Hidden Symmetries and Black Holes' Spacetimes

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By

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Dedicated to my Mom, my late Dad and my late brother,

Zamfira Pintilie, Panaite and Mihai Rugina

and to

my extended family in Romania, USA and elsewhere.

Especially to

Richard Watkins and his family of Wayland, MA, USA

and to

Peter Bauman and his family of Atlanta, GA, USA.

Figure 1: „SUSY in the sky", by artist Silvia Mihăilescu

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List of Publications

Published Papers:

1. **G. W. Gibbons, C. Rugina**, *"Goryachev-Chaplygin, Kovalevskaya and Brdička-Eardley-Nappi-Witten pp-waves spacetimes with higher rank Stäckel-Killing tensors"* , J. Math. Phys. **52** (2011) 122901, arXiv: gr-qc/1107.5987.

2. **K. Hinoue, T. Houri, C. Rugina, Y. Yasui** , *"General Wahlquist metrics in all dimensions"*, Phys Rev D **90** (2014) 024037, arXiv: 1402.6904.

3. **C. Rugina, A. Ludu**, *"Almost-BPS solutions in multi-center Taub-NUT"*, Grav. Cosmol. **23** (2017) 320, arXiv: 1307.2128.

4. **C. Rugina, A. Ludu**, *"Non-compact manifolds with Killing spinors"*, J. Geom. Phys. **151** (2020) 103641, arXiv: 1909.01780.

5. **C. Rugina**, *"The modular Dirac equation"*, Gen. Rel. Grav. **54** (2022) 125, arxiv: 2204.01428.

arXiv preprints:

1. **C. Rugina**, *"Hidden symmetries and Killing spinor of 5-dimensional minimal gauged supergravity solutions"*, arxiv: 1408.3487.

2. **A. Ludu, J. Morris, C. Rugina**, *"The counterfactual photo-electric effect"*, arxiv:1910.09364.

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Abstract

This book is about hidden symmetries of spacetimes. and gives a few examples of black holes' spacetimes, for instance, a view on a particular 11 dimensional black hole's spacetime compactified down to 5 dimensions. I start off by investigating the hidden symmetries of Goryachev-Chaplygin and Kovalevskaya gyrostats' spacetimes, as well as the Brdička-Eardley-Nappi-Witten pp-waves, and I find out that these spacetimes possess higher rank conformal Stäckel-Killing and Killing-Yano tensors, an important result for higher hidden symmetries. I then go into the realm of string theory, and I seek solu-tions in the framework of 11-dimensional N=2 theory, which I then compactify down to 5 dimensions and I find microstates of multiple collinear/one non-collinear black holes embedded in a multi-center Taub-NUT, spacetime are sought in 4 dimensions then lifted to 5 dimensions. A set of coupled ordinary partial differential equations are obtained and solved for almost-BPS states, where some supersymmetry is preserved in the context of N=2 supergravity in 5 dimensions. The regularity of solutions is being carefully considered and I ensure that no CTC (closed time-like curves) are present. Then I investigate the Chong-Cvetič-Lü-Pope black hole's spacetime, which is a solution of the 5-dimensional minimal gauged supergravity, endowed with a Sasaki structure deformed by torsion. I also explicitly construct a Killing spinor for the above mentioned specific 5 dimensional minimal gauged supergravity solution. Finally, I present a result in entangle-ment wedge reconstruction, retrieving the spin 1/2 in the center of the bulk via solving what is called here the modular Dirac equation. I also present as a background material a brief review of some aspects of the physics of classical and quantum black holes, since this book mainly deals with black hole solutions and also a brief review of some of the results in the literature on black holes' interiors, a chapter on

hidden symmetries and Killing spinors, one 5-dimensional minimal gauged/ungauged supergravity and a brief overview of aspects of the entanglement wedge reconstruction project.

Part I

Introductory Remarks

The Organization of this Book

Part I: Introductory Remarks

Chapter 1: Introduction for Lay Audience in Theoretical Physics

This part is aimed at people with high school level knowledge of Physics. It overviews briefly the History of Physics starting from Sir Isaac Newton, with an emphasis on Theoretical Physics results. It also presents some of the latest discoveries in Gravitation and String Theory, so that informally, I create the opportunity for everybody to gain access to the most important Theoretical Physics developments of this and the past century.

