Bose Liquid Theory for Unconventional Superconductors and Superfluids

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PREFACE

There has been a debate between wise men for a long time...

Which path to knowledge leads?

I'm afraid that a cry will be heard: “O ignoramuses,

The true path is not this and not that!”

Omar Khayyam

The discoveries of superconductivity and superfluidity in solids and quantum liquids (^4He and ^3He) have been a great progress in sciences and opened a new era in history of physics. The Fermi-liquid theory, proposed by Bardeen-Cooper-Schrieffer (BCS), describing the superconductivity in conventional metals was quite adequate for understanding this phenomenon in them. However, the discoveries of unconventional superconductivity and superfluidity in heavy-fermion and organic superconductors, superfluid ^3He and atomic Fermi gases, especially high-T_c cuprate superconductors completely changed our outlook on novel superconductivity and superfluidity in nature. Since the existing BCS-like theories of Fermi-liquid superconductivity and superfluidity were not adequate for description of the superconducting/superfluid states and properties of these unconventional superconductors and superfluids exhibiting pseudogap behavior in the normal state and a λ-like superconducting/superfluid transition at some critical temperature T_c. The pseudogap state and unconventional superconductivity (superfluidity) are observed in most condensed matter systems. Till now, many review articles and different books have been published on high-T_c cuprate superconductors and other superconductors and superfluids. Authors of these articles and books have used the different BCS-like pairing theories to describe the unconventional superconductivity and superfluidity. The BCS theory was quite successful in explaining the superconducting state and properties of conventional metals with large Fermi energies. But the high-T_c cuprates and other systems with low Fermi energies might be in the limit of bosonic Cooper pairs and superfluid Bose systems for which the validity of many competing theories proposed for explaining the pseudogap phenomena and unconventional superconductivity and superfluidity is obscure. In particular, for a long time the different BCS-like (*s*-, *p*-, and *d*-wave) pairing theories were used to describe the unusual superconducting/superfluid states and properties of these systems without clarifying the fermionic and bosonic nature of Cooper pairs and the relevant mechanisms of superconductivity and superfluidity. While the usual Bose-Einstein condensation (BEC) of an ideal Bose gas is irrelevant to the phenomena of superconductivity and superfluidity. Therefore, the purpose of this book is to present the well-founded and empirically adequate theory of Bose-type unconventional superconductors and superfluids. This book is needed for readers and researchers, who should be familiar with the fundamentals of the modern theory of unconventional superconductors and superfluids, since it is devoted to the new direction in physics, associated with the abandonment of the above inapplicable theories. In this theory the methodologies of current theory of solid state, quantum mechanics, statistical physics and mean-field methods are used. The new findings, concepts and principles of the theory of unconventional (Bose-liquid) superconductivity and superfluidity are presented. This theory is a real breakthrough beyond the usual physics of Fermi-liquid superconductivity and superfluidity. It would contribute to research on discovering the new promising high-T_c materials and room-temperature superconductors. The new theoretical results are compared with experimental findings in many specific cases.

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S. Dzhumanov
May 27, 2024

CHAPTER ONE

PROGRESS IN THE THEORY OF SUPERCONDUCTIVITY AND SUPERFLUIDITY

Introduction and Overview

In 1911, the phenomenon of superconductivity was discovered in mercury by Kamerling Onnes at very low temperature $T \simeq 4.2$ K [1]. Soon after, this phenomenon was also observed in other metals at some critical temperatures T_c below which the electrical resistance is dropped down to zero. It turned out that conventional metals have low critical superconducting transition temperatures $T_c \leq 10$ K. For a long time, the origin superconductivity in metals was remained unclear (see [2]). The important characteristic feature of the superconducting state is that the flow of the current electrons below T_c is resistanceless, that is, superfluid. Another key property of superconductors is their ability to expel a magnetic field completely or partially from their interior bellow T_c. Such an exclusion of the magnetic field from the bulk of a superconductor is called the Meissner effect, which was discovered by Meissner and Ochsenfeld [3]. Superconductors in which the magnetic field is excluded completely from its interior below T_c are called type I-superconductors. The first progress in understanding the superconductivity phenomenon was made in the early 1930s. First, Gorter and Casimer put forward in 1934 the idea of a two-fluid model [4] in which the electron gas within the superconductor has two superfluid and normal components. In 1935 after the discovery of the Meissner effect, F. London and H. London [5] using the Gorter and Casimer's two-fluid model and the Maxwell's electromagnetic equations, obtained their famous equations underlying the macroscopic electrodynamics of superconductors. In so doing, they gave an important phenomenological description of superconductivity. Then, London's phenomenological theory was generalized by Ginzburg and Landau [6] and by Pippard [7], who were introduced independly the concept of a coherence length ξ_c, which is different from the London penetration depth λ_L of a magnetic field into a superconductor and characterizes the spatial change of the superconducting order parameter. The Ginzburg-Landau theory provides a good phenomenological description of superconductivity near T_c. This theory was later developed by Abrikosov [8] and by Saint-James and DeGennes [9], who predicted the existence of the mixed state (or vortex state) in the type-II superconductors and the phenomenon of surface superconductivity.

The phenomenon of superconductivity is similar to superfluidity in liquid helium, ^4He, discovered by Kapitza in 1938 [10]. The liquid ^4He undergoes a λ-like second-order phase transition to the superfluid state at the temperature T_λ, since the specific heat anomaly near T_λ has a λ-like shape. The phenomena of superconductivity and superfluidity have one characteristic feature in common: the superconductivity is a frictionless flow of charged electrons (or holes) through the crystal lattice, whereas the superfluidity is a frictionless flow of helium atoms through thin capillaries. As mentioned above, the electron gas in a superconductor below T_c has two (superfluid and normal) components. A similar two-fluid model has been suggested by Tisza [11] for superfluidity in liquid ^4He. According to this model, liquid ^4He below T_λ has also two (superfluid and normal) components.

