Turbulent Two-Phase Jets of Mutually Immiscible Liquids

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Mixing and Heat Transfer

By

Ivan Kazachkov

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The basic symbols

$B_i(t)$ - function-indicator for the i-th phase in multiphase flow, $0 \le B_1(\eta) \le 1$

$B_1^{(n)}$ - piecewise continuous function-indicator of the first phase

p- pressure in a flow, ρ- density of fluid,

u, v- the longitudinal and transversal velocity components,

τ_i - turbulent stress for the i-phase

r_0 – radius of a nozzle,

u_{01}- velocity at the nozzle

a - potential core

b_1 - the internal sublayer containing an ejected liquid as a disperse phase

b_2 - the external sublayer containing the liquid outgoing from the nozzle as a disperse phase

$\tau_i = \rho_i \kappa_i \delta u_{mi} \partial u_i / \partial y$ - the "new" Prandtl's formula for turbulent stress in the phase

κ_i - the coefficient of turbulent mixing for the i-th phase

δ - the width of the mixing layer

$y = y_0 + \delta$ - total cross-section of a flow (potential core plus mixing layer)

$\eta = (y - y_0)/\delta$ - dimensionless transversal coordinate in mixing layer

$\overline{x} = x / r_0$ - dimensionless coordinate along the axis of the jet flow

$h(x) = \left(\partial^2 B_1 / \partial \eta^2\right)_{\eta=0}$ function responsible for the variation of B_1 by x, $h \le 0$

$y = y^*$- intermediate cross-section in mixing layer

$y=0$ - the jet's axis

ζ_i, δ_i - the dimensionless length of jet and its maximal radius (at the end of initial part)

$\overline{x} = \frac{x - x_t}{r_0}$, $\overline{u}_{mi} = \frac{u_{mi}}{u_{0i}}$, $\overline{u}_i = \frac{u_i}{u_{mi}}$, $i_0 = ns_0^2$, $\overline{y}_0 = y_0 / r_0$, $\overline{\delta} = \delta / r_0$,

$u_{m2} = s_0 u_{m1}$,

$\eta = (y - y_0)/\delta$, $\varsigma = \kappa_1 \overline{x}$, $s_0 = u_{02} / u_{01}$, $i_0 = ns_0^2$, $n = \rho_2 / \rho_1$,

$\kappa_{21} = \kappa_2 / \kappa_1$

κ_1, κ_2 - the coefficients of turbulent mixing in phases
x_t - the length of a transient part of jet flow
δ_t - radius of the jet at the transient cross section
h_0 and h_i – values of function h at the beginning and end of jet flow, respectively
B_{m1}- function-indicator at the axis of flow
u_{ci} - the velocity of the i-th phase in a confined jet's core
x_c- an initial cross-section of the confining channel
Q - Heat flux, W/m^2
t - Time, c
T - Temperature, K; θ - dimensionless temperature
u,v,w- velocity vector $\vec{v}$ components in Cartesian xyz or cylindrical $r\varphi z$ coordinate system, m/s
V - Volume, m^3
μ - Dynamic viscosity coefficient, $Pa{\cdot}s$
q_c - the coolant's flow rate
h_{lv} - heat of vaporization
T_{20}, T_2 - the initial and current temperatures of melt, respectively
M_2 - mass of melt in a pool cooled by volatile coolant from nozzles

Indices and mathematical symbols:

0 - parameters at the inlet
m - parameters of the axis of flow
w - parameters at the wall of channel
c - surrounding medium
$*$- parameters of the flow in some intermediate section, e.g. middle
s,l - Parameters of solid and liquid phases, correspondingly
Δ - difference of parameters

Devoted to a memory of my first mentor
Professor Alfred Ivanovych Nakorchevskii

INTRODUCTION

Multiphase multicomponent media are widely represented in various natural processes and areas of human activity. It is possible to confidently say that we deal with non-single-phase mixtures much more often than with single-phase ones. Therefore, the task of describing and studying such media is one of the most urgent and important problems of mechanics in general and the mechanics of continuous media in particular. Despite the constantly growing stream of publications on this topic and remarkable progress in separate specific multiphase flows, the mechanics of heterogeneous media is, without exaggeration, only at the beginning of its real development.

The fact is that heterogeneous media are characterized by an incredible diversity, mutual influence and complexity of effects arising due to non-uniformity. Such effects include phase transitions, chemical reactions, heat exchange, force interaction between phases, capillary effects, pulsational and chaotic movement of phases, deformation of phases, diverse other processes: collisions, crushing, coagulation of particles, etc.

The main problem of theoretical description and mathematical modeling of different multiphase and multicomponent flows and heat transfer is the construction of closed equations of motion of the mixture with given or varying physical and chemical properties of each phase and component separately, and given initial structure of the mixture.

The description of real heterogeneous mixtures is complicated for two main reasons:

- First, it complicates the description of processes in separate phases (such as compressibility, viscosity, thermal conductivity, etc.), which also occur in single-phase media.
- Secondly, there is a problem of describing the effects of interphase interaction (such as phase transitions, chemical reactions, capillary effects, exchange of momentum and energy at the interphase boundary).

Thus, the number of phenomena that must be reflected in the equations increases many times over. All this leads to the following situation: despite the fact that the general principles of building multiphase models were formulated more than 50 years ago, it is now clear that there are no any hopes of obtaining a universal equation of motion for an arbitrary multiphase medium, as it was possible to do for a homogeneous liquids (Navier-Stokes equation).

