

Formal Methods in Musicology

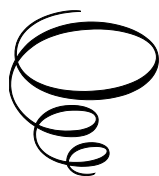
Formal Methods in Musicology:

Models and Computation

Edited by

Francis Knights and Pablo Padilla

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In Memory of Christopher Hogwood (1941-2014)

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CHAPTER 1

INTRODUCTION

Let us inspect the lyre, and weigh the stress
Of every chord, and see what may be gain'd
By ear industrious, and attention meet
—from *If by dull rhymes*, John Keats (1795-1821)

1.1 Analysis and musical style

The possibilities for the analysis of music – a subject defined simply by Nick Cook as ‘musical intelligence’¹ and by John White as the ‘understanding of musical style’² – through conventional and computational means are vast; any number of different questions could be asked of any type of music. Much of the recent literature concerns methods rather than musical repertoire, but this book starts with the music itself, asks questions concerning style, development, chronology, attribution and other aspects, and examines what new and what established methods (from within and outside music) seem appropriate to help answer those. The majority of the methods focus on specific and common components (for example, melody) and are thus widely applicable to music of many times and places; there have been a number of attempts at devising analytical methods capable of covering various styles and musical cultures,³ and from these it is clear that the fundamentals of music do share many commonalities across time and place.

The use of standard musical notation, or accurate transcription into it, is here assumed throughout,⁴ and the majority of the examples are drawn

¹ Cook (1996), p.vii. For a rounded discussion of the meanings of the term ‘analysis’, as well as the history of the subject, see Bent (2001).

² White (1976), p.1.

³ See Hijleh (2016), pp.8-10.

⁴ For analysis using sound files, see for example Weihs et al. (2016) and Müller (2015). The question as to what methods might be used for graphic scores is an interesting one – it would be difficult to determine which representations might be

from classical music, although both the concepts and methods used can be applied to many other genres, styles or repertoires.

Chapter 1 explains the musical and technical background to computational analysis, defined broadly, while ch.2 is a detailed study of the components that affect musical style, as well as an exploration of notion of ‘composing’, and a series of open questions related to music analysis. The computational tools are described in ch.3, including some of the different programming environments, while ch.4 explains how these are used, and what matters need to be considered in their real-world application; the latter repeats some of the definitions and explanations of the previous chapter, so that it can be used as a standalone tutorial. Ch.5 consists of numerous specialist case studies which explore the types of questions that can be asked and answered using these methods. The majority relate to specific repertoires or works from classical to jazz to pop,⁵ while others are more conceptual as regards what music is and how it can be understood through analysis.

The purpose of this book is to introduce both analytical insights and methods, for practical use by anyone wishing to explore the structures and functions of music. Those not familiar with computational techniques or the necessary computer programmes should work carefully through ch.3 and 4, and then prepare appropriate data for processing, bearing in mind the context laid out in ch.2; some of the case studies in ch.5 may provide specific models. The formulation of answerable questions is a very important skill, and it may be wise to start with explorations of simple data, such as patterns of intervallic distribution. These are likely to lead to more complex questions, and help understand the meaning of the final results. In most cases, the support of standard musicological analysis (see ch.5.3) may be a valuable check as to the validity of the results: a well-

meaningful. Some general issues related to the limitations of the study of non-Western musics which only later (if at all) reached notated form are implicit in Tomlinson (2007) and Munyaradzi and Zimidzi (2012).

⁵ Each field has attracted different levels of interest and different applications of methods from analysts, partly contingent on the extent to which it is expressed in standard music notation; pop music is one area that seems to have been relatively poorly served so far; see Griffiths (1999). Roads (1980), p.22, also notes, ‘the severe idiomatic constraints of the formula in most popular music have almost eliminated creative composition, since the compositional rule structure and sound lexicon has become ossified’. It is interesting to consider whether this statement has become more, or less, true over the past forty years.

formulated question asked of a specific repertoire should not normally produce a very unexpected result.

