The Trinity of Mass and Newton's Way

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From the Principia

By

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Preface

Opportunity to Pick Up the Subject

During the academic years 2016 - 2022, I had the privilege of teaching a series of lectures on differential equations as part of the BSc curriculum for aerospace engineering at Delft University of Technology. In this series, given this particular audience, I decided to include a few lectures on Kepler orbits, i.e. a treatment of the classical two-body problem. I was lucky enough to own a copy of Coddington's[1] *Introduction to Ordinary Differential Equations*. I don't recall if this book actually gave me the idea to include the two-body problem in my course, but Coddington's treatment of it was certainly inspiring because it shows how the Kepler problem can be exploited to *deduce*, to some extent, Newton's[2] Universal Law of Gravitation. This seemed highly relevant to my audience, as it provides insight into the origin, background and content of Newtonian gravity.

Reason to Start Digging

As I was motivated to deduce, and so to understand and explain, the full physical content of Newton's Universal Law of Gravitation, I was not entirely satisfied with Coddington's treatment: Coddington deduced the dependence on spatial coordinates of the gravitational field of a point mass, but not the dependence of the force of gravity on the masses of the bodies involved. Moreover, to extend Coddington's treatment and *deduce* the desired, well-known dependence on the masses proved to be non-trivial: too easily, an erroneous result was obtained. I was surprised to discover that the missing part of the derivation was not easily found in textbooks. This led me to consult the original source, i.e. Newton's *Philosophiae Naturalis Principia Mathematica*, which was first published in 1687 [42].

Meeting Newton's *Principia*

Illustrious readers, such as Lord Kelvin[3] [43] and Nobel laureate Subramanyan Chandrasekhar[4] [49], have honored Newton's *Principia*. I myself do not feel entitled to express my opinions of Newton's humbling work.

[1] Earl Alexander Coddington, 1920 - 1991.
[2] Sir Isaac Newton, 1643 - 1727.
[3] William Thomson, 1st Baron Kelvin, 1824 - 1907.
[4] Subrahmanyan Chandrasekhar, 1910 - 1995.

But I dare confess to believe that the influence of Newton on Western thought and civilization can hardly be overestimated.

Furthermore, what I can do is report the true fact that I first found what I was looking for in the *Principia*; more precisely, I found it in Newton's discussion of the proof of *Propositio VII* of *Liber Tertius*. It was that the key ingredient for the deduction of how the gravitational force depends on the mass of its source, lies in the *motus legem tertiam*, i.e. in Newton's Third Law of Motion.

It is also a true fact that, when I subsequently finalized my own documentation of the derivation, in twentieth century algebraic notation – so I could use this derivation to explain its subject on a chalk board during my lectures – I was astounded by one of its implications. This was that – contrary to the impression that I had been left with from academic courses and textbooks on classical physics – in Newton's physics, and in the *Principia*, there is really only *one* kind of mass genuinely *defined*, namely mass being an invariant attribute of a body measuring its inertia; and it can be *deduced* that this same quantity measures the impact of the same body in its acting as a source of gravitation.

Twentieth Century Perception of Newton's Physics

Contrary to the canonical twentieth century reading on the subject [33], the fact that *both* masses in Newton's Universal Law of Gravitation are *inertial* masses, is by no means some coincidence. On the contrary: close reading of Newton's *Principia* – at least my own reading – revealed it to be a *deduced* consequence of Newton's choice of concepts and of his theory, as applied to observed phenomena.

Subsequent interaction with colleagues – including a colloquium for mathematical and theoretical physicists, as well as including a peer review of a precursor to this book – showed that I wasn't the only one new to the above; on the contrary. And thus was the prelude to this monograph.