A specific combination of phases and components in multiphase multicomponent flow, and their structure, the methods of their interaction and diverse transformation in each specific problem requires efforts to obtain model equations specific for each such system. Moreover, even within the framework of one clearly formulated problem, different systems of defining equations have to be derived for different ranges of parameter values. Assessing the variety of types of heterogeneous media (suspensions, emulsions, gas suspensions, bubble media, flow of immiscible liquids like the oil and water) clearly shows the dramatic complexity of such flows. Also, the incredible variety of phenomena occuring in different multiphase flows require the construction of complex local models in each specific case. The study of the properties of heterogeneous media was concentrated within several world centers, each of which had its own specialization.

Within the framework of its specialization, each center achieved significant success, but due to a certain inertia, attempts to go beyond the limits outlined for its school were timid and episodic. The following example can be given, which the author had to face personally. For example, the Brussels school is well known in the world, successfully studying the nonlinear dynamics of reaction-diffusion processes in multicomponent media. With all the diversity of phenomena arising in chemical reactions, researchers from this school for a long time practically ignored the study of reaction processes under conditions of heat and mass transfer. On the other hand, there is the Perm hydrodynamic school, known in the world for its research in the field of thermal convection.

Despite the diversity of problems considered, the issues of convective stability of multiphase, multicomponent media were not often considered and in a very simplified formulation (binary mixture without reaction, reaction with fixed concentration of reagent, artificially imposed Arrhenius law, etc.).

At the same time, it is quite obvious that in the case of, for example, an exothermic reaction, heat is intensely released, which leads to the emergence of free convection. Convective motion, in turn, leads to mixing of reagents and intensification of the reaction. These two phenomena interact nonlinearly with each other in dynamics. Such interaction can obviously lead to completely new cross-phenomena.

The issues of controlling nonequilibrium processes in heterogeneous media are of particular interest. The presence of disturbance development mechanisms that are different in their physical nature makes heterogeneous convective flows sensitive to the effects of all kinds of external and internal factors. The study of instability mechanisms and characteristics in different situations is interesting not only from the point of view of fundamental concepts of modern hydrodynamics of multiphase media, but also in connection with the practically important task of controlling the stability of various states that arise in these media. There are two main control methods: without feedback and with feedback. The first method can be called "suppression" of an undesirable spatio-temporal dynamic regime. This method can include any external action on the system that produces the desired effect. For example, parametric action on a liquid by means of vibrations can lead to stabilization of convective movements or to their dynamic excitation (at a certain ratio between the amplitude and frequency of the force field modulation). The second method, "control", is more intelligent. Feedback in control theory is a process that results in the functioning of a system influencing the parameters on which the functioning of this system depends.

Many natural and technical processes deal with the turbulent mixing and heat transfer in the jets of mutually immiscible liquids, which represent an important class of the modern multiphase systems dynamics. In present monograph, the differential equations for axisymmetric two-dimensional stationary flow and the integral correlations in a cylindrical coordinate system are considered for the free and confined jets. The parameters of the turbulent mixing in a free jet and in two-phase flow in a chamber are modelled and analyzed.

The modelling algorithms and the results obtained may be of interest for some research and industrial tasks, where calculation of the parameters of multiphase turbulent mixing and heat transfer are important. The present work is devoted to development of the mathematical models and numerical procedures for simulation of the mixing and heat transfer features of mutually immiscible liquids in the two-fluid turbulent heterogeneous jet flow, as well as in more general case – turbulent multiphase jets.

The phenomenon of amplification of a jet's kinetic energy against the input energy of the first phase at the nozzle was revealed in numerical simulation and discussed. The experimental data with water and oil, as well as with liquid metals, confirmed the results of numerical computer simulation. One example was performed for simulation of the high-temperature melts' flow cooled by water in the hypothetic severe accidents at the nuclear power plant. The results obtained may be of interest for some research and industrial tasks, where the calculation of parameters of the multiphase turbulent mixing and heat transfer are important.

The method for mathematical modelling and simulation of the heterogeneous turbulent jets of immiscible liquids, as well as the physical experimental sensor were proposed by Prof. Nakorchevskii [1] in 1970th. It was further developed for different problems and reported in many publications since that time, e.g. [2-7] and the following ones:

- Nakorchevskii A.I., Vylegzhanin, A.N., and Gaskevich I.V. Mathematical modeling of convective heat and mass transfer in the drying of solid particles in a bed// Journal of Engineering Physics and Thermophysics, 1994, 67 (1-2), p. 721-726.
- Nakorchevskii A.I. Features and efficiency of interphase heat and mass transfer with a pulsation organization of the process// Journal of Engineering Physics and Thermophysics, 1998, 71 (2), p. 320–325.
- Nakorchevskii A.I., Basok B.I., and Chaika A.I. Pulsers with a variable geometry of working volume and the effect of processed composites on the dynamic characteristics of pulsers// Journal of Engineering Physics and Thermophysics, 1998, 71 (5), p. 757-766.
- Nakorchevskii A.I. Conjugate problems of unsteady heat and mass conduction under varying external conditions// Journal of Engineering Physics and Thermophysics, 1999, 72 (4), p. 755-765.
- Nakorchevskii A.I. Dynamics of a Pulsating Monodisperse Mixture// Theoretical Foundations of Chemical Engineering, 2000, 34 (1), p. 8-12. Translated from Teoreticheskie Osnovy Khimicheskoi Tekhnologii, 2000, 34 (1), p. 11-15.