The issues around the stylistic analysis of music are paralleled in many other arts, especially those whose meaning is comprised of abstract components. The semantic nature of language⁶ is treated differently (compare stylistic analysis in Shakespeare and his contemporaries, where contextual text-searching has provided many interesting and convincing results),⁷ but image components, like musical notes, can be subjected to contiguous comparison to explore (for example) an artist's colour palette. Architecture also consists of assembled components, often referencing previous styles or structures. Both subjects may have implications (as in music) for chronology or attribution. In the natural world, digital photographs of animal markings or clouds also yield potential classification information of a similar kind.⁸ In life, much of what humans experience through their senses⁹ is unconsciously compared with previous experiences, and computational analysis is merely one technical formulation of some of these experiences, such as those received through hearing.¹⁰

1.2 A History of music analysis

Music analysis – the systematic examination of musical scores in order to understand their content – has a long history. It has always been part of composition teaching, as it was through models that students learned to understand the rules, scope and style of the music of their time, especially in the time before textbooks. For example, Marpurg's influential *Abhandlung von der Fuge* (1753-54) includes a number of very detailed fugal analyses,¹¹ while Albrechtsberger's 1790 *Gründliche Anweisung zur Composition* includes an 'Explanation' of a fugue example which is a

⁶ It is interesting to compare indigenous whistled languages (which are partially analogous to music) with those having explicit word content; see Meyer (2021).

⁷ For the background to this, see Shannon (1951), Craig and Kinney (2009), Segarra et al. (2016) and Freebury Jones (2022).

⁸ Stevens et al. (2007).

⁹ The parallel case of touch is less obvious, but smell (for example, perfume design) may also be regarded as 'measurable', even if only recordable through chemical analysis.

¹⁰ Hatfield (2013). Aural analysis skills remain an underused part of undergraduate music education.

¹¹ Mann (1965), pp.180-184, 204-212.

structural analysis, and the student is then instructed to analyze subsequent examples in the same way, in order to understand them.¹² By the beginning of the 19th century Kollmann even produced by name *A Series of Analyzed Fugues*, Op.10.¹³ Such understanding through close study was similarly part of the way that those learning painting, sculpture, poetry, literature and other creative disciplines approached their subject – and with an expectation that it was the finest examples that had the most to teach.

As different styles developed over the centuries, eventually scholars wished to understand not only the music of their own time, but the music of the past, and this was also paralleled in the other arts from the early 18th century. Once no longer a living tradition, some of the maxims for historical composition were lost, and the only way for an 18th century musician to understand the music of (for example) the 16th century was to engage with it directly, with the assistance of such historical and partial works of theory that had survived. An appreciation of the changes in musical style over the centuries, coupled with the building of private and institutional collections of manuscript and printed scores, led to the creation of the first narratives of music history, as seen in the rival published volumes of Charles Burney (1776-1789) and John Hawkins (1776).¹⁴ At the same time, the living traditions of music continued with analysis for the purposes of teaching and understanding composition – it can be argued that the very structures of creative composition from Beethoven onwards were partly formed by such analytical insights¹⁵ – and the subject became part of both music history and a new formal discipline, music analysis.¹⁶ The transition can early be seen in Reicha's *Traité de mélodie* and Czerny's *School of Practical Composition*.¹⁷ Engagement with scores showed that simple observations about themes, key structures and so on were just a starting point, and rival approaches arose from the end of the 19th century, all claiming their own special insights; for some,

¹² Albrechtsberger (1855), pp.159-160. Taneiev's 1909 formal contrapuntal method (Taneiev (1962)) uses algebraic methods, and is in some sense a forerunner of later mathematically-driven methods, the most recent of which is Zagny (2022).

¹³ Kollmann (1809); see Kassler (2008).

¹⁴ Burney (1776-1789), Hawkins (1776).

¹⁵ In more recent times, Pierre Boulez described a compositional strategy called *analyse créatrice*, creative analysis, where 'analytical results are used as germs to create new compositions'; Boissière (2002). See also the discussion in Mazzola and Thalmann (2009) and the Beethoven-derived example in Mazzola et al. (2020), p.119-127.

¹⁶ For the 18th-century origins of music analysis, see Comen (2019).

