INTRODUCTION TO MOTION

1A. Vectors brief review

When dealing with topics such as motion and Newton’s mechanics, it is essential to build a good understanding of vectors and operations. This is because motion within the limits of Newton’s mechanics can be oriented in a three-dimensional space. Each dimension requires a value to describe a location or the change of that location along this dimension. Hence, being in a three-dimensional space requires three components to describe the motion through such space. Once any quantity requires more than one component to be represented, we could fundamentally call it a vector quantity.

Distance and displacement vector

Let us imagine the following scenario:

A student wanted to plan a trip from Chiang Mai to Bangkok. Being new to Thailand, he asked how far is it to travel from Chiang Mai to Bangkok? A local student answered quickly, “it is 700 km (about 435 mi)”. The first student thought for a moment then replied, Which direction? " As a teacher, you decided to interrupt without a doubt, “South?” The first student, being a sharp kid, replied, “Is it pointing directly to the south?” You were about to give him a fast answer looking at the map in figure 1.1 “South-East", but then you asked an important question:

“How much south and how much east?”

To reach a solution, you found yourself prone to set a coordinate system originated at Chiang Mai where x matches East and y matches North. Now, you can draw a 700 km long arrow that points directly from Chiang Mai to Bangkok. This arrow then forms a vector: Displacement vector of 700 km length and a direction as shown in figure 1.2.

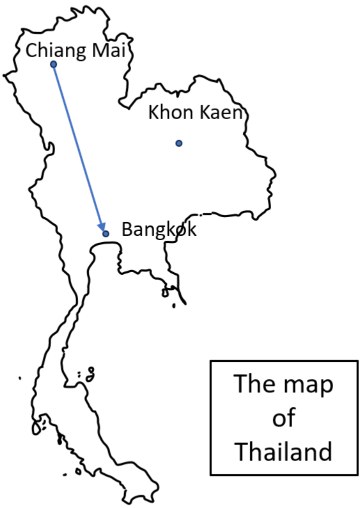

Figure 1.1. The map of Thailand at which the students are looking at. The map shows an arrow pointing from Chiang Mai to Bangkok. The length of the arrow is 700 km.

Vector representation

One can also say, to reach from Chiang Mai (point A) to Bangkok (Point B) you would need to move 669 km (about 415.7 mi) South then 204 km (about 126.76 mi) East. If we placed a two-dimensional coordinate system where North is along the y-axis and x-axis is along East, as in figure 1.3, then the South should have a negative sign. Or we simply say, move − 669 km (about 415.7 mi) on the y-axis, then move 204 km (about 126.76 mi) on the x-axis. In a vector form one writes the displacement vector as:

$$\overline{L} = (240km, -669km) \tag{1.1}$$

For this vector can be defined by

- Amplitude: $|\overline{L}| = \sqrt{204^2 + 669^2}$ = 700 km.
- Direction: $\theta = tan^{-1}(-669/204)$ = -73°.

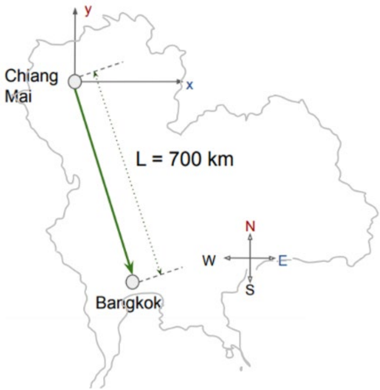

Figure 1.2. A displacement vector between two points: Chiang Mai and Bangkok. The vector has: A length that equals the distance between the two points and a direction pointing from the starting point to the ending point.

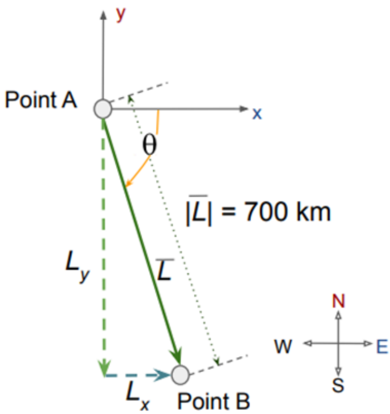

Figure 1.3. The displacement vector representation by a length L and angle θ that represents direction.