- Nakorchevskii A.I. Abrupt Outflow of Boiling Liquid// High Temperature, 2002, 40 (6), p. 919–925. Translated from Teplofizika Vysokikh Temperatur, 2002, 40 (6), p. 986–992.
- Nakorchevskii A.I. Effect of Low-Frequency Pulses of the Carrier Phase on Interfacial Heat and Mass Transfer// Theoretical Foundations of Chemical Engineering, 2002, 36 (1), p. 30–33. Translated from Teoreticheskie Osnovy Khimicheskoi Tekhnologii, 2002, 36 (1), p. 34–38.
- Nakorchevskii A.I., Basok B.I., and Ryzhkova T.S. Hydrodynamics of rotory-pulsatory apparatuses// Journal of Engineering Physics and Thermophysics, 2002, 75 (2), p. 338-351.
- I. Nakorchevskii, M. P. Martynenko, and B. I. Basok. Calculation of chamber-type pulsers// Journal of Engineering Physics and Thermophysics, Vol. 77, No. 1, 2004, p. 156-160.
- Nakorchevskii A.I. Dynamics of discharging of a heat accumulator in an infinite main massif// Journal of Engineering Physics and Thermophysics, 2005, 78 (6), p. 1119–1126.
- Nakorchevskii A.I. Evolutionary transformation of communal thermal-power engineering// Journal of Engineering Physics and Thermophysics, 2013, 86 (1), p. 229–241.

The analog of the Navier-Stokes equations in a boundary layer approach was derived and an algorithm for solution was proposed based on polynomial approximations of the profiles [1].

The main objective of this work is development of the mathematical model and computer program for processes of mixing and heat transfer in a two-fluid turbulent heterogeneous jet of mutually immiscible liquids. Many natural and technical processes (especially - chemical-technological ones) deal with this problem, e.g. oil-water turbulent jets, where it is important to know distribution of the phases in a flow, not only flow parameters. The application of method may be useful in some other important engineering problems, therefore the method is developed for a broader field, as well as the numerical approach and computer program for simulations were prepared.

The differential equations for the axially symmetrical two-dimensional stationary flow and the integral correlations in a cylindrical coordinate system are considered and analyzed for the jet of fluid going from a nozzle into a pool of another fluid immiscible with the first one. The original method is based on the introduction of the so-called function-indicator of the phases. It was well proven on the solution of some practical tasks and successfully compared with the experimental data. The goal is to further develop and substantiate the method and prepare easy tools for its application in practice.

The results may be of interest for experts in multiphase turbulent jets as far as it allows computing important characteristics, which cannot be obtained by existing methods. The lecture course by the modeling of such flows may be given too. The method was proven successful for a few important engineering problems during a few dozens of years but mainly all the results were published in Ukraine. I was the first who worked on the realization of the method under supervision of Prof. Nakorchevskii who invited me after graduation from university to work as a mathematical modeller and programmer. This was what I did for him in early 1980 as far as he did not possess solid knowledge in mathematics and programming. Then I was working in another field of fluid dynamics.

In 2016 Prof. Nakorchevskii passed away and I looked at his achievements in the application of the method to diverse engineering problems. I was mastering my mathematical and other skills during these years. Also intensive development of the applied mathematics, informatics and numerical methods during the last decades have shown that now the further development of the method is needed on the substantially higher level to make it applicable to many other engineering problems.

Chapter 1. STATEMENT BY THE MODELING OF MULTIPHASE TURBULENT FLOW

1.1 Multiphase flows of the immiscible fluids

Some liquids like oil and water are not miscible due to what they are flowing together like a multi-phase system. For example, in a turbulent jet of such two liquids one or other are in the flow present as the drops or small droplets inside the other liquid. Therefore, the common methods of fluid dynamics do not work in this case, and do not describe this kind of flow. But in a number of applications it is crucial to know how the phases are distributed in a volume of flow and how they are interacting with each other. Without this, computing flow parameters is very approximate and can be absolutely wrong.

We first met such a problem when we were developing the new jet type steel making machine in Ukraine in the 1980-th. It was proposed the new method for multiphase jet flows, which was applied and revealed successful. Also, a special two phase sensor for experimental study to measure both the parameters of flow and amount of specific phase at each point was invented and developed. During the last 30 years, Ukrainian science was financed at a very low level and this method was not developed for application in a broader field.

Now the progress in all directions of science and technology including micro electronics and computer science give the possibility to develop the method for easier and broader application in a number of problems for chemical technology, food industry, metallurgy, etc. Also it is available to develop and propose for industrial production special micro sensors for multiphase flows. We intend to develop the method and computer programs for its realization, to make a simple tool available for scientists and engineers to compute parameters of the flows of two immiscible liquids. In the long term it is possible to spread it for more than 2 phases.

The results of the project may be useful for some important industries so that will give profit. With this method we computed parameters of the new jet type steel making process, which was successfully proven but not realized due to lack of materials working in an aggressive high temperature medium.

If such materials are now available due to achievements of material science and technology, then steel making jet machines will be developed in cooperation with material scientists.

