Robust Control

Robust Control:

Supplementary Topics

By

Alexander Poznyak

Cambridge Scholars Publishing



Robust Control: Supplementary Topics

By Alexander Poznyak

This book first published 2024

Cambridge Scholars Publishing

Lady Stephenson Library, Newcastle upon Tyne, NE6 2PA, UK

British Library Cataloguing in Publication Data A catalogue record for this book is available from the British Library

Copyright © 2024 by Alexander Poznyak

All rights for this book reserved. No part of this book may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without the prior permission of the copyright owner.

ISBN: 978-1-0364-1664-5

ISBN (Ebook): 978-1-0364-1665-2



Contents

0.1	Introd	luction	1
	0.1.1	Overview of the Book	1
	0.1.2	Prerequisites	5
	0.1.3	Computational Tools	6
	0.1.4	The Relationship to Other Courses and	
		Books	6
	0.1.5	Acknowledgement	10
I M	lathen	natical Background and Linear Ma	ıt-
rix Ir	nequal	ities in Control Theory	13
rix Ir 1 Ma	\mathbf{nequal}	ities in Control Theory tical Background	13 15
rix Ir	\mathbf{nequal}	ities in Control Theory tical Background ratic Forms	13 15 16
rix Ir 1 Ma	\mathbf{nequal}	ities in Control Theory tical Background	13 15
rix Ir 1 Ma	nequal athema Quadi	ities in Control Theory tical Background ratic Forms	13 15 16
rix Ir 1 Ma	nequal athema Quada 1.1.1	tical Background ratic Forms	13 15 16

viii CONTENTS

		1.1.4 Nonnegative Definiteness of a Partitioned
		Matrix
	1.2	Finsler's Lemma and S - Procedure
		1.2.1 Finsler's Lemma
		1.2.2 S - Procedure (Lemma)
	1.3	Examples
	1.4	Exercises
2	Line	ear Matrix Inequalities 47
	2.1	Matrix Inequality With Respect to a Vector and
		a Matrix
	2.2	LMI's Feasibility 51
	2.3	Parametrization of all Solutions
	2.4	Nonlinear Matrix Inequalities Equivalent to LMI 56
		2.4.1 Matrix Norm Constraint
		2.4.2 Nonlinear Weighted Norm Constraint 57
		2.4.3 Nonlinear Trace Norm Constraint 57
		2.4.4 Lyapunov's Inequality
		2.4.5 Algebraic Riccati - Lurie's Matrix Inequality 58
	2.5	Appendix
		2.5.1 Some Simple Properties of Linear Matrix
		Equations
		2.5.2 Proofs of the Main Theorems on LMI's . 61
	2.6	Examples
	2.7	Exercises
3	Cha	aracteristics of LSS as LMIs 75
	3.1	LSS and their Transfer Function
	3 9	H. Norm

	3.3	Passivity and Positive-Real Lemma	78
	3.4	Nonexpansivity and the Bounded-Real Lemma .	81
	3.5	H_{∞} Norm	84
	3.6	Gamma Entropy	85
	3.7	Stability of Stationary Time-Delay Systems	86
	3.8	Hybrid Time-Delay Linear Stability	88
	3.9	Examples	89
4	Opt	timization Problems with LMI Constraints	97
	$4.\overline{1}$	Eigenvalue Problem (EVP)	98
	4.2	Tolerance Level Optimization	99
	4.3	Maximization of the Quadratic Stability Degree .	99
	4.4	Minimization of $\operatorname{Tr}(CPC^{\dagger})$ Under a Lyapunov-	
		Type Constraint	100
	4.5	The Convex Function log det $A^{-1}(X)$ Minimiz-	
		ation	102
	4.6	Numerical Methods for LMIs Resolution	103
		4.6.1 What Does it Mean "to Solve LMI"?	103
		4.6.2 Ellipsoid Algorithm	104
		4.6.3 Interior-Point Method	109
		Absolute Stability and ${ m H}_{\infty}$ Control: Gen	L -
er	aliz	ed Classical Problems	113
5	Abs	solute Stability	115
	5.1	Linear Systems With Nonlinear Feedbacks	116
	5.2	Generalized Sector Condition	117
		5.2.1 Multidimensional Case	117

