

Surveying for Engineers

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By

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INTRODUCTION

This basic book was prepared in support of a low-level civil engineering course the author taught at the university for years.

The idea is that construction projects require the participation of engineers, surveyors, and contractors. Whether it is a building, a bridge or a pipeline, customers have the expectation that the result is going to be precisely at the right location, orientation, slope or level, and elevation.

For that, engineers, surveyors, and contractors will have to interact in such a way that those expectations are met. The knowledge of specific basic words, concepts, and methods used by surveyors will greatly help engineers during this interaction.

This interaction can involve the following:

- The initial interpretation and/or preparation of survey maps and descriptions of parcels of land by the surveyor.
- The design of project detail by the engineer so that it fits those parcels or land.
- The establishment of a project reference system on the ground by the surveyor.
- The positioning of project detail within that system by an engineer, surveyor, or contractor.
- The building of project detail by contractors.
- The checking of contractors' work by an engineer or surveyor ("as-built" check).

The above-mentioned concepts, methods, and language, as presented in this book, will include the proper use of equipment, devices, methods, and procedures. Part of this effort will be to present how to collect field data with automatic levels and total stations, how to check the data, how to adjust them, and how to report them. This will help the reader recognize the purpose of these toolsets, to recognize sources of error, to evaluate measurement accuracies, and to recognize the purpose of methods. To

facilitate this, various toolsets (like tables) are given to the reader to use professionally.

Also presented is the following:

- Using levels to determine elevations, running level loops, assessing their accuracy, and calculating adjustments.
- Using total stations to measure the horizontal and vertical angles and distances that are part of a traverse and checking for errors and adjusting them.
- Using and converting between directions, angles, azimuths, and bearings.
- Calculating coordinates.
- Solving surveying problems from radial survey data.
- Calculating horizontal and vertical curves.
- Calculating areas and volumes from observations.
- Learn methods used in the construction surveys for building, pipeline, and road layouts.

CHAPTER ONE

LEVELING

1.1 Introduction

For starters, one must be clear about the meaning of certain words, as follows:

- **Plumb line:** This is a line that follows the direction of local gravity (normally assumed “vertical” and “straight”, even if it is not).
- **Horizontal or level plane:** A plane that is perpendicular to a local plumb line.
- **Horizontal line:** A line within a horizontal plane.
- **Datum:** A point from which coordinates are counted, like a zero coordinate or elevation, also called “origin”.
- **Horizontal datum:** A point holding a horizontal positioning grid, normally expressed in X, Y coordinates, such as 0,0.
- **Vertical datum:** A horizontal plane that has an elevation of zero (0.000)
- **Mean sea level:** A vertical datum based on a tidal gauge.
- **Geoid:** A vertical datum based on a specific gravitational potential (the basis for modern elevations).
- **Elevation:** The vertical distance along a plumbline from a vertical datum to a specific horizontal plane.
- **Vertical control:** A point with known elevation.
- **Benchmark (BM):** A permanent vertical control point having a published elevation (and maybe a horizontal position also).

The process of “leveling” is a procedure to determine the difference in elevation between points with the help of an instrument called “level”.

The purposes of leveling can be various:

- Determine the vertical shape of the land prior to designing construction projects (elevations, profiles, contour maps)
- Shape a project vertically according to design elevations (excavating or filling)
- Collecting information for the calculation of volumes for cut and fill operations (excavation, reclamation, etc.)
- Determine surface or subsurface drainage patterns

Leveling only works if someone has determined the elevation of a reference point (called “benchmark” because these markers often have the shape of a small bench that is inserted into a wall). Communities worldwide provide these benchmarks in support of local projects. The U.S. Federal Government is a principal source for nationwide reference systems that historically included the following:

- NGVD29 (National Geodetic Vertical Datum of 1929) is the first vertical datum established throughout the US with the help of mean sea levels.
- NAVD88 (North American Vertical Datum of 1988) is the first vertical datum established throughout the US with the help of worldwide GPS.
- GEOID03 is a vertical datum established with gravimeters.

1.2 Equipment for Leveling

At present, the equipment used for leveling consists of the following: An automatic level on a tripod (Figure 1-1) and a leveling rod that is extended (Figure 1-2)



Figure 1-1



Figure 1-2

The automatic level is called that because if it is close to being level (according to a built-in “bullseye” leveling bubble shown in Figure 1-3) the equipment finishes the leveling by itself (when half the bubble is within the circle).

The leveling rod reads zero feet at its bottom (where it touches the ground).