1.2 Introduction of the Function-Indicator for Multiphase Flows

All parameters $a^l(t)$ (density of liquid, flow velocity, temperature, etc.) of a mixture in the turbulent multiphase flow are considered in accordance with the method proposed by Prof. Nakorchevskyi [1], developed and reported in many publications, e.g. [2-7]. Analog of the Navier-Stokes equations in a boundary layer approach was derived in the following form [1]:

$$\sum_{i=1}^{m}\left[\frac{\partial}{\partial x}(y\rho_i B_i u_i)+\frac{\partial}{\partial y}(y\rho_i B_i v_i)\right]=0,$$

$$\sum_{i=1}^{m}\rho_i B_i\left(u_i\frac{\partial u_i}{\partial x}+v_i\frac{\partial u_i}{\partial y}\right)=-\frac{dp}{dx}+\frac{1}{y}\frac{\partial}{\partial y}\left[y\sum_{i=1}^{m}B_i\tau_i\right]_m, \qquad (1)$$

where $a^l(t)=\sum_{i=1}^{m}B_i(t)a_i^l(t)$, $\sum_{i=1}^{m}B_i=1$. The function-indicator $B_i(t)$ was introduced for the phases in multiphase flow as follows:

$$B_i(t)=\begin{cases}1, & \text{if } i-\text{phase occupies the elementary volume } \delta\mathrm{V}\\ 0, & \text{if } i-\text{phase is outside the elementary volume } \delta\mathrm{V}\end{cases}.$$

In the stationary equations (1) for the flow of incompressible liquids, written in a cylindrical coordinate system, are: *p*- pressure, ρ- density, *u*,*v*- the longitudinal and transversal velocity components, τ_i - turbulent stress for the *i*-phase. All parameters of the flow are averaged on the characteristic interval by time chosen. Index *m* belongs to the values at the axis of the flow (symmetry axis).

The function-indicator of a phase in multiphase flow may be considered as the mathematical expectation, in contrast to the other multiphase approaches [8-10], which are based on the introduction of the volumetric specific content of a phase in multiphase flow. Nevertheless, use of the function-indicator allows computing the volumetric specific content of the phases, which have been introduced by other multiphase approaches.

1.3 Physics of the turbulent two-phase jet of immiscible fluids

Development of the model for two-phase jet of immiscible fluids is done according to Fig. 1 and schematic representation in Fig. 2, where r_0 – radius of a nozzle, u_{01}- velocity at the nozzle.

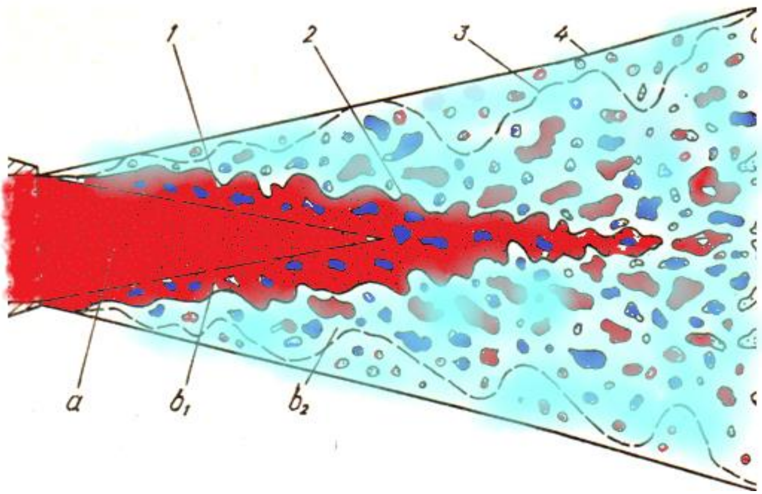

Fig. 1 General view of the multiphase turbulent jet in the pool of other liquid: formation of the phases of immiscible liquids in mixing zone

The conical surface 1 in Fig. 1 is a boundary of a homogeneous potential core *a*, the internal sublayer b_1 contains an ejected liquid as a disperse phase, while the external sublayer b_2 contains the liquid outgoing from the nozzle as a disperse phase. The internal and external sublayers are divided by the surface of phase inverse 2; and the surface 3 is dividing the turbulent and laminar flow zones 3, which is the most indefinite one; 4 is an external conical surface of the axially symmetrical mixing zone (conditionally smooth).

Due to limited regularity of the processes occurring in turbulent jets, the surfaces 2 and 3 in Fig. 1 are blurred into the corresponding regions of inversion and intermittency. The external boundary of the jet is the outer envelope surface 4 of the set of surfaces 3. The uniform velocity profile is assumed for the first liquid going from the nozzle. The surrounding liquid (phase 2) is in the rest before the first liquid starts flowing from the nozzle.

1.4 Schematic representation of the turbulent two-phase jet

The structural scheme for the mixing process in Fig. 2 is simplified:

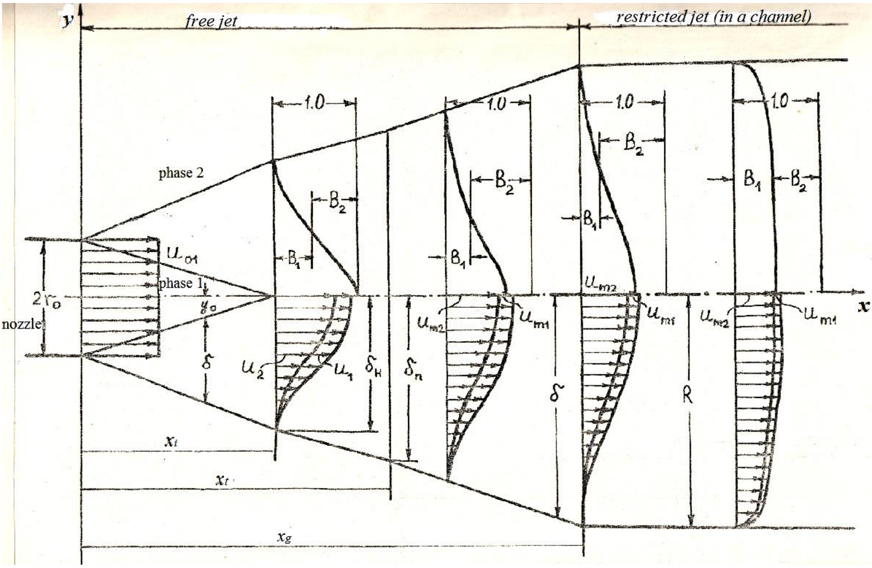

Fig. 2 Schematic representation of turbulent jet from the nozzle in surrounding immiscible liquid pool confined by the cylindrical channel at the distance x_g

the initial part of the length xi with the approximately linear boundaries for the conical surface (in cylindrical coordinate system) of the internal core of a first phase and mixing zone between internal and external boundaries of the jet. The turbulent zone contains fragments of the phases as far as immiscible liquids have behaviors like the separate phases, with their interfacial multiple surfaces. The first phase in a potential core is totally spent in an initial part of the mixing zone. Then a short transit area follows. Afterwards the main part of the two-phase jet begins, with the two phases well mixed across the entire layer of a jet.

Except for the parameters of phases, the function-indicator of phase $B_i(t)$ which shows the influence of i-th phase at each point of a space. Normally the spatial averaging of the conservation equations of mass, momentum and energy is performed for a description of multiphase flows based on the concept of volumetric phase content [8-10], which does not fit so well to the experimental study of a movement of the separate phases in a mixture. In contrast to this, an approach [1] with its special experimental technology and a micro sensor for the measurements in two-phase flows fits well for such flows. Actually all known methods of multiphase flows are well connected, and the parameters averaged by time [1] can be easily transformed to the ones averaged by space [8-10].

The external boundary of a mixing zone is determined by zero longitudinal velocity of the second phase and zero transversal velocity of the first phase (the second phase is sucked from an immovable surrounding into a mixing zone). The function-indicator of the first phase $B_1(t)$ is zero at the external interface because it is absent in the surrounding medium.

Similarly, the function-indicator $B_2(t)$ is zero on the boundary of the potential core, the interface of the first phase flowing from the nozzle. In a first approach, an influence of the mass, viscous and capillary forces are neglected. With account of the above-mentioned, the boundary conditions are [1]:

$$y=y_0,\ \ u_i=u_{0i},\ \ v_i=0,\ \ \tau_i=0,\ \ B_1=1,\ \ \partial B_1/\partial\eta=0\ ;$$

$$y=y_0+\delta,\ \ u_i=0,\ \ v_i=0,\ \ \tau_i=0,\ B_1=0. \qquad (2)$$

And dependence of the function-indicator B_1 from the longitudinal coordinate x is introduced through the second derivative of it at the boundary of a jet $y=y_0$: $\partial^2 B_1/\partial\eta^2=h(x)$.

Chapter 2. APPROXIMATION OF VELOCITY PROFILES AND FUNCTION-INDICATOR

The turbulent stress in the phase is stated by the “new” Prandtl’s formula $\tau_i=\rho_i \kappa_i \delta u_{mi} \partial u_i / \partial y$, where κ_i is the coefficient of turbulent mixing for the i-th phase, δ is the width of the mixing layer.

2.1 Polynomial approximations for velocity profiles of phases

Based on the structural scheme for the considered two-phase jet, we use the polynomial approximations for the velocity profiles across the mixing layer. The approximations have been obtained according to the boundary conditions (2) [1, 2]:

$$u_1 / u_{01} = 1 - 4\eta^3 + 3\eta^4, \qquad (3)$$

$$u_2 / u_{02} = 1 - 6\eta^2 + 8\eta^3 - 3\eta^4, \qquad (4)$$

where $\eta = \left(y - y_0\right)/\delta$ is the dimensionless coordinate across the jet mixing layer.

2.2 Polynomial approximations for the function-indicator of phases

The approximations for function-indicator across the mixing layer have been obtained based on the boundary conditions (2) [1, 2] as follows:

$$B_1 = B_1^{(0)} = 1 - \eta^3 + 0.5\eta^2(1-\eta)h(x), \quad h \in \left[-6, 0\right],$$

$$B_1 = B_1^{(1)} = 1 - 4\eta^3 + 3\eta^4 + 0.5\eta^2(1-\eta)^2 h(x), \quad h \in \left[-12, -6\right], \qquad (5)$$

$$B_1 = B_1^{(2)} = 1 - 10\eta^3 + 15\eta^4 - 6\eta^5 + 0.5\eta^2(1-\eta)^3 h(x), \quad h \in \left[-20, -12\right],$$

$$B_1 = B_1^{(3)} = 1 - 20\eta^3 + 45\eta^4 - 36\eta^5 + 10\eta^6 + 0.5\eta^2(1-\eta)^4 h(x), h \in \left[-30, -20\right],$$

$$B_1 = B_1^{(4)} = 1 - 35\eta^3 + 105\eta^4 - 126\eta^5 + 70\eta^6 - 15\eta^7 + 0.5\eta^2(1-\eta)^5 h(x),$$
$$h \in \left[-42, -30\right],$$

$$B_1^{(5)} = 1 - 56\eta^3 + 210\eta^4 - 336\eta^5 + 280\eta^6 - 120\eta^7 + 21\eta^8 + 0.5\eta^2(1-\eta)^6 h(x),$$
$$h \in \left[-56, -42\right],$$

$$B_1^{(6)} = 1 - 84\eta^3 + 378\eta^4 - 756\eta^5 + 840\eta^6 - 540\eta^7 + 189\eta^8 - 28\eta^9 +$$
$$+0.5\eta^2(1-\eta)^7 h(x), \qquad h \in [-72, -56],$$