Background of the Author

The author is a physicist by training, with a fairly broad experience in both industrial and academic applications of physics and applied mathematics. Apart from a basic one-year academic course in philosophy, he has no professional education or grade, nor career, in philosophy – e.g. epistemology or metaphysics – nor in the history of science. Neither are there any plans for a career change in either of those directions. That is to say: the excursions in this monograph into the history and philosophy of science are incidental, case-driven and based on spare time self-study, alongside a professional career in physics and applied mathematics. Consequently, given limited time and resources, and given the truly vast amount of material that has been published on the subjects touched upon in this monograph, it will be clear that any ambition of completeness would be illusionary. Yet I believe this monograph has something to add to the existing comments on its subject.

Envisioned Audience

The primary intended audience, or the reference audience, includes contemporary physicists and everyone who has an interest in Newtonian physics at the academic level: this provides a gauge as concerns expectations, interests and required prior knowledge. A knowledge of physics and mathematics at undergraduate level seems to be a prerequisite and should be sufficient.

Core: How Newton Arrived at his Universal Law of Gravitation

As it is, the core of this monograph consists of a derivation of Newton's Universal Law of Gravitation in twentieth century mathematical notation, i.e. in a format suitable for lecturing in a BSc course in physics or mathematics. Apart from using the standard mathematical notation of calculus and algebra – Newton's original is in Latin prose and doesn't use algebraic notation – the derivation of Newton's Universal Law of Gravitation presented in this monograph closely, but concisely, follows Newton's original argument. That is to say, since it is not our aim to give a faithful copy of Newton's treatise – after all, Newton's original is available to all – in our account, redundancies in Newton's evidence have been bridged. Our derivation in mathematical notation is inevitably more explicit and hence more lucid than Newton's original in Latin. Occasionally, our argument may even be somewhat more complete, in the sense of rendering a logically valid and consistent reasoning, while perhaps Newton's original Latin prose left some details to the reader.

At the same time, I dissociate myself from any claim that I have shown - or could have shown - flaws in the structure, formulation, or reasoning of the *Principia*, still less in regard to Newton's mastery of his subject. On the contrary, if Newton was occasionally less explicit about his argument, or about its consequences, I see only reason to think about the reasons he might have had for being so. And a substantial part of this monograph grew out of such considerations.

The Clouded View on Newton's Mass; a History of Perception

As hinted above, one of the explicit purposes of this monograph is to clarify what exactly is the interpretation of the masses that appear in Newton's Universal Law of Gravitation. An intriguing and driving question furthermore has been why this aspect does not simply belong to the collective ideas of modern physicists. For this reason, a multitude of descriptions and discussions of historical origins, context and developments can be found in this monograph. This includes many explicit quotations from the original source text of the *Principia*, and reflections on these. Based on these, it is pointed out that Newton did not highlight, let alone celebrate, one of the most astounding consequences of his deductions, i.e. the implied fact that inertia finds itself at the source of Newtonian gravitation. This observation alone would have called for a detailed review of the foundation, structure, terminology and methodology of the *Principia*, as well as for at least some view on its seventeenth century context, what preceded that, and what has happened since in terms of reading and interpretation of the *Principia*.

And thus a monograph has grown that will on the one hand be far from complete, but on the other hand can be of use to anyone who would like to learn about Newton's Universal Law of Gravitation, its origins, its foundation and its content. In passing, it also provides a view of the dynamics of the history, and perhaps the sociology, of this part of the body of thoughts of physics.

Relevance to Modern Physicists

The relevance of this to modern physicists lies in the fact that both quantum mechanics and Einstein's theory of gravitation do have, and must have, the corresponding Newtonian physics as a limit. Now, reconciliation of quantum physics and general relativity presents a notorious challenge. Therefore, it is all the more relevant to clearly see and understand how in the Newtonian limit, the mechanical and gravitational roles of mass are captured by one unified concept of mass.

It is remarkable and perhaps even confusing that the canonical twentieth century reading of classical physics seemingly suggested otherwise.

Therefore this monograph presents an attempt to expose and untangle the knot, in the hope that this may serve, if only amuse, its readers.

