System Modelling, Simulation, and Control

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TABLE OF CONTENTS

PREFACE

This book has been evolved through years of teaching the subject of System Dynamics and Control Systems for different engineering undergraduate and graduate students in different departments as introductory courses. The main objective of this book is to give a concise and short presentation of the two topics that are suitable for a semester worth of teaching. However, it is up to the instructors adopting this book should they feel it is worth two semesters of teaching. These types of textbooks (combining System Dynamics and Control Systems) are not commonly available in the market, for this reason I was inspired and thought it would be a good idea to create a brief reference textbook for engineers working in research and development, and anyone who is interested in this material to refresh their background on the subject. In doing this, the I would greatly appreciate those who are going to be utilizing this textbook to review the book's content for any possible improvements or corrections for the upcoming editions.

I would like to acknowledge the contribution received from my former colleagues while doing this book; Professor Dr. Atef Atta from the University of Alexandria in Egypt, and the late Professor Dr. Rini Akmeliawati from the University of Adelaide in Australia, their feedback, corrections, and additions in many parts of this book are what brought it to its finalized form. Furthermore, I would like to thank my daughters Maisoon and Yomna for helping me edit and modify multiple details while writing this book, and the editor Shelby Smith who spared many hours of her time to review the grammar and editing of this book. Finally, I would like to thank the College of Engineering in the University of Nevada Las Vegas in providing me the conducive environment that supported me in finalizing this book in the past year.

CHAPTER 1

INTRODUCTION AND BASIC DEFINITIONS AND CONCEPTS

System dynamics is the study of the dynamic or time-varying behaviour of a system and includes the following:

- Definition of the system, system boundaries, input variables, and output variables.
- Formulation of a dynamic model of the physical system, usually in the form of mathematical or graphical relationships determined analytically or experimentally.
- Determination of the dynamic behaviour of the system model's and the influence of system inputs on the system output variable of interest.
- Formulation of recommendations or strategies to improve the system performance through modification of the system structure or parameter values.

1.1 Basic Definitions

Systems are a set of interacting components connected in such a way that the variation or response in the state of one component affects the state of the others.

Static Systems have an output response to an input that does not change with time; i.e. the input is held constant. This means that the output has the same instantaneous relationship with the input.

Dynamic Systems have a response to an input that is not instantaneously proportional to the input or disturbance and may continue after the input is held constant. Unlike the static systems, dynamic systems can respond to input signals, disturbance signals, and initial conditions.

Inputs function of the independent variable of the differential equation, the excitation, or the forcing function to the system.

Outputs the dependent variables of the differential equation that represents the system's response.

Initial Conditions are the initial values of the system's dynamic variables.

Dynamic Variables are those variables whose dime derivatives appear in the governing equations.

Modelling is the process of identifying the principal physical dynamic effects to be considered in analyzing a system, writing the differential and algebraic equations from the conservation laws and property laws of the relevant discipline, and reducing the equations to a convenient differential equation form. A model can take several forms; physical, graphs or plots and mathematical.

Dynamic systems are found in all major engineering disciplines and include mechanical, electrical, fluid, and thermal.

1.2 Classifications of Dynamic Systems

1.2.1 Mechanical Systems

Systems that possess significant mass, inertia, and spring and energy dissipation components driven by forces, torques, and specified displacements are considered to be mechanical systems. An automobile is a good example of a dynamic mechanical system. It has a dynamic response as it speeds up, slows down, or rounds a curve in the road.

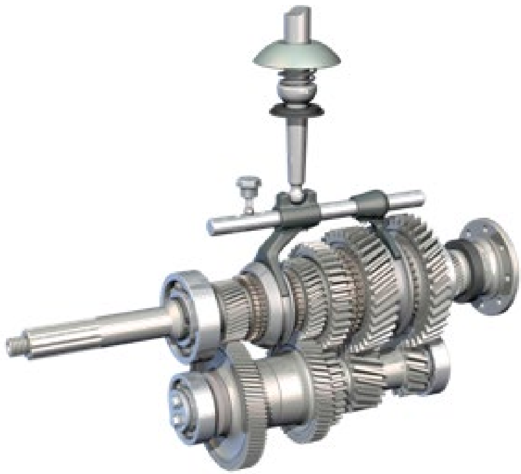

Figure 1.1 An Example of Mechanical System

1.2.2 Electrical Systems

Include circuits with resistive, capacitive, or inductive components excited by voltage or current. Electronic circuits can include transistors or amplifiers. A television receiver has a dynamic response to the beam that traces the picture on the screen.