Chapter 2: Introduction

I start off with this current general introduction for specialists in topics pertaining to classical and quantum black holes.

Part II: Building some Background

Chapter 3: The Tensors, Spinors and Operators of Hidden Symmetries

In this chapter, I lay the groundwork for presenting some aspects of things in the mathematical formalism that describes hidden symmetries: from the definitions and properties of Stäckel-Killing and Killing-Yano tensors, to Dirac type operators and Killing spinors. I overview a few of the properties of these mathematical objects in curved spacetimes with torsion.

Chapter 4: Black Holes' Interiors

One-sided and two-sided black holes' interiors are overviewed with an emphasis on the eternal Schwarzschild-AdS black hole, traversable wormholes and their symmetries, mirror operators and Tomita-Takesaki constructions.

Chapter 5: 5-Dimensional Minimal Gauged/Ungauged Supergravity

I present a number of gauged/ungauged minimal supergravity solu-tions of 5-dimensional supergravity and solutions for supergravity cou-pled to matter: Sasaki-Einstein solutions, AdS_5 solutions, 1/2 BPS so-lutions in AdS, black holes solutions (the CCLP solution - the Chong-Cvetič- Lü-Pope solution - the rotating charged non-extremal black holes with two independent angular momenta and the $\mathcal{N}= 2$ solution coupled to matter) and the maximally supersymmetric Gödel solu-tion. I also introduce a classification of minimal gauged/ungauged 5-dimensional supergravity based on bilinears formed from Killing spinors.

Chapter 6: Brief Overview of Aspects of Entanglement Wedge Reconstruction

A brief overview of entanglement wedge reconstruction (EWR) of the bulk and in particular of black holes interiors is presented. I start my short review with a view on what EWR means in the context of noisy universal recovery channels and entropy in similarity with concepts from quantum information theory using Petz maps and twirled Petz maps. Then brief considerations on the Ryu-Takayanagi (RT) surface and finding the quantum extremal surface (QES) via a maximin procedure follow. I then proceed with quantum error correction codes (QEC) explorations and the realization of that with various tensor network hyperbolic tiling. Then I introduce a mechanism by which the entanglement wedge can be seen and acted upon, as developed recently in the literature.

Part III: The Original Work

Chapter 7: Higher Rank Conformal Stäckel-Killing and Killing-Yano Tensors in Various Spacetimes

In this chapter, I find higher rank conformal Stäckel-Killing and Killing-Yano tensors for the Goryachev-Chaplygin and Kovalevskaya gyrostats spacetimes, as well as the Brdička-Eardley-Nappi-Witten pp-waves. I stress the fact that this is one of the few original works that constructs higher rank Stäckel-Killing tensors, which actually permits generaliza-tions of these integrable problems to a higher rank.

Chapter 8: Almost-BPS Solutions in 5 Dimensions

We lift to five dimensions previously found 4-dimensional fuzzball microstates, which deals with non-collinear almost-BPS multi-center Taub-NUT black holes. The context is that of 11-dimensional supergravity with 3 families of M2-M5 branes, compactified down to 4 dimensions with a Gibbons-Hawking base, then lifted to 5 dimensions.

Chapter 9: Hidden Symmetries of the Kerr-dS Black Hole Solution in 5 Dimensions

We prove here that the Chong-Cvetič-Lü-Pope (CCLP) black hole spacetime is an example of a Sasaki deformed by torsion spacetime and we construct a couple of new Killing-Yano and Stäckel-Killing tensors. We construct a Killing spinor for it, too.

Chapter 10: The Modular Dirac Equation

We introduce a new equation we dubbed the modular Dirac equation to see and reconstruct a spin 1/2 particle at the center of a nearly AdS_2 spacetime in the entanglement wedge reconstruction paradigm and we study hidden symmetries of this spacetime and various properties of the modular Dirac operator, too.

Chapter 11: Conclusions

We draw appropriate conclusions and we place our work in a more general context.