Later, Landau [12, 13] developed a successful phenomenological theory of superfluidity, which explains reasonably well the behavior of liquid ^4He at low temperatures [14]. According to this theory, the liquid ^4He is considered as a weakly excited quantum-mechanical system composed of a superfluid and a gas of elementary excitations or quasiparticles, which is called as a normal fluid. Further, Landau proposed an energy-momentum relationship for the elementary excitations in liquid ^4He. The most important features of the excitation spectrum of superfluid ^4He. suggested by Landau have been confirmed by neutron-scattering experiments [15]. Most importantly, Landau also derived the criterion for superfluidity. Any quantum liquid (including also electron liquid in a superconductor) will be a superfluid when the flow velocity of the liquid does not exceed some critical velocity determined by the Landau criterion. In particular, superconductivity phenomenon in condensed matter systems can be explained as the superfluidity of a Fermi or Bose liquid.

The next progress in understanding the superconductivity was made in 1950 when Ginzburg and Landau [6] proposed an important phenomenological theory of this phenomenon. Further, there have been great advances in understanding key features of conventional superconductors. Theoretical studies carried out by Fröhlich in 1950 [16] led to the prediction of the important role of the electron-phonon interaction in superconductivity. At the same time the so-called isotope effect on T_c

predicted by Fröhlich was discovered experimentally in 1950 by Maxwell [17] and Reynolds et.al.[18]. Later, the important step in understanding the microscopic mechanism of superconductivity in conventional metals was made by L. Cooper in 1956 [19], who put forward the idea of an instability of the normal state of an electron gas with respect to the formation of bound electron pairs. These so-called Cooper pairs are bound by a weak electron-phonon interaction. Based on this idea and the isotope effect on T_c, Bardeen, Cooper and Schrieffer (BCS) proposed in 1957 [20] their famous theory of Fermi-liquid superconductivity. This theory accounted for many of the experimental findings (e.g., the existence of an energy gap in the excitation spectrum of metals below T_c, the isotope effect on T_c and a second-order phase transition at T_c) in conventional superconductors and was capable of calculating the basic superconducting parameters, such as T_c, specific heat jump at T_c, the superconducting order parameter, the London penetration depth λ_L and critical magnetic field H_c.

Nevertheless, experimental studies on conventional superconductors have shown that various superconducting properties of some materials deviate from the predictions of the BCS theory. In particular, the values of the isotope effect exponent α observed in some metals are inconsistent with the BCS value of $\alpha = 0.5$ and are highly reduced or even negative [21, 22]. Such deviations of α from the value 0.5 have been explained in terms of a more pertinent theory of pairing interaction (which is the sum of the attractive electron- phonon interaction and the repulsive Coulomb interaction) between electrons developed by Bogoliubov, Tolmachev and Shirkov [23], Eliashberg [24] and McMillan [25]. The modifications of the BCS and Bogoliubov pairing were done by Eliashberg and McMillan using the experimental information about the crystal lattice vibrations. As a result, the Eliashberg-McMillan equations contain the effects of renormalization of quasiparticle excitations, which are not simply single electrons from broken Cooper pairs but dressed quasiparticles [26]. Therefore, these approaches serve as a basis for understanding the anomalous behaviors of various conventional superconductors, mostly transition metals [22, 27], in which the strong deviations from the BCS theory were observed. Over many years, most of the researchers believe that the BCS and Eliashberg theories extended to the strong electron-phonon coupling can be also used to explain the superconducting properties of high-T_c cuprates and other related materials, which were discovered after the 1980s. However, such BCS-like theories are applicable if Cooper pairs have fermionic nature. If the attractive pairing interaction between fermionic quasiparticles is comparable with their Fermi energy, then strongly correlated Cooper pairs behave like bosons and the BCS-like theories cannot explain the superconductivity and superfluidity of such bosonic Cooper pairs. Therefore, an alternative view is that the superconducting/superfluid states in most condensed matter systems are fundamentally different from such states of BCS-like Fermi liquids.

In 1972, Osheroff, Richardson and Lee discovered the superfluidity of liquid ^3He at temperatures below 3 mK [28] and observed two distinct superfluid states corresponding to the so-called A and B phases. Over many years, attempts to understand these superfluid states have been based on the two different BCS-like (p-wave) pairing models proposed by Anderson and Morel [29] and Balian and Werthamer [30] before 1972. However, these BCS-like pairing theories describe fairly well the formation of p-wave Cooper pairs, but fails to account for superfluidity in liquid ^3He. In particular, the first-order phase transition between the A and B phase of superfluid ^3He and the other unusual superfluid properties of liquid ^3He were remained a mystery to the above BCS-like theories. Therefore, the identification of the A and B phases of superfluid ^3He with Anderson-Morel and Balian-Werthamer states, respectively, is ill-founded. Further, the superfluidity discovered in atomic Fermi gases defies also a BCS description of this phenomenon (see [31]).

Before the discovery of high-T_c cuprate superconductors (until 1986), there have been some attempts to predict the possibility of high-T_c superconductivity in low-dimensional organic compounds on the basis of different BCS-like models [32, 33], but for many years this phenomenon was not found. The first organic superconductor $(TMTSP)_2X$ (where $X=PF_6$, ClO_4, etc, TMTSF is tetramethyltetra senafulvalene) with $T_c \simeq 1$ K was discovered by Jerome et al. [34] in 1980. Subsequently the superconductivity phenomenon was discovered in other organic compounds (see [15]) and fullerene superconductors K_3C_60 (with $T_c \simeq 18$K) [35] and $CsRbC_{60}$ (with $T_c \simeq 33$ K) [35]. The unusual superconducting properties of organic materials are similar to those of high-T_c cuprate materials and they belong to the class of unconventional superconductors [37, 38].

There is also other type of superconducting materials, such as $CeCu_2Si_2$, UBt_{13}, UPt_3, etc, which are known as the heavy-fermion superconductors [39]. In the end 1970s and in the early 1980s, superconductivity in $CeCu_2Si_2$, UBt_{13} and UPt_3 were discovered Steglich et al.[40], Ott et al.[41] and Stewart et al.[42]. Although, the critical temperatures of the superconducting transitions of these materials are very low, but their other properties are interesting. For example, the heavy-fermion superconductors possess huge values of critical magnetic fields. The unusual normal and superconducting properties of these unconventional superconductors are similar to such properties of high-T_c cuprate superconductors [38].