x CONTENTS

		5.2.2 Scalar Output-Control Case	118
	5.3	2,70	
		lute Stability	121
	5.4	Conjectures of Aizerman and Kalman	122
6	Abs	solute Global Stability in Time-Domain	125
	6.1	Scalar Feedback	125
		6.1.1 Lyapunov Function of the Lurie-Postnikov	
		Type	125
		6.1.2 Absolute Stability in the Time-Domain .	128
	6.2	Vector Feedback	129
	6.3	Exercise	132
7	Abs	solute Global Stability in Frequency-Domain	135
	7.1	On Equivalency of Hermitian and Quadratic forms	
	7.2	-	140
			140
		7.2.2 The Popov's Line	144
		•	145
8	\mathbf{H}_{∞}	Control	153
	8.1	Perturbations Attenuation in Linear Continuos-	
			153
	8.2		156
			156
			159
		8.2.3 Problem Formulation in the Hardy Space	
			162
	8.3		162

CONTENTS xi

		8.3.1 KYP - Lemma	162
	8.4	LMI Representation of the Perturbations Atten-	
		uation Problem	167
	8.5	Exercise	172
	- ,		
II	1 A	Attractive Ellipsoid Method	177
9	Con	aplete and Incomplete Information	181
	9.1	Complete Information Case: Classical Optimal	
		Control	182
		9.1.1 System Description	183
		9.1.2 Feasible and Admissible Control	184
		9.1.3 Problem Setting in the General Bolza Form	n185
		9.1.4 Specific Features of the Classical Optimal	
		Control	186
	9.2	Incomplete Information Case	187
	9.3	Robust Tracking Problem Formulation	
	9.4	What is the Effectiveness of a Designed Control .	191
	9.5	Ellipsoid Based Feedback Control Design	192
	9.6	Example and Exercise	194
10	Rob	oust State Feedback Control	197
	10.1	Proportional Linear Feedback	198
		10.1.1 Model Description	
		10.1.2 Problem Formulation	199
	10.2	Storage Function Method	200
	10.3	Attractive Ellipsoid	202
		10.3.1 Definition of an Attractive Ellipsoid	202

xii CONTENTS

		10.3.2 Attractive Ellipsoid for Proportional Lin-	
		ear Feedback	202
	10.4	Minimization of the Attractive Ellipsoid	204
	10.5	Conversion of NMI in to LMI Constraints	206
	10.6	Numerical Procedure	209
	10.7	Practical Stabilization	210
	10.8	Illustrative Example: an Inverted Pendulum	212
	10.9	Exercises	215
11	Rob	oust Linear Output Feedback Control	219
	11.1	System Description and Problem Formulation	220
	11.2	Special Orthonormal State Space Transformation	224
	11.3	Attractive Ellipsoid Design	225
	11.4	Optimal Output Feedback	230
	11.5	Example: Stabilization of a Discontinuous System	232
12	Obs	erver-Based Feedback Design	237
	12.1	State Observer and the Extended Dynamic Model	237
	12.2	Stabilizing Feedback Gains	239
	12.3	Minimization of Attractive Ellipsoid	249
	12.4	Adaptive Version of AEM	250
	12.5	Exercise	260
13	Dyn	amic Regulator	261
	13.1	Full Order Linear Dynamic Controller	261
	13.2	Attractive Ellipsoid for a Dynamic Controller	264
	13.3	Optimal Parameters of a Dynamic Controller	271
	13 /	Evoreigo	279

CONTENTS xiii

14	Rob	oust Stabilization of Time-Delay Systems	275
	14.1	Affine Systems with a Delay in State Variables .	276
		14.1.1 System Description and Problem Formu-	
		lation	276
		14.1.2 Lyapunov-Krasovskii's Functional and Sta-	
		bility Analysis	278
		14.1.3 Optimal Feedback Parameters	282
	14.2	Affine Systems with a Delay in Control Actions .	285
		14.2.1 System Description	285
		14.2.2 Prediction Approach and Unavoidable Sta-	
		bilization Error	286
		14.2.3 Attractive Ellipsoid	289
		14.2.4 Minimal Attractive Ellipsoid for the Ori-	
		ginal System	293
		14.2.5 Example and Exercise	296
15	Sam	pled-Data and Quantized Output	301
	15.1	Sampling and Quantization	302
	15.2	System Description and Problem Formulation	303
	15.3	Basic Assumptions	306
		Feedback Structure	308
	15.5	Stability Analysis	310
		15.5.1 The Lyapunov - Krasovskii Functional and	
		its Derivative	310
		15.5.2 Descriptor Form	
	15.6	Main Result	318
		Matrix Inequality Simplification	
	15.8	Optimal Feedback Parameters	322
		15.8.1 Evample	393