If one points the level to the rod, then the horizontal center crosshair that is inside its telescope is used to collect a reading. Figure 1-4 shows a reading of 5.45 feet.

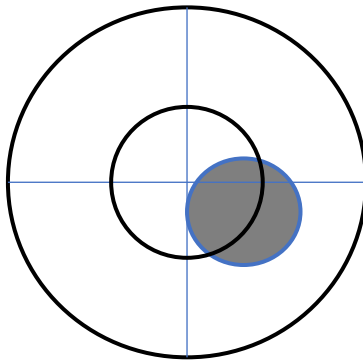


Figure 1-3

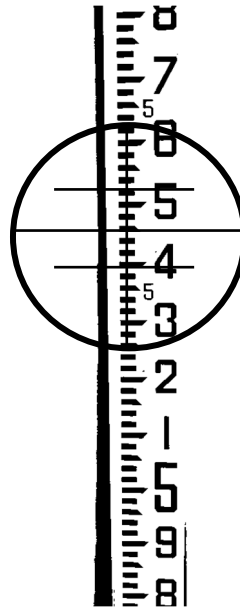


Figure 1-4

The four lines in the field of view of the telescope (as seen in Figure 1-4) are crosshairs that have various purposes:

1. The vertical crosshair is used for horizontal aiming
2. The center horizontal cross hair is used for most leveling work
3. The two other, shorter crosshairs can also be read to increase accuracy when the three readings are averaged. This also helps detect reading blunders.

4. If the bottom reading is subtracted from the top reading, and the difference is multiplied by 100, then the result is the distance in feet between the instrument and the rod. This is called “Stadia Surveying”.

1.3 The Procedure

The process of collecting consecutive readings is called “differential leveling” that is illustrated in Figure 1-5.

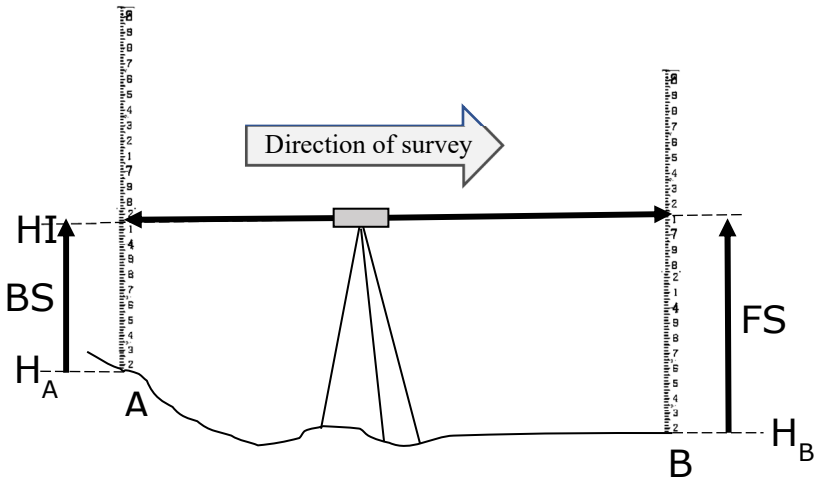


Figure 1-5

Imagine that one knows the elevation H_A of point A. One places the rod on that point and holds it vertically. Then one sets up the level at a convenient location, levels it, and looks back toward point A. One reads the value on the rod and records this “backsight” (BS). The “height of the instrument” HI (an elevation) is now known as $HI = H_A + BS$.

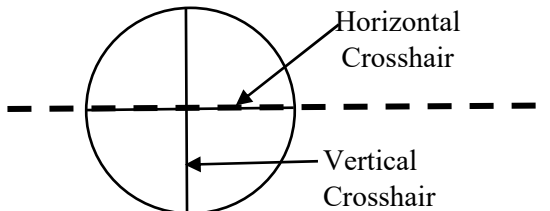


Figure 1-6

When the level is set up and “leveled”, its horizontal crosshair (as seen through the telescope’s eyepiece) and the telescope’s axis are located within a horizontal plane (dashed line). See Figure 1-6.

After collecting the backsight, one takes the rod to the desired point B, turns the level to it, and records that reading (“foresight” or FS).

Note that:

- The level establishes a horizontal plane that hits both rods.
- The rod is vertical at each point.
- The elevation of point A is “HA” (“height of A”).
- The “height of instrument” HI is $HA + BS$ (“backsight”).
- The elevation HB of point B is $HI - FS$.
- Or, $HB = HA + BS - FS$

This can be repeated along a string of points (a “traverse” as shown in Figure 1-7 below) with the purpose of transporting an elevation along a distance.