A general displacement vector can be written as

$$\overline{L} = (L_x, L_y) \tag{1.2}$$

The vector amplitude is defined as

$$|\overline{L}| = \sqrt{L_x^2 + L_y^2} \tag{1.3}$$

The vector direction is defined as

$$\theta = tan^{-1}(L_y/L_x) \tag{1.4}$$

For the general vector in figure 1.4, with length U and angle θ, we can write it as a summation of two vectors

$$\overline{u} = u_x\hat{x} + u_y\hat{y}. \tag{1.5}$$

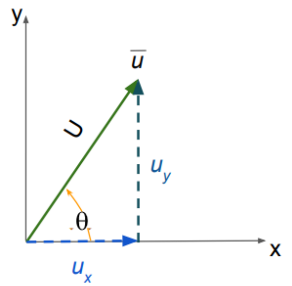

Figure 1.4. General vector of amplitude U and direction θ.

The vectors $\hat{x}$ and $\hat{y}$ are unit vectors in the x and y directions. They each have an amplitude of one. The vector can be then written in the form

$$\overline{u} = (u_x, u_y) \tag{1.6}$$

where u_x and u_y are the x and y components of the vector $\overline{u}$. We can obtain their values from the vector length U using Pythagoras theorem as follows.

$$u_x = U \cdot cos\theta \tag{1.7a}$$

$$u_y = U \cdot sin\theta \tag{1.7b}$$

The vector's direction is defined by the angle it makes with respect to the x-axis. From the geometry in figure 1.4, we can immediately realize that the ratio between u_y and u_x is the tangent of the angle.

$$\frac{u_y}{u_x} = \frac{sin\theta}{cos\theta} = tan\theta \tag{1.8}$$

Vector and scalar

In the first example, the vector given was the displacement vector. The amplitude of the vector is a scalar value that indicates the distance measured as a straight line between the starting and ending points. The vector's direction is represented by an angle to the x-axis formed between the starting point and the end point. The angle itself is a scalar value as it does not have a direction.

Vector operations

Back to the map. Imagine that there is a salesperson who travels from Chiang Mai to Bangkok then to Khon Kaen. What would be his net displacement? From the definition in the previous sections, we know that the displacement is a vector that is formed between the starting and ending points of the trip. In this case, the starting point is Chiang Mai, and the ending point is Khon Kaen

How do we represent this in vectors?

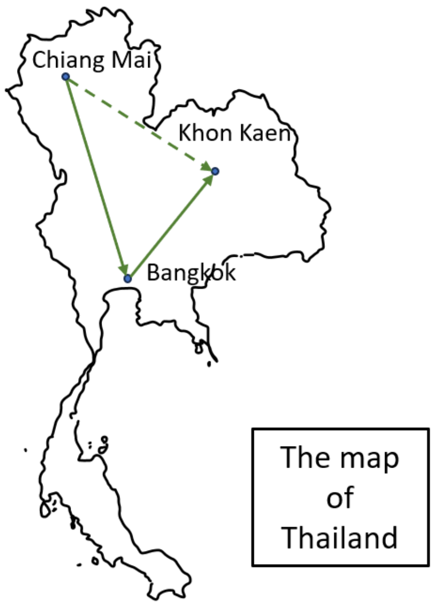

Figure 1.5. The salesman trip from Chiang Mai to Bangkok then to Khon Kaen and the net displacement vector in dashed arrow.

The total displacement can then be represented as a sum of the two displacements that the salesperson did. The first is from point A→B and the second is from B→C. Mathematically we write this as the following vector summaries

$$\overline{AC} = \overline{AB} + \overline{BC} = (AB_x, AB_y) + (BC_x, BC_y) = (AB_x + BC_x, AB_y + BC_y) \quad (1.9)$$

