Advanced Engineering Physics

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CHAPTER ONE

QUANTUM MECHANICS

1.1. Introduction

Till the end of the 18^{th} century, the growth of science and technology was based on Newtonian mechanics, the laws of thermodynamics, and Maxwell’s laws of electrodynamics, which fall under classical mechanics. The laws effectively illustrate the behavior of macroscopic entities, such as planets, stars, and objects of significant mass, size, or visibility. However, the same classical principles prove to be completely inadequate when attempting to account for the behavior of atoms and subatomic particles, including electrons and protons. In addition, it fails to provide explanations for various phenomena like the stability of an atom, discrete spectra of an atom, Raman effect, black body radiation, photoelectric effect, and Compton effect. During the early 20th century, a significant observation was made regarding the disparity between the behavior of macroscopic things and microscopic objects about governing rules. This observation subsequently prompted the emergence and advancement of quantum mechanics, currently recognized as the foundational theory of the natural world. The term "quantum" (plural: "quanta") originates from Latin and carries the connotation of "how much?" It pertains to the distinct entities of matter and energy that are both predicted and observed within the field of quantum physics. Even though space and time may seem to be continuous, they possess the smallest conceivable discrete values. The nomenclature "quantum mechanics" emerged as a result of the phenomenon of discreteness. Max Planck's seminal contribution to the field of quantum physics is commonly attributed to his 1900 work on black body radiation, which is considered a pivotal moment in the development of this scientific discipline. Renowned researchers such as Albert Einstein and Niels Bohr undertook subsequent breakthroughs in this specific area. Werner Heisenberg, Erwin Schrodinger, and numerous other scientists have made significant contributions to the field of quantum mechanics by employing Max Planck's quantum theory. This chapter attempts to project the overview of Planck's quantum hypothesis and its approach dealing with atomic particles.

1.1.1. Max Planck's Quantum Hypothesis

In 1900, Max Planck proposed a groundbreaking idea, commonly referred to as Planck's quantum theory, that effectively addresses the limitations inherent in classical theory. According to this theory:

- The emission of light occurs through the release of wave packets referred to as quanta.
- The emission of energy from molecules in the source occurs in discrete packets known as quanta, rather than in a continuous manner. The energy of quanta can be expressed as $E = nh\nu$, where n represents a positive integer (1, 2, 3...) known as the quantum number. The symbol 'h' denotes Planck's constant ($h = 6.632 \times 10^{-34}$ J-s) and ν represents the frequency of radiation. The oscillations exhibit a distinct frequency and propagate at the speed of light, denoted as c.

Einstein further developed Planck's quantum hypothesis, which proposes that radiant energy is released or absorbed discretely, taking the form of quanta. The packet of energy was assigned the new designation of "photon" by him. Hence, the quantization of radiant energy arises from the constraint that an atom can only occupy distinct energy levels, referred to as quantum states, characterized by a certain quantum number (n). Major differences between classical and quantum mechanics are tabulated in ***Table 1.1.***

Table 1.1 Classical mechanics versus Quantum mechanics.

S. No.	**Classical Mechanics**	**Quantum Mechanics**
1.	It applies to macroscopic bodies.	It applies to atomic particles.
2.	According to classical theory, light is considered to be an electromagnetic wave that is produced as a result of the acceleration of charged particles. When charges undergo oscillation at a constant frequency ν, they generate an electromagnetic wave with the corresponding frequency ν.	As per the principles of quantum physics, it is understood that light is emitted in the form of a stream of photons, whereby each photon possesses an energy denoted by the equation $E = h\nu$.

S. No.	Classical Mechanics	Quantum Mechanics
3.	The Energy of the wave is a function of amplitude.	The energy of a photon varies depending on the frequency (ν) of the light radiation.
4.	Radiant energy can be received or emitted continuously or randomly.	The radiant energy is absorbed /emitted in discrete units.

1.2. Properties of Photon

(i) **Energy:** The energy of a photon depends according to its frequency, as described by the equation E = hν.

(ii) **Velocity:** Similar to electromagnetic waves, photons constantly propagate at the constant velocity of light, denoted as 'c'.

(iii) **Mass:** The rest mass of the photon is zero ($m_0 = 0$). Since a photon is incapable of being at rest and travels at the speed of light, denoted as 'c', its relativistic mass can be expressed as follows:

$$E = mc^2 \Rightarrow m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{h}{c^2} \times \frac{c}{\lambda} = \frac{h}{c\lambda} \qquad \text{---------} \ (1.1)$$

(iv) **Linear momentum:** Photons possess linear momentum, given by

$$P = mv = mc$$

substituting the mass of photon from eq.(1.1), we get:

$$P = \frac{h}{c\lambda} \cdot c = \frac{h}{\lambda} \qquad \text{---------} \ (1.2)$$

(v) **Nature:** Photons are electrically neutral.

(vi) Photons are neither deflected by an electric field nor a magnetic field.