Also, after collecting a string of points one can return to the starting point BM1 as shown in Figure 1-8. This is then called a “closed traverse”.

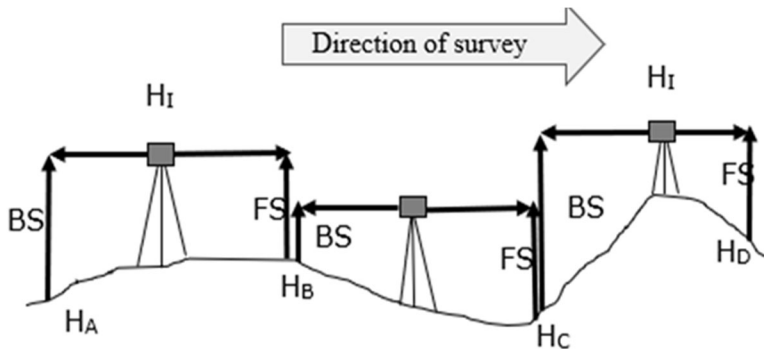


Figure 1-7

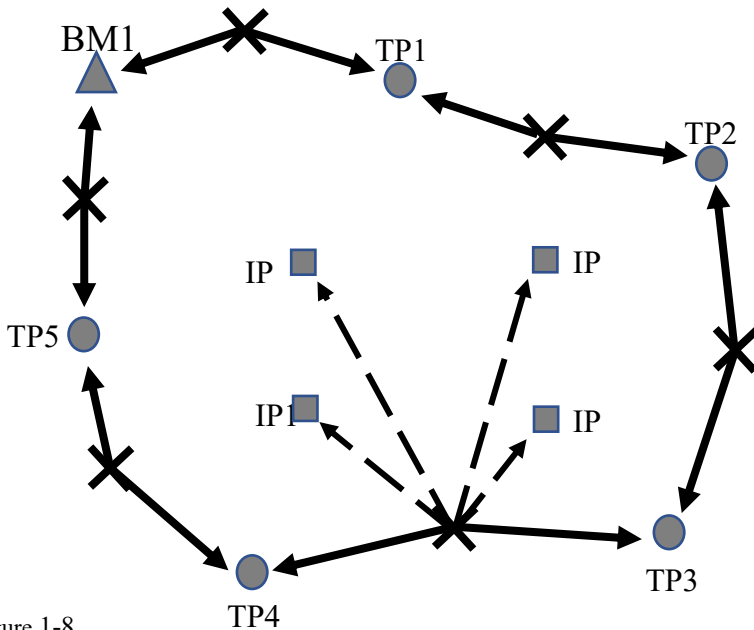


Figure 1-8

Note that:

- The starting benchmark with a known elevation is BM1 (triangle).
- Level instrument locations are marked with an “X”.
- Backsights and Foresights are marked with solid arrows.
- Rod locations are marked with circles and identified as “TP” or “turning points” (that is where the rod is first read for a foresight, and then turned to be read for the backsight). These points are numbered and recorded in the direction of survey (clockwise).
- Note that other points outside the traverse were also collected (squares). These are identified as IP for “intermediate point”. They are also called “side shots”. They could, for example, be the corners of a building. Thus, it is possible to bring the elevation to the site of a project (the traverse) and then collect site information (the side shots for, say, the corners of that building).

1.4 Recording Measurements

There is a standard method to record readings. It follows the steps mentioned under “The Procedure” above. This is best illustrated with a simple Excel table that looks like this:

Sta	BS	HI	ADJ.H I	FS	ELE V	ADJ.ELE V

Where:

- Point names go into the first column (**station or Sta**)
- Backsights are written into the second column (**BS**).
- Foresights are recorded in the fifth column (**FS**).
- The third column holds Heights of Instrument that are calculated (**HI**).
- The sixth column holds calculated elevations (**ELEV**).
- Columns **ADJ. HI** and **ADJ.ELEV** will be explained later in this chapter.

Referring back to the plan of the traverse, the starting point with the known elevation is BM1. That name goes into the first column, top row, and its known elevation (say, 5000.00') goes into the “Elevation” field.

Sta	BS	HI	ADJ.H I	FS	ELEV	ADJ.ELE V
BM1					5000.0 0	

Then the first backsight to BM1 is measured and recorded (say, 5.43'), and we can calculate the Height of Instrument ($5000.00' + 5.43' = 5005.43'$) and we put that into the “HI” column.