Acknowledgments

Acknowledgments are due to the host of my original lectures on the subject, the Faculty of Aerospace Engineering at Delft University of Technology, and to the students that attended and actively participated in these lectures. Furthermore, I am indebted to Dr. Paul Visser, for the invitation to contribute a lecture to his yearly Christmas colloquium and to the assembled physicists and mathematicians that made this event so alive, inspiring and welcoming. Acknowledgments are also due to anonymous reviewers of a precursor to this manuscript.

An expression of appreciation is also appropriate to the publisher and to all of its staff: in turbulent times – including when it comes to media – they defend and uphold an invaluable tradition of publication of scientific work in book form. I consider this as no less than supporting the independent voice of science. It has been an honor to me to be allowed to contribute to this.

The author wishes to express his deepest gratitude to Dr. Theodoor Gerkema (Royal Dutch Institute for Sea Research) for shared sources and interest, in-depth discussions and encouragement. The author wishes to express gratitude to his father, for reading various versions of the manuscript and shared enthusiasm for the subject and findings of this study. Gratitude is also due to my further friends and family, all of whom were patient enough to listen to my thinking-aloud about the developing content of the manuscript. Last, I wish to acknowledge Des for all of our spending time together and for sharing his views and wisdom and B, for lifelong friendship.

Ramses van der Toorn

The Hague, June 6, 2024.

Chapter 1

The Trinity of Mass and Newton's Revolutionary Way

1.1 One Central Subject: the Concept of Mass in Newton's Physics

1.1.1 Entrance: Newton's legacy

One concept central to this monograph will be the notion of mass in 'Newtonian physics'. We shall explore how mass figured in Newton's *Principia* [42] and how this relates to the conception of 'mass in Newtonian physics' as it eventually emerged, crystalized and settled, up until the present. We shall develop and defend a view according to which *the modern canonical reading concerning 'mass in Newtonian physics' does no justice to Newton's legacy*.[1]

1.1.2 The Canonical, Twentieth Century Reading: a Triad of Masses

According to the canonical, twentieth century conception [33, Ch. 10]:

[1] We tend to denote what Newton actually delivered as 'Newton's', while we tend to use the qualifier 'Newtonian' for what became of it under the hands of later generations.

> **Quote 1:** "*Newtonian mechanics, strictly speaking, distinguishes three different kinds of mass:*
>
> 1. *inertial mass (…),*
> 2. *active gravitational mass (…) and*
> 3. *passive gravitational mass (…).*"

Inertial mass (1) is a dynamical attribute of a body; see furthermore Sections 3.6 and 7.3.1. "*Active gravitational*" or *gravitating* mass – item (2); see further Section 2.1.2 – measures the property that parameterizes a body's ability to *generate* a gravitational field [11]. "*Passive gravitational*" mass or simply *gravitational* mass – item (3) and see Section 2.1.1 – measures the quantitative attribute of a body that parameterizes its *susceptibility* to a gravitational field.

Regarding *relations* between these kinds of mass in Newtonian physics, the canonical view is that [33, Ch.10, p.126]

> **Quote 2:** " *(…) the proportionality between inertial and passive gravitational masses is a purely empirical and purely accidental feature in classical physics.*"

At the same time "the proportionality of active and passive gravitational mass" – i.e. of types (2) and (3) – is reported to be "deeply rooted in the very principles of Newtonian mechanics" [33, Ch. 10]. A little reflection quickly reveals that this conception has a disputable aspect. It is that, of the three kinds of mass mentioned, only the first belongs to the core of Newtonian mechanics, i.e. *dynamics*. The second and third kind of mass are – at first sight – rather like attracting and attracted charges, similar to charges in the electrostatic Coulomb force [32, 37]. The explanation of their proportionality [33, p. 126] resides on an argument that would belong to statics, rather than dynamics. This suggests that even the proportionality of masses of type (2) and (3) is not so deeply rooted in Newtonian mechanics, after all. As a consequence, the Newtonian cohesion of the above triad of masses would be even less firm than is suggested by the canonical reading.

