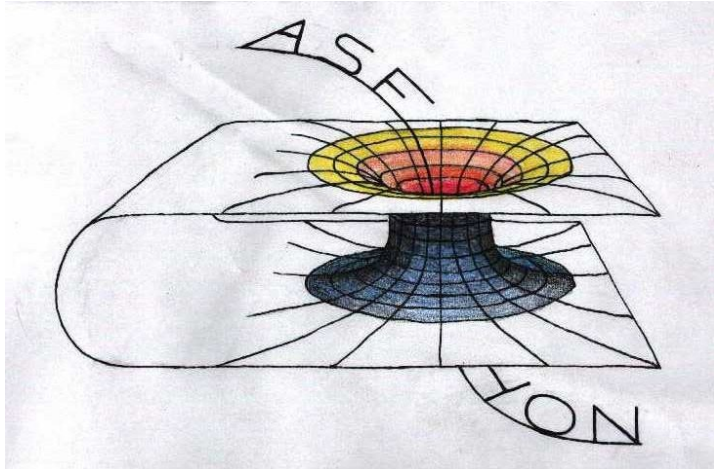


# Everything You Need to Know About Black Holes





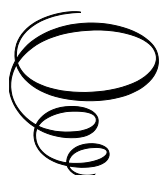
# Everything You Need to Know About Black Holes:

*Shadow of an Extra Dimension*

By

Marten Slagter and Reinoud Slagter

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Shadow of an Extra Dimension

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# Foreword

In the early 1960s, life was still simple. The amount of information you could get from your surroundings, was very limited. I was 17 years old when I graduated from high school and I was deciding what to choose as my next step. We didn't have a radio or television at home yet, and there was no World Wide Web or cell phones from which you could find information. My father was a maths teacher and astronomy amateur.

An impressive story he told me once, was about the confiscation of his telescope during World War II. It was made of copper and the Germans desperately needed this metal for their weapon industry. Many evenings I wan-



*The dissertation defense of my PhD in 1986 with Professors Gaemers, 't Hooft and van den Heuvel.*

dered around his study and read a lot about the wonders of the sky. He told me I should study astronomy. Together with a good friend, we start studying physics at Utrecht University. During the first year of my study, however, I became interested in theoretical physics, partly because of the impressive lectures by Nobel laureate Martinus Veltman, together with Gerard 't Hooft, on field theory. 't Hooft was a student of Veltman at that time. He acquired a professorship in Utrecht at a young age. I joined his lectures on quantum field theory in the following years and learned a lot. I finally completed my master with 't Hooft in 1979 on a study on the Mixmaster quantum cosmological model. It was a great privilege to get so many ideas from 't Hooft. First in real time and later through his huge amount of publications. He also became my co-supervisor while completing my PhD thesis. In 1980, I went to the University of Amsterdam and joined a 'cosmo'-group, which at that time was active around Karel Gaemers at Nikhef and Ed van den Heuvel at the Anton Pannekoek institute. I started my PhD on several topics in theoretical physics. In particular, multiple scale analysis for wave-like problems in general relativity attracted my attention. Over time, I switched to cosmic strings and the possible proof of their existence by observing alignment of the rotational axis of quasars in large quasar groups. Eventually, I fell under the spell of quantum black holes. It is widely believed that the research of physics near the horizon of a (Kerr) black hole will sooner or later contribute significantly to the solution of a quantum gravity model.

To understand the essence of black holes, one must first consider the physics underlying our understanding of Nature. I have tried to present the fundamentals of theoretical physics as clearly as possible, with a minimum of formulas. Unfortunately, however, one does not always escape this challenge. I present some mathematical details using *intermezzos*. It is not necessary to follow these extensions in order to keep track of the results of this book. Now it is difficult to briefly list all aspects of physics in one book. Physicists took roughly 200 years to arrive at the Standard Model of particle physics and to fathom gravity via Einstein's General Theory of Relativity (GRT).