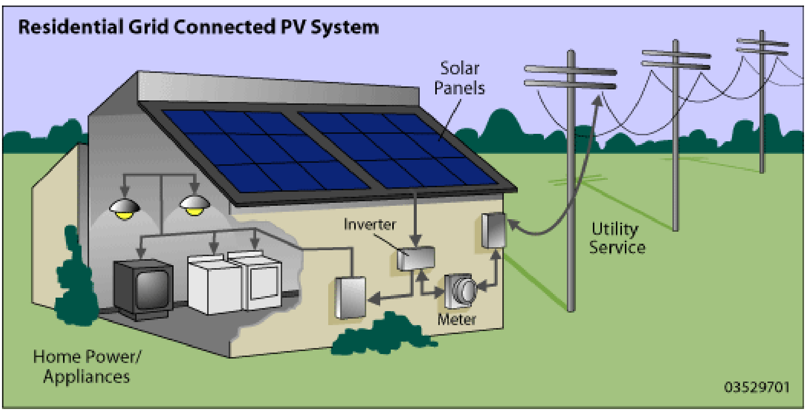

Figure 1.2 An Example of An Electrical System

1.2.3 Fluid Systems

Fluid systems employ orifices, restrictions, control valves, accumulators (capacitors), long tubes (inductors), and actuators excited by pressure or fluid flow. A city water tower has a dynamic response to the water height as a function of the amount being pumped into the tower and the amount the citizens use. Other examples are garden hoses and water pumps.

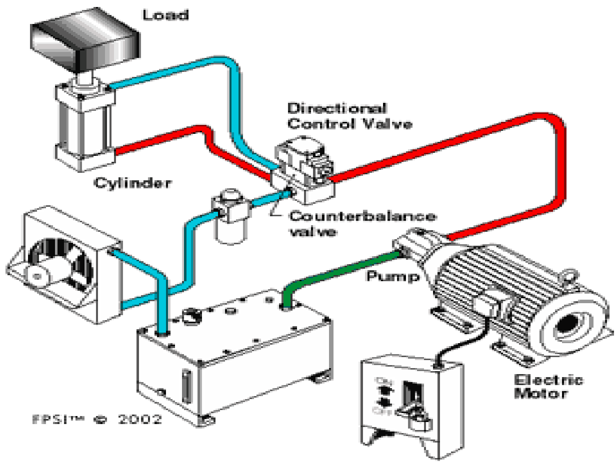

Figure 1.3 An Example of Fluid System

1.2.4 Thermal Systems

Thermal systems have components that provide resistance (conduction, convection, or radiation) and capacitance (mass and specific heat) when excited by temperature or heat flow. A heating system warming a house has a dynamic response as the temperature rises to meet the set point on the thermostat.