(vii) Photons do not ionize matter.

(viii) Photons can be created or destroyed when radiation is emitted or absorbed, respectively.

(ix) Photons can have particle-like collisions with other particles, such as electrons.

(x) Like other particles they carry linear momentum and energy.

1.3. Compton Effect

The Compton Effect is a significant experiment in the field of quantum mechanics, typically observed within the X-ray or γ-ray portion of the electromagnetic spectrum. The phenomenon was initially seen and documented by Arthur Holly Compton in the year 1923, ultimately leading to his recognition and receipt of the Nobel Prize in 1927.

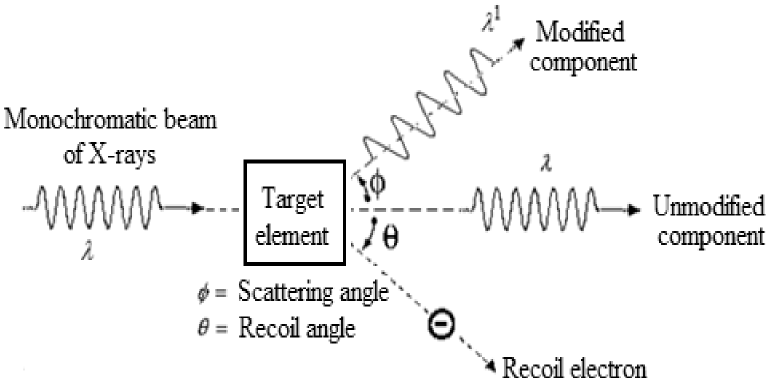

Fig.1.1: Illustration of Compton effect.

Based on this phenomenon, when a monochromatic beam of high-frequency radiation, such as X-rays or γ-rays, interacts with a target element, such as graphite or an element with a higher atomic number, the resulting scattered radiation splits into two components: the modified component and the unmodified component. Additionally, recoiled electrons are also observed, as depicted in ***Fig. 1.1.*** The unmodified component (λ) refers to the component that has the same wavelength as the incident wavelength, whereas the modified component *(*λ^1***)*** refers to the component that has a wavelength greater than that of the incident wavelength. This phenomenon is commonly referred to as the Compton effect. The difference in wavelength between the modified and unmodified components is known as the ***Compton shift***, denoted by $\Delta \lambda = \lambda^1 - \lambda$. Compton shift was found to only depend on the scattering angle ϕ and is independent of the nature of the target and incident wavelength.

1.3.1. Experimental Arrangement of Compton Effect

The following assumptions were made by Compton while experimenting –

- The interaction between X-rays and electrons has a striking resemblance to the collision characteristics observed between two material particles.
- The electrons present in the scattering material are regarded as free due to the very negligible kinetic energy possessed by the valence electrons within an atom, in comparison to the energy carried by incident X-ray photons. Despite the electrons being confined to the nucleus, a minute amount of energy, denoted as hν, is necessary to liberate the electron. This work function is considered to be insignificantly small in comparison to the energy possessed by the incident photon. Consequently, the electrons are regarded as effectively free.
- Free electrons are initially at rest. When a photon collides with a stationary free electron, a part of the photon energy is transferred to the electron. On account of this, the electron gets recoiled and the photon scatters with energy less than that of the incident photon. The energy acquired by the electron after the collision is tremendous hence, its initial energy can be theoretically ignored and considered to be zero and thus at rest.
- Two-body collision is elastic i.e., the principle of conservation of energy and momentum applies to the collision between an electron and a photon.

1.3.1.1. Experimental Set-Up

The experimental setup for the Compton Effect is illustrated in ***Fig. 1.2.*** A monochromatic X-ray beam is emitted from the X-ray source and subsequently directed via the collimators to produce a narrow beam of photons. When these X-rays strike the target element, they are diffracted (scattered) by the crystal in different directions. Bragg's X-ray spectrometer with an ionization chamber acts as a movable detector, which measures the intensity of the scattered beam in different directions. The measurements of the scattered X-ray wavelengths and their related intensities were recorded as a dependent variable of the scattering angle ϕ.

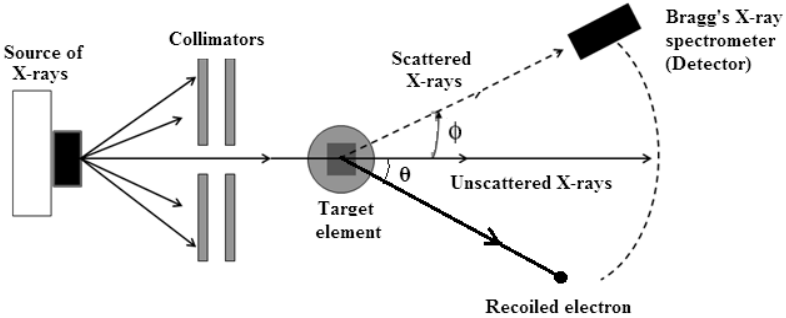

Fig.1.2: Experimental setup of Compton effect.