Readers may wonder why a book of more than 300 pages is needed to explain the black hole. Black holes are objects in our universe that are predicted by Einstein's famous GRT. They could arise from stars that have reached the end of their existence. The object eventually becomes so compact that not even light can escape its gravity. The boundary of the black hole is called the horizon. Everything that takes place within this horizon would remain hidden from the outside world. So there is no need to worry about the processes that take place there, such as the singularity in the center. All the mass would be at this point  $r = 0$ , a nasty point for physicists. A point with infinite mass density is physically not tenable. The famous physicist Hawking discovered that a black hole is not completely black after all. It emits radiation, albeit not

very quickly. According to the laws of thermodynamics, this radiation should be thermal, which means that it should not contain information. Nowadays, it is thought that the information will return with the radiation. Conservation of information is a fundamental property of quantum mechanics (QM). The radiation will have to be pure and entangled. But then the reality that the observer experiences within the horizon will not remain hidden either. What does the interior look like? There are wild ideas about the interior. Does one enter another universe through a wormhole? Or a white hole? It is clear that quantum effects will play a role. No one yet knows what such a quantum gravity model should look like. Quantum mechanics, the theory that perfectly describes the behaviour of the micro world, is completely different in nature from GRT. The theory does not make definitive predictions about phenomena, but rather provides more or less statistical results. However, the predictive power of QM is extremely accurate. The predicted results are always found in experiments. But this is also true for GRT. However, things change dramatically when both theories must be combined, such as in the vicinity of a black hole. Both theories are in fact incompatible. Should one of the theories be modified and adapted? Please specify which one! But the history of science illustrates that progress in such cases will simply take different directions and even completely different avenues. Most scientists opt for an adaptation of the GRT, i.e., a quantified version. There are also theorists who doubt the sustainability of QM. It may be necessary to revise the topology of our spacetime in order to take a step further in the unification process. In this book, we describe a new model of a black hole, in which an extra dimension is used and a new topological description of the spacetime. Extra-dimensional models play nowadays an important role in theoretical physics. There is yet another mystery that needs solving. Black holes and galaxies are observed in the very early beginnings of the universe. These primordial black holes could not possibly have formed from stars, which must have been a few billion years old. How did these objects come into being? Probably through an instanton. And what will remain when a black hole has evaporated? There is also an astrophysical explanation. In the early universe, when there were no elements heavier than helium, only very heavy stars, known as Population III, with hundreds of solar masses, could form. These stars only lived for a few million years and could form black holes. In star clusters, they could merge into supermassive black holes in a relatively short period of time. Detailed observations can provide a definitive answer.

In this book, we have attempted to discuss all aspects necessary to understand a new black hole model. The book is organized in such a way that it is self-contained. This means that I do not refer to parts that cannot be discussed. All aspects related to black holes will be addressed. Some parts are technical, such as the standard model of particle physics. Nevertheless, the reader



should familiarize themselves with the beauty of the standard model. What is matter made of? It is said that the model cannot be reduced any further. The model consists of mathematics that reaches the limits of fundamental group theory.

In Chapter 1, we provide a concise overview of the current state of affairs in theoretical physics. In Chapter 2, we discuss quantum mechanics and its strangeness. Nevertheless, we will see that the theory is extremely effective in calculating quantum systems. In Chapter 3, we discuss the standard model (SM) of particle physics. This is no easy task. Readers who work their way through it will gain a complete picture of the wonderful SM. In Chapter 4, we discuss Einstein's special theory of relativity, the basis of relativistic quantum mechanics. Chapter 5 deals with GRT in all its glory. The emphasis is, of course, on black holes, the intriguing solutions of GRT. But strange solutions are also discussed. These solutions are at the cutting edge of what is physically acceptable. Chapter 6 discusses our new model, a black hole solution in a five-dimensional conformally invariant space-time. Chapter 7 deals with the quantum aspects of the model and everything related to it. Is there such a thing as a quantum black hole? Finally, we finish with a chapter on metaphysics, a controversial subject. Nevertheless, it is instructive to discuss some aspects that do not fall within the realm of 'hard' physics. We will consider some extraordinary ideas put forward by Penrose, Bell, Bohm, Gödel and Dirac. Not just anyone.

The reader can easily skip certain technical parts. Nevertheless, I have managed to explain all the relationships with other branches of physics. This is necessary to gain a complete understanding of the mystery of black holes. I hope the reader will finally understand how incredibly law-like our universe is. This book is a follow up version of my book 'Black Holes and Cosmic Strings Revisited, (Slagter, 2024). We need some parts of it to understand the new black hole solution. My son Marten is a philosopher and science fiction specialist. He draws some pictures and contributed to chapter 8.