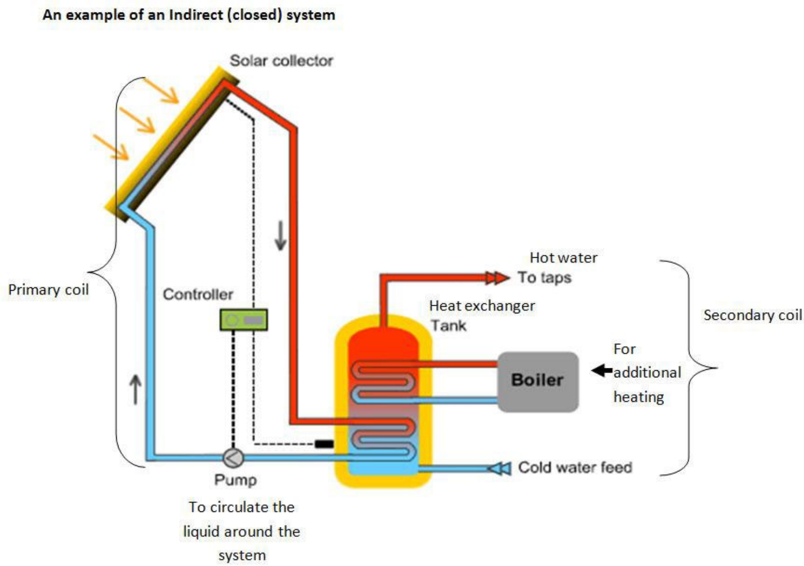

Figure 1.4 An Example of Thermal System

1.2.5 Mixed Systems

Some of the interesting dynamic systems use two or more of the previously mentioned engineering disciplines, with energy conversion between the various components.

- **Electro-mechanical**: Systems employing an electromagnetic component that converts a current into a force generally have a dynamic response. Examples are loudspeakers in a stereo system, a solenoid actuators, and an electric motor.

- **Fluid-Mechanical**: Hydraulic or pneumatic systems with fluid-mechanical conversion components exhibit dynamic behaviour. Examples are a hydraulic pump, a valve-controlled actuator, and a hydraulic motor drive.

- **Thermo-Mechanical: A** combustion engine used in a car is thermo-fluid-mechanical. It converts thermal energy into fluid power and then into mechanical power.

- **Electro-Thermal**: A space heater that uses an electric current to heat a filament, which warms the air.

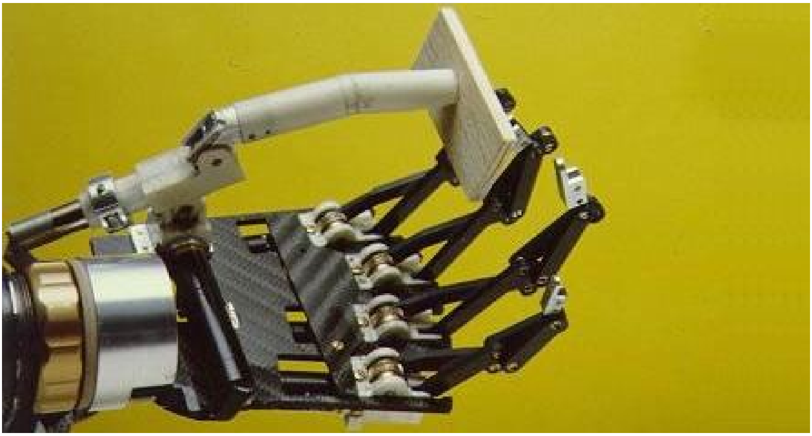

Figure 1.5 An Example of Electromechanical System

1.3 Definitions Related to Dynamic Systems Analysis

Dynamic systems can be represented by differential equations in different forms and depending on these forms different solutions are available.

Types of Dynamic System Representations

1.3.1 Classical Differential Equation

Involves terms in the dependant variable (the equated response) and some of its derivatives, summed or differenced together, with the result equated to the input function.

$$\frac{d^2x}{dt^2} + a_1\frac{dx}{dt} + a_0x = f(t) \qquad \textbf{(1.1)}$$

Subject to the initial conditions:

$$x(0) = x_0$$

$$\dot{x}(0) = \dot{x}_0$$

Where:

x(t) is the response variable (output variable)

f(t) is the external inputs

$a's$ are constants that depend upon the system parameters.

x_0 and x_0 represent the initial state and the initial rate of the system just before t=0.