In our example AB_y is pointing south and hence has a negative sign, while BC_y is pointing north which makes it positive. The summation $AB_y + BC_y$ is the subtraction of the two amplitudes and the result is a shorter distance as illustrated in figure 1.6. The x components however, AB_x and BC_x are both pointing to the east. Hence, the summation gives us the addition of the two amplitudes. In general, adding two vector quantities is simply the summation of each corresponding component individually, e.g., adding the x component of each vector to form the x component of the final vector. One can do similar proof for vector subtraction using a similar example. Though we can write the two operations as follows:

$$\overline{u} + \overline{v} = (u_x + v_x, u_y + v_y) \quad (1.10)$$

$$\overline{u} - \overline{v} = (u_x - v_x, u_y - v_y) \quad (1.11)$$

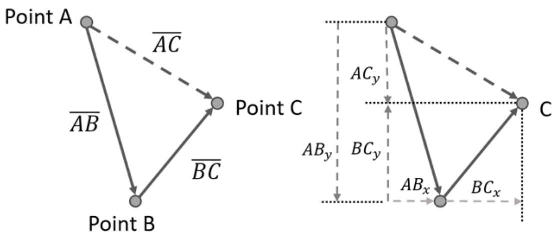

Figure 1.6. Displacement vector between point A and C as sum of two displacements $\overline{AC} = \overline{AB} + \overline{BC}$.

When multiplying a vector u by a scalar b, we simply multiply each component by the scalar.

$$b \cdot \overline{u} = (b \cdot u_x, b \cdot u_y) \quad (1.12)$$

$$b \cdot (\overline{u} + \overline{v}) = b \cdot \overline{u} + b \cdot \overline{v} \quad (1.13)$$

Dot product of vectors

So, what about vector-vector multiplication? There are two types of vector multiplication: One that returns a scalar value, dot product, and another returns a vector, cross product. We will be presenting here only the dot product and cross product will be introduced in chapter 9. Dot product returns a scalar value, and it is defined mathematically as

$$\overline{u} \cdot \overline{v} = u_x v_x + u_y v_y \qquad (1.14)$$

If we assume the two vectors have lengths of U and V respectively, then we can use Pythagoras theorem to write the vectors as $\overline{u} = U \cdot cos\theta_u \hat{x} + U \cdot sin\theta_u \hat{y}$ and $\overline{v} = V \cdot cos\theta_v \hat{x} + V \cdot sin\theta_v \hat{y}$. Using this representation, we can write equation 1.14 as

$$\overline{u} \cdot \overline{v} = U \cdot V \cdot (\cos\theta_u \cos\theta_v + \sin\theta_u \sin\theta_v) \qquad (1.15)$$

To simplify the expression inside the parentheses one needs to revisit some basics of trigonometry in the next section.

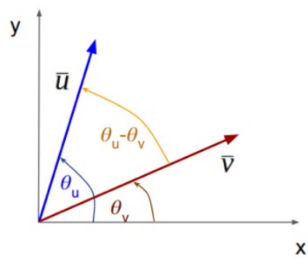

Figure 1.7. Dot product of two vectors

1B. Mathematical review

Brief review of trigonometry

For now, we need to remember the following two relations by heart. This will make derivations much easier when we try to simplify the dot product expression in equation 1.15b.

$$cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right) \tag{1.16a}$$

$$sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right) \tag{1.16b}$$

We can use equations 1.16 to expand the sines and cosines in equation 1.15b.

$$sin\theta_u sin\theta_v = \frac{1}{2i}\left(e^{i\theta_u} - e^{-i\theta_u}\right) \cdot \frac{1}{2i}\left(e^{i\theta_v} - e^{-i\theta_v}\right) \tag{1.17a}$$

$$= \frac{-1}{4}\left(e^{i(\theta_u+\theta_v)} + e^{-i(\theta_u+\theta_v)} - e^{i(\theta_u-\theta_v)} + e^{-i(\theta_u-\theta_v)}\right) \tag{1.17b}$$

We can arrange the equations as

$$sin\theta_u \cdot sin\theta_v = \frac{1}{4}\left(e^{i(\theta_u+\theta_v)} + e^{-i(\theta_u+\theta_v)}\right) - \frac{1}{4}\left(e^{i(\theta_u-\theta_v)} + e^{-i(\theta_u-\theta_v)}\right) \tag{1.18}$$

Comparing the two parentheses in equation 1.18 to equations 1.16, we can write the following relation

$$sin\theta_u \cdot sin\theta_v = \frac{1}{2}(cos(\theta_u + \theta_v) - cos(\theta_u - \theta_v)) \tag{1.19}$$