Bussum, August, 2025

*Reinoud Jan Slagter & Marten Mirk Slagter*

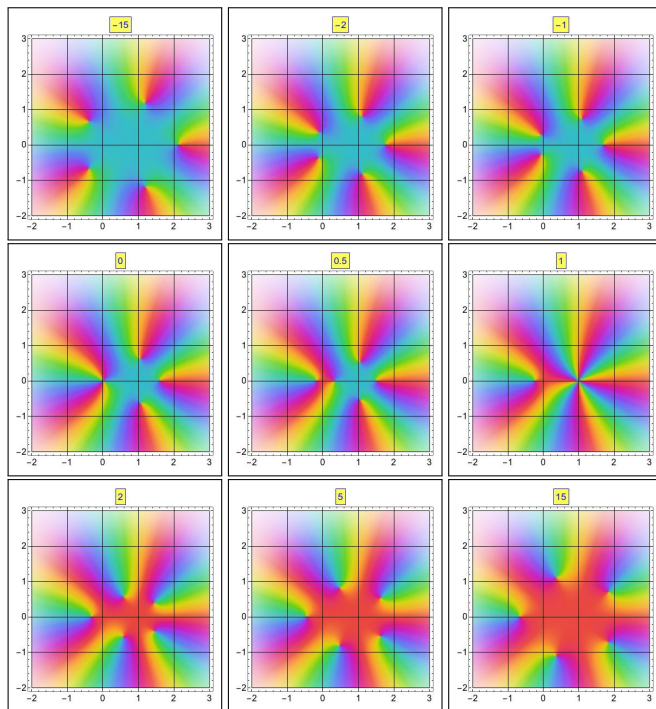




# Acknowledgment

I am grateful to all the people who have inspired me during my life and studies in theoretical physics. I have met many people during my visits to conferences and workshops, who have encouraged me to explore the beauty of our universe. I would like to mention: Dr Barto Oranje, Dr Pieter Miedema, Dr Gerard Bauerle, Ir Derek Masselink, Dr Supriya Pan, Dr Ed van den Heuvel, Dr Karel Gaemers, Dr Eugen Radu and many others. In particular, I mention Dr Gerard 't Hooft as my former teacher. He is a great inspiration through his publications and encouraged me to pursue theoretical physics. Last but not least, I thank my family for the support. My wife Titia, children Taco, Marten, Soetje and grandchildren Pau, Bobby, Mick, Nina, Felice, Veive and Berenice.

This book is dedicated to my mother Martha Adriana van de Fliert, who sadly passed away too soon.



*'Dance of the roots' in the complex plane of the singular points of the black hole spacetime with one extra space dimension. In chapter 7 we will study this exceptional behavior of the 'horizons' of this 'quantum'-like black hole model.*

### **Cover pictures:**

Top left: The new antipodicity in the 5D spacetime, with the double cover of the Klein surface. Top right: The shape of the inside of the black hole. Bottom: A curve with a cusp singularity, looks in a dimension higher every where regular.

# Chapter 1

## Theoretical physics in a nutshell

We are at a turning point in the history of physics. The fundamental principles of physics are being shaken to their very core. These are Quantum Mechanics (QM) and Einstein's General Theory of Relativity (GRT). The former describes small-scale physics extremely accurately, for example the behaviour of subatomic particles or electromagnetic radiation. The latter describes large-scale phenomena, such as the movement of celestial bodies by gravity or the expansion of the universe. The mathematical languages of the two theories are very elegant, yet unexpectedly different. One option is to accept this fundamental difference and continue researching both fields. Most physicists do not tend to worry about this difference. However, some physicists cannot resist trying to understand this difference. This quest is the 'holy grail', in other words, the 'theory of everything' (TOE). To date, there is no TOE. Unified mathematics should describe a new field of research: quantum gravity. In the 20th century, the greatest discovery was the fact that gravity could be described by