$h(x) = \left(\partial^2 B_1 / \partial\eta^2\right)_{\eta=0}$ is responsible for the variation of B_1 by coordinate x. It may vary in the range $h \le 0$ due to the requirements of its nature: each phase in multiphase flow may have content from 0 to 1.

The first approximation $B_1(\eta)$ in (5) reveals restricted application in the range $h \in [-6, 0]$, while outside of this region it does not satisfy the boundary conditions (2) and the condition $0 \le B_1(\eta) \le 1$, $\forall \eta \in [0,1]$. Therefore, all the next approximations $B_1(\eta)$ in (5) were obtained as a transition of the piece-wise continuous function-indicator $B_1^{(n)}$ to its next approximation following the condition that the derivative by η with respect to a point $\eta = 1$ is zero up to (n+1)-th order (such condition of the smooth variation for the function-indicator).

2.3 Peculiarities of the piecewise approximations for function-indicator

These functions $B_1^{(n)}$ have the breaks at the transition points of the permanent characteristic function $B_1^{(n)}(\eta, h)$ from the one regional approximation to the other one (a first derivative has a break at those points). The advantage of such approximations is that all functions $B_1^{(n)}$ are smoothly transforming from one region by *h(x)* to the next one as shown in Fig. 3, where all polynomial approximations (3)-(6) are presented. For the function u_1 the approximation on a main part of a jet is as follows (see Fig. 2):

$$u_1 / u_{m1} = 1 - 3\eta^2 + 2\eta^3 . \qquad (6)$$

2.4 Real profiles for the multiphase flow

The real profiles for the multiphase flow are represented as a product of the corresponding profile above considered multiplied by its function-indicator.

They are shown in Fig. 4 for a few regions of variation by *h*(*x*).

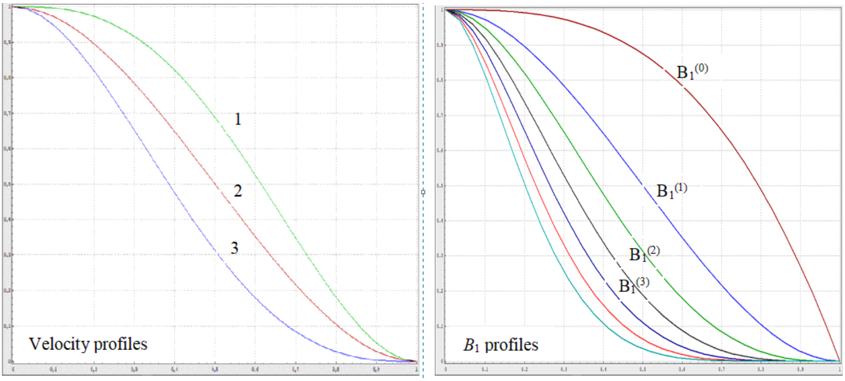

Fig. 3 Velocity and function-indicator $B_1^{(n)}$ profiles across the layer: $1\text{-}u_1/u_{01}$, $2\text{-}u_1/u_{m1}$, $3\text{-}u_2/u_{02}$

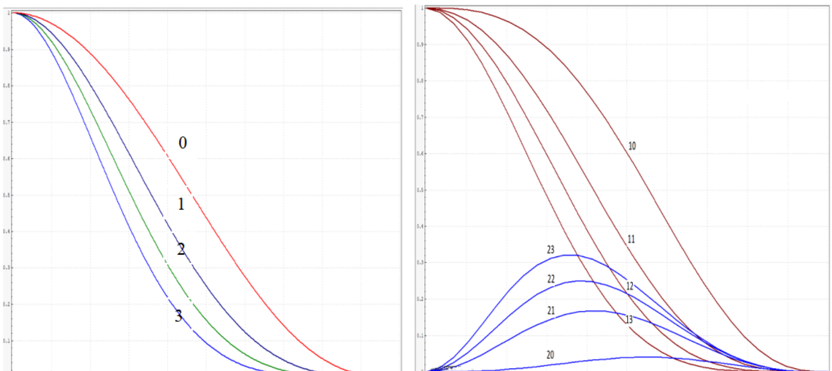

Fig. 4 The real velocity profiles of the phases for a few values of function *h*(*x*): 0, -6, -12, -20 to the right 10-13: B_1u_1/u_{01}, 20-23: B_2u_2/u_{02}; to the left 0-3: B_1u_1/u_{m1}

Each function $B_1^{(n)}$ exactly coincides with the previous one $B_1^{(n-1)}$ at the conjugation boundary, where the $B_1^{(n-1)}$ ends and the $B_1^{(n)}$ starts. Physical meaning of the varying approximations $B_1^{(n)}$ is determined by dependence of the phase distribution in the mixing layer on the density ratio of the mixing phases: the higher is density of the surrounding liquid, the shorter is penetration of the first, lighter phase, into the mixing layer. Disadvantage of such approximations is a growing complexity of the functions $B_1^{(n)}$ in analytical calculation, which is nevertheless easily fought by computer symbolic calculations. It is impossible to get a common approximation for $B_1(\eta, h)$ satisfying boundary conditions (2) in all ranges by parameter i_0 (due to requirement of variation B_1 from 0 to 1). The polynomial approximations for u_2, B_1 remain the same on a main part of the jet.