We can follow a similar approach to write the following expression

$$cos\theta_u \cdot cos\theta_v = \frac{1}{2}(cos(\theta_u + \theta_v) + cos(\theta_u - \theta_v)) \tag{1.20}$$

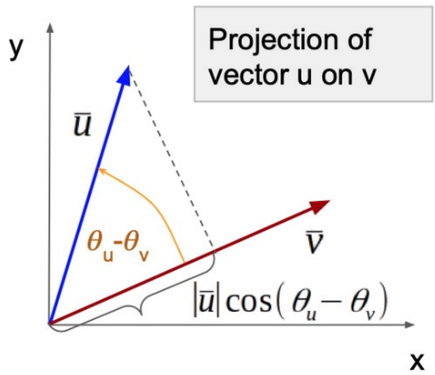

Figure 1.8: projection of $\overline{u}$ on $\overline{v}$

Adding equations 1.19 and 1.20 we obtain the following relation

$$sin\theta_u \cdot sin\theta_v + cos\theta_u \cdot cos\theta_v = cos(\theta_u - \theta_v) \quad (1.21)$$

Dot product as vector projection

If we substitute equation 1.21 into 1.15b we obtain a more known representation of the vector dot product.

$$\overline{u} \cdot \overline{v} = UV \cdot cos(\theta_u - \theta_v) \quad (1.22)$$

Or in a general form

$$\overline{u} \cdot \overline{v} = |\overline{u}||\overline{v}|cos(\theta_u - \theta_v) \quad (1.23)$$

Notice here we write the length of the vector, or amplitude, as $|\overline{u}|$. Equation 1.23 represents the projection of one vector on the other as illustrated in figure 1.8. We can write the projection of the vectors as follows:

- Projection of u on v : $\overline{u} \cdot \overline{v}/|\overline{v}| = |\overline{u}|cos(\theta_u - \theta_v)$
- Projection of v on u : $\overline{u} \cdot \overline{v}/|\overline{u}| = |\overline{v}|cos(\theta_u - \theta_v)$

Review of complex numbers

Generally, a complex number z is defined as $z = a + ib$. Here, a and b are real values. The symbol i is for imaginary and it has a numerical value of $i = \sqrt{-1}$. Complex numbers can be represented in a way similar to vector representation. However, instead of x and y coordinates, we set the real and imaginary coordinates as shown in figure 1.9. The amplitude of the complex number is:

$$|z| = \sqrt{a^2 + b^2} \tag{1.24}$$

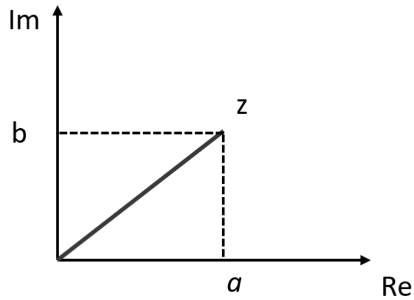

Figure 1.9. Complex number vector representation.

As in figure 1.9 and in a similar way to vector representation, we can write the real and imaginary parts of the vector in terms of cosine and sine functions as in equations 1.17.

$$a = |z|cos\phi \tag{1.25a}$$

$$b = |z|sin\phi \tag{1.25b}$$

Here, the angle $\phi = tan^{-1}(b/a)$. The complex number z can be written then as

$$z = |z|(cos\phi + i\ sin\phi) \tag{1.26}$$

The term in the parentheses in equation 1.26 can be written in an exponential form

$$cos\phi + i\ sin\phi = e^{i\phi} \tag{1.27}$$

The complex number z can be written as

$$z = |z|e^{i\phi} \tag{1.27b}$$

The constant ϕ is commonly called the phase.

Derivative, change and rate

Derivative means change and change can be defined as the difference between the conditions at two different states. The two different states can be two different times, locations in space, or two different temperatures for example. Let us consider the following scenario. You decided to put a thermometer inside an oven and started to record the measured temperature every 10 seconds. From the records, you managed to produce the graph in figure 1.10. Keep in mind that this graph is hypothetical and does not represent actual oven behavior.