The great progress in the theory of unconventional superconductivity has been stimulated by the experimental discoveries of high-T_c cuprate superconductors. The advances in finding new superconductors with higher T_c values from 1911 to 1986 were achieved very slowly. Only the superconductor Nb_3Ge which was discovered by Gavaler in 1973 (see [2]) had the highest $T_c \simeq 23$ K. In the following years, all attempts to find materials with higher superconducting transition temperatures were unsuccessful up to 1986. Such a slow progress in discovering the new superconductors with substantially higher T_c values inclined most of researchers to think that superconductivity is a low-temperature phenomenon. This pessimistic conclusion was also confirmed by the BCS expression for T_c. Only the discovery of superconductivity below 35 K in the ceramic hole-doped copper oxides (cuprates) $La_{2-x}Ba_xO_4$ by Bednorz and M$\ddot{u}$ller in 1986 [43] has changed this view-point completely. Soon after, Chu et al. [44] discovered the new cuprate superconductors with $T_c \simeq 93$ K. Later, other families of the cuprate superconductors with an even higher T_c were discovered [45, 46]. The *Hg*-based cuprate superconductors $HgBa_2Ca_2Cu_3O_{8+\delta}$ ($Hg - Ba - Ca - Cu - O$) discovered by Schilling et al. in 1993 [47] has the record T_C values of 133 K at ambient pressure, starting from 1993, there has been considerable interest in the increase of T_c in these materials to a maximum value at different applied pressure and the record values of $T_c \simeq 153 - 164$ K reported for $Hg - Ba - Ca - Cu - O$ [48, 49].

The observed electronic properties of high-T_c cuprates in the underdoped and optimally doped regimes are very unusual in many respects, in both the normal and superconducting states. Since the behaviors of these high-T_c materials strongly deviate from the predictions of the standard theories of the normal Fermi liquid and the BCS-type Fermi superfluid. Therefore, after discovery of high-T_c superconductivity in doped cuprates, many distinctly different theoretical models have been proposed for description of this remarkable phenomenon (see, e.g.,[50, 51, 52, 53, 54, 55, 56, 57, 58, 59]). At present, theoretical research on high-T_c superconductivity is an enormous field and it would be quite difficult to discuss the vast literature related to the subject. It is important, however, to discuss briefly some specific theoretical ideas and models, which are often considered as possible candidates to explain high-T_c superconductivity in the cuprates, and the main merits and shortcomings of these theoretical proposals.

One of the early ideas of high-T_c superconductivity was the so-called bipolaronic mechanism of superconductivity. In polar cuprate materials, the self-trapping of doped hole carriers at their strong interaction with phonons leads to the formation of polarons and the superconductivity of bound polaron pairs (bipolarons) proposed by Chakraverty, Ranninger, Alexandrov, Emin and others (see [50, 51, 53, 56, 57]), might be expected at low densities of doped charge carriers. Since the pairing of polaronic carriers in real space, in contrast to their Cooper pairing in k-space takes place in the low carrier concentration limit and results in the formation of small or large bipolarons. The binding energy of a small bipolaron would be much larger than that of a large bipolaron, so that small bipolarons tend to be localized rather than mobile. Therefore, Emin argued [59] that large bipolarons play an important role in high-T_c superconductivity and will condense into a superfluid state in cuprates. The binding energy of large bipolarons decreases and tends to zero when the doping level increases towards the underdoped region. As a result, large bipolarons can exist in the lightly doped cuprates, which are insulators. It follows that the possibility of realizing large-bipolaronic superconductivity in underdoped cuprates becomes problematic.

Other ideas of high-T_c superconductivity in the cuprates were based on the so-called excitonic and plasmonic mechanisms of the BCS-like pairing of charge carriers at their interaction with electronic excitation whose energy is much higher than the phonon energy. Long before 1986, Little [32] and Ginzburg [33] put forward the ideas of the excitonic mechanisms for Cooper-pair formation in the quasi-one-dimensional and quasi-two-dimensional organic conductors which could possibly lead to high-T_c superconductivity. Later, interest in exciton-mediated pairing mechanisms was increased after the discovery of high-T_c cuprate superconductors (see [60, 61]). The possibility of the BCS-like pairing of charge carriers in high-T_c cuprates due to the exchange of electronic excitations has also been discussed by Ginzburg and other authors [62, 63].

The alternative BCS-like models based on the localized electron-bag, spin-bag and correlation-bag mechanisms of Cooper pairing have been proposed by Weinstein [64], Schrieffer et al.[65] and Goodenough and Zhou [66]. The above BCS-like theories of Fermi-liquid superconductivity have serious difficulties in describing the superconducting transition in high-T_c cuprates, which is a λ-like transition and not BCS transition, and other superconducting properties of these materials.