Chapter 1

Introduction in Theoretical Physics

The mysteries of nature are hard to decipher and through out the evolution of humanity a variety of scientific and philosophical methods were adopted to probe and understand reality: empirical (experimental) scientific methods, theoretical methods (i.e. models and theories which represent more or less approximately a portion of reality), philosophical and spiritual methods of knowing the world and even the insight of an artist is useful in understanding our perceptions about reality.

This introduction for lay audiences is part of a book in Theoretical Physics, to be more precise in String Theory and Gravitation, and so it is expected that this short informal introduction in the mysteries of nature is to be centered more on the description and presentation of some concepts from String Theory, Gravitation (with subjects like black holes, dark energy and dark matter), and some cosmology. We will also review the main directions and accomplishments in Physics from Isaac Newton on, in a short and non-comprehensive history of the field.

We can end up understanding some of the most hidden secrets of the universe (maybe not all, but, let's say, some) even if we are not theoretical physicists. Nowadays String Theory and Gravitation, so the science of the fundamentals of matter and spacetime has progressed very much, so this modest introduction in this field wants to help spread some of these spectacular results.

To understand a few things from String Theory, the unified theory of matter and spacetime, we will first take a look at some fundamental concepts in physics from Newtonian Classical Mechanics of 300 years ago, then at Maxwell's Electromagnetic Theory from the XIX-th century and Quantum Mechanics of the 20s in the last century. We will then briefly overview E instein's Special Relativity and General Relativity from the beginning of the XX-th century.

One can say that Newton's theory lays the foundation of modern physics. It is true that Newton's theory recorded in *Principia* published in 1687, has as precursors the effort in science among others of Galileo Galilei and Giordano Bruno. In any case, *Principia* states the laws of classical mechanics, defines t he notion of mass of a body, the acceleration and also the action-reaction principle (if a body acts on another body with a force (action), then the second body acts on the first with an equal force but opposite to the first (reaction)). However, perhaps the most important law enounced by Newton is the law of universal attraction of bodies:

$$F = G\frac{m_1 m_2}{R^2} \tag{1.1}$$

where F is the force between the two bodies, G is Newton's constant, m_1, m_2 are the two masses of the two bodies and R the distance between the two bodies. This law is the celebrated law of universal gravitational attraction, which helped with the understanding of the movement of celestial bodies at that moment in time, but also later on, together with the laws that govern the motion of the planets stated by Johannes Kepler. For almost three hundred years until Einstein's theory, this law defined and dominated the theory of gravitation, together with the notions of absolute space and time introduced also by Sir Isaac Newton. This time, in the realm of mathematics, Isaac Newton also laid the foundations of real differential and integral calculus. His theories and contributions are still used to this day, in high schools, but also in NASA's space programs.

Another giant in Physics is James Clerk Maxwell (if Newton was an English nobleman, Maxwell is from a family of baronets from Scotland). Maxwell will write in 1873 an electricity and magnetism treatise, which ties in together the works of other great physicists such as

Ampére or Faraday. In these treatise, Maxwell states the electromag-netic laws with partial derivatives, presenting in an unified form, the laws of electricity and magnetism. One can say this is the first grand unification of fields in physics, the notion of unification of forces takes shape more and more in physics and it culminates with String Theory towards the end of the XX-th century.

Light and theories that explain light involving phenomena preoccu-pied Newton, too, but also Huygens, Fresnel and then Maxwell, later on in the XVII to XIX-th centuries. The speed of light appears in the electromagnetic equations and that's not surprising, since light is a very special electromagnetic wave. Research has been carried on ever since Newton's time on phenomena in geometrical optics of light rays, such as reflection and refraction, but also on optical phenomena in which the wave aspect of the light is more pronounced, such as the dispersion of light on a prism and the rainbow phenomenon. The de-velopment and understanding of the physics of the lenses and mirrors helped build telescopes, which oriented towards the sky, led to the de-velopment of astronomy and the dream of humanity to conquer cosmic space. The theories of light took a new turn at the end of the XIX-th century when an older theory, the aether theory catches ground - the aether is an omnipresent, rigid medium which allows for the propagation of light. Lord Kelvin himself, another pioneer in physics, with notable results in thermodynamics, formulates an aether theory. However, later on, a series of experiments designed by Michelson and Morley invalidate the existence of the aether and establish the speed of light constant with the value of 3×10^8 m/s. These experiments laid a solid foundation for a revolution in physics which the beginning of the XX-th century was set forth to bring on.