1.1.3 An Alternative Triad, Reflecting Newton's *Principia*

Although the canonical reading described in Section 1.1.2 may serve didactic purposes – to be addressed in Section 2.1.1 – in this monograph we will propose an alternative view. It is that, only *one* concept of mass featured in Sir Isaac Newton's *Principia* [42], in three different *roles*. We shall argue, substantiated with quotes from the *Principia*, that – rather than three distinct kinds of *mass* – the following three fundamental notions can be considered characteristic for Newton's conception of mass and gravitation in the *Principia*:

1. inertia,

2. the gravitational force is subject to Newton's Third Law, – the 'action equals reaction' law – i.e. *over a distance*,

3. gravitational acceleration is universal.

As we shall see in this monograph, with this numbering these notions can be viewed as counterparts of the three canonical types of mass, as listed and numbered in Section 1.1.2. We shall see that Newton posed, upheld and exploited these three notions in the *Principia* [42] and that they effectively implied a unification – right at their very genesis – of the three canonical kinds of mass. The *Principia* offers a *trinity* of mass, rather than a triad. The three "kinds" of mass were actually only distinguished by the later canonical reading presented in Section 1.1.2.

1.2 The Value of Newton's Physics and Arguments to Modern Science

As speaks from both Quote 1 and from the triad presented in Section 1.1.3, our subject spans Newtonian mechanics and Newton's Universal Law of Gravitation, including their underpinning; together this forms 'classical Newtonian physics', for short.

Both Newton's mechanics and Newton's Universal Law of Gravitation are still anchor points to physics. Indeed, any more modern theory of physics must include, and perhaps correct, Newtonian physics as a limit case. For this reason it is still of value, to any physicist, to know and understand what exactly the content of Newtonian physics is, including the arguments that underpin it. The meaning and splendor of Newton's theory of mechanics and gravitation, as Newton developed it in his *Principia* speaks – perhaps more than from the familiar modern formulation of it – from Newton's arguments that underlie it.

This monograph aims to contribute to, or facilitate, knowledge and understanding of Newton's classical physics, and to provide a view on how Newton, through the *Principia*, reported to have arrived at it.

1.3 Motivation: the Nuances of Mass, Gravitation and Newton's Way in the *Principia*

1.3.1 Nuances of Mass Remained Buried in the *Principia*

The initial motivation for this monograph was rooted in the observation that actually *neither* of the nuanced views on the classical concept of mass, as presented in Section 1.1, is commonly taught, nor supported in any tangible manner, by academic textbooks on *classical* physics or mechanics[2]. Rather, in physics textbooks, the proportionality of inertial mass and passive gravitational mass is associated with Einstein's[3] theory of gravitation [35, IX.11] or, more specifically, with the "*Principle of Equivalence*" [34], an idea that Einstein [25, p. 57] used to illuminate one of the merits of his theory of general relativity.

The lack of coverage, in physics textbooks, of the nuances of mass in Newtonian physics may very well be a consequence of the canonical perception on the matter. Indeed, if – as we noticed in Section 1.1.2 – the picture is that there is little cohesion among the various roles of mass in Newtonian dynamics and gravitation, then it might seem that little of the subject is worthwhile teaching about.

What we hope to convey with this monograph however is that the picture is mistaken. Furthermore, as we shall also show in this monograph, the mentioned lack of general recognition of the character and status of mass in Newtonian physics can be traced back to redactional choices that Newton himself made in the *Principia*. These redactional choices camouflaged, rather than exhibited, key results that Newton *did* deduce; key results that actually established a firm cohesion among his concept and roles of mass.