xiv CONTENTS

IV	SLIDING MODE CONTROL	335
16 Ab	out Sliding Mode Control	337
16.1	1 Tracking as Stabilization	. 338
16.2	2 State Space Desired Dynamics	. 340
16.3	B ODE with Discontinuous Right-Hand Side	. 344
	16.3.1 Why ODE with DRHS are Important in	1
	Robust Control	. 344
	16.3.2 ODE with DRHS and Differential Inclusion	0ns 352
17 Slic	ding Mode Control	357
17.1	1 Sliding Mode Surface as Desired Dynamics	. 357
	17.1.1 First-Order Tracking System	. 361
	17.1.2 Stabilization of a Second Order Relay-Sys	
17.2	2 Equivalent Control Method	. 367
	17.2.1 Equivalent Control Construction	
	17.2.2 Sliding Mode Motion	
	17.2.3 Low-Pass Filtering	. 373
	17.2.4 The Realizable Approximation of the Equ	
	valent Control	. 374
17.3	3 Exercise	. 380
18 Slic	ding Mode Observers	381
18.1	1 General Observer for Nonlinear Systems	. 381
18.2	2 SM Observations for the Class of Mechanical Mod	1-
	els	. 386
	18.2.1 Model of the System	. 386
	18.2.2 Main Assumptions	
	18 2 3 Observer Structure	

CONTENTS xv

		18.2.4 Equivalent Control Concept Application .	388
	18.3	Exercises	390
		18.3.1 Exercises	390
19	Inte	gral Sliding Mode	393
	19.1	Main Idea	393
	19.2	Problem Formulation in General Affine Format .	398
		19.2.1 Control Design Objective	400
		19.2.2 ISM Control Design	401
	19.3	Exercises	403
20	ASC	G-Version of ISM Control	407
	20.1	Model Description and Problem Setting	407
		Accepted Assumptions	409
		Desired Dynamics and Its Properties	412
		20.3.1 Auxiliary sliding variable $s(t)$	412
	20.4	Main Theorem on ASG Robust Controller	416
		Exercise	420
21	Twi	st and Super-Twist Controllers	421
		Problem Formulation	421
	21.2	Twist Controller	424
		21.2.1 Lyapunov Function Analysis	
		21.2.2 Method of Characteristics for the Lyapunov	
		Function Design	
	21.3	Super-Twist Controller	431
		21.3.1 Lyapunov Function Analysis	431
	21.4	Super-Twist Observer and Differentiator	433
		21.4.1 Super-Twist Observer	

xvi CONTENTS

		21.4.2 Super-Twist Differentiator 43	35
	21.5	Exercises	37
22	Ada	aptive SMC 43	9
	22.1	The σ -Adaptation Method	10
	22.2	The Dynamic Adaptation Based on ECM 44	13
		22.2.1 The Simple Motivating Example 44	13
		22.2.2 Multidimensional Case	19
		22.2.3 Super-Twist Control with Adaptation 45	4
	22.3	Exercises	6
\mathbf{V}	Eı	ngineering Examples 46	1
23	Aut	onomous Vehicles (AV) Avoiding Obstacles 46	5
		AV Description and Coordinate Systems 46	
		Kinematic Constraints of the AV	:8
	23.3	Euler-Lagrange Equations for Nonholonomic Dy-	
		namics	0
		23.3.1 The Hamilton's Principle 47	0
		23.3.2 Kinetic Energy Calculation 47	3
		23.3.3 The Euler-Lagrange Model in an Open	
		Format	5
		23.3.4 A Kinematic Relation 47	7
		23.3.5 Modified Euler-Lagrange Equation 47	7
	23.4	Problem Formulation	31
		23.4.1 Descriptive Problem Formulation 48	31
		23.4.2 Obstacles Description 48	31
		23.4.3 Main Assumptions	33

CONTERNIEC	
CONTENTS	XV11
0011121112	12 1 1 1

		23.4.4 Cost Function with Penalty Terms 4	185
		23.4.5 Mathematical Problem Formulation 4	186
	23.5	ASG Method	187
		23.5.1 Desired δ -Dynamics	187
		23.5.2 Modified Lyapunov Function and the Ro-	
			189
	23.6	Numerical Simulation	194
		23.6.1 Selected Functions and Parameters 4	194
		23.6.2 Simulation Results and Discussions 4	197
	23.7	Conclusions	198
24	Gui	dance Control of Underwater Vehicle 5	01
	24.1	UAV Mathematical Model	502
		24.1.1 Kinematic Model	504
		24.1.2 Dynamic Model	505
		24.1.3 Complete Dynamics	507
	24.2	Guidance Laws by ISM Control	508
		24.2.1 Problem Statement in Descriptive Form . 5	508
		24.2.2 Backstepping Concept	509
	24.3	Numerical Simulation Results	521
	24.4	Conclusions	525
	24.5	Appendix	528
25			35
	25.1		536
			537
			539
	25.2	Problem Statement of the Motion Cueing Control	
		Design	540