However, before we go on to the physics of the XX-th century, it is important to mention the great physicists who took on thermodynam-ics and statistical physics, when the notions of microscopic energy, work, heat, temperature and entropy (the measure of disorder in na-ture) were carefully studied. The industrial revolution of the XIX-th century is partly due to the study of thermodynamics, of thermal phenomena and of the engines studied initially in the framework of thermodynamics by Diesel and Carnot. Based on these theoretical models the first steam engines were built, so that at the end of the XIX-th century to reach the gas motor car. Lord Kelvin, originally from Ireland, defines the Kelvin temperature scale and the notion of absolute zero on the temperature scale and to get a feel of how low this temperature really is, we can mention the fact that 0K is about $-273°$ C. Stefan Boltzmann, an Austrian physicist has genius contributions in statistical physics, too, and writes the law which will also be engraved on his tomb stone in Vienna:

$$S = k_B \log \mathrm{W} \tag{1.2}$$

where S is the entropy of a system, k_b is Boltzmann's constant, log means logarithm and W, the number of possible microstates associated with a macroscopic state given to a physical system.

The most spectacular results in physics started to come about at the end of the XIX-th century, but especially in the beginning of the XX-th century. During this period they laid strong foundations for a new direction in physics: theoretical physics(one can say that it is the field in physics that is closest to mathematics and that developments in theoretical physics led to revolutions in mathematics and vice-versa). A remarkable year for this new discipline is 1905, when Albert Einstein publishes 4 papers, each revolutionizing physics. One paper deals with the photoelectric effect, another one with Brownian motion, another one develops Special Relativity and the fourth establishes the famous relation:

$$E = mc^2 \tag{1.3}$$

where E is the energy of the particle or body, m is its moving mass and c is the speed of light in vacuum. The photoelectric effect establishes the corpuscular nature of light and together with Max Planck, Einstein establishes the fact that the light is formed of quanta, tiny particles, with no mass, which are the photons. The energy of a photon is given by:

$$E = \hbar\nu \tag{1.4}$$

where E is the energy of the photon, $\hbar$ is Planck's constant divided by 2π and ν is the frequency of vibration of the photon.

This paper opens subsequently, together with Max Planck's and others' activity, the path towards Quantum Mechanics of 20 years later. To be noted the fact that the light is at the same time a particle (the photon), but also an electromagnetic wave, as it was determined earlier and this fact was named the wave-particle duality, an appreciated theory in Quantum Mechanics and extended by Louis de Broglie to all particles via the relation which ties the momentum (mass multiplied with the velocity of a particle) to the wavelength:

$$p = \frac{h}{\lambda} \tag{1.5}$$

So p is the momentum and it characterizes the corpuscular properties of the particle and λ is the wavelength (the distance between two ‚peaks' of the wave) of the wave associated to the particle, h is Planck's constant (see figure (1.1)).

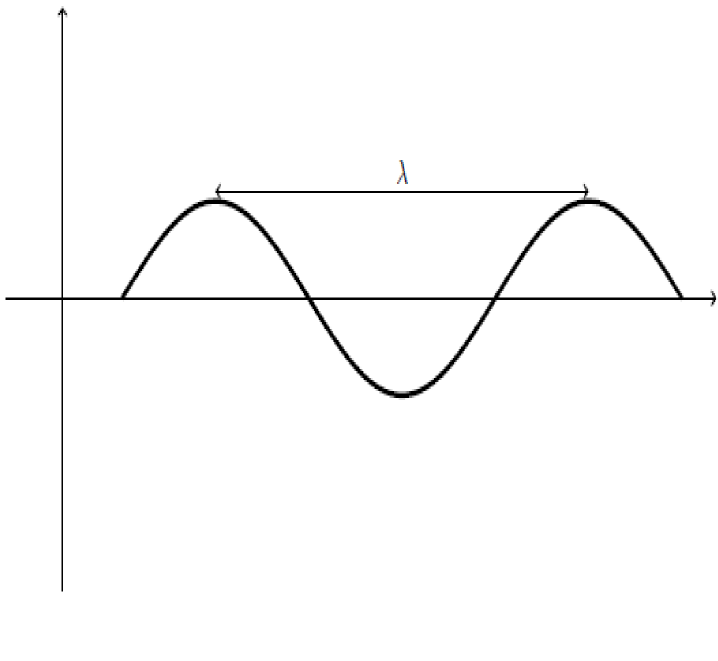

Figure 1.1: The wavelength λ of a wave.