Another theoretical model, in which a mixture of preformed local pairs or real-space pairs (small bipolarons) and free electrons (or holes) and possible effects resulting from boson-fermion interactions, was first proposed by Ranninger and Robaszkiewicz [67] and then rediscovered in a phenomenological approach by Friedberg and Lee [68]. This two-component (boson-fermion) model combines the ideas of the BCS-like pairing of electrons and the Bose-Einstein condensation (BEC) of an ideal Bose gas of preformed real-space pairs. It was argued that superconductivity of mixed boson-fermion systems can be described by using a two-band model consisting of a broad electronic (fermionic) band and a narrow bosonic band. Later the boson-fermion model was developed by Geshkenbein et al.[69] and other authors [70]. The preformed bosons are assumed to be in a bound state (when the bosonic level lies below the fermionic band) or resonant state (when the bosonic level lies inside the fermionic band) in which preformed pairs spontaneously decay into pairs of electrons (or holes). It was also assumed that when the bosonic level lies inside the fermionic band localized bosons (preformed pairs) due to the boson-fermion interaction can acquire itinerancy as the temperature is lowered towards T_C and then Bose condense in the background of a Fermi liquid [69, 71]. In this case, a BCS-type Fermi-liquid superconductivity is believed to be induced simultaneously in the fermionic subsystem, while the localized bosons play the role of phonons in the conventional BCS theory. Below T_c, both bosons and fermions are assumed to be superfluid. When the bosonic level is low compared with Fermi level, the fermions will flow into the bosonic band forming bosons and the superconductivity in the entire system occurs due to the usual BEC of preformed pairs. According to this boson-fermion scenario, the superconducting transition temperature is determined by the BEC of bosons. In the boson-fermion model [71], the pseudogap formation in high-T_c cuprates is explained by the presence of localized preformed pairs (i.e., bipolarons) above T_c. The relevance of the above boson-fermion models to high-T_c cuprate superconductors is questionable for the following two reasons: (i) in doped polar cuprate materials the free electrons (or holes) will be absent due to their strong interaction with phonons and spontaneous self-trapping, so that the existence of a mixture of preformed local pairs and free electrons (or holes) in polar cuprates is unlikely; (ii) the usual BEC of an ideal Bose gas of preformed pairs is irrelevant to the phenomenon of superconductivity.

The so-called marginal and nearly antiferromagnetic (AF) Fermi-liquid models have been proposed by Varma et al.[72, 73] and Pines et al.[74, 75] for the explanation of the anomalous electronic properties of high-T_c cuprate superconductors. According to the marginal Fermi liquid model [73], the breakdown of the standard Fermi-liquid theory occurs in the region of the electronic phase diagram of doped cuprates around a quantum critical point (QCP). It was assumed that the anomalous normal and superconducting states are controlled by fluctuations around this QCP and the superconducting region in the temperature-doping phase diagram is surrounded by the usual Fermi-liquid, singular Fermi-liquid and pseudogap regions. The natures and origins of the pseudogap and superconducting states of high-T_c cuprates in the marginal Fermi-liquid model have not been fully understood.

An alternative Fermi-liquid approach to the theory of high-T_c cuprates is based on the idea of the occurrence of AF correlations in doped cuprates. In the nearly AF Fermi-liquid model, charge carriers are dominantly scattered by spin fluctuations. It was argued [75] that the AF spin fluctuations are responsible for the formation of pseudogap state in underdoped cuprates. The nearly AF Fermi-liquid model predicts the existence of weak and strong pseudogap regimes and two crossover temperatures T^{cr} (the upper crossover temperature marks the onset of AF correlations) and T^* (the lower crossover temperature associated with the opening of a spin pseudogap) in the normal state [75]. In this model two types of quasiparticles emerge: hot quasiparticles, which have high anomalous behavior, and cold quasiparticles, which are not strongly connected by AF spin fluctuations. It was assumed that the pseudogap opens in the hot quasiparticle spectrum and superconductivity emerges when the cold quasiparticles become gapped, independent of hot quasiparticles. When the doping level in the cuprates increases towards underdoped region, the AF order is completely destroyed [76] and therefore, the applicability of the nearly AF Fermi-liquid model to the underdoped cuprates remains questionable. Further, the high-T_c cuprates undergo a λ-like superconducting transition at T_c and their unusual superconducting state and properties cannot be understood within the any Fermi-liquid theory.

The relevance of other theoretical approaches, such as the resonating valence bond model [52], the superconducting phase fluctuation models [76, 77], the different BCS-like (s- and d-wave) pairing models [57, 75, 78, 79, 80], the two-stage Fermi-Bose liquid model [53, 81, 82] and other models based on the BCS-BEC crossover [83, 84, 85], to the high-T_c cuprate superconductors will be discussed below (in the following sections).

Landau Criterion for Superfluidity

We have already mentioned that a quantum fluid flowing with velocity $v < v_c$ (where v_c is the critical flow velocity of the fluid, determined from the Landau criterion for superfluidity) becomes superfluid. Landau formulated [12, 13] that under the condition $v < v_c$ the quantum fluid should flow as a superfluid without any friction and the superfluidity is destroyed at $v > v_c$. We now discuss the question of the critical velocity of superfluid and consider the processes which might lead to the destruction of superfluidity (or superconductivity). We consider an excitation-free superfluid flowing through a long tube (or crystal lattice) with velocity ν relative to the immobile tube (or laboratory system of coordinates) at $T = 0$ K. If we go over into the system of reference in which the superfluid is at rest, the walls of the tube are moving with respect to the superfluid with velocity $-\nu$. When the flow velocity of the fluid approaches to ν_c the drag friction between the tube and the superfluid arises. Therefore, the viscosity of the fluid will appear and the creation of an excitation in the superfluid becomes possible. This causes a loss of the superfluid's kinetic energy, which is ultimately converted into heat. As a consequence, the superfluid begins to slow down. We now determine the minimum value of the flow velocity at which a single excitation (or quasiparticle) can appear. Suppose that a single excitation with energy $\varepsilon(p)$ and momentum p appeared in the superfluid. Because of the recoil, the velocity of the tube is then changed and become equal to $-\nu_1$. According to the principles of conservation of energy and momentum, one can write

$$\frac{M\nu^2}{2} = \frac{M\nu_1^2}{2} + \varepsilon(p) \tag{1.1}$$

and

$$-M\vec{\nu} = -M\vec{\nu}_1 + \vec{p} \tag{1.2}$$

where M is the mass of the tube.

Combining (1.1) and (1.2), we obtain

$$\varepsilon(p) + \vec{p}\vec{\nu} + \frac{p^2}{2M} = 0 \tag{1.3}$$

from which it follows that

$$\varepsilon(p) + \vec{p}\vec{\nu} = \varepsilon(p) + p\nu cos\theta < 0 \tag{1.4}$$

where θ is the angle between $\vec{p}$ and $\vec{\nu}$.