Sta	BS	HI	ADJ.HI	FS	ELEV	ADJ.ELEV
BM1	5.43				5000.00	
		5005.43				

Then the foresight to point TP1 is measured and recorded under “FS” (say, 6.54’). Now we can calculate the “Elevation” of TP1 ($5005.43' - 6.54' = 4998.89'$).

Note that the HI values occupy their own row between corresponding points, since the instrument is positioned between them.

Sta	BS	HI	ADJ.HI	FS	ELEV	ADJ.ELEV
BM1	5.43				5000.00	
		5005.43				
TP1				6.54	4998.89	

This process is repeated for TP2. First, one measures the BS to TP1 (say, 7.65’), and that is recorded opposite TP1. This gives us a new HI ($4668.89' + 7.65' = 5006.54'$).

Sta	BS	HI	ADJ.HI	FS	ELEV	ADJ.ELEV
BM1	5.43				5000.00	
		5005.43				
TP1	7.65			6.54	4998.89	
		5006.54				

Then we measure the FS to TP2 and record that (say, 4.76’). From this we calculate the elevation for TP2 ($5006.54' - 4.76' = 5001.78'$).

Sta	BS	HI	ADJ.H I	FS	ELEV	ADJ.ELE V
BM1	5.43				5000.0 0	
		5005.4 3				
TP1	7.65			6.54	4998.8 9	
		5006.5 4				
TP2				4.76	5001.7 8	

This is repeated until the whole traverse is measured, and we collected the last FS back to BM1. The completed survey could look like the table on the next page.

The side shot data have been intentionally left out, and this table only shows the main traverse data. The side shots will be added after the adjustment.

Sta	BS	HI	ADJ.HI	FS	ELEV	ADJ.ELEV
BM1	5.43				5000.00	
		5005.43				
TP1	7.65			6.54	4998.89	
		5006.54				
TP2	3.24			4.76	5001.78	
		5005.02				
TP3	4.14			6.52	4998.50	
		5002.64				
TP4	3.54			2.54	5000.10	
		5003.64				
TP5	5.68			4.25	4999.39	
		5005.07				
BM1				4.6	5000.47	

1.5 Checking the Measurements

After “closing” the traverse (coming back to the starting point) we can perform the first check. The calculated elevation for BM1 turns out to be 5000.47 and not 5000.00. The difference of 0.47 feet is called a “misclosure” and is due to an accumulation of rod reading errors along the traverse. Note that this misclosure is also equal to the sum of all BS minus the sum of all FS.

Sta	BS	HI	ADJ.H I	FS	ELEV	ADJ.ELE V
BM1	5.43				5000.0 0	
		5005.4 3				
TP1	7.65			6.54	4998.8 9	
		5006.5 4				
TP2	3.24			4.76	5001.7 8	
		5005.0 2				
TP3	4.14			6.52	4998.5 0	
		5002.6 4				
TP4	3.54			2.54	5000.1 0	
		5003.6 4				
TP5	5.68			4.25	4999.3 9	
		5005.0 7				
BM1				4.6	5000.4 7	
				MISC L	0.47	
				NR HI	6	
				CORR	-0.078	

Can this error be eliminated from the table? Of course. If the error is large, then one would have to re-measure the traverse. What is “large”? A misclosure of 0.02 feet per TP is acceptable. The corresponding “correction” is the misclosure divided by the number of HI, or $0.47 / 6 = -0.078$ feet (note the intentional sign change). See the calculation at the bottom of the table. This is a bit large, but we will accept it this time.

1.6 Adjustment

The adjustment process consists of adding the correction 1 times to the first **HI**, 2 times to the second, etc., and the results are placed into the **Adj.HI** column. For example, for the fourth HI, $5002.64 + 4 \times (-0.078) = 5002.33$.

Then, the elevations are re-calculated using these Adj.HI and placed into the **Adj.Elevation** column. See the table below.

Note that now the elevations of BM1 at start and end are the same since the error was spread around the traverse successfully.