xviii CONTENTS

	25.2.1 Cueing Dynamic Model	541
	25.2.2 Problem Formulation	542
25.3	Control Designing	544
25.4	Partial Case: Two-link Robotic Arm Motion Sim-	
	ulator	549
	25.4.1 Description of the Model	549
	25.4.2 Kinematic Model and Acceleration Time	
	Dependence	551
	25.4.3 Dynamic Model of the 2-link Robotic Arm	552
25.5	Numerical Evaluations	555

List of Figures

5.1	Linear systems with a nonlinear feedback	117
5.2	Sector condition	119
5.3	Strict sector-condition	120
7.1	Popov's lines for $q=0,q<0$ and $q>0.$	145
7.2	The class of the systems is absolutely stable: the	
	Popov's plot admits the existence of a Popv's line	
	with $q > 0$	146
7.3	The class of the systems is absolutely stable: the	
	Popov's plot admits the existence of a Popv's line	
	with $q < 0$	147
7.4	The class of the systems can not be considered as	
	absolutely stable: a Popov's line does not exist,	
	such that the	148
10.1	Inverted pendulum	213
	Inverted pendulum	

10.3	Attractive ellipsoid	217
11.1	States dynamics	234
	Attractive ellipsoid	
11.2	Troctacon to empsora	200
14.1	Trajectories of the controlled system	298
14.2	Convergence into the attractive ellipsoid	299
	Sampled and quantized output signal	305
	Measurable output $y(t)$	326
15.3	Controlled trajectories $x_1(t), x_2(t), \dots$	327
15.4	Robust control action $u(t)$	328
15.5	Convergence of trajectories to the attractive el-	
	lipsoid	329
	4(0)	
	A function with the property $f(0) = 0$	345
	A function with $f_0 \neq 0$	346
	Signum function	350
	The chattering effect	352
16.5	Differential inclusion $\dot{x}(t) \in -\text{sign}(x(t)) \dots$	356
1 7 1		
17.1	The sliding surface and the rate vector field at	950
1 = 0	the point $x = 0$	359
	The velocity vector on a sliding surface	360
	A tracking system	362
	The finite time tracking to the surface $s = e = 0$.	364
	The finite time tracking the tragectories $r(t)$	365
17.6	The sliding motion on the sliding surface $s(x) =$	
	$r_0 + cr_1$	366

17.7 The amplitude-frequency characteristic of the filter.374
23.1 AV description
23.2 The 2D trajectory in plain 497
23.3 Control signal
23.4 Lyapunov function convergence to zero practic-
ally from the beginning of the process 499
23.5 The cost function $F(\delta(t))$ converging to its min-
imum
24.1 UV and its coordinates 503
24.2 Diagram of control structure 520
24.3 Displacement in surge axis (x) for UAV 523
24.4 Displacement in surge axis (y) for UAV 524
24.5 Displacement in surge axis (z) for UAV 525
24.6 UAV movement in three-dimensional space 526
24.7 $\ \Delta_1\ $ for UAV
24.8 Voltage v_u
24.9 Voltage v_q
$24.10 \text{Voltage } v_r. \dots \dots \dots \dots \dots \dots \dots \dots \dots $
25.1 Motion cueing scheme
25.2 The robotic arm with a pilot's cabin on the end-
effector
25.3 2-links robotic arm
25.4 Geometrical restrictions on end-effector's motion
in linear space

25.5	Non-perturbed (left) and perturbed (right) accel-	
	eration tracking on the x-axis for the 1-st order	
	of SM, PID and ISM controllers	555
25.6	Non-perturbed (left) and perturbed (right) accel-	
	eration tracking on the y-axis for the 1-st order	
	of SM, PID and ISM controllers	556
25.7	Non-perturbed (left) and perturbed (right) $\ \delta\ $	
	and $\ \Delta\ $ comparison ($\varepsilon = 0.1$) for the 1-st order	
	of SM, PID and ISM controllers	557
25.8	Non-perturbed (left) and perturbed (right) tra-	
	jectory tracking on the x-axis for the 1-st order	
	of SM, PID and ISM controllers	558
25.9	Non-perturbed (left) and perturbed (right) tra-	
	jectory tracking on the y-axis for the 1-st order	
	of SM, PID and ISM controllers	559
25.10	ONon-perturbed (left) and perturbed (right) eval-	
	uation of the control absolute value ($\varepsilon = 0.1$) for	
	the 1-st order of SM. PID and ISM controllers	559

0.1 Introduction

0.1.1 Overview of the Book

What does Robust Control mean in control theory?

