Principles and Analysis of Historical Masonry Structures

Principles and Analysis of Historical Masonry Structures:

The Strength of the Past

By

Giovanni Castellazzi
and Alberto Taliercio

Cambridge Scholars
Publishing

Principles and Analysis of Historical Masonry Structures:
The Strength of the Past

By Giovanni Castellazzi and Alberto Taliercio

This book first published 2025

Cambridge Scholars Publishing

Lady Stephenson Library, Newcastle upon Tyne, NE6 2PA, UK

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

ISBN: 978-1-0364-5633-7
ISBN (Ebook): 978-1-0364-5634-4

DEDICATION

To Our Families

EPIGRAPH

Homo qui erranti comiter monstrat viam,
quasi lumen de suo lumine accendat facit.
Nihilo minus ipsi lucet cum illi accenderit.

"One who kindly shows the way to someone who is lost
is like a man who lights another's lamp from his own:
it still shines just as brightly for himself after lighting the other's."

– inspired by Seneca, *Epistulae Morales ad Lucilium*, VI.5

CONTENTS

List of Symbols

a	dimension of an element
A	area
b, B	width
c	cohesion (simple shear strength)
C	elasticity flexibility matrix/tensor
D	elasticity stiffness matrix/tensor, macroscopic strain rate
e	eccentricity, distance
E	Young's modulus, macroscopic strain
f_c, f_t	compressive/tensile strength
f_k	characteristic strength
F	force
g	gravity acceleration, weight per unit area (shells)
G	shear modulus, macroscopic strength domain, gravity center, centroid
h, H	height, depth
J	moment of inertia
k	coefficient
l, L	length
M	bending moment
n	unit normal vector, number of specimens
N	axial force (beams), membrane force (shells)
p	distributed load (shells), pressure
P	point load
q	distributed load (beams), weight per unit area (shells), generalized strain
Q	resultant force, generalized stress
r	radial coordinate, ratio
R	radius

s	standard deviation
t	thickness
u	displacement
v	displacement, velocity
V	shear force
W	weight, virtual work
x, y, z	Cartesian coordinates
α, β	non-dimensional parameters
γ	unit weight, shear strain
δ	coefficient of variation, virtual variable, normalized displacement
Δ	displacement
ε	strain
θ	angle of rotation, longitude
λ	load multiplier, first Lamé's parameter
μ	friction coefficient
ν	Poisson's ratio
ρ	mass density
σ	stress, normal stress
$\sigma_I, \sigma_{II}, \sigma_{III}$	principal stresses
Σ	macroscopic stress
τ	shear stress
φ	angle, colatitude
ψ	angle
ω	strain energy density
∇	Nabla symbol (differential operator)
$\{a\}$	column matrix
$[a]$	square or rectangular matrix
$\underline{a}$	vector (1st-order tensor)
$\underline{\underline{a}}$	2nd-order tensor
$\cdot$	scalar (dot) product of two vectors ($\underline{a} \cdot \underline{b} = \sum_i a_i b_i$)
$\otimes$	tensor (dyadic) product of two vectors ($(\underline{a} \otimes \underline{b})_{ij} = a_i b_j$)
$:$	scalar (double dot) product of two 2nd-order tensors ($\underline{\underline{a}} : \underline{\underline{b}} = \sum_{i,j} a_{ij} b_{ij}$)

List of Abbreviations

- CDP – Concrete Damaged Plasticity
- CST – Constant Strain Triangle
- d.o.f. – degree of freedom
- FE – Finite Element
- FEM – Finite Element Method
- GSF – Geometrical Safety Factor
- LT – Line of Thrust
- MoC – Method of Cells
- NT – no-tension
- RC – reinforced concrete
- RNTM – Rigid, No-Tension Material
- RVE – Representative Volume Element

PREFACE

The analysis of the mechanical behaviour of historical masonry structures has been a central focus of our research ever since we graduated. A significant part of our scientific work has concentrated on the structural analysis of these structures and the attempt to mathematically describe the mechanical behaviour of masonry. Recently, both of us have been lecturing in post-graduate courses on historical buildings at our respective universities: Politecnico di Milano and the University of Bologna in Italy. Over the years, we independently accumulated a substantial amount of material (texts, scientific articles, including some authored by ourselves) used in our courses, which we attempted to reorganize individually for teaching purposes. This effort eventually inspired the idea of writing a shared textbook that could serve advanced university courses focusing on the assessment and retrofitting of historical masonry buildings (typically in Civil or Building Engineering and Architecture).