Sta	BS	HI	ADJ.HI	FS	ELEV	ADJ.ELEV
BM1	5.43				5000.00	5000.00
		5005.43	5005.35			
TP1	7.65			6.54	4998.89	4998.81
		5006.54	5006.38			
TP2	3.24			4.76	5001.78	5001.62
		5005.02	5004.79			
TP3	4.14			6.52	4998.50	4998.27
		5002.64	5002.33			
TP4	3.54			2.54	5000.10	4999.79
		5003.64	5003.25			
TP5	5.68			4.25	4999.39	4999.00
		5005.07	5004.6			
BM1				4.6	5000.47	5000.00
				MISCL	0.47	0.00
				NR HI	6	
				CORR	-0.078	

1.7 Dealing with Side Shots

Now that the traverse is adjusted, one can insert the side shots IP1 to IP4. These IPs are located between the third and the fourth HI (the fourth instrument setup). From this one calculates their elevation using the same Adj.HI from the row above. For example, for IP2, $5002.327 - 4.11 = 4998.217$.

Sta	BS	HI	ADJ.HI	FS	ELEV	ADJ.ELEV
BM1	5.43				5000.00	5000.00
		5005.43	5005.35			
TP1	7.65			6.54	4998.89	4998.81
		5006.54	5006.38			
TP2	3.24			4.76	5001.78	5001.62
		5005.02	5004.79			
TP3	4.14			6.52	4998.50	4998.27
		5002.64	5002.33			
IP1				3.25		
IP2				4.11		
IP3				3.84		
IP4				1.57		
TP4	3.54			2.54	5000.10	4999.79
		5003.64	5003.25			
TP5	5.68			4.25	4999.39	4999.00
		5005.07	5004.6			
BM1				4.6	5000.47	5000.00
				MISCL	0.47	0.00
				NR HI	6	
				CORR	-0.078	

1.8 Last Comments on Leveling

This completes the adjustment. Note that the above procedure gave you several things:

1. A tool to record measurements in the field.
2. A tool to check measurements.
3. A tool to adjust measurements.
4. A tool to deliver data to a customer.
5. A standard record for information safekeeping.

1.9 - Instrument Error Detection and Correction

But what can one do if the instrument is faulty? The principal source for this type of error is when rough handling of the instrument causes the crosshairs to be knocked out of position. In Figure 1-9 below, this causes the crosshair reading to be too low (or too high) and is called “collimation error”.

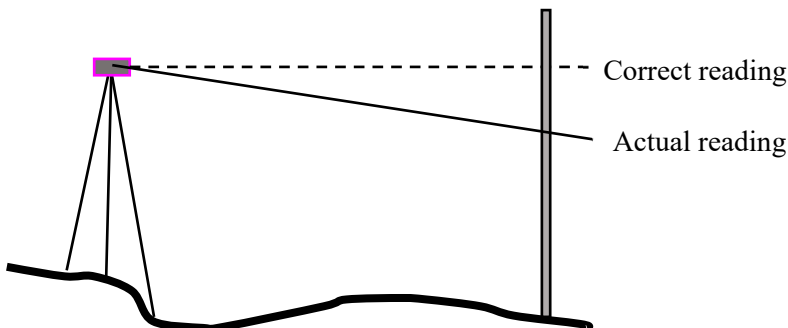


Figure 1-9

How can one determine if the instrument has such an error, and how big? This is done with a procedure called the “peg test”. See Figure 1-10. A distance is laid out in a straight line between two points A and B, and it is split into three segments of the same length (say, 25 feet each).

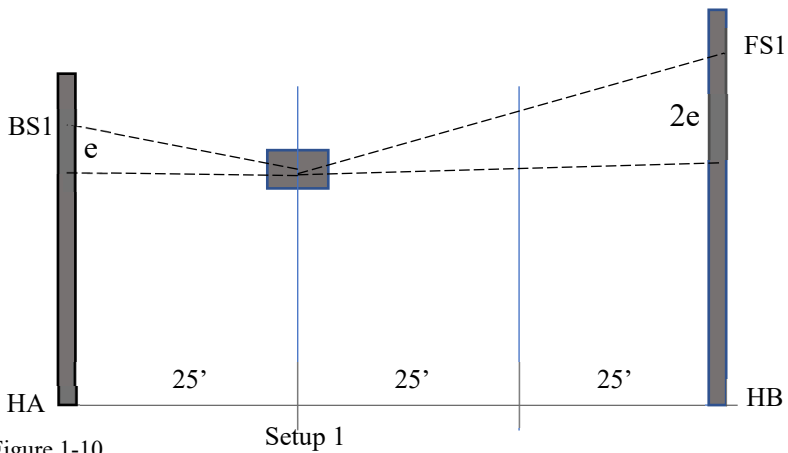


Figure 1-10

The instrument is set up at 1/3 of the distance (Setup point 1), and the elevation of B is calculated as follows: $HB = HA + BS1 - FS1$. One inserts a possible error amount e into the equation: $HB = HA + (BS1 - e) - (FS1 - 2e)$.