¹⁷ Reicha (1813), Czerny (c.1849).

the highly technical aspects of these new traditions were part of their attraction. The more ‘scientific’ such studies became, the more divorced they seemed from the realities of practical music studies,¹⁸ and elements of this divergence remain in many university curricula to this day. The danger also remains of a pedagogical tendency that one can understand music ‘by reading textbooks rather than going to the music itself’.¹⁹

The view that music was a subject for scientific study originally arose as a result of the Enlightenment, where post-Aristotelian collectors and taxonomists started to systematically categorize many aspects of the world in order to help understand it; the French *Encyclopédie ou dictionnaire raisonné des sciences, des arts et des métiers* (1751-1772) and the works of botanist Carl Linnaeus (1707-1778) are key examples.²⁰ Textbooks for historical composition originated in the early 18th century, most obviously for archaic but respected styles that were still part of compositional training (Fux on Palestrina,²¹ for example), and from the early 19th century were followed by similar studies of fugue (there are numerous examples, some intended as textbooks for the newly-created conservatoires of France, Germany and Britain, following the proto-conservatoires of Baroque Italy). Such explicit analysis as is included there is to justify the rules presented to students (‘Palestrina does not do that’, ‘Bach does this’), but further studies of music showed that the examination of components within the music could be a fascinating subject in its own right. Priority was given to Classical and Romantic repertoire – especially the masterpieces of the late 18th and early 19th centuries – partly with a view to explaining (or even justifying) their greatness.²² Some analysts focused on themes and thematic transformation, whereas others were interested in key

¹⁸ See, for example, Lester (1998).

¹⁹ Cook (1996), p.144.

²⁰ There were earlier categorization-driven examples in music, such as Burmeister’s structured five-step analysis of the Lassus motet ‘In me transierunt irae tuae’ (Joachim Burmeister, *Musica Poetica*, 1606): 1) investigation of the mode; (2) investigation of the melodic genus; (3) investigation of the type of polyphony; (4) consideration of the quality; (5) sectioning of the piece into affections of periods; see Comen (2019), pp.15-16.

²¹ Johann Joseph Fux, *Gradus ad Parnassum* (Vienna, 1725), trans Mann (1965).

²² For examples, see Bent (1994). One of Schenker’s early essays is even entitled ‘The Mission of German Genius’ (Clark (2007), p.142). ‘Bad’ music is invariably neglected, but may equally benefit from analytical investigation, on the grounds that understanding why a piece of music does *not* work well might provide insights equal to the study of music that does work well.

structures and tonal relationships;²³ however, those things that were not so amenable to these analytical methods, such as timbre,²⁴ texture or gesture,²⁵ tended to be downgraded in favour of reductionist, anatomical, approaches, where a movement could be graphically or musically represented in compressed form on a printed page²⁶ – the musical skeleton of the work. However, the discussion of such a reduction could itself be extensive: see for example the four-page demonstration analysis by John White of a Rameau minuet only one page in length.²⁷

From the early 20th century until about the 1980s, concertgoers and gramophone society members alike represented additional classes of readers and listeners looking for musical education or training in what was then called ‘music appreciation’, and specially marked-up scores²⁸ showed them how to listen – in the sense of what part to follow – for those that could read music. Publishers like Boosey & Hawkes also provided analytical information to accompany some of their miniature scores, such as the concise four-page insert in Britten’s String Quartet No.2.²⁹ Donald Tovey’s descriptive analyses³⁰ fulfilled an important purpose for those categories of informed listener, but for serious analysts of a later period these did not penetrate the ‘truth’ of the music (that is, Tovey was more interested in the surface detail and points of change, not the underlying compositional structures). The broadening of the repertoire from about 1900 demanded further specialized tools – for example, in serial music – while early music was and still is something of a Cinderella repertoire. This is partly because modal content was less amenable to the strongly tonal methods of analysis that then predominated, and partly because the early music revival³¹ was more performer-driven, with active scholars

²³ Cook (1987), p.13.

²⁴ See White (1976), pp.157-166 on timbre.

²⁵ White (1976), p.15 also considers the idea of ‘growth’, where the options include repetition, development, variation and use of new material.

²⁶ One practical example of this which is still (in some sense) ‘complete’ is the vocal score with piano reduction of an opera or oratorio, a musical equivalent of jpeg compression; see Capdevila (2017).

²⁷ White (1976), pp.19-24.

²⁸ See Weir (1935), where themes and other items of interest are indicated by arrows.

²⁹ Stein (1943). The same insert system was also used in some Heugel miniature scores, such as that provided by Gustave Bret for Haydn’s Symphony No.92 (Paris, 1951).

³⁰ Tovey (1944).