The book begins with an overview of historical building typologies and the materials used across centuries. It then focuses on the mechanical behaviour of masonry and its mathematical modelling, both in the elastic range and at failure, using a phenomenological (macroscopic) perspective as well as micromechanical approaches (homogenization theory). It also examines the mechanical response of various structural masonry members, including walls subjected to in-plane and lateral loads, compressed walls and columns prone to buckling, arches, arch bridges, domes, and vaults. Classical limit analysis is adapted for no-tension materials, such as masonry, and applied to estimate the load-bearing capacity of walls and arches. Typical faults observed in masonry structures are described, along with their potential causes. The book concludes with a review of the main testing techniques for masonry samples and structural components.

To the best of our knowledge, the existing bibliography does not encompass as broad a scope as this textbook. We have striped to address historical constructions in their entirety, rather than limiting the focus to a particular historical period. While other books are often descriptive though exhaustive, they do not explore the mechanics of historic constructions as deeply as we hope to have done. With this book, we aim to make the study of historical masonry constructions – so fascinating and challenging from an analytical perspective – more accessible to students and professionals alike.

Giovanni Castellazzi
Alberto Taliercio

ACKNOWLEDGMENTS

We would like to thank all the colleagues who, in different ways, have contributed to this book. Many topics have been rearranged from the works of Mario Como, one of the leading Italian authorities in the field of masonry constructions and formerly a professor of Structural Mechanics at the University of Tor Vergata in Rome, whose contributions are extensively referenced throughout the book. The book's layout, intended as the organization into chapters, was inspired by a textbook in Italian by Renato Sante Olivito, formerly a professor of Structural Mechanics at the University of Calabria.

Many colleagues from various Italian universities, including those where we teach, generously allowed us to use several pictures and photographs. Their contributions are acknowledged wherever appropriate in the book. Finally, we would like to thank those senior colleagues who, through their teaching and passion, introduced us to the study of historical constructions. In particular, we extend our gratitude to Luigia Binda and Giannantonio Sacchi Landriani, both formerly professors at the Politecnico di Milano, and Angelo Di Tommaso, formerly a professor at the University of Bologna.

INTRODUCTION

Historical buildings

What is a "historical building"? There is no consensus among researchers and art historians regarding the definition of a historical building. According to the new protocol for historical buildings, recently developed by the Italian Green Building Council (2016), a "historical building" is defined as an artifact that holds value as "material evidence of civilization". This value can only be acknowledged after the completion of the historical cycle in which the artifact was built. For Europe, the last historical cycle is considered to have ended with the advent of industrialization in the construction industry, which conventionally began in 1945. Before that date, buildings were constructed using pre-industrial processes, phases, operations, and materials.

Based on the above definition, many concrete buildings dating back to the first half of the 20th century would be considered historical buildings. However, this textbook will primarily focus on constructions made of materials other than concrete, such as masonry, and to a much lesser extent, iron. These were the typical materials used in the construction industry until the first decades of the 20th century. Figure I displays the number of residential buildings in Italy categorized by the year of construction, as reported in a 2011 survey conducted by ISTAT – the Italian National Institute of Statistics. The data reveals a significant presence of "historical" buildings: approximately one out of four buildings in use in 2011 was constructed before 1945, with 15% dating back even further to before 1919. This distribution is likely to be similar in many other European countries. In certain regions, the proportion of historical buildings is even higher. Figure I illustrates also the age of surveyed buildings in a north-western Italian region (Piemonte),

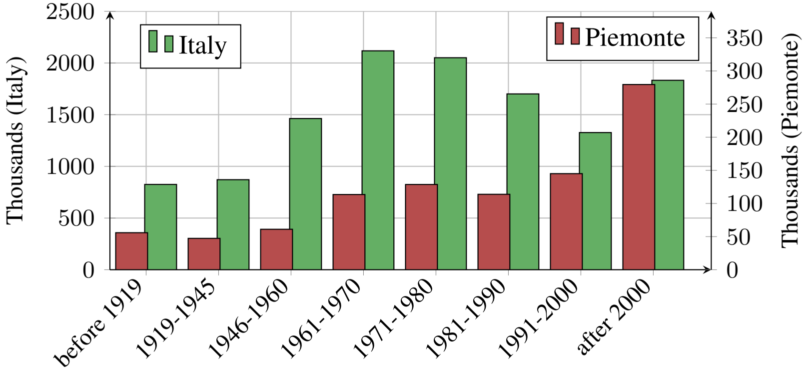

Figure I: Comparison of residential buildings in Italy and Piemonte according to the year of construction (ISTAT - Istituto Nazionale di Statistica, 2011).