The above differential equation can be written in another form as:

$D^2x + a_1Dx + a_0\, x = f(t)$ or in compact form as

$(D^2 + a_1D + a_0\ )x = f(t)$ **(1.2)**

Where D is a linear operator and is defined by:

$$Dx = \frac{dx}{dt}$$

$$D^2x = \frac{d^2x}{dt^2}$$

1.3.2 Laplace Transform Differential Equation

It is often desirable to express a system response variable normalized by the input variable as output-input ratio. For linear systems, the transfer function is defined as the ratio of the output to the input of the system with as determined from the Laplace transform:

$$(S^2 + a_1S + a_0\ )X(s) = F(S) \qquad \textbf{(1.3)}$$

The Laplace transform converts the system equation from time domain (Differential Equation) to S-domain (algebraic equation). Solving Eq. (1.3) for the output-input ratio gives the transfer function.

$$\frac{X(S)}{F(S)} = \frac{1}{[S^2+a_1S+a_0\ ]} \qquad \textbf{(1.4)}$$

Or in time domain

$$\frac{x(t)}{f(t)} = \frac{1}{[D^2+a_1D+a_0\ ]} \quad \textbf{(1.5)}$$

1.3.3 State-Space Differential Equations

Form a set of simultaneous first-order differential equations to be solved by numerical techniques easily and accurately. The state variables are the dependent variables of each first-order differential equation and represent the dynamic response variables of the system. For example:

$$x = a_1x + a_2y + f(t) \quad \textbf{(1.6)}$$

$$y = a_3xy\ sin\ (x) + g(t)$$

$$z = a_4xz + a_5e^{-yt} + h(t)$$

$$x(0) = x_0$$

$$y(0) = y_0$$

$$z(0) = z_0$$

1.4 Order and Degree of Differential Equations

(a) Ordinary differential equations, are the difference of the highest and lowest derivatives in the equation. For example, Eq. (1.1) is of second order.

(b) System of simultaneous equations, are the number of independent derivatives in all of the equations. For example, Eq. (1.6) represents third-order system. Consider the following differential equations.

$$\boldsymbol{\frac{d^3y}{dx^3} - (\frac{dy}{dx})^2 + 4y = 4e^x\, cos\, x}$$

Ordinary differential equation only one independent variable involved: x

$$\rho C_p \frac{\partial T}{\partial \theta} = k(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2})$$

Partial differential equation: more than one independent variable involved: x, y, z, θ

3rd order O.D.E and 1st degree O.D.E.

$$(\frac{dy}{dx})^2 + 4y = 4e^x \cos x$$

1st order O.D.E and 2nd degree O.D.E.

Differential equations are said to be non-linear if any products exist between the dependent variable and its derivatives, or between the derivatives themselves.

$$\frac{d^3y}{dx^3} - (\frac{dy}{dx})^2 + 4y = 4e^x \cos x$$

Product between two derivatives ---- non-linear

$$\frac{dy}{dx} + 4y^2 = \cos x$$

Product between dependent variables themselves ---- non-linear.

Linear and Nonlinear Systems

A linear function has the following two properties:

$$f(ax) = af(x) \quad \textbf{(1.7)}$$

$$f(x_1 + x_2) = f(x_1) + f(x_2) \quad \textbf{(1.8)}$$

Equation (1.7) indicates that a multiple of the input results in a multiple of the output, while the second means a sum of the input results in a sum of the outputs. Eq. (1.7) can be written in the form:

$$f(a_1x_1 + a_2x_2) = a_1f(x_1) + a_2f(x_2) \quad \textbf{(1.9)}$$

- Linear Combination: If we have

$$L = ax + by \quad \textbf{(1.10)}$$

Where a *and b* are constants, L is said to be a linear combination of x and y.

- Linear Differential Equation: is an equation formed by a linear combination of the derivatives of the system.

$$L = ax + by + cz \tag{1.11}$$

where $a, b\ and\ c$ are constants.

- Linear System: is described by linear algebraic or differential equations.
- Nonlinear System: has nonlinear combination of variables and their derivatives (product of two variables, square of variable, trigonometric) as explained previously for the differential equations.

Types of Solution

There are two types of solutions: analytical and computational.

- Analytical Solution: is the mathematical expression of the dependent variable as a function of time. It requires knowledge of the initial conditions and the inputs as explicit functions.

- Computational Solution: can be found by numerical integration, which is the process of computing an approximate solution to the integral of a derivative function by a numerical algorithm.