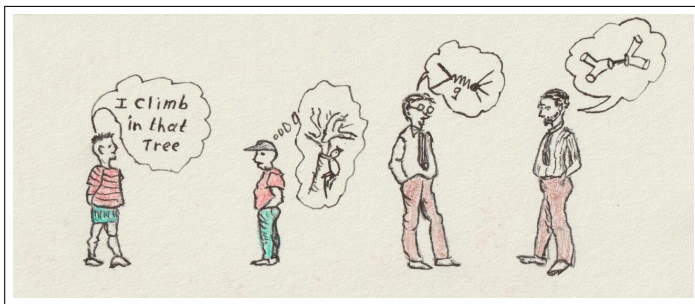


Fig. 1.1: *Theoretical physicists usually employ complicated mathematical language. It is therefore difficult to explain the results in plain language, such as 'I climb in that tree'.*

the curvature of spacetime. It was Einstein's greatest achievement, based on the pioneering work of Riemann and Minkowski. In this respect, the theory of gravity is separate from

the theories describing the other fundamental forces in nature, namely, the electromagnetic force and the weak and strong nuclear forces. These forces could be united in a so-called Grand Unified Theory (GUT). It is a natural extension of the Standard Model (SM) of particle physics, which describes the electromagnetic and weak nuclear forces so well. The dynamics is called Quantum Electrodynamics (QED). The strong force is described by Chromodynamics (QCD). It replaces the proton and neutron as fundamental particles with the quarks and its carrier, the gluon. Together with the carriers of electromagnetism, the photon, and the carriers of the weak force, the three  $W^+$ ,  $W^-$  and  $Z$  bosons, we have three types of so-called gauge particles. In a unified model, we should also have the graviton as a gauge particle. All these gauge particles, along with the Higgs boson (except for the graviton), have been experimentally verified. The Higgs boson was discovered at CERN in 2012. It was a great achievement. This particle, the ‘God particle’, had been predicted by theorists for almost forty years. It acquires mass to elementary particles via the Higgs mechanism. The combined theory for a GUT will manifest at very high energy, or very small scale and a TOE at an even smaller scale, namely the Planck scale. This region can be characterized by particle energies of about  $10^{19} \text{ GeV}$  or  $10^9 J$ . They manifest themselves in time intervals of about  $10^{-43} \text{ s}$ , i.e., on length scales of  $10^{-35} \text{ m}$ . Of course, one would want to test such a model experimentally with particle accelerators or with space instruments. An obvious question is why it is so difficult to realize these unifications mathematically. In this book, we attempt to answer this question, at least in part. Many leading theoretical physicists have attempted. Many theoretical physicists have taken on this extraordinarily complicated task. Initially, they used an approximation scheme, which is not always the most effective method. We also know that QM is the best model we have for describing the micro world. It is a robust model that accurately explains the behaviour of elementary particles. A logical next step is therefore to try to incorporate gravitational effects into a comprehensive quantum model. The discovery of the graviton, the intended carrier of the gravitational force similar to the photon in QM, made this approach plausible. However, there is a significant difference between these force carriers. Firstly, gravity operates at all scales. In other words, it has a ‘long tail’, whereas the other forces have a short range. Secondly, when quantizing gravity, one has to deal with nonlinear effects. The mathematics is completely different. For example, if an electron emits a photon, can it also emit a graviton? After all, an electron has mass. In other words, the electron travels on a curved spacetime background, which causes curvature itself. This is a very nasty problem. Thirdly, GRT is a deterministic theory, whereas QM deals with probabilities. Small-scale reality is created by the observer. The next step in the approximation process involves performing calculations on a fixed background. It is quite normal for physicists to make approximations in order to gain insight. The deviation from this background is described by gravitational waves. This is known as the semi-classical approach, which combines the quantum behaviour of the graviton on a background that is handled by GRT. But what happens when one approaches the horizon of a black hole, where the curvature is extremely high? In this case, the constant background method breaks down. Some people say that understanding the processes near a black hole is the same as understanding quantum gravity. Another approach originates from pure mathematics and is based on group theory. The forces caused by the gauge carriers possess certain symmetries. These symmetries are formulated in terms of a symmetry group. A simple example is the circle group. Any point on a circle can be rotated through any angle. It transpires that Maxwell’s electromagnetic model is invariant under this group. The unitary group  $U(1)$  is the symmetry group of electromagnetism. The electroweak model, the unification of electromagnetism and the weak force, has the symmetry group  $SU(2) \times U(1)$ . This describes the formal operations that can be applied to the field equations without altering the system’s dynamics. The larger the unification, the larger the