where around 30% of residential buildings existing in 2011 originated from the beginning of the previous century or earlier. The number of historical buildings in use remains notable even in regions where they constitute only a small fraction of the overall existing buildings, owing to the construction market boom during the mid-20th century. In Lombardy (northern Italy), for instance, over 216,000 buildings out of a total of nearly 1.5 million were identified as over a century old as of 2011. Also the number of historical bridges still in use is far from insignificant. Figure II(a) depicts the various types of railway bridges in Europe, based on a project sponsored by the European Union in 2007 (Bień et al., 2007). Masonry arches, constructed from either brickwork or stone, account for more than one-third of the nearly 220,000 railway bridges surveyed. Of the arch bridges, 85% are made of masonry, with the remaining 15% composed of reinforced concrete. Figure II(b) illustrates the age distribution of the surveyed bridges, revealing that nearly 35% of them are over a century old. This percentage increases to 65% when considering only arch bridges

Masonry

The construction materials used to build most historical buildings and other artifacts are wood, iron or steel, and, above all, *masonry*: this is the material

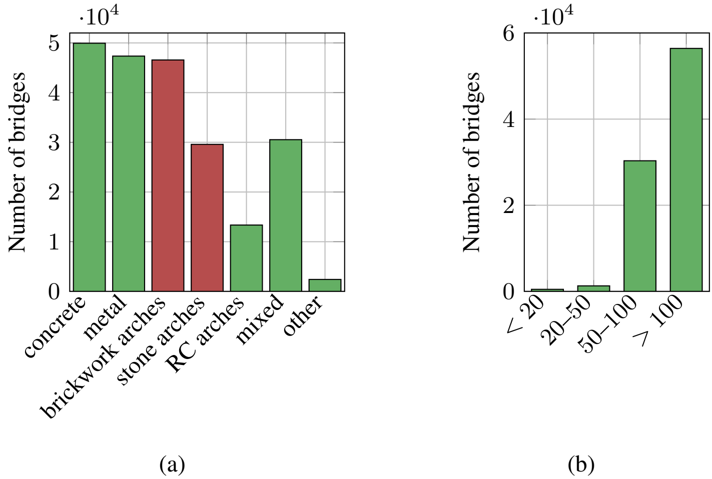

Figure II: Railway bridge statistics in Europe: (a) bridge types and (b) age (Bień et al., 2007).

to which this textbook will be devoted.

According to the Encyclopaedia Britannica, masonry is "the art and craft of building and fabricating in stone, clay, brick, or concrete block. Construction of poured concrete, reinforced or unreinforced, is often also considered masonry". This broad definition includes both constructions built in ancient times using natural materials (see Fig. III(a,b)) and modern constructions employing man-made materials (see Fig. III(c,d)). However, this textbook will focus solely on the former category of masonry constructions. Details on the materials and techniques used to create masonry buildings in ancient times are provided in Chapter 1.

The materials used, the quality of the mortar and workmanship, and the pattern in which the units are assembled can significantly affect the performance and durability of masonry constructions. When executed properly, masonry assemblies represent a highly durable form of construction. In fact, particularly in Europe, masonry structures constitute the majority of historical buildings (as pointed out in the previous section) and many of the most important monuments of its architectural heritage.

Due to the variety of masonry typologies, proposing a single, unified ap-

(a): solid brick masonry

(b): stone masonry

(c): hollow clay block masonry

(d): hollow concrete block masonry

Figure III: Ancient (a,b) and modern (c,d) types of masonry.

proach to analyze all types of material constructions seems unrealistic. Following the classification proposed by D'Altri, Sarhosis, Milani, Rots, Cattari, Lagomarsino, Sacco, Tralli, Castellazzi, and de Miranda (2020), the following modeling strategies can be considered for masonry structures:

- Block-based Models (BBM): These models take the actual masonry bond into account, explicitly considering the presence of blocks or units. The blocks can be modeled as either rigid or deformable, with their interactions mechanically formulated.

- Continuum Homogeneous Models (CHM): Masonry is treated as a homogeneous material, ignoring the distinction between blocks and joints. Constitutive laws are derived either directly (e.g., from experiments) or through multi-scale *homogenization*, where the global (or *macroscopic*) mechanical behavior is derived from the local behavior of the masonry constituents.

- Macroelement Models (MM): Structures are divided into larger panel-scale components, like piers and spandrels, to capture their mechanical or phenomenological response. These models differ from continuum approaches by focusing on panel-scale behavior rather than material-scale behavior.
- Geometry-based Models (GBM): These models represent structures as rigid bodies, focusing on geometry rather than block details. Equilibrium and collapse are analyzed using methods such as static or kinematic *limit analysis*, emphasizing rigid-body assumptions.

These four modeling strategies are concisely illustrated in Fig. IV and thoroughly reviewed in D'Altri et al. (2020), where the limitations and possibilities of each approach, along with their numerous variants, are discussed.