1.5 Modelling of Dynamic Systems

The main objective of the modelling technique is to obtain the differential equation representing the system before going to simulation. Valuable information can be extracted from the differential equation of the system in terms of linearity, degree, order, continuity, time-varying and so on. The modelling procedure is summarized in Figure 1.6 as:

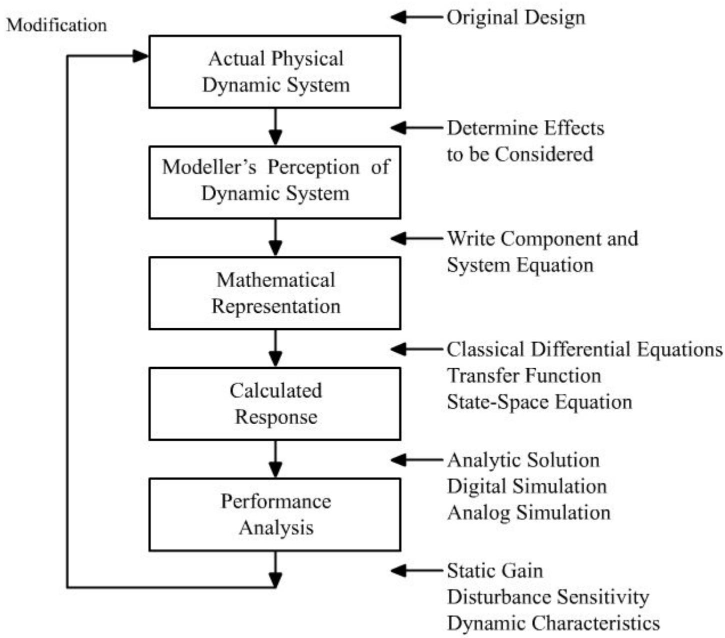

Figure 1.6: Modelling of Dynamic Systems.

The dynamic systems can be classified based on the equation of motion as follows:

TABLE 1.1 Classifications of System Models:

Type of Model	Classification Criterion	Type of Model Equation
Nonlinear	Principle of superposition does not apply	Nonlinear differential equation
Linear	Principle of superposition applies	Linear differential equations
Distributed	Dependent variables are functions of partial spatial coordinates and time	Partial differential equations
Lumped	Dependent variables independent of spatial variables	Ordinary differential equations
Time-varying	Model parameters vary in time	Differential equations with time-varying parameters
Stationary	Model parameters constant in time	Differential equations with constant parameters
Continuous	Dependent variables defined over continuous range of independent variable	Differential equations
Discrete	Dependent variables defined only for distinct values of independent variables·	Difference equations

1.6.1 What is Simulation?

Simulation is the imitation of a dynamic system using a computer-based model developed in order to evaluate and improve system performance. In practice, simulation is usually performed using commercial simulation software, such as MATLAB that have modelling constructs specifically designed to capturing the dynamic behaviours of systems.

1.6.2 Why Simulate?

Trial-and-error approaches are expensive, time consuming, and often disruptive. Rather than trusting design decisions to chance, simulation provides a way to validate whether or not the best decisions are being made. Simulation avoids the expensive, time-consuming, and disruptive nature of the trial-and-error approach.

By using a computer to model a system before the latter is built, or to test operating policies before they are implemented, many of the pit-falls, which are often encountered in the start-up of a new system or the modification of an existing system, can be avoided.

1.6.3 Performing Simulation

Simulation is nearly always performed as part of a larger process of system design or process improvement. Alternative solutions are generated and evaluated, and the best solution is selected and implemented. Simulation comes into play during the evaluation phase.

The procedure for performing simulation follows the scientific method:

- Formulating a hypothesis
- Setting up an experiment.
- Testing the hypothesis through experimentation.
- Drawing conclusions about the validity of the hypothesis.