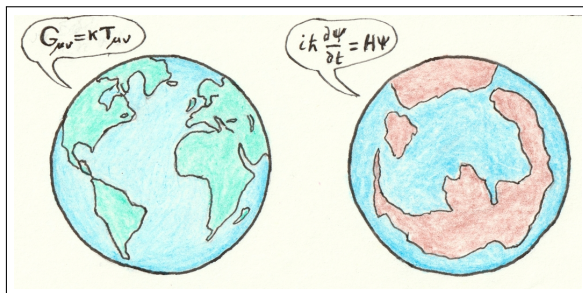


Fig. 1.2: An advanced alien civilization would develop the same mathematics to describe nature.

gauge group. The ultimate symmetry group appears in super-string theory. It is said to unify all forces. String theories require extra dimensions of space and time for mathematical consistency. In bosonic string theory, spacetime is 26-dimensional; in super-string theory, it is 10-dimensional; and in M-theory, it is 11-dimensional. Elementary particles are no longer considered to be point particles, but rather one-dimensional ‘strings’. Therefore, to describe real physical phenomena using string theory, one must consider scenarios in which these extra dimensions would never be observed in experiments. The question is whether such a model is tenable. We will see that there are less exotic possibilities. Could nature not choose the simplest representation of the laws of physics? I prefer the famous quote by the 14th-century English philosopher, William of Ockham (Occam’s razor): ‘*It is vain to do with more what can be done with less*’. Sometimes, this new, highly mathematical language is difficult to grasp. Some readers may skip some of the technical details. Nevertheless, I hope they will get some impression of the formidable task that theoretical physicists face. Some physicists doubt the usefulness of highly mathematical language in describing nature. The author believes that this mathematical framework is universal. An advanced civilization would develop the same Lie groups, simply because they are the most fundamental way of understanding the blueprint of our universe. They will also discover that  $e^{i\pi} = -1$  and that the fine-structure constant is approximately  $1/137$ . As we have already said, GRT is by far the most successful theory developed by theoretical physicists. It has been subjected to rigorous experimental testing and underpins our understanding of the universe. Alongside quantum field theory, general relativity lies at the heart of modern physics. GRT predicts the existence of black holes, which are the final stage in the evolution of a massive star that has collapsed. These have masses up to a few hundred solar masses. On the other hand, in the nuclei of galaxies, there are black holes with masses up to  $10^9$  solar masses! These black holes are related to real physical objects, at least as observed from the outside. Movements of stars around the center of our galaxy, Sagittarius A, predict the presence of a black hole, 26,000 light years away from the solar system and with a mass of 4.1 million solar masses. It is now believed that at the center of almost all galaxies there is a rotating black hole, a Kerr black hole, probably along with a quasar. Most supermassive black holes at the centres of galaxies are not quasars but active galactic nuclei (AGN). Quasars form a small subclass of AGN’s, with exceptionally high brightness. A stationary black hole is characterized by its mass, angular momentum and electric charge. This is called the ‘no-hair’ theorem. These quantities are determined by fields outside the black hole. No hair means

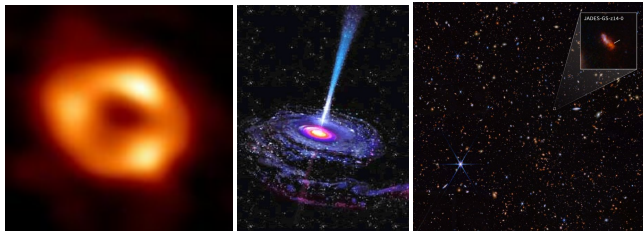


Fig. 1.3: *Left: This is the first image of a black hole at the center of a galaxy. This photograph shows radio waves originating just beyond the horizon. Middle: An artist's impression of a Kerr black hole at the center of a galaxy. It is thought that in the center there is a quasar, which radiates an enormous amount of energy along the axis of rotation. Right: Recently the James Webb telescope discovered a primordial galaxy, the MoM-z14 ('Mother of all Milky ways') at redshift  $z = 14.44$ , just 282 million years after the Big Bang. That is a very young galaxy (Retrieved August 10, 2025 from <https://science.nasa.gov/resource/first-image-of-a-black-hole/>; <https://arxiv.org/html/2505.11263v1>).*

that all other properties were destroyed during formation. Some scientists speculate about 'hairy' black holes. The additional parameters of the black hole could then be its magnetic charge and the amount of information that can be stored. The reader must realize that nobody still know what exactly happens inside the horizon of the black hole. A singularity in the center? That is a nasty point where a huge amount of mass is concentrated in a point. Physicist try to avoid singularities. Probably, quantum effects will play a prominent role at the horizon and physicist have no clue how to handle a quantum gravity model.