The term "robust" comes from the Latin word "robustus," which means "strong." A durable product is one that does not readily break. As a result, a resilient operating system is one in which any particular program may fail without disrupting the operating system or other applications. Thus, robust design focuses on improving the fundamental function of the process.

Control theory is a branch of Applied Mathematics dealing with the use of feedback to affect a system's behavior in order to achieve a certain objective.

Definition 0.1 Robust control is a controller design feedback strategy that emphasizes the control algorithm's dependability (robustness). Robustness is commonly described as the minimum requirements that a control system must meet in order to be helpful in a real-world situation.

When one or more of a system's output variables must follow a certain reference throughout time, a controller manipulates the system's inputs to achieve the intended impact on the system's output. So, a feedback controller measures a process output and then changes the input as needed to drive the process variable toward the desired setpoint, i.e., a controller reacts to setpoint variations initiated by operators as well as random disturbances to the process variable caused by external forces. Dealing with the control design of any actual dynamic system, a researcher attempts to meet *three basic requirements* in order to make the control process more easy and appealing from a practical standpoint:

- first, the mathematical model of a plant to be controlled may be inexactly known or contain some uncertain parameters or structure-elements;
- second, the controlled system should be able to work satisfactorily in the presence of external perturbations (even bounded and not necessarily "smooth");
- third, the controller should be the simplest form that allows for straightforward implementation: a linear state-feedback regulator (despite the fact that the considered plant is nonlinear) appears to be the most suitable option.

Clearly, any traditional optimum control approaches (such as the Pontryagin Maximum Principle and Bellman Dynamic Programming) developed for control design under complete and precise plant information are inapplicable in such uncertain scenarios. Recent research and actual implementations have demonstrated that the most appropriate strategies for the control design of various types of uncertain systems include

- Robust Control Theory (Zhou-Doyle-Glover, 1996), primarily concerned with the H_∞ - approach and its several variations, such as

Robust Adaptive (Ioannou-Sun, 1996) and Robust Adaptive Controls (Hong, 2008);

3

- Attractive Ellipsoid Method (for example, Poznyak-Polyakov-Azhmyakov, 2014);
 - Sliding Mode Control (Utkin, 1992).

Content of the book

This course consists of five parts:

1. Part I: Mathematical Background and Linear Matrix Inequalities in Control Theory.

The fundamental characteristics of quadratic forms are addressed in the first lecture. Then the positive definitiveness of partitioned matrices is investigated using Schur's complement lemma. Finsler's lemma is provided, as well as the so-called S - method, which deals with extra restricting quadratic forms.

2. Part II: Absolute Stability and H_{∞} - Control.

The stability theory of the group of nonlinear systems with sectorial restrictions is considered. The generalized the Lurie-Postnikov type Lyapunov function with a vector feedback is applied in time and frequency domains to analyze the absolute stability property. Also the problem of perturbations attenuation in linear continuos - time systems (H_{∞} -control) is analyzed based on the Kalman - Yakubovich - Popov´s (KYP) frequency lemma.

3. Part III: Attractive Ellipsoid Method (AEM).

It includes the design technique of state and output feedbacks, the full-order dynamic feedback, feedbacks in systems with delay and sample-data with quantized output feedbacks.

4. Part IV: Sliding Mode Control (SMC).

Sliding Mode Control is a nonlinear control approach that changes the dynamics of a nonlinear system by applying a discontinuous control signal (or, more precisely, a set-valued control signal) that causes the system to "slide" along a cross-section of its desired behavior. The state-feedback control law is not a time-dependent function. Instead, depending on where it is in the state space, it can flip from one continuous structure to another.

5. Part V: Engineering Examples.

This part contains 3 examples:

- Autonomous Vehicles (AV) moving in 2D and avoiding obstacles;
- Guidance Control of Underwater Autonomous Vehicle;
- Acceleration Control for Pilots and Astronauts Simulator.

The bibliographic list of references is given in the end of each part.