1.3.1.2. Experimental Results

Graphs depicting the relationship between intensity (I) and wavelength (λ) of scattered X-rays are presented in ***Fig. 1.3***, for various values of ϕ.

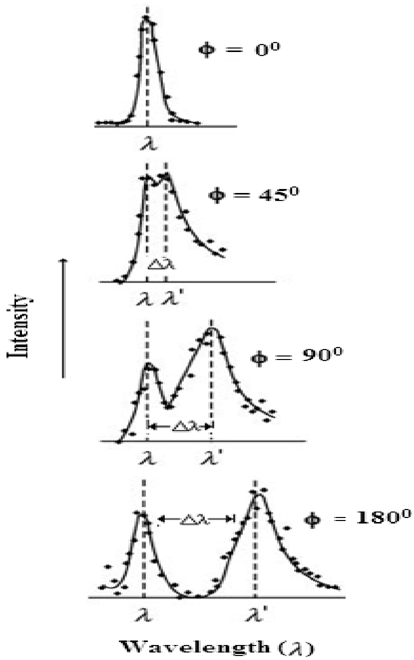

Fig 1.3: Variation of Compton shift with scattering angle ϕ

The following are the conclusions drawn from this experiment:

- The incident X-rays have a single wavelength (λ), while the scattered radiation has two wavelengths (λ and λ^1). The component with a wavelength same as that of the incident wavelength is known as the unmodified component (λ) and the component with a wavelength greater than that of the incident wavelength is known as the modified component (λ^1).
- The position of the unmodified component (λ) remains the same while the position of the modified component (λ^1) changes with the scattering angle 'ϕ'.
- The difference in wavelength between the modified and unmodified components is known as the Compton shift, denoted by $\Delta\lambda = \lambda^1 - \lambda$.
- Compton shift (Δλ) increases with the scattering angle ϕ according to the relation.

$$\Delta\lambda = \frac{h}{m_0 c}(1 - \cos\phi)$$

- Compton shift (Δλ) varies from 0 to $\frac{2h}{m_0 c}$ when ϕ is varied from 0^0 to 180^0.
- The intensity of scattered radiation (modified and unmodified) depends on the target element.

1.3.2. Why Modified and Unmodified Components?

Any target element consists of both valence and bound electrons. Out of a large number of photons incident on the scattering material, some photons collide with valence or free electrons, while others collide with the bound electrons.

Case 1: When a Photon Collides with a Valence Electron

When the incident photon interacts with a free or weakly bound electron, the entire energy of the photon is transmitted to the electron. The electron utilizes this energy to liberate itself. However, it cannot travel with the remaining energy, as else it will be moving with speed more than the speed of light which is not possible. Therefore, it releases energy in the form of photons and experiences recoil throughout the emission process. The photon that is emitted undergoes scattering with reduced energy, resulting in a greater wavelength compared to the incident photon, as depicted in ***Fig.1.4***.

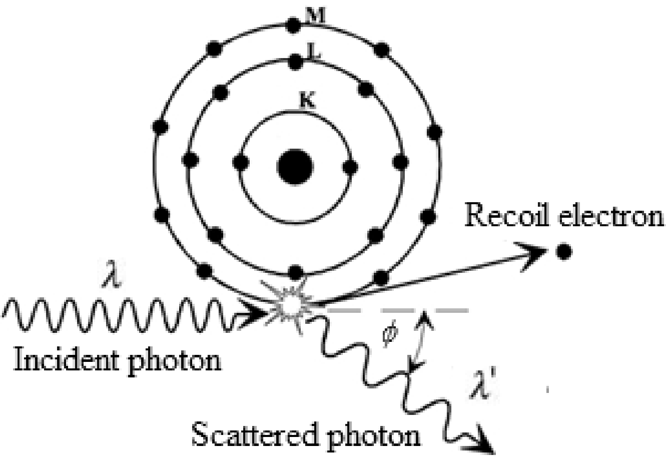

Fig.1.4: Illustration of modified component.

The Compton shift in this case is given by:

$$\Delta\lambda = \frac{h}{m_0 c}(1 - \cos\phi)$$

As the m_o of the electron is less, $\lambda^1 - \lambda \neq 0$. Thus, the scattered wavelength is not the same as that of the incident wavelength ($\lambda^1 \neq \lambda$). An alternative approach to the clarification of the cause of modified components is by the relation $\lambda = \frac{hc}{E}$, which states that when the energy of a scattered photon diminishes, the wavelength corresponding to such a scattered photon must exceed that of the input photon. Therefore, the modified component (λ^1) is produced as a result of a photon colliding with an unbound electron.