The James Webb telescope discovers regularly very young, ancient galaxies. The record holder (2025) is the MoM-z14. See Fig. (1.3). Scientists previously didn't think the universe should have reionized as early as galaxies like MoM-z14 suggest it did. There is also the unexplained star formation. Could exotic, mysterious forms of energy, like dark matter or dark energy, be involved? Maybe. Another explanation could be delivered by primordial black holes, as we already suggested in the foregoing. It looks like a progenitor of some kind of a massive global cluster, which are usually very compact and contains lots of stars. But other early galaxies appear mature, probably with a black hole. It indicates that the growth must have a different signature. The relationship with instantons and therefore black holes, as in our model, is then a possibility.

The prediction of gravitational wave-like properties in the weak field approximation of GRT was recently observed by the Laser Interferometer Gravitational Wave Observatory (LIGO). The source was a pair of colliding black holes. This massive event released an energy of about  $10^{50}$  Joules in a fairly short time (Fig. 1.4). Detectors for gravitational waves will provide new observational data in the coming decades. To date, LIGO has observed more than 150 binary black holes. These data are badly needed to explain the mysteries encountered in the very early stages of our universe. Black holes also show a deep connection with thermodynamics. Quantities such as temperature, pressure and entropy can be related to black holes. In 1974, Hawking predicted in an epic work, that a black hole emits



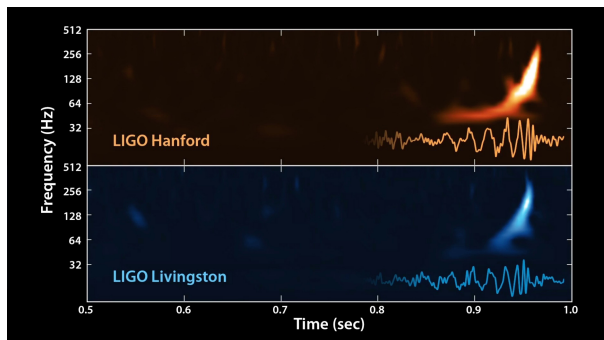


Fig. 1.4: ‘The event’. The violent burst of gravitational energy caused ripples in spacetime, which reached earth on 14 September 2015. They were detected by the two LIGO observatories in Livingston, Louisiana, and Hanford, Washington. The signals came from two merging black holes, each about 30 times the mass of our sun, located 1.3 billion light years away. Clearly, they fit together (Retrieved August 10, 2025, from <https://www.space.com/31894-gravitational-waves-ligo-search-complete-coverage.html>).

thermal radiation. At the horizon of the black hole, pair production will take place, because of its immense curvature. For example, if an electron-positron pair ( $e^+$ ,  $e^-$ ) pops out of the

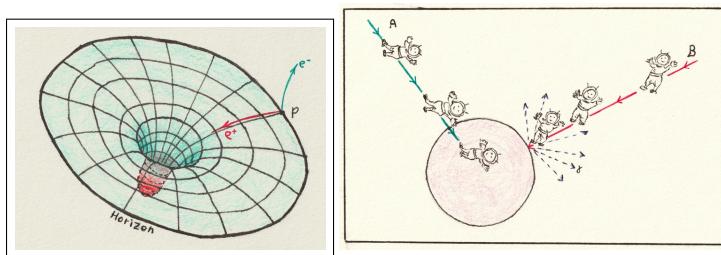


Fig. 1.5: Left: Pair production at a point  $P$  at the event horizon (in this example, an electron-positron pair) means that a black hole is not completely black. We will see that this phenomenon, predicted by Hawking, causes many paradoxes in the description of the black hole. In this violent place, quantum mechanics and gravitational theory ‘meet’ each other. Right: The complementary nature of GRT: an observer  $A$  falling into a black hole will perceive something completely different from observer at a great distance. He will see  $A$ ’s image ( $B$ ) ‘freeze’ towards the horizon, while  $A$  will not notice any difference when passing the horizon.