The left-hand side of (1.4) is minimal when $\theta = \pi$ or $\vec{p}$ and $\vec{\nu}$ are antiparallel, so that $\varepsilon(p) - p\nu < 0$. Consequently, the condition for the appearance of an excitation in the superfluid is written as

$$\nu > \frac{\varepsilon(p)}{p} \tag{1.5}$$

The minimum value of v at which an excitation can appear in the superfluid is equal to

$$v_c = min\left[\frac{\varepsilon(p)}{p}\right] \tag{1.6}$$

If $v < v_c$, the excitation cannot be appeared in the superfluid, which will flow through a tube or a crystal lattice with any dissipation. Thus, the Landau criterion for superfluity is written as

$$v < v_c = min\left[\frac{\varepsilon(p)}{p}\right] \tag{1.7}$$

Extremum of the function $\varepsilon(p)/p$ are determined from the equation

$$p\frac{d\varepsilon(p)}{dp} = \varepsilon(p) \tag{1.8}$$

These relations presented are also valid for $T \neq 0$. The Landau theory of superfluidity is based on the excitation spectrum of liquid ^4He and the equation (1.8) has two solutions corresponding to different regions of the spectrum (see, e.g., Ref. [15]). One solution corresponds to the origin and all points of the phonon region of the spectrum, whereas the second solution corresponds to the point near the minimum in the roton region. From Eq.(1.7) it follows that the critical velocity v_c determines the threshold below which the existence of superfluidity in the system is possible. The theoretical calculations of v_c for the quantum and electron liquids, which have the different energy-momentum relationships, are very important. According to (1.7), any liquid with the energy-momentum relationship of the form $\varepsilon(p) = v_S p$ (where v_s is the sound velocity) is a superfluid (or superconducting (SC)). If the energy-momentum is given by the ideal-gas relationship $\varepsilon(p) = p^2/2m$, then the critical velocity would be zero. In this case, any velocity of the flowing fluid is greater than v_C. That is, the quantum (or electron) liquid would not be a superfluid (or SC). This is a very important result, for it brings out very clearly the fact that the specific interatomic and interelectron interactions in quantum and electron liquids, which give rise to an excitation spectrum different from the one characteristic of the ideal gas, play a key role in superfluidity and superconductivity. In particular, the criterion (1.7) can be satisfied when the excitation spectrum of the Fermi-and Bose-liquid has an energy gap at $p(=$

$\hbar k) = 0$. Although, an ideal Bose-gas does undergo the phenomenon of Bose-Einstein condensation (BEC), it bears no a relation to the phenomenon of superfluidity and superconductivity [14, 53, 81, 86, 87]. As already pointed out by Landau [12, 13], who argued that the BEC of an ideal Bose-gas should not be confused with the superfluidity. We shall discuss this question in more detail in chapters 5 and 8.

The Criterion for the BCS-Type Fermi-Liquid Superconductivity

The Landau criterion for superfluidity applied to the excitation spectrum of weakly-bound Cooper pairs allows us to understand the phenomenon of conventional superconductivity, which is associated with the coherent motion of Cooper pairs. This criterion should be applied to the excitation spectrum (energy-momentum relation) $\varepsilon(\vec{K})$ of moving Cooper pairs (with center - of – mass momentum (CMM) $\vec{K}$). The critical velocity of the superfluid BCS condensate is often determined from the criterion

$$v_c = min\frac{E(\vec{k})}{\hbar(\vec{k})} > 0 \tag{1.9}$$

where $E(\vec{k})$ is the excitation spectrum of independent BCS (Bogoliubov) quasiparticles and not moving Cooper pairs. Actually, the energy spectrum $\varepsilon(\vec{K})$ of moving Cooper pairs should be used to determine the condition of their superconductivity. Therefore, we define the criterion for superfluidity of Cooper pairs as

$$v_c = min\frac{\varepsilon(\vec{K})}{\hbar(\vec{K})} > 0 \tag{1.10}$$

The criterion (1.10) is satisfied for the system of the moving Cooper pairs with the linear energy-momentum relation [88]

$$\varepsilon(\vec{K}) = \frac{1}{2}\hbar v_F\vec{K} + 0(K^2), \tag{1.11}$$

which is obtained from the solution of the Cooper problem in the weak electron-phonon coupling regime for a special case of the Cooper model interaction.

Let us now consider the pairing of two carriers in k-space above the filled Fermi sea in heavily overdoped cuprates and the formation of Cooper pairs with zero- and non-zero CMM. The Schrödinger equation for two carriers interacting via the potential V above the filled Fermi sea is written as

$$\left[-\frac{\hbar^2}{2m^*}(\nabla_1^2+\nabla_2^2)+V(\vec{r}_1+\vec{r}_2)\right]\Psi(\vec{r}_1,\vec{r}_2)=\left(\varepsilon+2\varepsilon_F^f\right)\Psi(\vec{r}_1,\vec{r}_2) \quad (1.12)$$

where m^* is the effective mass of charge carriers, $\Psi(\vec{r}_1,\vec{r}_2)$ is the two-carrier wave function, ε is the energy defined relative to the Fermi level of quasi-free carriers ($2\varepsilon_F^f$).

Defining the center of mass coordinate $\vec{R}=\frac{1}{2}(\vec{r}_1+\vec{r}_2)$ and the relative coordinate $\vec{r}=\vec{r}_1-\vec{r}_2$, one can write the Eq.(1.12) and wave function $\Psi(\vec{r}_1,\vec{r}_2)$ in terms of these coordinates. Then, the wave function $\Psi(R,r)$ is a product of the CMM wave function $\Omega^{-1/2}\exp(i\vec{K}\vec{R})$ and a bound-state wave function $\psi(\vec{r})$ that describe the structure of the Cooper pair, so that

$$\Psi(\vec{R},\vec{r})=\frac{1}{\sqrt{\Omega}}exp(i\vec{K}\vec{R})\,\psi(\vec{r}) \quad (1.13)$$

where Ω is the volume of the system.

