Robust Control

Robust Control:

Supplementary Topics

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To Mexico with love.

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0.1 Introduction

0.1.1 Overview of the Book

What does *Robust Control* mean in control theory?

The term "*robust*" comes from the Latin word "*robustus*," which means "*strong.*" A durable product is one that does not readily break. As a result, a resilient operating system is one in which any particular program may fail without disrupting the operating system or other applications. Thus, robust design focuses on improving the fundamental function of the process.

Control theory is a branch of Applied Mathematics dealing with the use of feedback to affect a system's behavior in order to achieve a certain objective.

Definition 0.1 ***Robust control*** *is a controller design feedback strategy that emphasizes the control algorithm's dependability (robustness). Robustness is commonly described as the minimum requirements that a control system must meet in order to be helpful in a real-world situation.*

When one or more of a system's output variables must follow a certain reference throughout time, a controller manipulates the system's inputs to achieve the intended impact on the system's output. So, a feedback controller measures a process output and then changes the input as needed to drive the process variable toward the desired setpoint, i.e., a controller reacts to setpoint variations initiated by operators as well as random disturbances to the process variable caused by external forces.

Dealing with the control design of any actual dynamic system, a researcher attempts to meet *three basic requirements* in order to make the control process more easy and appealing from a practical standpoint:

- first, the mathematical model of a plant to be controlled may be inexactly known or contain some uncertain parameters or structure-elements;
- second, the controlled system should be able to work satisfactorily in the presence of external perturbations (even bounded and not necessarily "smooth");
- third, the controller should be the simplest form that allows for straightforward implementation: a linear state-feedback regulator (despite the fact that the considered plant is nonlinear) appears to be the most suitable option.

Clearly, any traditional optimum control approaches (such as the Pontryagin Maximum Principle and Bellman Dynamic Programming) developed for control design under complete and precise plant information are inapplicable in such uncertain scenarios. Recent research and actual implementations have demonstrated that the most appropriate strategies for the control design of various types of uncertain systems include

- Robust Control Theory (Zhou-Doyle-Glover, 1996), primarily concerned with the H_∞ - approach and its several variations, such as

Robust Adaptive (Ioannou-Sun, 1996) and Robust Adaptive Controls (Hong, 2008);

- Attractive Ellipsoid Method (for example, Poznyak-Polyakov-Azhmyakov, 2014);
- Sliding Mode Control (Utkin, 1992).

Content of the book

This course consists of five parts:

1. **Part I: Mathematical Background and Linear Matrix Inequalities in Control Theory.**

   The fundamental characteristics of quadratic forms are addressed in the first lecture. Then the positive definitiveness of partitioned matrices is investigated using Schur's complement lemma. Finsler's lemma is provided, as well as the so-called S - method, which deals with extra restricting quadratic forms.

2. **Part II: Absolute Stability and $\mathbf{H}_\infty$- Control.**

The stability theory of the group of nonlinear systems with sectorial restrictions is considered. The generalized the Lurie-Postnikov type Lyapunov function with a vector feedback is applied in time and frequency domains to analyze the absolute stability property. Also the problem of perturbations attenuation in linear continuos - time systems (H_∞-control) is analyzed based on the Kalman - Yakubovich - Popov´s (KYP) frequency lemma.

3. **Part III: Attractive Ellipsoid Method (AEM).**

It includes the design technique of state and output feedbacks, the full-order dynamic feedback, feedbacks in systems with delay and sample-data with quantized output feedbacks.

4. **Part IV: Sliding Mode Control (SMC).**

   Sliding Mode Control is a nonlinear control approach that changes the dynamics of a nonlinear system by applying a discontinuous control signal (or, more precisely, a set-valued control signal) that causes the system to "slide" along a cross-section of its desired behavior. The state-feedback control law is not a time-dependent function. Instead, depending on where it is in the state space, it can flip from one continuous structure to another.

5. **Part V: Engineering Examples.**

   This part contains 3 examples:

   - Autonomous Vehicles (AV) moving in 2D and avoiding obstacles;

   - Guidance Control of Underwater Autonomous Vehicle;

   - Acceleration Control for Pilots and Astronauts Simulator.

The bibliographic list of references is given in the end of each part.