vacuum, the  $e^+$  can be absorbed by the black hole, while the  $e^-$  escapes. As a result, the black hole loses mass. Eventually, the black hole will completely vaporise. This will take a long time. It is said that quantum mechanical effects are introduced in Einstein's GRT: quantum gravity is born. Now it turns out that this theory of quantum gravity involves serious paradoxes. If a black hole is formed from a pure state and will eventually evaporate into a mixed state, one encounters a serious problem, i.e., a violation of QM. One could say that black holes are the most suitable playing fields for exploring a theory of quantum gravity. In fact, you cannot understand quantum gravity without a thorough understanding of the properties of black holes. In the future, it will probably be possible to collect observational data from gravitational wave signatures. Several attempts have been made in recent decades to overcome problems arising from quantum gravity, in particular the information-, the complementarity- and the firewall problem. Since information cannot stay in the black hole after evaporation, it must somehow get out with the Hawking radiation. Further, to restore unitarity, one would have to conclude that the Hawking radiation is not really thermal. Many attempts have been made to resolve these paradoxes. The most interesting one has been formulated by considering the complementarity between the inside and outside of the black hole, i.e., information is simultaneously reflected and transmitted through the horizon. But then a new problem arises. The particles will be entangled. An outgoing particle at time  $t$  must be entangled with all previously emitted radiation. It was inevitable then to introduce a 'firewall' of extreme energy close to the horizon. However, an in falling observer will not notice the firewall at all, because of the equivalence principle: he will perceive spacetime as Minkowski. Another possibility would be that the in falling observer is burned at the firewall. However, all these models ignore the gravitational interaction in a dynamical environment. The energy of particles near the horizon will become comparable to the Planck mass. Furthermore, gravitational waves will also emerge and the surface gravity needs to be investigated. We will see that the problem can be addressed by the cor-

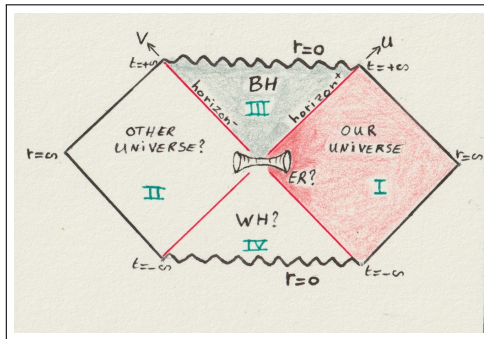


Fig. 1.6: Maximally extended Penrose diagram of the Schwarzschild black hole. Penrose introduced a clever coordinate system, the so-called Kruskal-Szekeres ( $U, V$ ) coordinates, to visualize all the features of this spacetime in a compact way. The square on the right, region I, represents our universe. Region III is the black hole. The new regions II and IV can represent an 'other universe' and a 'white hole'. Even an Einstein-Rosen (ER) bridge, or 'worm-hole', could exist. We will see later on that there are other possibilities.

rect interpretation of the so-called Penrose diagram. In general relativity, one can freely choose in which coordinate system one wants to study physics. One is thus not limited to Cartesian coordinates  $(t, x, y, z)$ . For the Schwarzschild black hole solution, we will see that one usually chooses a coordinate system that describes the physics of singularities in the most transparent way. However, one then often has to deal with an extended range of the new coordinates. The maximally extended black hole solution is often difficult to interpret. Some scientists suggest that one enters a parallel universe via an Einstein-Rosen bridge, or wormhole. See the Figs. (1.6) and (1.7). Perhaps a completely different presentation is

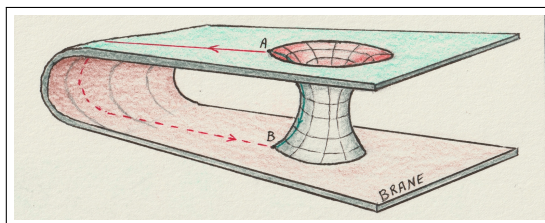


Fig. 1.7: A hypothetical wormhole. The wormhole is said to exist in an additional fifth dimension. It is exciting to imagine a transition through the wormhole. However, the extra dimension has not yet been found at CERN or in space. The throat is likely to be very unstable and one needs negative energy to keep it open. The question is where one ‘comes out’. In an other universe, a white hole, or in the same spacetime? Recently, Maldacena constructed a solution on a warped spacetime, where the journey from A to B in our world (‘brane’) takes millions of years, while the journey through the throat takes a few seconds! We will see in due course that there are another theories, maybe less spectacular.