Chapter 3. INTEGRAL CORRELATIONS FOR THE INITIAL AND MAIN PARTS OF THE JET

3.1 Initial part of the jet flow

Based on (3)-(5), the integral correlations have been derived for the two-phase turbulent jet [1, 2] integrating the mass and momentum conservation equations (1) with the boundary conditions (2) for the total cross-section of a flow $y=y_0+\delta$, as well as the momentum conservation for $y=y^*$, respectively:

$$u_{01}\left(r_0^2-y_0^2\right)=2\delta\int_0^1 B_1u_1\left(y_0+\delta\eta\right)d\eta\,,\quad B_1+B_2=1\,,$$

$$\rho_1u_{01}^2\left(r_0^2-y_0^2\right)=2\delta\int_0^1\left(\rho_1B_1u_1^2+\rho_2B_2u_2^2\right)\left(y_0+\delta\eta\right)d\eta\,,$$

(7)

$$\rho_1u_{01}\left(u_{01}-u_1^*\right)y_0y_0'+\frac{d}{dx}\delta\int_0^{\eta^*}\sum_{j=1}^2\rho_jB_ju_j^2\left(y_0+\delta\eta\right)d\eta-$$

$$-\sum_{j=1}^2u_i^*\frac{d}{dx}\delta\int_0^{\eta^*}\rho_iB_iu_i\left(y_0+\delta\eta\right)d\eta=\left(y_0+\delta\eta^*\right)\sum_{j=1}^2\rho_jB_j\kappa_ju_{0j}\frac{\partial u_j^*}{\partial\eta}.$$

The first equation of the system (7) was to integrate the mass conservation equation by y, the second and the third ones – integrating the momentum conservation for the total flow of a two-phase mixture for $y=y_0+\delta$ and $y=y^*$, respectively. Parameters at $\eta=\eta^*<1$ are marked by star *.

3.2 The main part of the jet flow

The integral correlations for the main part of a jet are [1]:

$$2\int_0^{\delta}B_1u_1ydy=u_{01}r_0^2,\qquad 2\sum_{j=1}^2\int_0^{\delta}\rho_jB_ju_j^2ydy=\rho_1u_{01}^2r_0^2\,,\qquad (8)$$

$$\frac{d}{dx}\sum_{j=1}^2\int_0^{y^*}\rho_jB_ju_j^2ydy-\sum_{j=1}^2u_j^*\frac{d}{dx}\int_0^{y^*}\rho_jB_ju_jydy=y^*\sum_{j=1}^2B_j^*\tau_j^*\,,$$

where the first is the equation of mass conservation for the first phase, the second and the third – the momentum conservation equations for the total and for the part of the cross section, respectively, according to the methodology [2]. The integral correlations for the main part of a jet were obtained similarly to the initial part. The momentum equations for the total and for the part of the cross section, respectively, were obtained according to [11]. The momentum equation on the jet's axis (y=0) is used too:

$$\sum_{j=1}^{2} \rho_j B_{mj} u_{mj} \frac{du_{mj}}{dx} = 2\sum_{j=1}^{2} \left[\frac{\partial}{\partial y}\left(B_j \tau_j\right) \right]_m . \qquad (9)$$

The Mathematical model (7)-(9) including the ordinary differential equations by longitudinal coordinate x are used for analysis and numerical simulation of the basic features of turbulent stationary two-phase jets of two immiscible liquids. The function-indicator B_1 shows how much is a presence of the first phase in a selected point of mixing zone, which can be directly compared to an experimental data by two-phase sensor [1]. Therefore, a solution of the task may give both parameters of the flow together with their belonging to a particular phase.

Chapter 4. TRANSFORMATION OF THE MATHEMATICAL MODEL TO DIMENSIONLESS FORM

4.1 The initial part of a jet flow

The equation array (7) for the initial part of the jet is transformed to the following dimensionless form with the scales r_0, δ, u_{0i} for the longitudinal and transversal coordinates and velocity, respectively:

$$y_0^2 + 2\delta\sum_{j=1}^{2} y_0^{2-j}\delta^{j-1}a_j = 1, \qquad y_0^2 + 2\delta\sum_{j=1}^{2} y_0^{2-j}\delta^{j-1}\left(a_{j+2} + i_0 b_{j+2}\right) = 1,$$

$$\left(1-u_1^*\right)y_0\frac{dy_0}{d\varsigma} + \frac{d}{d\varsigma}\delta\sum_{j=1}^{2} y_0^{2-j}\delta^{j-1}\left(a_{j+2}^* + i_0 b_{j+2}^*\right) - \qquad (10)$$

$$-\frac{d}{d\varsigma}\delta\sum_{j=1}^{2} y_0^{2-j}\delta^{j-1}\left(a_j^* u_1^* + i_0 b_j^* u_2^*\right) = \left(y_0 + \delta\eta^*\right)\sum_{j=1}^{2} B_j^*\left(\frac{\partial u_j}{\partial \eta}\right)^*\left(i_0\kappa_{21}\right)^{j-1}.$$