³¹ See Haskell (1988), Sherman (1997) and Haynes (2007).

tending to be more interested in editing than in analysis: the ‘what’ of the music rather than the ‘how’. Nevertheless, many recent analytical works show the possibilities for earlier repertoires, such as those by Karol Berger, David Collingwood, Julie Cumming, Laurence Dreyfus, Cristle Collins Judd, Joel Lester and Peter Schubert,³² even if there is a great deal of unfinished (and unstarted) business remaining. There is also the problem of using analytical approaches which diverge so far from historical conceptions of composition that they may lack the validity they claim.³³ Similarly, much non-serial contemporary music (such as electronic music) and world music awaits proper analytic investigation, as well as the development of some of the tools with which to do it.³⁴

Although historically from the Ancient Greeks onwards there were numerous debates as to what music was *for* (including spiritual enhancement, moral improvement, education and entertainment), equally important discussions as to how it *works* (music perception) and how it is *created* (analysis) are of relatively recent date. Performers have always been aware that they are ‘communicating’ music, and listeners have always been aware of the ways in which music is experienced, but the intellectual desire to dissect in order to understand and appreciate (analysis) came later. The scientific basis on which analysts worked, and the formal methods of their descriptions, helped create (as was the intention) the idea that such analysis was objective, and academic study thereafter seemed to focus on comparative methods rather than on the intrinsic truth or value of such analyses; they easily became an end in themselves. At the same time, an analytical canon of works (drawn mostly from the period of Bach to Brahms) grew up, and works such as Bach’s Prelude in C, BWV846/i, Mozart’s A major piano variations K331/i and various of Chopin’s Mazurkas remain a very familiar part of that canon. The continued relevance of such core works can be seen in textbooks such as Nick Cook’s *Analysis through composition: Principles of the Classical Style*³⁵ and William Caplin’s *Analyzing Classical Form*.³⁶ And the existence

³² Aldrich (1969), Berger (1980), Schulenberg (1985), Lester (1992), Schubert (1994), Dreyfus (1996), Judd (1998), Collingwood (2008), Schubert and Cumming (2015). For recent work specifically on fugue, see Giraud et al. (2015) and Rydén (2020).

³³ Lester (1998).

³⁴ See Stock (1993) for the application of Schenkerian methods to non-Western music traditions; and Cohen and Katz (2013) for analytical techniques applied to Middle Eastern folk music,

³⁵ Cook (1996).

³⁶ Caplin (2013), which was preceded by Caplin (1998).

of an analytical canon itself raises a suspicion that some analysts are (unlike working musicians) more interested in methods and applications rather than understanding specific repertoires.³⁷ In particular, an excessive focus on formal structures may be misplaced: ‘Too many theorists seem to believe that only when we find hidden structures or non-obvious aspects of structure are we doing “real” analysis. This focus causes us both to concentrate too often on hidden aspects of music, and at the same time to avoid other obvious areas that rarely seem to catch our attention, even though they are quite often central to performers’.³⁸ Peter Schubert points out that ‘It is a problematic method that synthesizes a theory partly on the basis of an author’s work and then questions that work in terms of other theory or a perception of practice’.³⁹ It should also be remembered that analysis does not always have to take the form of ‘whole work’ study, and that editorial examination of specific components such as textual variants and errors is also important.⁴⁰

The many different methods – Schenkerian, semiotic and set theory, among others – include those of Hugo Riemann (1849-1919), Heinrich Schenker (1866-1935), Rudolph Reti (1885-1957), Leonard Meyer (1918-2017), Allen Forte (1926-2014), Jean-Jacques Nattiez (1945-) and Joel Lester (1945-).⁴¹ A fuller list can be found in Cook’s *Guide to Music Analysis*,⁴² and the reader should familiarize themselves with at least the basic principles of each method.⁴³ Interestingly, some of these methods are more than a century old, while none is more recent than forty years; the arrival of computational methods of analysis, with their ability to deal comparatively with larger amounts of data and with multiple perspectives, thus seems well timed.

³⁷ For a discussion of the neglected topic of personal musical taste, see Greenberg et al. (2015) and Gasser (2019); in that respect, it is also worth noting that Schäfer et al. (2013) assert that the two main reasons why people listen to music are to ‘regulate arousal and mood’, and to ‘achieve self-awareness’; these aspects ought also to be the concern of music analysts: not so much what the music is, but how it is heard and used. For music recommendation software, see Seaver (2022).

³⁸ Lester (1998).

³⁹ Schubert (1994), p.7.

⁴⁰ See, for example, Hughes (2007).