According to the author opinion, this text covers several subjects that have never been considered in other related books:

- The necessary conditions for the existence of an LMI solution.
- the extension of Schur's lemma to nonnegative (not required strictly positiveness) matrices,
- the dynamic feedback controller and its design using the AEM application,
 - the AEM for time-delay systems.
- robust control designing for systems with Sampled-Data and Quantized Output;
- the SMC method in unitary representation including Averaged Subgradient Method and Integral Sliding Mode approach,
- and the analysis of Absolute Stability for vector nonlinear feedbacks are among them.

0.1.2 Prerequisites

This course is aimed at graduate students (Masters and Doctorate) of the Electrical and Mechanical Engineering faculties, studying Control Theory and Mecatronics, and, who wish to learn more about how the modern robust control theory solves different problems that arise in the real world.

We will assume familiarity with systems theory at the basic level, including:

- Math language and logic;
- Real and complex mathematical analysis;
- Linear algebra (Vectors, Matrices, and Least Squares approach);
- Linear control systems theory (Linear system theory and design, Feedback systems).

This book is a research-oriented material with a heavy emphasis on original sources. Participants should be confident in their ability to locate, read, and comprehend conference and journal publications well enough to duplicate and/or explain results to a colleague.

0.1.3 Computational Tools

The analytical skills we learn in class may be used in formal control system thinking. Real-world systems, on the other hand, rarely allow for pen-and-paper study, therefore we rely heavily on computational tool outputs in practice. As a result, this book will stress both analytical and computational methods, as well as their benefits and drawbacks.

Python is now the most favorite computational toolkit; it is free, open-source, cross-platform, and full-featured. I'll encourage the participants to use Python tools by publishing sample code, homework assignments, and homework answers. For their course work, the readers are free to use any computational tool (for example, my favorite is MATLAB with Control System Toolbox and Simulink).

0.1.4 The Relationship to Other Courses and Books

Of course, this book includes fresh ideas that might serve as **supplements** to the older works that are still in print and well-liked among Modern Control Theory experts. These books are

7

 Maciejowski, J.M., 1989, Multivariable Feedback Design. Addison Wesley.

The optimization of the feedback parameters is not considered. The book respectively old.

 Grimble, M.J., 1994, Robust Industrial Control. Prentice Hall International.

Some specific systems such as sample-data quntized output feedbacks, Time-Delay, Implicit and Switched Structer Systems are not considered.

- Zhou, K., Doyle, J. & Glover, K., 1996, Robust and Optimal Control, Prentice Hall

The book treated only \mathbb{H}_2 and \mathbb{H}_{∞} control problems.

- Kurzhanski, A., Valyi, I. (1997), Ellipsoidal Calculus for Estimation and Control: 2, Systems & Control: Foundations & Applications, Boston, MA: Birkhauser.

The book basically deals with the ellipsoidal calculus and analysis of reachable sets.

- Mahmoud, M. S., Robust Control and Filtering for Time— Delay Systems, Marcel Dekker, 2000.

This book concerns only time-delay sistems.

- Blanchini, F. & Miani, S. 2008, Set Theoretic Methods in Control. Systems & Control: Foundations & Applications, Boston, MA: Birkhauser.

- This book makes the detail analysis of set-stability but does not touch the output feedback design methods.
- Haddad, W. & Chellaboina, V. 2008, Nonlinear Dynamical Systems and Control, Princeton University Press, Princeton.
 - It does not considere uncertainty effects and set-stability properties.
- Dullerud, G.E.; Paganini, F. (2000). A Course in Robust Control Theory: A Convex Approach. Springer Verlag New York. ISBN 0-387-98945-5.
 - This course deals basically with \mathbb{H}_2 and \mathbb{H}_{∞} concepts in robust control and the realization of LMI's technique for \mathbb{H}_{∞} designing of robust controllers.
- Bhattacharya; Apellat; Keel (2000). Robust Control-The Parametric Approach. Prentice Hall PTR. ISBN 0-13-781576-X.
 - This book presents the complete account of the available results in the field of robust control under parametric uncertainty only.
- Zhou, Kemin; Doyle C., John (1999). Essentials of Robust Control. Prentice Hall. ISBN 0-13-525833-2.
 - The book considers Gap metric, V-gap metric, model validation and real μ -sinthesis in the frame \mathbb{H}_{∞} theory.
- Morari, Manfred; Zafiriou, Evanghelos (1989). Robust Process Control. Prentice Hall. ISBN 0-13-782153-0.