Therefore, Eq.(1.12) is rewritten as

$$\left[-\frac{\hbar^2}{m^*}\nabla_{\vec{r}}^2+V(\vec{r})\right]\psi(\vec{r})=\left(\varepsilon+2\varepsilon_F^f-\frac{\hbar^2K^2}{4m^*}\right)\psi(\vec{r}) \quad (1.14)$$

When the wave function $\psi(\vec{r})$ is expressed as

$$\psi(\vec{r})=\sum_{k>k_F}a_k\exp(i\vec{k}\vec{r}), \quad (1.15)$$

the equation (1.14) has the form

$$\left[\frac{\hbar^2K^2}{4m^*}+\frac{\hbar^2k^2}{m^*}-\mathcal{E}-2\mathcal{E}_F^f\right]a_{\vec{k}}+\sum_{\vec{k}'}a_{\vec{k}'}\langle\vec{k}|V(\vec{k},\vec{k}')|\vec{k}'\rangle=0 \quad (1.16)$$

where $V(\vec{k},\vec{k}')=\int V(\vec{r})exp\left[-i(\vec{k},-\vec{k}')\vec{r}\right]d^3\vec{r}$. In order to find the solution of Cooper-pair equation (1.16) we choose the Cooper model interaction potential in the form [88]

$$V(k, k^{'}) = \begin{cases} -V & if \;\; k_F < \left|\vec{k} \pm \frac{\vec{K}}{2}\right|, \vec{k'} \pm \frac{\vec{K}}{2} < \sqrt{k_F^2 + k_c^2} \\ 0 & otherwise, \end{cases} \quad (1.17)$$

where k_F and k_c are the Fermi and characteristic cut off wave vectors defined by $k_F = \sqrt{2m^* \varepsilon_F^f / \hbar}$ and $k_c = \sqrt{2m^* \varepsilon_c / \hbar}$, ε_c is the characteristic cut off energy for the interaction potential $V(k, k^{'})$.

For the potential (1.17) the Cooper pair equation becomes

$$\frac{1}{V} = \sum_{\vec{k}}{}' \frac{1}{\frac{\hbar^2 k^2}{m^*} + \frac{\hbar^2 K^2}{4m^*} - E} = \sum_{\vec{k}}{}' \frac{1}{2\left(\varepsilon(\vec{k}) - \varepsilon_F^f\right) + \varepsilon(\vec{K}) + \Delta_K} \quad (1.18)$$

where $E = \varepsilon + 2\varepsilon_F^f$, $\varepsilon(\vec{k}) = \hbar^2 k^2 / 2m^*$, $\varepsilon(\vec{K}) = \hbar^2 K^2 / 4m^*$, $\Delta_K = -\varepsilon$.

The prime on the summation sign implies the restriction $\left|\vec{k} \pm \frac{\vec{K}}{2}\right| > k_F$. The condition in Eq.(1.17) is rewritten as

$$k_F < \left[k^2 \pm \vec{K}\vec{k} cos\Theta + K^2/4\right]^2 < \sqrt{k_F^2 + k_c^2} \quad (1.19)$$

where θ is the angle between wave vectors $\vec{k}$ and $\vec{K}$.

Replacing the sum in Eq.(1.18) by an integral and taking into account the condition (1.19), after some algebra we obtain

$$\frac{1}{V} = \frac{\sqrt{2}(m^*)^{3/2}}{(2\pi)^2 \hbar^3} \int_0^{\frac{\pi}{2}} sin\theta d\theta \int_{\varepsilon_{min}}^{\varepsilon_c} \sqrt{\xi + \varepsilon_F^f} \frac{d\xi}{2\xi + \varepsilon_K + \Delta_K} \quad (1.20)$$

where

$$\varepsilon_{min} = \hbar^2 \vec{K} k_F \cos\theta / 2m^* .$$

If the inequality $\varepsilon_c < \varepsilon_F^f$ is satisfied, then in Eq.(1.20) the integration with respect to ξ may be performed by expanding the expression $\sqrt{\xi + \varepsilon_F^f}$ in powers of ξ / ε_F^f. Thus, at $\varepsilon_c >> \Delta_K$ and $K \to 0$, we obtain on integrating and after some algebraic manipulation the following expression for the binding energy of Cooper pairs:

$$\Delta_K = \Delta_0 - \frac{\hbar \nu_F \vec{K}}{2\left[1 - \frac{\Delta_0}{4\varepsilon_F^f}\right]} = \Delta_0 - \varepsilon(\vec{K}). \tag{1.21}$$

where $\varepsilon(\vec{K}) = \hbar \nu_F \vec{K}/2\left[1 - \frac{\Delta_0}{4\varepsilon_F^f}\right]$ is the excitation energy of Cooper pairs. Consequently, the total energy of the Cooper pair in 3D systems is given by

$$E = 2\varepsilon_F^f - \Delta_K \tag{1.22}$$

We see that the energy of Cooper pairs $\varepsilon(\vec{K})$ in the weak coupling limit ($\Delta_0 << \varepsilon_F^f$) is given by the Eq. (1.21) [88, 89].

The linear dispersion relation for Cooper pairs in the 3D system is also obtained within the BCS approach [88]. Therefore, the excitation spectrum of Cooper pairs in the weak coupling limit satisfies the Landau criterion for superfluidity and the BCS condensate of quasi-fermionic Cooper pairs in the heavily overdoped cuprates just like in conventional superconductors is a superfluid Fermi-liquid. One can expect that the underdoped, optimally doped, and moderately overdoped doped cuprates fall in intermediate and strong electron- phonon coupling regimes and the preformed Cooper-like polaron pairs in these systems are composite bosonic particles and have the quadratic dispersion relation $\varepsilon(\vec{K}) = \hbar^2 K^2/4m_p$ in the non-interacting particle approximation, where m_p is the mass of a polaron. The Landau criterion for superfluidity is not satisfied for an ideal Bose-gas of preformed (bosonic) Cooper pairs with the quadratic dispersion relation, so that Bose-Einstein condensate of such Cooper pairs is not superfluid condensate.