⁴¹ See Riemann (1877), Riemann (1887), Schenker (1932), Salzer (1962), Reti (1951), Meyer (1973), Forte (1973), Nattiez (1990), Lester (1989) and Lester (1992).

⁴² Cook (1987).

⁴³ Cook (1987), ch.7 offers a useful general introduction for beginners.

Analytical methods vary in their effectiveness depending on the history and style of the works being examined: the proverbial hammer must fit the nail. As noted above, tonal analysis methods work well for tonal music, but not for Medieval or serial pieces.⁴⁴ An understanding of both the selected material and its context is important, at both high level and close-reading level (where the meaning of a single note can vary according to its location in a chord or melody). Essentially, the analyst must understand both the tools to hand, and know what to ask of them and of the music under investigation. Similarly, single-work analysis is a different matter from comparative analysis, or where tools are used to quantitatively study stylistic trends and features in music (authorship, chronology, stylistic development, and so on – see ch.2.3). It may also be used to assess global features such as unity or coherence (however defined),⁴⁵ but analysis is rarely simple measurement, or indeed quality assessment (see ch.2.9). For example, ‘unity’ can be a positive quality in a work, but it is not a measure of the success of a piece; nor (for example) is skill in handling larger musical forms a guarantee of success for a composer; there are numerous cases of composers stretching their natural musical gifts and inclinations beyond such limitations.⁴⁶ A work designed to be excessively repetitive (Ravel’s *Bolero*, Satie’s *Vexations*) also poses challenges as to what analysis can achieve. Christopher Hatch and David W. Bernstein suggest a series of questions than can be asked of each analytic method: ‘Does it tell us something about the piece that we did not know before? Is that “something” a recognizably important component in the piece? Does the analysis explain anomalies? Does it provide intellectual justification for what we have already felt about the piece? And does it use a method that can be fruitfully applied to fruitfully applied to other pieces?’.⁴⁷ Finally, all analytical methods necessarily have significant limitations, and that a work is difficult to analyse using a given method may be a failing of the method rather than the work in question.⁴⁸

⁴⁴ White (1976), p.9 observes that ‘music which is totally predetermined by some sort of schema is ... difficult to analyze’.

⁴⁵ White (1976), p.15 defines ‘organic unity’ as the ‘binding relationship among all parts of a composition’.

⁴⁶ It is interesting to consider whether some pieces of music have an optimal length with regard to their content.

⁴⁷ Hatch and Bernstein (1993), p.4.

⁴⁸ Interestingly, both Salzer (1952), pp.261-263 and Cook (1987), ch.10 contain sections titled ‘Problematic compositions’ and ‘Some Problem pieces’ respectively.

As well as being an independent subject of study, analysis is also traditionally described as a performer's tool; the idea is that a thorough understanding of a work's content will aid good performance ('all interpretation can be regarded as inherently analytical', in the words of Arnold Whittall).⁴⁹ And as early as the turn of the 18th century Roger North noted that a player that does not understand harmony 'can scarce be a good performer; for how should he fill his sounds with proper emphasis, when he knows nothing of the matter?'.⁵⁰ How this analysis-informed performance is actually to be done is rather more complicated,⁵¹ and while some players seem to intuitively grasp the structure of a work, and through that some of the composer's apparent intentions, others find that excessive analytical input into the decision-making processes that lead to an interpretation can be a problem in itself: the 'paralysis of analysis'. The question also needs to be asked as to which of its many components a performance should best focus on at any one time – melodic, harmonic, structural, gestural? This raises the issue of how such decisions are even manifested in performance: the actual tools for the expression of such content are surprisingly limited – dynamic, agogic, registral and so on.⁵² What would a performance of a Mozart piano sonata that explicitly prioritized the motivic content over the harmonic, or vice-versa, actually *sound* like?⁵³ (Is it – a standard criticism of the Schenkerian approach – even possible to 'express' an *Urlinie*⁵⁴ through performance?). In other words, the apparent larger scale content of the performance is determined through the details, and the relationships between those details. In that sense, one job of the performer is to guide the listener through the piece, 'explaining' its musical and dramatic content, repetitions, transformations and so on. This is where analysis is of particular value – a musician who 'understands' a work is more likely to be able to 'explain' it in performance, provided that they can keep a balance between the wood and the trees (the background and the foreground, in Schenkerian terms).

⁴⁹ Whittall (1993), p.318.

⁵⁰ Wilson (1959), p.74.