We shall argue that Newton may have felt compelled to these choices, in view of the attitude of his peers in the seventeenth century, with respect to his theory of gravitational attraction in the first place. An implication could be that, the coarse manner in which the concept of mass is conceived

[2] See Section 2.2.
[3] Albert Einstein, 1879 - 1955.

and taught in our times bears marks of the intellectual atmosphere of the seventeenth century.

1.3.2 Newton's New Attitude, Approach and Way of Proceeding, Regarding Physics.

The full title of Newton's treatise, published in 1687, was '*Philosophiae Naturalis Principia Mathematica*', or '*Mathematical Principles of Natural Philosophy*'. The work is commonly known for Newton's Laws of Mechanics and for Newton's Universal Law of Gravitation.

In full accordance with the title of the work however, Newton's *Principia* brought something even more important and lasting than his theories of mechanics and gravitation: Newton's attitude, approach and way of proceeding, regarding physics. We only but deliberately avoid the term 'method' here because even Newton's way does not provide a genuine algorithm for generation or distillation of scientific theories. Rather, as we will see, with the *Principia* Newton presented a demonstration of how to rationally *do* physics.

Because of the pivotal position of Newton's *Principia* in the history of western science, philosophy and perhaps civilization indeed, this aspect of the *Principia* is of general interest.

1.3.3 The Intertwined Issues of Newton's Mass and Newton's Way.

As we mentioned so far: with the *Principia*, Newton introduced profound novelties, to say the least, both to *physics* and to the *way of doing physics*.

Well: in this monograph we shall argue that the confusion that arose around the concept of mass in 'Newtonian physics' – as outlined in Section 1.1 – can perhaps best be understood in terms of confusion caused by the combination of these two novelties. We shall argue[4] that the confusion about Newtonian mass and gravitation is intimately connected with *expectations* that Newton's readers may have had of *any* formulation of a theory in physics. We wish to distinguish two kinds of expectations here.

Firstly, expectations may well have been fed and encouraged by the *format* that Newton chose for the *Principia*. It seems, at least at first sight, to be presented as a system of *definitiones*, *axiomata*, *propositiones* and *corrolaria*, supported by deductive proofs, supplemented with reflections

[4]See Sections 3.1, 7.3.2 and 8.2.3.

in *scholia*. This may very well have caused readers of the *Principia* to tend to interpret it as an Aristotelian axiomatic system, i.e. conform the classical recommendations for argumentative science in Aristotle's *Organon* [6]. But as we shall see, Newton actually pressed beyond this traditional format, and precisely this opened a way to progress.

Clues hinting at confusion caused by expectations of this first kind can be found in how much, and what, has been written about Newton's 'definition' of mass [33]. Actually, Newton's *definitiones* and *axiomata* in general have been critically questioned as to the soundness and definiteness of the foundation they present [38]. This may very well indicate reasons why Newton's heirs have not developed a very clear picture of what Newton's *Principia* actually entails. We will argue that what has been labeled as shortcomings of Newton's *definitiones* and *axiomata* rather can be taken to reflect Newton's revolutionary approach of *physics*. And thus, clarifying Newton's attitude, approach and way of proceeding can also clarify the nature, status and cohesion of Newton's physical concepts.

Secondly, a classical philosophical tradition known as *essentialism* may well have interfered with receptive reading of the *Principia*, both in and since Newton's times. Because in this monograph we shall aim to concentrate on Newton's physics, we shall avoid to explore and import any more philosophical embedding and alternatives than would be strictly instrumental to our purposes. Nevertheless, Appendix B gives a, necessarily incomplete, sketch of the historical and contextual embedding of Newton's *Principia*, that may serve the reader to obtain a first glimpse on, and references to, these matters.

The aspects mentioned above seem to have been overlooked by Newton's earlier readers. For us, to recognize this sheds light on how Newton's inheritance evolved. Furthermore we thus also refresh the question of how to read and interpret the *Principia*.