Case 2: When a Photon Collides with a Bound/Inner Shell Electron

When the incident photons strike the ***bound electron*** in an ionic core of the scattering block, electrons cannot recoil as they are tightly bound to the nucleus (i.e., the binding energy is more) and hence, atom as a whole is involved in the process of collision. The incident photon does not impart energy or momentum to the atom, resulting in the scattered photon having an identical wavelength to that of the input photon. In the given circumstances, it is necessary to substitute the rest mass of the electron (m_0) with the mass of the atom (M). Therefore, the equation for the Compton shift can be simplified as:

$$\Delta\lambda = \lambda^1 - \lambda = \frac{h}{M\,c}(1 - \cos\theta) \;,$$

As M >> m_o, hence, $\Delta\lambda = \lambda^1 - \lambda = 0 \Rightarrow \lambda^1 = \lambda$

Therefore, unmodified component λ arises when a photon strikes bound electrons.

1.3.3. Which Component is Intense? (The Modified or the Unmodified)

The intensity of modified and unmodified components depends on the target element considered i.e., whether it is a lower atomic number element or higher atomic number element. Let us study the following two cases:

Case 1: Lower Atomic Number Elements

If the target element is a lower atomic number element (such as Lithium or Carbon), then almost all the electrons are valence electrons which can be considered to be free. When X-rays interact with these free electrons they can be easily detached from the atom. In this scenario, a portion of the energy carried by the input photon is utilized by the electron to achieve liberation, while the remaining energy is released as a scattered photon. Consequently, the electron experiences a recoil effect after the emission process. The changed component will exhibit greater intensity due to the higher abundance of free electrons in comparison to bound electrons.

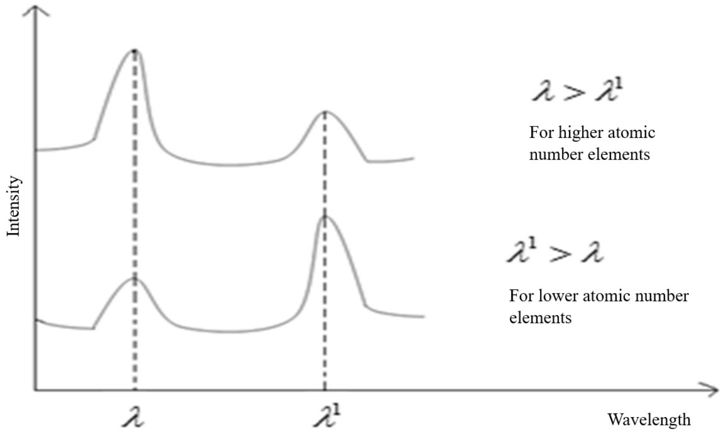

Fig.1.5: Modified and unmodified components in various target elements.

Case 2: Higher Atomic Number Elements

In the case of heavy elements or higher atomic number elements (such as Gold or Chromium), the bound electrons can be considered to be more as compared to valence electrons. When X-rays collide with these bound electrons, then the interaction of the photon takes place with the whole atom. In this instance, it is observed that the photon is unable to impart a fraction of its energy to the electrons to induce recoil. Consequently, the Compton shift becomes insignificant, resulting in no change in the wavelength. However, as the number of bound electrons is more as compared to the free electrons, the unmodified component will be more intense as shown in ***Fig.1.5.*** Thus, it can be concluded that the modified component (λ^1) is more intense in case of lower atomic number elements as compared to higher atomic number elements.

1.3.4. The Compton Theory: Quantitative Explanation

According to Compton's explanation, the interaction between particles of light (X-ray photons) and electrons in the target material is just like a billiard ball collision. Let us consider a single photon-electron collision quantitatively as shown in ***Fig.1.6***. When a photon with a wavelength λ interacts with a stationary electron, the photon undergoes scattering at an angle ϕ and its wavelength increases to λ^1. Simultaneously, the electron recoils with a velocity v at an angle θ in a distinct direction. In the context of an elastic collision, it is imperative to uphold the principles of energy and momentum conservation for both the electron and photon involved, both before and after the collision event. These fundamental parameters are duly demonstrated in **Table 1.2**.

Table 1.2: Energy and momentum of electron and photon before and after collision.

Quantity	Particle	Before Collision	After Collision
Energy	Photon	$h\nu$ or $\frac{hc}{\lambda}$	$h\nu^1$ or $\frac{hc}{\lambda^1}$
	Electron	m_0c^2	mc^2
Momentum	Photon	$\frac{h}{\lambda}$	$\frac{h}{\lambda^1}$
	Electron	0 (Rest)	Mv

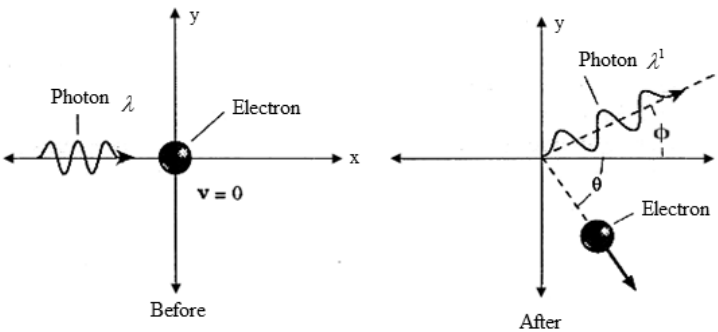

Fig.1.6: Geometry of Compton scattering before and after collision.