Here the star marked values are taken by $\eta = \eta^*$. Normally it is adopted $\eta^* = 0.5$. The other assignments are as follows:

$$\bar{y}_0 = y_0 / r_0,\ \bar{\delta} = \delta / r_0,\ \eta = \left(y - y_0\right)/\delta,\ \bar{x} = x / r_0,\ \varsigma = \kappa_1\bar{x},$$

$$s_0 = u_{02}/u_{01},\ i_0 = n s_0^2,\ n = \rho_2/\rho_1,\ \kappa_{21} = \kappa_2/\kappa_1,\ a_i = a_{i1} + a_{i2}h,$$

$$a_i = \int_0^1 B_1\bar{u}_1\eta^{j-1}d\eta,\ b_i = \int_0^1 B_2\bar{u}_2\eta^{j-1}d\eta\ (i=1, 2);\ a_i = \int_0^1 B_1\bar{u}_1^2\eta^{j-1}d\eta,$$

$$b_i = b_{i1} + b_{i2}h,\quad b_i = \int_0^1 B_2\bar{u}_2^2\eta^{j-1}d\eta\ (i=3, 4);\ j=1, 2. \qquad (11)$$

4.2 Integral parameters of the initial part

The computed values of the integral parameters in (11) have been done in a range of variation of the function $h(x)$ according to the approximation of the velocity profiles (3), (4) and the function-indicator of the phase B_1 (5). As shown below mostly the region by parameter $h(x)$ is covered in (5) for the bright enough density ratio of the mixing liquid phases. The coefficients a_{ij}

, b_{ij} according to (11), (3)-(5) are presented in the Table 1 and Table 2. Integral correlation for the part of the mixing layer in the system (10) was considered at $\eta = 0.5$. The corresponding coefficients a_{ij}^*, b_{ij}^* for this middle section of mixing layer are given in the Tables 3, 4:

Table 1 – Integral parameters a_{ij} of the model for different regions of the function-indicator B_1

Value a_{ij} for h:	a_{11}	a_{12}	a_{21}	a_{22}	a_{31}	a_{32}	a_{41}	a_{42}
$h \in [0, -6]$	0.5464	0.0208	0.1667	0.0101	0.4604	0.0139	0.1198	0.0059
$[-6, -12]$	0.4857	0.0107	0.1333	0.0046	0.4250	0.0080	0.1027	0.0030
$[-12, -20]$	0.4310	0.0062	0.1065	0.0023	0.3884	0.0049	0.0866	0.0017
$[-20, -30]$	0.3844	0.0038	0.0859	0.0013	0.3542	0.0032	0.0728	0.0010
$[-30, -42]$	0.3455	0.0025	0.0703	0.00078	0.3237	0.0022	0.0614	0.00064
$[-42, -56]$	0.3128	0.0017	0.0583	0.00049	0.2969	0.0016	0.0522	0.00042
$[-56, -72]$	0.2853	0.0013	0.0490	0.00032	0.2734	0.0012	0.0447	0.00028

Table 2 – Integral parameters b_{ij} of the model for different regions of the function-indicator B_1

Value b_{ij} for h:	b_{11}	b_{12}	b_{21}	b_{22}	b_{31}	b_{32}	b_{41}	b_{42}
$[0, -6]$	0.0179	-0.0101	0.0095	-0.0042	0.0054	-0.0048	0.0023	-0.0016
$[-6, -12]$	0.0429	-0.0060	0.0214	-0.0022	0.0149	-0.0032	0.0059	-0.00097
$[-12, -20]$	0.0690	-0.0038	0.0327	-0.0012	0.0625	-0.0023	0.0098	-0.00062

$[-20, -30]$	0.0939	-0.0025	0.0424	-0.00076	0.0390	-0.0016	0.0141	-0.00042
$[-30, -42]$	0.1167	-0.0018	0.0506	-0.00049	0.0515	-0.0012	0.0180	-0.00029
$[-42, -56]$	0.1371	-0.0013	0.0573	-0.00032	0.0637	-0.00092	0.0215	-0.00021
$[-56, -72]$	0.1552	-0.00096	0.0629	-0.00022	0.0752	-0.00072	0.0246	-0.00015

Table 3 – Integral parameters a_{ij}^* of the model for different regions of the function-indicator B_1

Value a_{ij}^* for h:	a_{11}^*	a_{12}^*	a_{21}^*	a_{22}^*	a_{31}^*	a_{32}^*	a_{41}^*	a_{42}^*
$h \in [0, -6]$	0.4436	0.0110	0.1029	0.0038	0.4103	0.0094	0.0902	0.0032
$[-6, -12]$	0.4206	0.0071	0.0941	0.0024	0.3912	0.0062	0.0831	0.0020
$[-12, -20]$	0.3920	0.0062	0.0836	0.0015	0.3673	0.0042	0.0745	0.0012
$[-20, -30]$	0.3619	0.0033	0.0730	0.00097	0.3416	0.0029	0.0657	0.00084
$[-30, -42]$	0.3327	0.0023	0.0631	0.00064	0.3164	0.0021	0.0574	0.00056
$[-42, -56]$	0.3069	0.0016	0.0548	0.00043	0.2936	0.0015	0.0503	0.00039
$[-56, -72]$	0.2814	0.0012	0.0468	0.00030	0.2711	0.0011	0.0434	0.00027

Except the above, for the dimensionless parameters we retain the same notations as for the dimensional ones. Only here in (11) it is stated for clarification of the dimensionless notations.