⁵¹ For further discussion, see Clarke (1999) and Rink (2002).

⁵² Scott (2014).

⁵³ Colour might be creatively used in musical notation for analytical guidance to a performer; certainly, indicating some hierarchies of meaning could be useful (compare the concept of 'depth' in Graph Theory and the use of 'Weighted trees' in dendrograms).

⁵⁴ See Clark (2007).

While analysis can categorize components and explain their relationships, the discipline is less often focused of the aesthetic result of these elements, and critics can provide an additional sense of what is intended (by the composer) or what is experienced (by the listener). For example, Eduard Hanslick's acidic review of the premiere of Bruckner's Symphony No.8 in 1892 not only identifies one aesthetic vision of the work, but some of the technical means through which that is expressed, and the (as he sees it) unsatisfactory final result:

*The nature of the work consists ... in applying Wagner's dramatic style to the symphony ... he seems even to have accepted certain Wagnerian pieces as models for symphonic construction ... Bruckner begins with a short chromatic motive, repeats it over and over again, higher and higher in the scale and on into infinity, augments it, diminishes it, offers it in contrary motion, and so on, until the listener is simply crushed under the sheer weight and monotony of this interminable lamentation.*⁵⁵

Traditional analysis could of course measure and describe motivic use in this work, but would not likely make any comment as to its effectiveness (or indeed, excessive usage); in some ways, it is an insufficiently 'critical' form of text reading. As Ian Bent notes, 'In general, analysis is more concerned with describing than with judging'.⁵⁶ And it must be remembered that the apparent 'scientific' certainty of data analysis applied to music may leave out the key perceptual feature of human response to musical sound, 'the way in which the human psyche transforms sensory input into meaning'.⁵⁷

Different forms of analysis focus on different components, principally form, melody, harmony and rhythm. Their very notation systems determine what is and what can be presented to the reader, whether using figured bass, Schenkerian notation, semiotics, set theory or melodic contours (the Charles Adams method),⁵⁸ or indeed any combination of these (for example, figured bass-plus-Schenker). However, there is a limit to the quantity and type of information that can be presented in static, two-dimensional forms - which is partly a reason for the intrinsically reductionist nature of analysis - nor can they express complex component interrelationships of the kind that real music represents. For example, a

⁵⁵ Pleasants (1963), p.288.

⁵⁶ Bent (2001). See also the discussion in Horton (2001).

⁵⁷ Thomson (1966), p.91.

⁵⁸ Cook (1987), p.196.

straightforward harmonic analysis⁵⁹ would likely align significantly with a gestural analysis (and probably to one based on dynamics also), but combination harmonic-and-thematic analysis would be very difficult to express in useful hierarchical terms. From moment to moment, what is most significant in music (and what is perceived as such by a listener) shifts, with a fluidity and complexity that resists fixed notation or simple explanation. Similarly, the balance between stability and tension⁶⁰ – which the ear is very aware of – is hard to precisely notate. It is likely that some form of scrolling visual analysis in colour – showing stratified components and ‘quantities’, perhaps including some music notation – is possible, but, existing in time (like music), it would be both difficult to present or to annotate. This may be returning to the old saw that the best analysis of a piece of music is a piece of music (an experiment that itself needs to be tried further than Hans Keller’s Wordless Functional Analysis method).⁶¹

It is also worth considering whether there are things analysis should *not* be used for; to give two examples, Mark Hijleh argues that global music analysis ‘ought not to be aimed in any sense at demonstrating the intrinsic superiority of any musical culture, system, or style over any other’,⁶² and James Grier notes that analysis ought not be used to ‘correct’ a composer, as in Jacques-Louis Monod’s editorial changes to the score of Schoenberg’s *A Survivor from Warsaw*.⁶³

1.3 Musical style: analysis and computation

The problem of identifying musical styles using quantitative and qualitative analytical tools is central not only in musicology and music theory, but also in applications to music pattern recognition and automated music generation in a particular idiom. An overview of the key ideas follows, with a description of how these methodologies can be used to

⁵⁹ Sophisticated automated harmonic analysis is proving to be a fairly intractable problem; for one method, see Temperley (1997) and, more recently, Micchi, Gotham and Giraud (2020). Context is one of the main concerns, as it is in perceptual studies; see Tillmann, Bigand and Pineau (1998). For an automated chord labelling system, see Tymoczko et al. (2019).