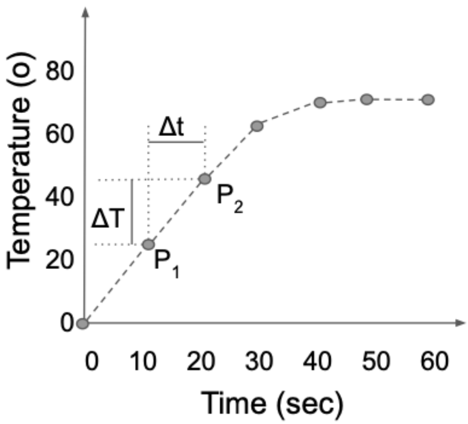

Figure 1.10: The produced graph of temperature versus time inside the oven.

Now, let us pick two points on the graph: P_1 and P_2. As shown by the values in table 1.3, between points P_1 and P_2, the temperature has increased by

$$\Delta T = 45^o - 22^o = 23^o$$

and the time has increased by

$$\Delta t = 20s - 10s = 10s.$$

Hence, we can say that the rate of change between the two points is 23 degrees in 10 seconds.

Table 1.3: Time and temperature values of two points in figure 1.10

Point	Time (s)	Temperature (°)
P_1	10	22
P_2	20	45

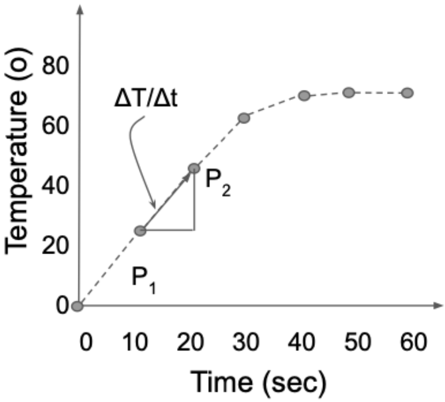

Figure 1.11: Rate of change of temperature versus time.

The rate of change can be defined as

$$Rate = \frac{\Delta T}{\Delta t} = \frac{\text{Difference in temperature}}{\text{Difference in time}} \quad (1.28)$$

According to equation 1.28, The unit of Rate is degrees per second. For example, for the values here, the rate of change in the oven's temperature between the two points is

$$Rate = \frac{23^o}{10s} = 2.3\ \text{Degrees/sec} \quad (1.29)$$

Notice, that if you measure while cooling the oven, then the difference in temperature would be negative. That will give you a negative Rate. Now you have decided to connect the thermometer with a very fast computer that reads temperature values with a very small amount of time difference. In other words, the time intervals, Δt, became very small or infinitesimally small. In this condition we can replace the phrase "difference" by differentiation or

$$\text{When } \Delta T \rightarrow dT \text{ then } \Delta t \rightarrow dt \tag{1.30}$$

The Rate is now written as

$$Rate = \frac{dT}{dt} = \frac{\text{Differentiation in temperature}}{\text{Differentiation in time}} \tag{1.31}$$

In other words, we could say that the rate is the derivative of temperature with respect to time when we use infinitesimally small-time steps.

Functions

With very small-time intervals, one can say that the temperature can "almost" be estimated at any time value. We can think of the graph as the temperature represented in a continuous form rather than a set of discrete points. The graph in figure 1.12 shows the temperature as a continuous function of time. Mathematically, when we want to describe temperature as a function of time, we write T(t). That reads as temperature at time t. How can we now think of the rate of change of a continuous function?

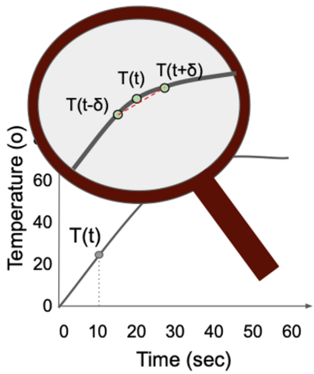

Figure 1.12: Rate of change of temperature versus time.

We know that for very small intervals the rate of change was defined as the ratio of the differentiation of temperature over the differential of time. This was, however, calculated between two discrete points. In a continuous form we have a function T(t) that gives us the temperature for each time value. To get a better understanding of that, we need to zoom at a selected region in the continuous graph as shown in figure 1.12. We can think that the rate