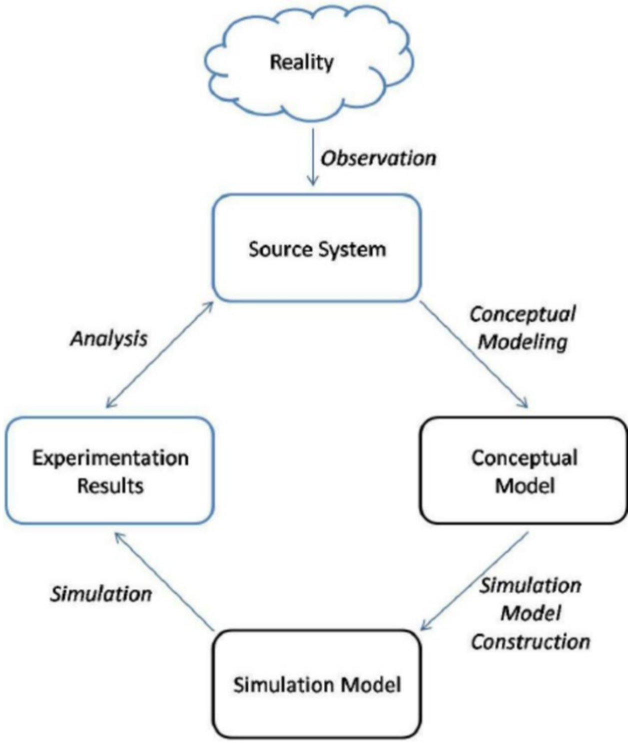

Figure 1.7: Simulation Provides a Virtual Way of Doing System Experimentation

The simulation procedure can be summarized in the following figure:

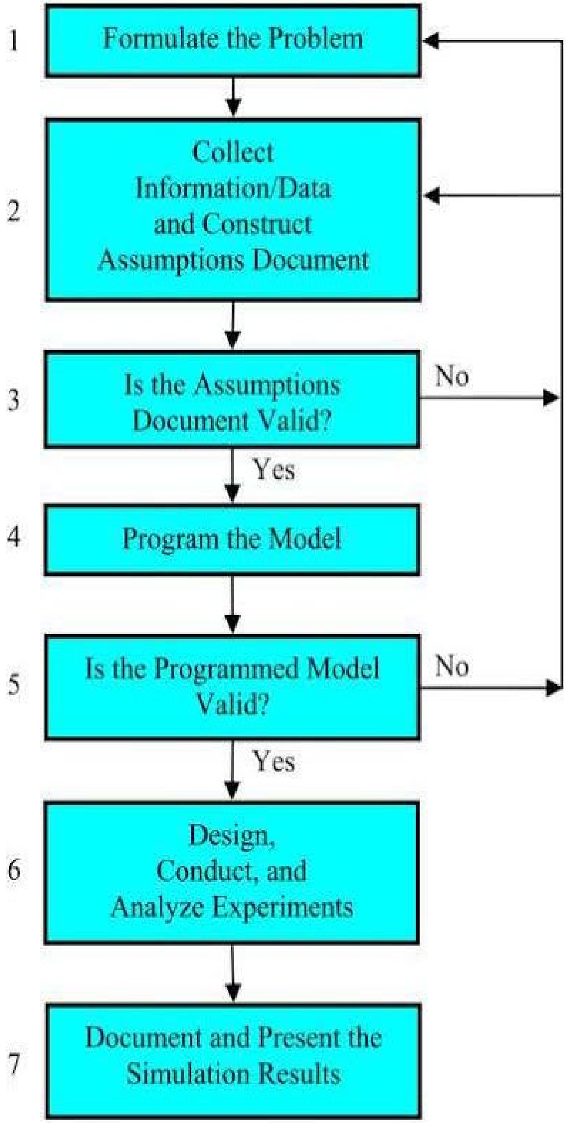

Figure 1.8 Simulation Procedure

Utility of Modelling and Simulation:

The major advantages of modelling a system and analysing its response are:

1. It allows us to predict the behaviour of the system before it is built (virtual prototyping).

2. It can analyse the performance of an existing system with the intent of improving the static or dynamic behaviour of the system.

3. It can determine what might happen to a system with an unusual input or condition without exposing the actual system to danger.