The Validity of the Other Proposed Theories of Unconventional Superconductivity and Superfluidity

The usual band theory has been successful enough in describing the normal state of conventional metals with large Fermi energies $\varepsilon_F > 1$ eV [90, 91], while the theory of superconductivity proposed by Bardeen-Cooper-Schrieffer (BCS) [20] was quite adequate for describing Fermi-liquid superconductivity in these systems. However, unconventional superconductivity (superfluidity) and pseudogap phenomena discovered in doped high-T_c copper oxides (cuprates) [43, 44, 92, 93, 94, 95] and other systems (e.g., liquid ^{3}He, heavy- fermion and organic compounds, Sr_2RuO_4 and possibly H_3S, $LaH_{10\text{-}x}$ and *C-S-H* systems and ultracold atomic Fermi gases) [28, 34, 40, 41, 96, 97, 98, 99, 100, 101] turned out the most intriguing puzzles in condensed matter physics. The normal state of high-T_c cuprates exhibits many unusual properties [54, 55], which are assumed to be closely related to the existence of a novel pseudogap (as distinct from Mott's pseudogap in amorphous semiconductors). The existence of such a pseudogap in high-T_c cuprates was predicted first theoretically [53, 76, 77, 102, 103] and then observed clearly experimentally [92, 93, 94, 95]. The normal state of other exotic superconductors [98, 100, 104, 105] and superfluids [31] also exhibits a pseudogap behavior above the superconducting/superfluid transition temperature T_c. Further, the normal state of recently discovered high-T_c superconductors LaH_{10} [106] and *C-S-H* [101] seems to exhibit pseudogap behavior above T_c, since the temperature- dependent behavior of their resistance $R(T)$ above T_c closely resembles the anomalous behavior of the in-plane resistivity of high-T_c cuprates in the normal state. The formation of a normal-state gap (pseudogap) in high-T_c cuprates and other systems manifests itself in the suppression of the density of states at the Fermi level and the pseudogap appears at a characteristic temperature T^* above T_c without the emergence of superconducting order. Most importantly, the high-T_c cuprates in the intermediate doping regime exhibit exotic superconducting properties inherent in unconventional superconductors [99, 107, 108, 109] and quantum liquids (^3He and ^4He) [15, 110, 111, 112], while the heavily overdoped cuprates are similar to conventional metals [57, 113].

In the case of high-T_c cuprates which are prototypical unconventional superconductors/superfluids and are of significant current interest in condensed matter physics and beyond (e.g., in the physics of low-density nuclear matter [114]), our understanding of pseudogap phenomena and unconventional superconductivity is still unsatisfactory and incomplete. Aside from early theoretical ideas [53, 68, 71, 76, 77, 102, 103, 115, 116, 117], later other competing theories have been proposed for explaining the origins and the nature of the pseudogaps and high-T_c superconductivity in these materials (for a review see Refs. [54, 56, 57, 118, 119, 120, 121, 122]). The high-T_c cuprates exhibit not only the pseudogap behavior in the normal state, but also many unusual superconducting properties, such as λ-like superconducting transition at T_c, reduced and nearly vanishing isotope effects on T_c, first-order phase transition in the superconducting state, half-integer magnetic flux quantization, gapless excitations below T_c, kink-like anomalies in the temperature dependences of all superconducting parameters and two-peak specific heat anomalies. After the discovery of high-T_c cuprate superconductivity, a great deal of attention has been focused on the Anderson's idea of spinon-holon phenomenology and other ideas of purely electronic mechanisms of superconductivity in high-T_c cuprates [115, 123]. Therefore, many competing theories have emerged based on the different extended Hubbard models (including one- and three-band models), *t*-*J* models, d-wave BCS-like pairing models and magnetic interactions (see, e.g., Refs. [57, 58, 115, 124]). While the strong electron-phonon interactions and polaron phenomena, which are so ubiquitous in the cuprates, were often ignored by many advocates of non-phononic mechanisms without any justification. However, the inadequacy of magnetic mechanism proposed for high-T_c cuprates has been pointed out by Chakraverty et al. [50] on the basis of the experimental results. Later, other experiments have provided evidence for the existence of strong electron-phonon interactions and polaronic effects in doped cuprates [56, 120, 125, 126]. Theoretical results have also shown [56, 71, 120, 125, 127] that strong electron-phonon interactions are more relevant to these polar materials. The key experimental results of Lanzara et al. [128] draw the same conclusion and rule out the magnetic interactions in high-T_c cuprates. It follows that the relevant electron-phonon interactions must be included in any microscopic theory of pseudogap phenomena and high-T_c cuprate superconductivity.

It is natural to believe that the unconventional interactions between quasiparticles in high-T_c cuprates and other related systems may take place, leading to new and unidentified states of matter. In particular, the vortex-like, pseudogap and diamagnetic states observed above T_c [57, 119, 121, 129] and the unusual superconducting state (see Fig. 1 in Refs. [103, 130]) and quantum critical point (QCP) (at $T = 0$) discovered below T_c [131, 132, 133, 134] in high-T_c cuprate superconductors are natural consequences of such interactions. The origin of the pseudogap state in these systems has been debated for many years, being attributed to the different pairing effects in the electronic subsystem [53, 56, 76, 77, 103, 120] and spin subsystem [115, 116, 135] or to other effects associated with different competing orders which have nothing to do with superconductivity (see Refs. [57, 120]). Pseudogap phenomena and high-T_c superconductivity in the cuprates are often discussed in terms of quantum phase fluctuation (i.e., superconducting phase fluctuation) theories [76, 77, 120, 136]. The first proposed theory argues [76] that the superconducting phase fluctuation scenario is justifiable only for temperatures well below the onset temperature of Cooper pairing T^* in the normal state of high-T_c cuprates. Other superconducting phase fluctuation theories are believed to be less justifiable (see, e.g., Refs. [57, 118]). It was assumed [57, 77, 136] that the superconductivity is destroyed at T_c by the phase fluctuation, whereas superconducting Cooper pairs persist well above T_c or even up to the characteristic temperature $T^* >> T_c$. It was also speculated that the BCS-like (s-or d- wave) gap represents the superconducting order parameter below T_c and then persists as a superconductivity-related pseudogap above T_c. So far, the phenomenological Ginzburg-Landau and Kosterlitz-Thouless (KT) theories are used to determine the superconducting phase fluctuation region above T_c and the actual T_c [57, 76, 129]. Quantum phase fluctuations are believed to be important in underdoped cuprates as the superconductor-insulator transition (SIT) is approached [76, 77] and result in the decrease of the superfluid density, effectively lowering T_c until it vanishes near the SIT. As a consequence, the reduced T_c in these systems can be much lower than the mean-field T_c. The universal jump condition of KT theory was used to determine the actual T_c in underdoped cuprates. The expected influence of quantum phase fluctuations on T_c in irradiated high-T_c cuprate superconductors has been carefully studied experimentally [137]. These studies indicate that such phase fluctuations are important only in determining the actual T_c in the highly damaged (i.e. disordered) region near the SIT. Actually, the jump of superfluid density at T_c characteristic of the KT transition was not observed in high-T_c cuprates; instead the observed temperature dependence of the superfluid density is mean-field-like in a wide temperature range even for underdoped cuprates (see Ref. [118]). Further, the experimental data show that the pseudogap in high-T_C cuprates is unrelated to superconducting fluctuations [138] and the superconducting transition is a λ-like transition [112, 139, 140] and it is characterized by a narrow fluctuation region ($T - T_c \lesssim 0.1T_c$) above T_c [57, 118]. Actually, the pseudo-gap state in high-T_c cuprates has properties incompatible with superconducting fluctuations [57, 118, 138] and behaves as an anomalous metal above T_c [57, 103, 141, 142].