In the context of an elastic collision, it is observed that both the law of energy conservation and the law of momentum conservation hold for the system. Following the principle of energy conservation, the combined energy of the system comprising the electron and photon before their collision is equivalent to the combined energy of the system after the collision. By utilizing the principle of the conservation of energy, we can get the following equation for the system.

$$\text{Energy of the system before collision} = \text{Energy of the system after collision}$$

$$h\nu + m_oc^2 = h\nu^1 + mc^2 \quad \text{---------} (1.3)$$

Let m_0 represent the rest mass of the electron, and m denotes its relativistic mass. To write m in terms of m_0, let us consider the relation between two masses:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad \text{--------- (1.4)}$$

Squaring, eq. (1.4), we get:

$$m^2 = \frac{m_0{}^2 c^2}{c^2 - v^2}$$

$$m^2c^2 - m^2v^2 = m_0^2c^2$$

Multiplying throughout the equation by c^2, we get:

$$m^2c^4 - m^2v^2c^2 = m_0^2c^4$$

$$\Rightarrow m^2c^4 = m_0^2c^4 + m^2v^2c^2$$

$$E = \sqrt{m_0{}^2c^4 + p^2c^2} \qquad \left(\because E = mc^2 \quad \text{and} \quad P = mv\right) \qquad \text{--------- (1.5)}$$

Substituting eq. (1.5) in eq. (1.3), we get:

$$h\nu + m_0c^2 = h\nu^1 + \sqrt{m_0{}^2c^4 + p^2c^2} \qquad \text{--------- (1.6)}$$

Given that momentum is a vector quantity, we shall examine the x and y components of the particles' momentum before and following the impact. By utilizing the principle of the conservation of momentum, we may analyse this collision in the x-direction.

$$\frac{h}{\lambda} + 0 = \frac{h}{\lambda^1}\cos\phi + mv\cos\theta \qquad \text{--------- (1.7)}$$

By utilizing the principle of the conservation of momentum in the y-direction, we can get the following equation:

$$0 + 0 - \frac{h}{\lambda^1}\sin\phi - mv\sin\theta \qquad \text{--------- (1.8)}$$

We aim to find the Compton shift Δλ caused by the scattered photons, so among the five variables λ, λ^1, ϕ, θ, v that appear in eq. (1.6), (1.7), and (1.8), let us eliminate θ, v, which deals solely with electrons. Solving these equations leads to a simple result as given below:

$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos\phi) \qquad \text{--------- (1.9)}$$

Eq. (1.9) shows that the Compton shift.

i) The equation does not include the incidence wavelength 'λ', hence Compton shift is independent of the incident wavelength established.
ii) The equation does not include the atomic number, thereby making Compton shift independent of the nature of the scattering substance.
iii) The Compton shift is entirely determined by the scattering angle (ϕ).

Case (I): If ϕ = 0^0, then Δλ= 0 i.e. change in wavelength is zero. Hence, Compton shift is minimum at ϕ = 0^0

Case (II): If ϕ = 90^0, then Δλ is equal to Compton wavelength λ_c, given by

$$\Delta\lambda = \frac{h}{m_0 c} = \lambda_C = \frac{6.626\times10^{-34}(J.s)}{9.1\times10^{-31}(kg)\times3\times10^{8}(m/s)} = 0.02427\text{Å}$$

The numerical value of Compton shift Δλ for ϕ = 90^0 is called Compton wavelength (λ_c).

Case (III): If ϕ = 180^0, then Δλ can be found as follows:

$$\Delta\lambda = \frac{2h}{m_0 c} = \frac{2\times6.626\times10^{-34}(J.s)}{9.1\times10^{-31}(kg)\times3\times10^{8}(m/s)} = 0.04854\text{Å}$$

In this case, Compton shift is maximum. Thus, the Compton shift changes from 0 to 0.04854 Å when the value of the scattering angle ϕ changes from 0^0 to 180^0.

1.3.4.1. The Kinetic Energy of the Recoiled Electron

The electron recoils with a certain energy after absorbing the energy from the incident photon. Since the collision is treated as elastic;

$E_{\text{recoil electron.}}$ = energy of incident photon – the energy of the scattered photon

$$E = h\nu - h\nu^1 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda^1}\right) = \frac{hc\Delta\lambda}{\lambda\lambda^1} \qquad \text{where} \quad \Delta\lambda = \lambda^1 - \lambda$$

--------- (1.10)

Eq. (1.10) represents the required expression for the kinetic energy of the recoiled electron.