⁶⁰ LaRue (1992), p.45.

⁶¹ See Keller (2001). Some composers, such as Alexander Goehr, have even provided analyses with their scores; and see also the marking of note-row themes, inversions and retrogrades in Stravinsky’s *In Memoriam Dylan Thomas* (1954).

⁶² Hijleh (2016), p.4.

⁶³ Grier (1996), pp.137-138.

systematically to study stylistic changes in different contexts by incorporating probabilistic and statistical tools as well as ideas coming from information theory, Artificial Intelligence and machine learning.

One of the most interesting problems in musicology, the history of music and musical analysis is the identification of a particular style. As early as 1956, and with reference to new technical methods, Leonard Meyer had asked the question, ‘Is it possible to develop an accurate mathematical picture of musical style which could serve as a basis for the quantification and measurement of musical information?’.⁶⁴ Developing a systematic methodology to identify and classify stylistic trends has deep theoretical implications not only in analysis, composition and musicology, but is also relevant in specific applications such as automated music generation and authorship validation. The problem is not new and has been addressed in different ways; see the work of Gerhard Nierhaus for a historical perspective,⁶⁵ and the article by David Hardoon and others using machine learning and information theoretical tools,⁶⁶ as well as ch.4. Yet the question is in many ways difficult to formulate in a precise way, if for no other reason than the notion of ‘style’ itself is elusive and escapes simple characterisations. Many books have been written on musical style and analysis, yet there is no generally accepted definition; according to several authors,⁶⁷ style could be understood as the conscious or unconscious repetition of certain patterns. If this is taken as a starting point, it is natural to look for regularities in the choice of these patterns. It makes sense to expect that pattern recognition tools, e.g. neural networks and other classification approaches, might be applied first to identify them, and then statistical methods used to characterize these regularities. There are in fact numerous ways in which analytical methodologies from outside music can be applied to its study, such as the discussion of the application of fractals.⁶⁸ In the context of automated music generation in a specific idiom much has already been done, particularly using Markov chains.⁶⁹ Tom Collins and Robin Laney’s recent study of computer-generated stylistic composition gives an account of the problem, especially as regards new

⁶⁴ Meyer (1956), p.422.

⁶⁵ Nierhaus (2009).

⁶⁶ Hardoon, Saunders and Shawe-Taylor (2004).

⁶⁷ See, for example, Meyer (1989).

⁶⁸ See Hsü and Hsü (1990).

⁶⁹ A Markov chain, a concept devised by Andrey Markov (1856-1922), is a sequence of possible events where the probability of each depends on the state of the previous event

approaches dealing with the complexities of music as a global temporal process.⁷⁰ However, in the past few years AI-generated music has become more and more common and new developments take place frequently, making it difficult to present a up-to-date account of the subject.⁷¹

Nevertheless, one important point to be noted is that AI generated music has ceased to be an academic matter only and now has many commercial applications, to the extent that – almost unnoticed – a great deal of music heard in public spaces, waiting rooms and similar environments (‘elevator music’) is generated by computers (see ch.1.6). With this trend, and for obvious reasons, since commercial interests conflict with public accessibility of the methods and tools developed by companies, it is difficult to give an account of the subject; much of the current activity in this field is only documented on the webpages of composers and sound designers using AI methods in their creative work.⁷² Another parallel development is live coding, the creation from scratch by a computer programme, generating music as part of a public performance. This is relevant to the discussion for at least two different perspectives: first, because live coding naturally raises question as to whether it is possible to develop and identify different programming styles, and to what extent this is reflected in a personal musical style in the music generated; and second, that even if in the more traditional version of live coding no AI resources are used, recently and in extended practices involving even traditional performers, these tools are being more and more frequently encountered.

1.4 The origins of musical notation

While complex musical staff notation eventually became the medium through which Western music was actually composed, it originally had a simpler function, as a notated reference to solve the limitations of human memory: ‘Unless sounds are held by the memory of man they perish, because they cannot be written down’ (Isidore of Seville, c.600).⁷³ The earliest forms of Western music notation used shaped symbols to indicate melodic contours, thereafter refining this to a more precise system which indicated notes or note groups on a stave, where each line and space

⁷⁰ Collins and Laney (2017).

⁷¹ For a survey on the early history of the subject from its origins to 1980, see Roads (1980), while Meredith (2016) brings some of this up to date.