In alternative theoretical scenarios, the unconventional (i.e. non-superconducting) Cooper pairing can be expected in the low carrier concentration limit at $T = 2T_c$ in superconducting semiconductors [143] and in underdoped cuprates in a wide temperature range above T_c [53, 103, 142]. Such a Cooper pairing of fermionic quasiparticles (e.g., polarons) in the normal state of high-T_c cuprates can occur in the BCS regime and lead to the formation of a non-superconductivity-related pseudogap. Since this pseudogap appearing below T^* and the superconducting order parameter appearing below T_c may have different origins and coexist below T_c [103], and the high-T_c cuprate superconductors might be in the bosonic limit of Cooper pairs [82].

Attempts to understand the different pseudogap regimes in high-T_c cuprates have been based on the different temperature-doping phase diagrams showing only one pseudogap crossover temperature [73, 115, 131] or two pseudogap crossover temperatures [75, 135, 142, 144] above T_c and a QCP under the superconducting dome at $T = 0$ [73, 131, 142, 144]. However, a full description of the distinctive phase diagrams and the pseudogap, quantum critical and unusual superconducting states of these intricate materials in the different competing theories is still out of reach (see, e.g., question marks in the proposed phase diagrams of the cuprates [115, 119, 145]).

Many theoretical scenarios for high-T_c superconductivity are based on the BCS-like pairing correlations and on the usual Bose-Einstein condensation (BEC) of an ideal Bose- gas of Cooper pairs and other bosonic quasiparticles (e.g., bipolarons and holons). But the validity of the BCS-like and Migdal-Eliashberg theories of superconductivity in high-T_c cuprates and new high-T_c materials (e.g., in high-T_c hydrides [101, 106]) remains questionable. So far, most researchers have been misled that the BCS-like (s-, p- and d-wave) pairing theories are sufficient for understending of unconventional superconductivity and superfluidity in condensed matter systems. Other researchers believe that the transition from BCS-type condensation regime to BEC regime would occur in an interacting Fermi system with increasing the strength of attracting interaction between fermions or with decreasing the density of these fermions. Therefore, other attempts to understand the high-T_c superconductivity in the cuprates [56, 146, 147] have been based on the BCS-BEC crossover [84, 85, 143]. However, the usual BEC of an ideal Bose gas of real-space pairs and Cooper pairs is irrelevant to the superconductivity (or superfluidity) according to the Landau criterion [15]. It was also argued by Evans and Imry [148] that the superfluid phase in ^4He is not described by the presence of BEC in an ideal or a repulsive Bose gas of ^4He atoms.

Successful solutions of complex problems posed by high-T_c cuprate superconductors may provide new insights into the microscopic physics and thus contribute toward a complete understanding of unsolved problems of other unconventional superconductors and superfluids. The high-T_c cuprates are very similar to the superfluid ^4He and they may be superfluid Bose systems so that their superfluid properties cannot be understood within the BCS-like theories. Further, unconventional superconductivity in other systems and superfluidity both in ^3He and in atomic Fermi gases (with an extremely high superfluid transition temperature with respect to the Fermi temperature $T_F \simeq 5T_c$ [31]) cast a doubt on any BCS-like pairing theory as a theory of these phenomena. At present, there is no successful microscopic theory capable of describing all the pseudogap features and the mysterious superconducting/superfluid properties of high-T_c materials and other systems.

From the above considerations, it follows that the understanding of the new physics of high-T_c superconductors and other unconventional superconductors and superfluids requires a consistent and correct microscopic theory for describing not only the relevant pairing mechanisms of fermionic quasiparticles but also the relevant mechanisms of superconductivity and superfluidity in these systems.

In the following chapters, the well-founded and empirically adequate microscopic theory of novel pseudogap phenomena and unconventional (Bose-liquid) superconductivity and superfluidity in different condensed matter systems is presented. This theory explains essentially all the emerging pseudogap behaviors and unusual superconducting/superfluid states and properties of most condensed matter systems. Such a workable theory describes the formation of tightly-bound (bosonic) Cooper pairs above T_c and the subsequent condensation of such Cooper pairs into a Bose superfluid at T_c resulting in the appearing of the pseudogap state and the novel Bose-liquid superconductivity and superfluidity in high-T_c cuprates and other condensed matter systems (e.g., heavy-fermion and organic compounds, ruthenate Sr_2CuO_4, high-T_c hydrides, quantum liquids (^3He and ^4He), atomic Fermi gases and low-density nuclear matter). The Bose-liquid theory for unconventional superconductors and superfluids is capable of giving the new predictions and the full and adequate description of the emergent behavior found in the superconducting/superfluid state of high-T_c cuprates and other condensed matter systems.

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