1.3.4.2. Velocity of the Recoiled Electron

The velocity of the recoil electron can be determined by utilizing the expression as follows.

$$E = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2E}{m}}$$ --------- (1.11)

1.3.5. Compton Shift cannot be Experimentally Observed in Visible Light

The statement ***"Compton shift cannot be experimentally observed in visible light"*** can be proved by calculating the percentage shift observed in X-rays and Visible radiation as follows:

Case (I): Compton shift in visible light:

$$(\%\ \text{Shift})_{Vis} = \frac{\Delta\lambda}{\lambda_{Vis}} \times 100 = \frac{0.02427\ \text{Å}}{5500\ \text{Å}} \times 100 = 4.4 \times 10^{-4}\ \% \approx \text{negligible}$$

The percentage shift is negligible and hence, the Compton shift cannot be detected.

Case (II): Compton shift in X-rays:

$$(\%\,\text{Shift})_{X-rays} = \frac{\Delta\lambda}{\lambda_{X-rays}} \times 100 = \frac{0.02427\,\text{Å}}{1\,\text{Å}} \times 100 = 2.427\,\%$$

The percentage shift is appreciable and hence, the Compton shift can be detected. Thus, Compton shift cannot be observed experimentally in visible light but can be observed with X-rays as their wavelength is small. Generally, the Compton shift will be of the order of 10^{-2} Å.

1.3.6. Explanation of Compton Effect Based on Quantum Theory

Compton utilized Planck's quantum hypothesis and Einstein's theory to explain his findings from experiments. He considered that the incoming X-ray beam not only behaves as a wave, but also as an assembly of photons of energy $E = h\nu$, carrying momentum $P = h/\lambda$. Hence, the interaction between photon and electron in a material can be treated as an elastic collision, which obeys the law of conservation of energy and momentum. Photons experience billiard ball-like collisions with the free electrons in the scattering target. Thus, when a photon encounters a stationary free electron, part of the incident photon momentum will be imparted to the electron, which thereby gains some kinetic energy. To conserve total energy in this process, the incident photon transfers some of its energy to the electron and hence, the wavelength of the scattered photon must be greater than that of the incident photon. When a photon encounters a bound electron, which is tightly bound to the nucleus, the atom as a whole is recoiled in the process of collision and hence, the scattered photon has the same wavelength as that of the incident photon.

1.3.7. Failures of Classical Theory to Explain Compton Effect

X-rays, as per classical wave theory, are electromagnetic waves that possess a specific wavelength (λ) or frequency (ν). When these waves strike the target element, the electrons in the target element oscillate with the same frequency (ν) and re-radiate electromagnetic waves in all directions at the same frequency or wavelength as that of the incident radiation as pictorially represented in ***Fig.1.7.*** Thus, classical theory can explain the existence of unmodified wavelength successfully, but fails to explain the existence of modified wavelength in the scattered beam.

Fig.1.7: Classical representation of Compton effect.

Further, the electrons in the material radiate waves uniformly in all directions, and hence the wavelength of the scattered radiation should not depend on the scattering angle ϕ. According to the wave theory, (i) the scattered radiation should have the same wavelength as the incident radiation, and (ii) the wavelength of the scattered radiation should not depend on the scattering angle (ϕ). Thus, the presence of scattered wavelength cannot be understood if the incident X-rays are regarded as electromagnetic waves. So, the Compton shift and hence Compton Effect is a purely quantum effect, that is not expected to occur based on classical physics.

1.3.8. A Free Electron Cannot Absorb a Photon

In the Compton Effect, the interaction of photons is with the free electrons and hence there occurs only a partial transfer of photon energy to the free electrons. This interaction should also obey the law of conservation of energy and momentum. Let us assume that a stationary free electron can absorb an incident photon completely as shown in ***Fig.1.8***. Energy and momentum of electron and photon before and after collision is given in ***Table 1.3***. Applying the law of conservation of energy:

$$hv + m_o c^2 = 0 + mc^2 \quad \text{---------} (1.12)$$

Substituting the value of m in terms of m_0 in eq. 1.12, we get:

$$hv + m_o c^2 = 0 + \sqrt{m_0{}^2 c^4 + p^2 c^2} \quad \text{---------} (1.13)$$

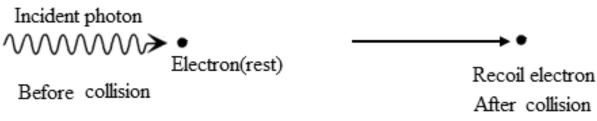

Fig.1.8: A free electron absorbing a complete photon.

Table 1.3: Energy and momentum of electron and photon before and after collision.