The paper about the Brownian motion underlines the fact that on a microscopic scale the particles (the atoms and molecules) display a chaotic, rapid, and continuous motion dubbed by Einstein Brownian motion.

The paper on Special Relativity lays the foundation of a new direction in thinking in Theoretical Physics, a vision complementary to the absolute space and time, as seen by Newton. In Special Relativity, things are relative as you can imagine: time and space are relative. Moreover, on this occasion, a new unification in Physics takes place, namely space and time are on equal footing and one talks about a spacetime continuum (namely in Special Relativity the time coordinate of a reference system is treated alike with the three spatial coordinates). Coming back to the notion of relativity: the length of a meter stick measured by you in a lab reference frame can look shorter in a reference frame which travels at a high speed, for example 0.3c (c is naturally the speed of light). Also time is relative, too, namely if you stay in a lab for 1h, in a system of reference which travels with 0.85c it seems as if you stayed in the lab two hours! So in Special Relativity length can be contracted and time dilated when we study things with respect to systems of reference which travel very fast. Using mathematical formulae, length contraction is written as:

$$l = l_0\sqrt{1 - \frac{v^2}{c^2}} \tag{1.6}$$

where l is the contracted length of a real object of length l_0, and contracted length is measured in a system of reference which travels at speed v.

Similarly, one can find the fact that time can be dilated in this way:

$$\tau = \frac{\tau_0}{\sqrt{1 - \frac{v^2}{c^2}}}, \tag{1.7}$$

where τ is the time measured in a system of reference which travels with a speed v, when the time measured by you in the university lab is τ_0. The twins paradox is well known in this context of time: there are two twins of 20 years old, and while one remains on Earth and the other one leaves on a one year journey with a high speed space shuttle, then when they meet after a year again, according to the time measured by the twin in the space shuttle, the one on Earth is already 80 years old! Time on Earth flows differently from the very high speed space shuttle.

However, relativity is not limited to space and time, the mass of a body is modified too, in reference systems which travel with very high speeds, according to the formula below:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}, \tag{1.8}$$

where m is the moving mass, measured in a reference system which travels with speed v and m_0 is the rest mass of the body (of the respective particle). In principle, energy formula is modified too, in relativistic systems (in reference to the energy formula in Newtonian mechanics) and takes the value according to (1.3). To be noted that the relativistic systems, where Special Relativity applies, are systems that travel with very high speeds, comparable with the speed of light of 300,000 km/s.

Special Relativity opened even more the way for Les Grandes Idées in Theoretical Physics: the beginning was marked at the end of the XIX-th century. After Special Relativity, Einstein's General Theory of Relativity appears in 1915, we will come back to it a little later, and it is one of the greatest accomplishments of human thinking of all times.

Undoubtedly Quantum Mechanics remains one of the greatest revolutions in physics. There exists a relatively large group of sacred personalities who fathered and developed the Quantum Mechanics of tiny particles smaller than the atomic scale even before 1920. Among these: Schrödinger, Heisenberg, Bohr, Dirac, Compton, but also Planck, Pauli, de Broglie, Mme. Curie and Einstein (this last one a pioneer in the field with the photoelectric effect, but who spent the second half of his life in a big dispute with the Copenhagen School led by Niels Bohr, trying to invalidate the probabilistic outlook of Quantum Mechanics saying that „God doesn't play dice").

Quantum Mechanics is complementary to Newton's Classical Mechanics, which deals with the mechanics of macroscopical objects. Quantum Mechanics deals with the microscopic scale of nature, from atoms and molecules to smaller entities in the mathematical framework of the theory of probabilities. Relativistic Quantum Mechanics goes along very well with Einstein's Special Relativity and gives birth in a number of years from its inception to the relativistic field t heory (Dirac is one of the pioneers of this field). Q uantum M echanics i n i ts initial