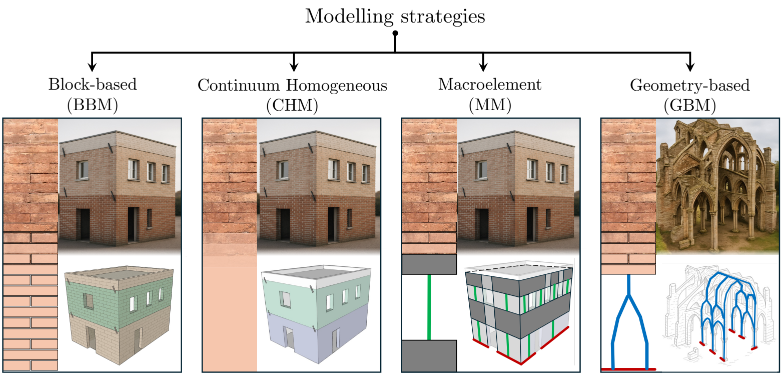

Figure IV: Modeling strategies for masonry structures, (adapted from D'Altri et al. (2020))

In this textbook, special attention will be devoted to CHMs, as the goal in several instances will be to derive the macroscopic elastic and strength properties of masonry based on those of the units and joints (Chapters 2 and 4). While BBMs are challenging and informative for the analysis of small-scale structures, they are impractical for real-scale masonry buildings and, as such, will not be considered. MMs are particularly suitable when the ultimate load-bearing capacity of the structure is sought and will be employed in conjunction with limit analysis in Chapters 3 and 6. Finally, GBMs will be briefly addressed in Chapter 3, where the global stability of masonry walls will be discussed.

Book layout

The layout of this textbook is structured as follows:

- Chapter 1 provides an overview of the main building typologies throughout the centuries and the materials utilized in historical buildings.

- Chapter 2 focuses on the mechanical behavior of masonry, exploring various modeling techniques.

- Chapter 3 addresses the analysis of masonry walls under vertical and lateral loads, with specific attention given to common failure mechanisms.

- Chapter 4 is dedicated to predicting the macroscopic mechanical properties of brick masonry in the elastic field and at failure, utilizing the homogenization theory for heterogeneous periodic media.

- Chapter 5 delves into the buckling analysis of compressed masonry walls and columns, highlighting the impact of masonry's limited tensile strength on load-carrying capacity.

- Chapter 6 explores the analysis of masonry arches and bridges, demonstrating the application of classical limit analysis to structural elements without tensile strength.

- Chapter 7 extends this analysis to masonry domes, while

- Chapter 8 focuses on masonry vaults.

- Chapter 9 describes common faults observed in masonry buildings and discusses their likely origins.
- Finally, Chapter 10 presents the main testing techniques employed for masonry specimens and structural elements.

Chapter 1

Masonry: An Overview

1.1 Introduction

Masonry is a composite, heterogeneous material whose mechanical properties are strongly influenced by its components: natural or artificial *units* arranged in various patterns, and, except in the case of dry-stone masonry, *mortar*.

A wide range of units can be used in masonry structures, including artificial units (commonly bricks) and natural blocks obtained by cutting rock, such as tuff, sandstone, and limestone. Unlike historical masonry, modern artificial units typically have standardized sizes, while natural units can vary greatly in both size and shape.

Mortar not only binds the units together, but also creates a deformable layer that accommodates the irregularities of adjacent surfaces. As a result, the units are supported across their entire surface rather than at just a few contact points. However, many historical structures are made from *dry-stone masonry*, which consists of large blocks stacked without any binding material.

The geometry of masonry patterns and construction techniques in historical buildings evolved slowly over centuries. Understanding this evolution helps us appreciate how the internal structure of masonry and the geometry of the buildings were gradually refined, resulting in the diverse range of architectural heritage we see today.

Masonry construction techniques have varied worldwide due to several factors, including the cultural development of societies, the availability of suitable materials, regional topography, and climatic conditions. As a result, the sophistication of masonry construction is not always indicative of a civilization's overall level of advancement. Nomadic peoples, for example, rarely practiced masonry construction due to the scarcity of stone and other materials in their regions, instead using wood where forests were present.

Masonry structures can be classified in various ways: by the materials used, the size and arrangement of the components, or the presence of binding materials between them. The stability of masonry structures without binding materials (also known as dry-stone constructions) relies on the static equilibrium of the carefully arranged units. The introduction of binding materials, such as mortar, enhances the stability of the structure by providing cohesion between adjacent elements. This allows for greater flexibility in the size, shape, and selection of constituent materials. Binding materials were introduced later in masonry construction; however, advanced civilizations were still able to construct remarkable dry-stone masonry walls. Notable examples include the impressive pre-Inca and Inca monuments, renowned for their grandeur and precision (Fig. 1.1).

(a) (b)

Figure 1.1: (a) Dry-stone Inca wall in Cuzco, Peru, featuring the famous 12-sided stone; (b) Dry-stone Inca walls at Tambo Machay, Peru (15th century).