The level is then moved 1/3 of the distance to Setup point 2 as in Figure 1-11. The equation now is this one: $HB = HA + (BS2 - 2e) - (FS2 - e)$.

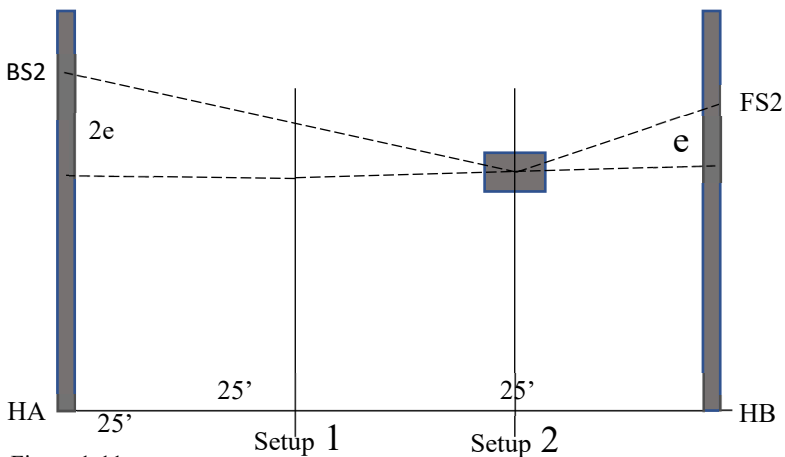


Figure 1-11

Subtracting equation 2 from equation 1 gives:

$0 = 0 + BS1 - e - FS1 + 2e - BS2 + 2e + FS2 - e$, and re-arranging, one gets $e = (BS2 - BS1 - FS2 + FS1)/2$.

1.10 Reciprocal Leveling

When one needs to transfer elevations across a large distance, as in the case of a river, highway, or lake, then one collects measurements in both directions to minimize the effects of collimation errors. This is like the above method for “e”.

Figure 1-12 shows the readings from both sides of a lake. The equations from each end are: $EB = EA + BS1 - (FS1 - e)$ and $EB = EA + (BS2 - e) - FS2$. Adding the two equations up, one gets $EB = (2 \cdot EA + BS1 - FS1 + BS2 - FS2)/2$.

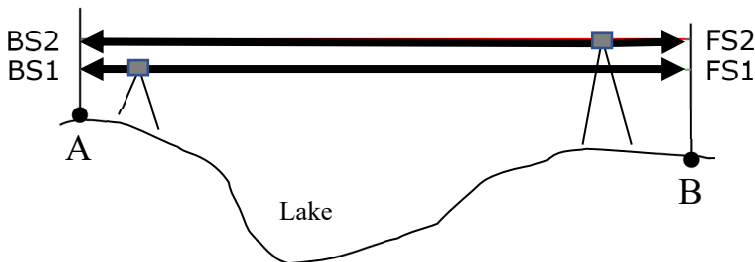


Figure 1-12

1.11 Trigonometric Leveling

Another type of leveling is used when big differences of elevation are present, and one could not measure this with a level.

In Figure 1-13 one sees the use of a “total station” (a theodolite capable of measuring angles and distances). This instrument is placed at point A, and a vertical angle “ α ” and a slope distance SD are observed to a reflector that is standing on point B. For this one needs to know the height of instrument “hi” and the height of the reflector “hr”. Alternatively, the instrument may give not α but the zenith angle “z”.

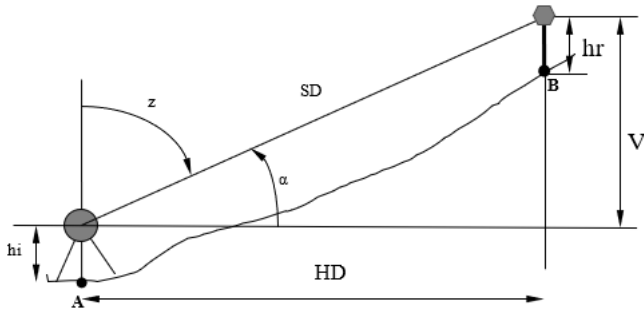


Figure 1-13

The related equations are:

$$V = SD \cdot \sin(\alpha) = SD \cdot \cos(z) \quad \text{and} \quad HD = SD \cdot \cos(\alpha) = SD \cdot \sin(z)$$

$$\text{Elevation B} = \text{Elevation A} + hi + V - hr$$

It must be stressed that a key value in this exercise is the recording of the height of the instrument (hi). It normally is in the range of about four to seven feet. Instrument operators often forget to measure this for every instrument setup, forcing them to repeat the survey.