⁷² For a recent series of studies, see Johnson, Rodríguez-Fernández and Rebelo (2023).

⁷³ Barney et al. (2006), p.95.

located a specific pitch or its chromatic alteration. Such notation also allowed for the addition of sung text, meaning that memorization of (for example) a complete annual cycle of liturgical chant was no longer necessary, and that the notated music could be more easily taught, both by ear and by learning the notation. The addition of further aligned staves and parts made the notation and composition of multiple voices possible, leading to complex polyphony. As with the writing of words, such means of notation allowed the contents of works to be fixed; thereafter, composers' views of performers' agency with regard to written music varied considerably from person to person, time and place – from notation being a compositional medium broadly interpreted by the performer, to an absolutely precise instruction as to how and what to play (compare the differences in attitude to their scores of Couperin, Debussy and Stravinsky). From the middle of the 18th century there came an increase in the quantity of 'instructional' content: to the basic Renaissance notational content of pitch and rhythm were added dynamics, ornaments, tempo markings, agogics, articulation, bowings, and so on.⁷⁴ The reasons for this include the adoption of the very flexible graphical method of engraving for publication, the wider distribution of music to places where local or national musical traditions were not known, the increasing number of works for amateurs and students, and also a sense of the composer's greater 'ownership' of their works. While some have insisted that the 'music' is not in (or not just in) the notes themselves, the composer nevertheless creates music through the notation,⁷⁵ and the score can thus be considered a valid document for analysis.⁷⁶ Although staff notation has major weaknesses in terms of representing rhythmic subtleties (for example, jazz syncopations and slides, or intentional off-the-beat pop singing), it can with care still be used for transcription of such music from recordings (see ch.5.20), and for non-Western styles and animal music (see ch.2.5).

⁷⁴ For the history of classical musical notation, see Apel (1944) and Rastall (1983).

⁷⁵ An increasing 20th-century tendency saw the introduction of chance elements, or non-specific representations of music such as graphic scores; having over-specified the content in the past, composers perhaps again wanted to be surprised by what they heard in their music.

⁷⁶ For a discussion, see Crilly (1996). It is possible to exaggerate the difference between the score data and a performance; Roger North's comment (c.1726) that Matteis's published scores contained 'nothing of his manner of playing' likely refers to his ornamentation style, not the validity of the texts themselves; Wilson (1959), p.310.

1.5 Musical data: score and sound

There are at least two ways in which music can be analyzed. The first is to consider as raw material the sound, that is an acoustic signal stored via a recording process and accessible in different possible audio formats, such as mp3. This approach enables the use of all the tools of digital signal processing, and the ability to manipulate acoustic data directly; this methodology has been successfully followed by numerous researchers.⁷⁷ However, if a notational system is chosen (such as the standard Western musical notation using staves), such a symbolic representation provides a natural way to analyze music. This perspective – symbolic or formal – is used here since it lends itself more easily to abstract questions and also because, very importantly, it allows the separation of melodic, rhythmic, harmonic and other components through the selection and editing of the score data. The symbolic approach also enables the use of methods and ideas from logic and the theory of formal languages, graph theory and so on. These provide suitable methodological frameworks through which to pose questions related to melodic, rhythmic and harmonic issues and their interplay in a more direct way than if an acoustic signal is processed directly. Moreover, issues of a more abstract nature, such as ‘recurrence’ in music or the identification of motifs or modular structures, musical form and so on, seem to be easier to express in a symbolic setting. Nevertheless, in two ch.5 case studies audio signals are used to address both stylistic questions, including Schumann’s various musical styles, and timbral issues in orchestration.

A key issue related to the use of any musical data is the actual volume of data involved. Contrary to many recent musical projects in which massive amounts of data are processed (the so-called ‘big data’ approach),⁷⁸ for many of the questions explored here, only limited and specific data is available: and typically, many musicological questions require a ‘small data’ approach. This is due to the fact that specific questions related to chronology, attribution or other stylistic issues require comparisons among limited corpora due to genre or period constraints. For instance, in studying the contested attribution of a piece (ch.5.3 and 5.5) or the stylistic evolution of a composer (ch.5.11), only particular comparison pieces by the same composer or their contemporaries can be used. In general, experience has shown that even further restrictions can be necessary and

⁷⁷ See Weihs et al. (2016).

⁷⁸ For one example relating to historical music publications, see Rose, Tuppen and Drosopoulou (2015).