CHAPTER 2

MODELS OF DYNAMIC SYSTEMS AND SYSTEMS' SIMILARITY

2.1 – Formulation of Models for Engineering System

As we mentioned previously, to simulate or control any dynamical systems, the equations of motion or the mathematical model of the system must be derived. These mathematical models for dynamic systems can be derived from the conservation laws of physics and the engineering or material properties of system components. The preferred forms are:

1. The classical representation of a single n^{th}-order differential equation (analytical solution approach).

2. The transfer function, which gives the output in terms of the input (Laplace transform solution for control systems).

3. The state-space representation of n-simultaneous first order differential equations (computational solution approach).

2.1.1 – Governing Laws of Engineering Systems

1 – Linear Momentum Principle

The principal of linear impulse-momentum states that the translational momentum is equal to the applied impulse. Newton's second law of motion was derived from this principal which can be applied to the motion of a rigid body or a fluid. In the time-differential format, the sum of all forces acting on a mass particle causes a rate of change in the momentum with respect to time in the form:

$$\sum F_{net} = \frac{d}{dt}(mv) \tag{2.1}$$

which represents the mathematical form of the Newton's second law of motion. Using D'Alembert principle for fixed-mass, it yields:

$$\sum F_{net} - \frac{d}{dt}(mv) = 0 \tag{2.2}$$

2 - Angular Momentum Principle

This principle is similar to the principle of linear momentum but it can be applied for angular motion. The principle of angular impulse-momentum states that; the angular impulse is equal to the rate of change of angular momentum with respect to time. For rigid bodies, the angular momentum equals the product of the mass moment of inertia J and the rotational velocity both taken with respect to a fixed axis of rotation.

In time-differential format, the sum of all torques and moments about an axis through the centre of mass of a body or about a body's fixed axis of rotation is equal to the rate of change, with respect to time, of the angular momentum for that axis. Mathematically, this concept represents Newton's second law for angular motion as:

$$\sum T_{net} = \frac{d}{dt}(J\omega) \tag{2.3}$$

where T_{net} is the net external torque and/or moment, and ω is the angular velocity.

Similarly, using D' Alembert principal yields:

$$\sum T_{net} - \frac{d}{dt}(J\omega) = 0 \tag{2.4}$$

3 - Conservation of Charge

This law is usually applied to electrical circuit systems. Kirchhoff's law states that the sum of all current at a node of an electric circuit is equal to the rate at which charge is being stored at the node.

$$\sum I_{node} = \frac{dQ}{dt} = C\frac{de}{dt} \quad \textbf{(2.5)}$$

where C is the capacitance. Given we consider the charge can be stored only in capacitor C

$$\sum I_{node} - C\frac{de}{dt} = 0 \quad \textbf{(2.6)}$$

4 - Conservation of Mass

This principle can be applied to fluid systems which states that; the net mass flow rate at a location is equal to the rate of change, with respect to time, of the mass at that location.

$$\sum \dot{m} = \frac{d}{dt}(\rho V) = \rho\dot{V} + \dot{\rho}V \quad \textbf{(2.7)}$$

Where ρ is the mass density and V is the volume. If we consider the mass is stored in a fluid capacitor, then:

$$\sum \dot{m} - \frac{d}{dt}(\rho V) = 0 \quad \textbf{(2.8)}$$

5 - Conservation of Energy

This law can be referred to as the first law of thermodynamics which states that the sum of all power (heat transfer, mechanical power, and the thermal power) in and out of the system is equal to the rate at which energy is being stored in a control volume of the system.

$$\sum Q_h - \dot{W} - \dot{m}_{net}[h + \frac{v^2}{2g} + z] = \frac{d}{dt}(mu + \frac{mv^2}{2} + mz)_{cv} \quad \textbf{(2.9)}$$

For no significant energy exchange and no energy is being stored, the above equation is reduced to Bernoulli equation along stream line:

$$\frac{P}{\rho} + \frac{v^2}{2g} + z = \text{constant} \tag{2.10}$$

2.1.2 – Property Laws for Engineering Systems

The property laws are derived from the various special properties of each discipline in engineering, and may also be derived from geometric or mathematical properties as well.