As a possible answer, we will show that the item of Newton's concept of mass and the item of Newton's way of working can effectively and fruitfully be addressed – surprisingly perhaps – *together*. In short, contrary to what the format of the *Principia* suggests, Newton's concept of mass is *not* fully specified in the opening 'definition'. Nor was it meant to be. Rather, Newton's trinity of mass – as it is announced in Section 1.1.3 – is the outcome of Newton's process of deducing the Universal Law of Gravitation, *throughout* the *Principia*.

1.4 History: Newton's Peers and his Later Readers Shaped his Legacy

As announced in the previous sections, in this monograph we shall revisit both Newton's physics as presented in the *Principia* and its epistemology. This latter item concerns how Newton developed his physics in the course of the *Principia* and the consequences of this for the structure and status of the presented concepts and theory.

This would have been a straightforward case of one hand washing the other, were it not that, on close inspection, Newton's *writing* appears to have been affected by the intellectual atmosphere in his own times. Subsequently, Newton's *legacy* of course was also subject to the attitudes of his *later* readers.

This introduces three additional aspects to this monograph. Firstly, we shall pay close attention to *how* Newton reported on his work and results, through the *Principia*, down to the level of his redactional choices. Secondly – because Newton's redactional choices turn out to raise stark questions – we shall have to face the question of *why* Newton made certain redactional choices. We shall consider a possible explanation in terms of the context of Newton's times. Thirdly, to gain some idea of how Newton's legacy evolved, it is of interest to consider the attitudes and expectations of some of the later commenters and criticasters of the *Principia*.

As we shall see, close reading reveals that in the *Principia* a trinity of mass emerges as a *consequence* of Newton's laws of dynamics and of Newton's applications of these, including his analysis of celestial mechanics. As we will argue, however, in the *Principia* the most startling conclusions of Newton's analyses remain *concealed*, as a result of Newton's choice of terminology and presentation. We will consider what Newton's motivations, if any, may have been in these matters. This is where the general attitude of Newton's peers may have played a role.

As concerns Newton's legacy: as is well known, Newton's contributions outlived their originator. But, as we shall see – and this is perhaps less part of the common awareness – the positions of Newton's criticasters outlived theirs.

1.5 First Reading Aid

1.5.1 Four Interwoven Threads

As was introduced by Sections 1.1 to 1.4, this monograph has four interwoven threads, or aspects. First, there is a thread of Newtonian physics. Second, there is the thread of Newton's scientific attitude, approach and way of working, and the consequential epistemology. Third, there is the aspect of how Newton presented his material in the *Principia*; this involves Newton's redactional choices, down to the level of his choice of terminology. Fourth, there is the thread of the context and history of Newton's work and legacy, and of the social-historical aspects of these. Given this, as different readers of this monograph may have different interests and priorities, a reading guide seems to be appropriate and we aim to present one here, in this Section 1.5.

1.5.2 Navigation Aids

Section 2.1 presents a further introduction into the core subjects and motivation for this monograph. Section 2.2 presents a survey of how 'mass' is conceived and taught in post-Newtonian textbooks on mathematics, mechanics and physics.

Section 2.3 compares the post-Newtonian conception of 'Newtonian mass' with the actual status that mass has in Newton's *Principia*. This comparison reveals a gap that the present monograph aims to address. Section 2.4 further specifies the goals of our review, and it gives a preview of results that can be expected.

Section 2.5 presents a further, more detailed outline of the rest of the monograph: this section can be consulted as a detailed reading guide for Chapters 3 to 8. Alternatively, the reader may start reading the summary that is presented by Section 8.1. This summary contains detailed references to sections in which results are established. Hence, Section 8.1 can be used as a navigation aid for the whole of this monograph as well.

1.5.3 The Physics Trail

Readers who are primarily interested in our reconstruction of the content of Newton's physics in modern notation may give priority to Chapters 5 to 6. Detailed references in these sections will specify which concepts and results from earlier sections are used, so the reader can easily consult