According to the author opinion, this text covers several subjects that have never been considered in other related books:

- The necessary conditions for the existence of an LMI solution,

- the extension of Schur's lemma to nonnegative (not required strictly positiveness) matrices,

- the dynamic feedback controller and its design using the AEM application,

- the AEM for time-delay systems,

- robust control designing for systems with Sampled-Data and Quantized Output;

- the SMC method in unitary representation including Averaged Subgradient Method and Integral Sliding Mode approach,

- and the analysis of Absolute Stability for vector nonlinear feedbacks are among them.

0.1.2 Prerequisites

This course is aimed at graduate students (Masters and Doctorate) of the Electrical and Mechanical Engineering faculties, studying Control Theory and Mecatronics, and, who wish to learn more about how the modern robust control theory solves different problems that arise in the real world.

We will assume familiarity with systems theory at the basic level, including:

- Math language and logic;

- Real and complex mathematical analysis;

- Linear algebra (Vectors, Matrices, and Least Squares approach);

- Linear control systems theory (Linear system theory and design, Feedback systems).

This book is a research-oriented material with a heavy emphasis on original sources. Participants should be confident in their ability to locate, read, and comprehend conference and journal publications well enough to duplicate and/or explain results to a colleague.

0.1.3 Computational Tools

The analytical skills we learn in class may be used in formal control system thinking. Real-world systems, on the other hand, rarely allow for pen-and-paper study, therefore we rely heavily on computational tool outputs in practice. As a result, this book will stress both analytical and computational methods, as well as their benefits and drawbacks.

Python is now the most favorite computational toolkit; it is free, open-source, cross-platform, and full-featured. I'll encourage the participants to use Python tools by publishing sample code, homework assignments, and homework answers. For their course work, the readers are free to use any computational tool (for example, my favorite is MATLAB with Control System Toolbox and Simulink).

0.1.4 The Relationship to Other Courses and Books

Of course, this book includes fresh ideas that might serve as **supplements** to the older works that are still in print and well-liked among Modern Control Theory experts. These books are

- Maciejowski, J.M., 1989, *Multivariable Feedback Design.* Addison Wesley.

  The optimization of the feedback parameters is not considered. The book respectively old.

- Grimble, M.J., 1994, *Robust Industrial Control.* Prentice Hall International.

  Some specific systems such as sample-data quntized output feedbacks, Time-Delay, Implicit and Switched Structer Systems are not considered.

- Zhou, K., Doyle, J. & Glover, K., 1996, *Robust and Optimal Control*, Prentice Hall

  The book treated only $\mathbb{H}_2$ and $\mathbb{H}_\infty$ control problems.

- Kurzhanski, A., Valyi, I. (1997), *Ellipsoidal Calculus for Estimation and Control*: 2, Systems & Control: Foundations & Applications, Boston, MA: Birkhauser.

  The book basically deals with the ellipsoidal calculus and analysis of reachable sets.

- Mahmoud, M. S., Robust Control and Filtering for Time–Delay Systems, Marcel Dekker, 2000.

  This book concerns only time-delay sistems.

- Blanchini, F. & Miani, S. 2008, Set Theoretic Methods in Control. Systems &Control: Foundations & Applications, Boston, MA: Birkhauser.

This book makes the detail analysis of set-stability but does not touch the output feedback design methods.

- Haddad, W. & Chellaboina, V. 2008, Nonlinear Dynamical Systems and Control, Princeton University Press, Princeton.

  It does not considere uncertainty effects and set-stability properties.

- Dullerud, G.E.; Paganini, F. (2000). A Course in Robust Control Theory: A Convex Approach. Springer Verlag New York. ISBN 0-387-98945-5.

  This course deals basically with $\mathbb{H}_2$ and $\mathbb{H}_\infty$ concepts in robust control and the realization of LMI's technique for $\mathbb{H}_\infty$ designing of robust controllers.

- Bhattacharya; Apellat; Keel (2000). Robust Control-The Parametric Approach. Prentice Hall PTR. ISBN 0-13-781576-X.

  This book presents the complete account of the available results in the field of robust control under parametric uncertainty only.

- Zhou, Kemin; Doyle C., John (1999). Essentials of Robust Control. Prentice Hall. ISBN 0-13-525833-2.

  The book considers Gap metric, V-gap metric, model validation and real μ-sinthesis in the frame $\mathbb{H}_\infty$ theory.

- Morari, Manfred; Zafiriou, Evanghelos (1989). Robust Process Control. Prentice Hall. ISBN 0-13-782153-0.