1 – Mechanical Systems

By mechanical system we mean any combinations of mass, spring and damper in translation or rotation motions. Due to the relative motion between these moving and rotating parts, two types of friction exist:

- **Viscous Friction:** represents damping and may be linear or nonlinear.
- **Coulomb Friction:** this kind is always nonlinear and may be relatively difficult to handle analytically.
- **Spring Stiffness:** the relation between stress and strain determines the value of spring stiffness for a given size and diameter.
- **Mechanical Inductance:** this can be represented by the mass or rotational inertia property.

2 – Electrical Systems

- **Resistance:** is caused by the resistivity property of the material.
- **Capacitance:** is due to the dielectric properties of the material.
- **Inductance:** is caused by the magnetic properties of wire wrapped in a coil.

3 – Fluid Systems

- **Resistance:** fluid flow causes pressure losses due to viscous shear (Hagen-Poiseuille flow) or velocity head losses in orifices (Bernoulli equation).

- **Capacitance:** is due to the compressibility of the fluid or by a compliant container.

- **Inductance:** is caused by the inertial properties of the fluid as it accelerates in a pipe.

4 – Thermal Systems

- **Resistance:** is represented by conduction, convection, and radiation.

- **Capacitance:** is due to the specific heat property of the mass.

- **Inductance:** NO Thermal Inductance.

2.2 - State-Space Representation of Differential Equation

The state-space representation of differential equations consists of a set of simultaneous first-order differential equations.

State-Variables are defined as the system variables that appear as first derivatives and can be thought of as the minimum number of variables needed to define the state of the system. For a given system there is no unique set of variables; a variety of system variables could be used.

EXAMPLE 2.1:

Consider the spring-mass-damper system of Figure 2.1 in which the motion of the mass is described by x_1 and the motion of the free end of the spring is described by x_o. (See Chapter 3 for the modelling techniques used for this mechanical system).

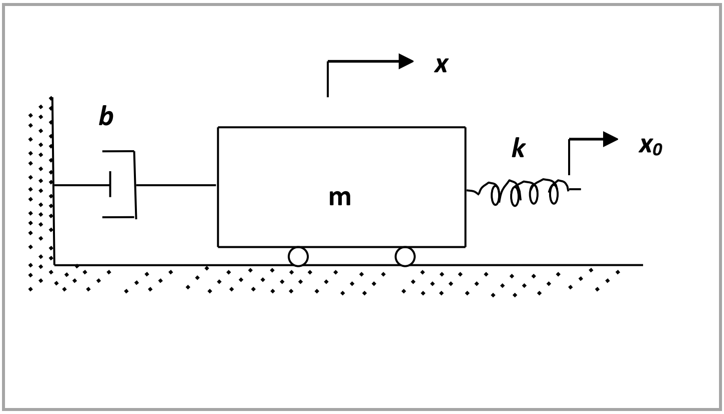

Figure 2.1: Mechanical Systems.

The second-order equation, which describes the system, is:

$$m\ddot{x} + b\dot{x} + kx = kx_0(t) \qquad \textbf{(2.11)}$$

By taking note of the relationship between the position x_1 and the velocity v_1, i.e., $v_1 = \dot{x}_1$, we can write:

$$v_1 = \dot{x}, \text{ and } \quad \dot{v}_1 = \ddot{x} \qquad \textbf{(2.12)}$$

Observation of Equations (2.30) and (2.31) reveals that there are two variables whose first derivatives occur in the formulation, and thus, two state variables can be defined as the position x and the velocity v_1. Arranging the equations into two first-order differential equations yields:

$$\dot{x} = v_1 \qquad \textbf{(2.13)}$$

$$\dot{v}_1 = -\frac{k}{m}x - \frac{b}{m}v_1 + \frac{k}{m}x_0(t) \qquad \textbf{(2.14)}$$

The two state-variable equations in this example are coupled together because both state variables appear in the equations. The input to the set of equations is $x_0(t)$.