CHAPTER TWO

DISTANCES

Distances are an important topic in surveying, in relation to how they are measured, calculated, and reported. This involves tools, methods, and the law.

2.1 Instruments

Distances are measured with physical tapes or with electronic instruments. See two such instruments below.



If measured with tapes, additional equipment is needed, as follows:

1. A “fish scale” helps apply the proper tension to a tape to minimize sag.



2. A “hand level” helps keep the tape horizontal.



3. A “plumb bob” helps transfer a point vertically so that the surveyor can lift the tape from the ground to a convenient height.



4. Wooden “stakes” (not shown) that are driven into the ground to witness points placed on the ground, to hold instructions or data, and often to hold a nail on the top for various purposes.
5. Long “range poles” that help mark points so that their position can be seen from a distance (as when a surveyor needs to stay on a long straight line).

This indicates some of the needs that the user of a long tape faces, such as:

- “Lining in” or staying along a straight line for multiple tape lengths.
- Applying proper tension (more of that later).
- Keeping the tape horizontal (leveling it).
- “Plumbing” or using a plumb bob.
- Correct reading of the tape.
- Marking tape lengths on the ground (usually with nails).
- Recording Distances.

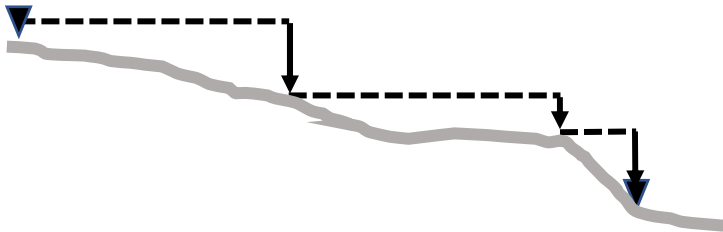
2.2 Cadastral distances

These are distances that are measured along legal land boundaries. By law, all these distances must be converted to horizontal distances, after correcting them for various things (see next section). Slope distances cannot be recorded with “plats” (legal and official descriptions of land parcels, as used by the courts).

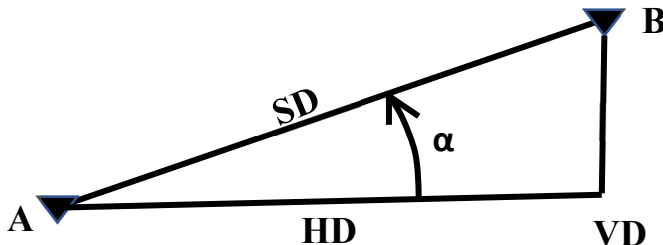
2.3 Horizontal vs. sloped measurements

If one encounters sloped ground, there are two ways that allow the collection of a horizontal distance.

One is the piecemeal measurement of horizontal pieces with a tape (dashed lines shown below) and a plumb bob (black vertical arrows) that are then added up. This is best done downhill.



Another method is the use of a “total station” (a device that measures angles and distances electronically). As shown on the next page, a slope distance **SD** and a vertical angle α are converted into a horizontal distance **HD** with this formula: $HD = SD * \cos(\alpha)$, and a vertical distance $VD = SD * \sin(\alpha)$.



2.4 Taping errors and their corrections

The principal errors affecting taping are twofold, namely operator blunders and taping errors, as follows.

The operator blunders are:

- Poor horizontal alignment (subsequent tapings are not done in a straight line).
- Tape not horizontal (no use of a hand level).
- Improper plumbing (swinging plumb bob, lack of care).
- Faulty marking.
- Incorrect reading and/or recording.
- Combinations of the above.

The taping errors are:

- C_L Incorrect length of tape due to manufacturing error or other reason.
- C_T Stretching or contracting due to temperature changes.
- C_P Stretching due to Incorrect pull.
- C_s Sagging (only correction that is always negative)

A detailed discussion of each follows.