Quantity	Particle	Before Collision	After Collision
Energy	Photon	hν	0
	Electron	m_0c^2	mc^2
Momentum	Photon	$\frac{h\nu}{c}$	0
	Electron	0	mv

Applying the law of conservation of momentum to this system, we get:

$$\frac{hv}{c} + 0 = 0 + mv \Rightarrow hv = cp \qquad (\because mv = p)$$ --------- (1.14)

Substituting (1.14) in (1.13), we get:

$$pc + m_o c^2 = \sqrt{m_0{}^2 c^4 + p^2 c^2}$$ --------- (1.15)

Squaring on both sides, we get:

$$p^2c^2 + m_o{}^2c^4 + 2\ pc\ m\ _oc^2 = m_o{}^2c^4 + p^2c^2$$ --------- (1.16)

$$\Rightarrow 2\ pc\ m_0c^2 = 0$$

Substituting (1.14) in (1.16), we get:

$$2\ hv\ m_0c^2 = 0$$

This result implies that either hv =0 (or) m_oc^2 =0, which is impossible. Thus, our earlier assumption that a free electron can absorb a complete photon is wrong. Hence, we can conclude that a free electron cannot absorb a complete photon.

1.3.8.1. Scientific Merits of the Compton Effect

- Proved that photon description applies not only to visible light but also to X-rays and γ-rays.
- Proved that photons carry energy and momentum like other material particles.

- The Compton Effect provides compelling evidence that electromagnetic waves also exhibit particle-like characteristics.

1.3.8.2. Limitations of the Compton Effect

- Compton shift is observed only in the X-ray and γ-ray region of the EM spectrum.
- The Compton Effect can be explained only based on quantum theory.

1.4. De Broglie's Hypothesis

In 1924 Louis de Broglie, a French physicist was the first to state that, like radiation, matter also exhibits dual nature i.e. sometimes matter behaves as a wave and sometimes as a particle, but not both simultaneously. He put forth a hypothesis regarding the wave nature of matter called as de Broglie hypothesis.

1.4.1. Wave-Particle Duality

Light exhibits the phenomenon of interference, polarization, diffraction, reflection, refraction, scattering, Photoelectric Effect, Compton Effect, Raman Effect, etc. A phenomenon like polarization, diffraction, and interference can be explained only based on the wave theory of light in which light possesses wave nature. On the other hand, phenomena like the Photoelectric Effect, Compton Effect, and Raman Effect cannot be explained based on the wave theory of light but can be explained only based on the quantum theory of light in which light is assumed to possess particle nature. This indicates that light possesses a dual nature known as wave-particle duality.

It was observed that at lower frequencies, wave nature dominates and at higher frequencies particle nature dominates. This dualism between wave nature and particle nature can be justified based on the frequency of the radiation as follows: At low frequencies; the radiation has a very large wavelength i.e. it occupies a large region in the space as shown in ***Fig.1.9***. Hence, at these frequencies; the wave nature of radiation is dominant. In the EM spectrum, the spatial spread of waves decreases to an enormous extent with an increase in frequency. At higher frequencies; waves have very small wavelengths and hence they are crowded over a small region of space, where this can be assumed to resemble a particle. Therefore, at high frequencies, the particle nature prevails. The visible portion of the electromagnetic spectrum is the transitional region where radiation can be observed to exhibit either particle-like or wave-like behavior.

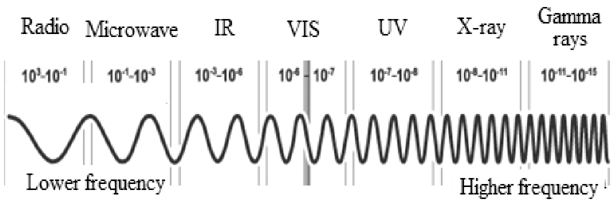

Fig.1.9: Spatial spread of electromagnetic waves.

To understand the wave-particle duality, it is necessary to know what is a particle. and what is a wave? A particle is characterized by mass(m), velocity(v) and hence momentum ($p=mv$), energy ($E=1/2mv^2$), a definite position in space, etc., while a wave is characterized by frequency(ν), wavelength (λ), Phase (φ), amplitude (a), Intensity (I) and has no definite position. By considering the above facts, it appears difficult to accept the conflicting ideas that radiation has a dual nature. But it is to be accepted because, like radiation, matter also exhibits a dual nature, depending on the circumstances. Let us examine the two equations:

$$P = \frac{h}{\lambda} \qquad\qquad E = h\nu$$

The parameters on the left-hand side are related to a particle while the parameters on the right-hand side are related to a wave i.e. the wave and particle nature of matter are related to each other by Planck's constant.

1.4.2. Matter Wave

According to de Broglie, every moving particle is associated with a wave known as de Broglie wave or matter wave, characterized by the wavelength given by,

$$\lambda = \frac{h}{mv} = \frac{h}{p} \qquad \text{--------- (1.17)}$$

Where h is Planck's constant, p is the momentum of a particle, m is mass and v is the velocity of the particle associated with the wave.