C_L - Tape length correction: What does one do if a surveyor's tape is marked to be 100.00 feet long, but it actually is 100.23 feet long? Surveyor tapes need to be tested for correct length. In that case, tape measurements need to be corrected using the formula below, where “ l ” is the nominal length, “ l' ” is the actual length, “ L ” is the total measured distance and C_L is the correction for the total distance.

$$C_L = \left(\frac{l-l'}{l'} \right) L$$

C_T - Temperature correction: All tapes react to temperature changes. Surveyor tapes also do that. They are made from steel and are produced in a controlled environment. In addition, these tapes come with a statement that indicates at what temperature they were made (their standard temperature “ T ”, which normally is 68°F or 20°C). This is compared to the actual temperature during measurement “ T_1 ”. The change in length

depends on the thermal expansion coefficient k , which is either $0.00000645 / ^\circ\text{F}$ or $0.0000116 / ^\circ\text{C}$. The correction applies to multiple tape lengths in m or ft.

$$C_T = k (T_1 - T) L$$

C_P - Pull correction: When pulled, surveyor steel tapes change in length. The manufacture is done under a standard tape tension "P" of 7 kg or 15.4 lbs. If one uses a fish scale to maintain this standard tension during measurement, then no correction is necessary. If a correction is needed, it is based on the actual tension (or pull) applied during measurement "P₁". The change in length will depend on the tape cross-section "A" and the modulus of elasticity of steel E (29 000 000 lb / sqin or 2 000 000 Kg / cm²). "L" is the total measured distance in m or ft.

$$C_P = (P_1 - P) \frac{L}{AE}$$

C_S - Sag correction: Unless supported by the ground, all tapes sag, and steel tapes even more so. Sag is a measure of how much the center of the tape length sinks down (it is a vertical event that causes a change in horizontal length). The correction involves the weight of the tape per unit length ("w" in kg/m or lb/ft), the unsupported tape length during measurement ("L_s" in m or ft) and the amount of pull applied ("P" kg or lb).

$$C_S = - \left(\frac{w^2 L_s^3}{24P^2} \right)$$

Combination of Corrections

To combine the corrections as needed, one uses the following relationship, all in the same units of m or ft:

$$L_{\text{corrected}} = L_{\text{measured}} + C_L + C_T + C_P + nC_S$$

Where "n" is the number of tape lengths used to measure the distance (only used for the sag correction).

2.5 Example of a correction

A 100 m steel tape (standardized at 20°C and supported throughout) was found to be 100.015 m under tension of 7 Kg. The tape has a cross-sectional area of 0.05 cm² and a weight of 0.045 Kg/m. This tape was used to measure a 450 m slope distance. Calculate the horizontal distance if:

- $T_1 = 25^\circ\text{C}$

- Tension = 9 Kg
- The tape was supported only at the ends (it sags)
- The elevation difference between points was 10.25 m.

Recap of values:

$$l = 100.000 \text{ m} \quad l' = 100.015 \text{ m}$$

$$P = 7 \text{ kg}$$

$$P_1 = 9 \text{ kg}$$

$$T = 20^\circ\text{C}$$

$$T_1 = 25^\circ\text{C}$$

$$\Delta El. = 10.25 \text{ m} \quad k = 0.0000116$$

$$w = 0.045 \text{ kg/m}$$

$$E = 2000000 \text{ kg/}$$

$$\text{cm}^2$$

$$L = 450.000 \text{ m} \quad A = 0.05 \text{ cm}^2$$

Calculations: $C_l = \left(\frac{100.015 - 100.000}{100.000} \right) * 450 = 0.067 \text{ m}$

$$C_t = 0.0000116 * (25 - 20) * 450 = 0.026 \text{ m}$$

$$C_p = (9 - 7) \frac{100}{0.05 * 2000000} * 4 = 0.008 \text{ m}$$

$$C_p = (9 - 7) \frac{50}{0.05 * 2000000} = 0.001 \text{ m}$$

$$C_p = 0.008 + 0.001 = 0.009$$

$$C_s = \left(\frac{0.045^2 * 100^3}{24 * 9^2} \right) 1.042 \text{ m (correction per every 100m)}$$

$$C_s = \left(\frac{0.045^2 * 50^3}{24 * 9^2} \right) = 0.130 \text{ m (correction per 50m)}$$

$$\text{Total } C_s = 4 * 1.042 + 0.130 = -4.298 \text{ m}$$

$$L_{\text{corrected}} = 450.000 + 0.067 + 0.026 + 0.009 - 4.29 = 445.804 \text{ m}$$

$$H = \sqrt{L_{\text{cor.}}^2 - \Delta El.^2} = \sqrt{445.804^2 - 10.25^2} = 445.686 \text{ m}$$