1.4.2.1. Proof

De Broglie derived the correlation between particles and waves by utilizing Planck's quantum theory of radiation and Einstein's mass-energy equivalence. According to Planck's theory, the energy of a photon is given by:

$$E = h\nu = \frac{hc}{\lambda} \qquad \text{---------} \ (1.18)$$

Einstein's mass-energy relation is given by:

$$E = mc^2 \qquad \text{---------} \ (1.19)$$

Equating (1.18) and (1.19), we get:

$$\frac{hc}{\lambda} = mc^2 \Rightarrow \lambda = \frac{h}{mc} = \frac{h}{mv} = \frac{h}{p} \qquad \text{---------} \ (1.20)$$

Eq. (1.20) represents the de Broglie wavelength in terms of velocity. This relation is now universally applicable to photons as well as to any other material particles like electrons, α-particle, etc.

1.4.3. De Broglie Wavelength in Terms of Kinetic Energy

Consider a particle of mass m, moving with velocity v. Its momentum is p = mv and the de Broglie wavelength associated with it is:

$$\lambda = \frac{h}{p}$$

The kinetic energy of the particle is given by:

$$E = \frac{1}{2}mv^2 = \frac{1}{2m}m^2v^2 = \frac{1}{2m}p^2 \Rightarrow p = \sqrt{2mE} \qquad \text{---------} \ (1.21)$$

Substituting eq. (1.21) in eq. (1.20), we get:

$$\lambda = \frac{h}{\sqrt{2mE}} \qquad \text{---------} \ (1.22)$$

1.4.4. De Broglie Wavelength in Terms of Accelerating Potential

Consider a charged particle, subjected to an electric field, accelerated through an electric potential difference V. It acquires kinetic energy E, and hence velocity v, given by:

$$E = \frac{1}{2}mv^2 = eV \Rightarrow \frac{1}{2m}m^2v^2 = eV \Rightarrow \frac{p^2}{2m} = eV$$

$$\Rightarrow p = \sqrt{2meV} \qquad \text{--------- (1.23)}$$

Substituting the value of p from eq. (1.23) in the eq. (1.20), the expression for the de Broglie wavelength of a particle in terms of accelerating potential V reduces to:

$$\lambda = \frac{h}{\sqrt{2meV}} \qquad \text{--------- (1.24)}$$

Substituting the values of for an electron as h = 6.626 x10^{-34} J-s, m= 9.1x 10^{-31} kg, e = 1.609 x10^{-19}C, eq. (1.24) can be written as:

$$\lambda = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times (9.1 \times 10^{-31}) \times (1.6 \times 10^{-19})}} \times \frac{1}{\sqrt{V}}\ m$$

$$= \sqrt{\left(\frac{150}{V}\right)}\ \text{Å} = \frac{12.24}{\sqrt{V}}\ \text{Å} \qquad \text{-------- (1.25)}$$

1.4.5. Properties of Matter Waves

- The wavelength is inversely proportional to the mass of a particle.
- The wavelength is inversely proportional to the velocity and thus, momentum of a particle.
- The wavelength is independent of the charge of the particle; this proves that the matter waves are not electromagnetic.
- The velocity of matter waves is variable and depends upon the velocity of the associated matter particle.
- The lighter the particle, the greater the wavelength associated with it.
- Matter exhibits a dual nature.
- The wave nature of matter introduces an uncertainty in the position of the particle.
- The wavelength of a matter wave is inversely proportional to the kinetic energy of the particle.

- The frequency of matter waves is directly proportional to the kinetic energy of the particle.
- The velocity of matter wave is greater than the velocity of light.

1.4.5.1. Proof:

Wave velocity $(w) = \nu\lambda$

On equating the eqs. $E = h\nu$ and $E = mc^2$, we get $h\nu = mc^2$

$$\Rightarrow \nu = \frac{mc^2}{h}$$

$$\therefore \text{Wave velocity} = \nu\lambda = \frac{mc^2}{h} \times \frac{h}{mv} = \frac{c^2}{v} \Rightarrow w.v = c^2$$

The velocity of the particle (v) cannot exceed the velocity of light (c), hence, wave velocity *w* is greater than the velocity of light, c.

Differences and Similarities between Matter Wave and Electromagnetic Wave

Table 1.4: Differences between matter wave and electromagnetic wave.

S. No	Matter-Wave	Electromagnetic Wave
1	It is generated by any moving particle, whether charged or uncharged.	It is generated by only charged particles, oscillating in the electric and magnetic fields.
2	The wavelengths of these waves are determined by the mass and velocity of the moving particle, according to the equation $\lambda = h/mv$.	The wavelengths of these waves are determined by the frequency of oscillation of charged particles, according to the equation $\lambda = c/\nu$
3	The velocity of the waves depends on the velocity of the particle.	The velocity of the waves remains constant.
4	The velocity of matter wave is greater than the velocity of light.	The velocity of electromagnetic waves is equal to the velocity of light.
5	The wave function does not have any physical significance.	The wave function has physical significance as it reveals the complete information about the waves.