possible for the relationship between the two regions of the maximally extended black hole. This could possibly be the so-called antipodal picture, first introduced by Schrödinger. He called the method the elliptical interpretation of spacetime near the horizon. This approach hangs on the topology of spacetime. Spacetime within the horizon is removed so that the edges are glued together by identifying antipodal points. In a sense, it can be compared to a Möbius strip. See Fig. (1.8). The inside surface is the same as the outside surface. This can also be done in one dimension higher: the Klein bottle. There is no inside volume! The self intersection disappears in 4 dimensional space. Such a space is, however, hard to be imagined. Our brains work in 3 dimensions. We enter the realm of topology. We will return to the theory of topology in more detail later on. There is then no ‘hidden’ area in the Penrose diagram. It is said that area II in the diagram is the charge-parity-time (CPT)-transformed (quantum) copy of area I. It turns out that this antipodal boundary condition can be used to study the gravitational back-reaction in this spacetime. After all, spacetime is not Minkowski close to the horizon. What will a local observer experience? We will see that he (she) will never can get access to the central point. The aforementioned black hole paradoxes could be solved by using this antipodal identification. Recently, ‘t Hooft applied this method of ‘cut-and-past’ to circumvent the need for a firewall. The Hawking radiation

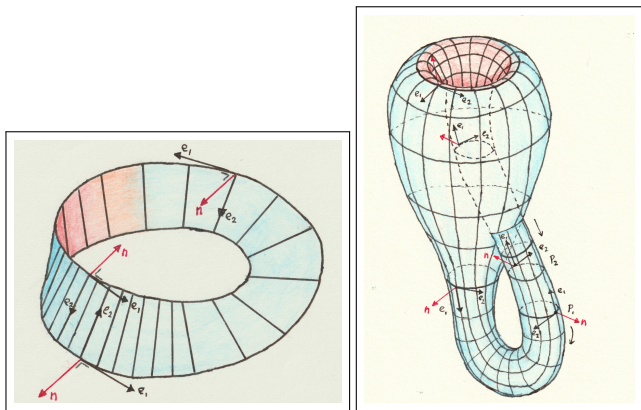


Fig. 1.8: Left: A Möbius strip. This ‘space’ is not orientable, as you can see by following the unit vectors of the surface. An orthonormal field is not allowed. Right: the Klein surface (or Klein bottle), an alternative hyper-surface and also not orientable. However, this surface cannot be embedded in our 3-space, as the space of Möbius can, because there is a self intersection. It may be well possible embedded in the 4-space  $(x, y, z, y_5)$ . This means that a 5-dimensional spacetime is appropriate. Remember that we still have a time as a coordinate. The behavior of the entangled Hawking particles, emerging from a black hole in this spacetime, could shed new light on the controversial ‘spooky action at a distance’ in QM, as Einstein formulated.



Fig. 1.9: Left: In our world, a right hand turns into a left hand through a mirror. Right: Illustration of an antipodal mapping, not conceivable in our world.

is usually treated by using the effective laws of black holes, or low-energy physics, whose rules are well documented by the standard model of particle physics. One also uses a fixed background, which is untenable at energies above those of the validity the standard model. In a clever way, 't Hooft can deal with the inevitable high-energy particles (‘hard’ parti-

cles) that are sure to make their appearance somewhere between the formation of the black hole and its final stage. In this Planckian area, the problems crop up. Usually, QM works with a Hilbert space. A Hilbert space is a mathematical construct and not the space that we normally have in mind. In mathematics, space means a set of vectors that interacts in a certain way. It has a defined set of operators, i.e., addition, subtraction, etc. One also needs a metric, a function to measure the distance between two vectors. A new boundary condition allows all states to remain in Hilbert space. In fact, one avoids the Planck region by using a set of evolution law on eigenstates, via high spherical modes. The number of orthonormal eigenstates can then be found to reproduce Hawking's results. However, a cut-off is needed for the spherical modes. It is conjectured that the antipodal boundary condition can be extended to a 5-dimensional warped spacetime. It turns out that a natural cut-off can then be provided by assuming that the Hawking particles stay on this hyper surface for a while. The particles become hard in a different way. In the extra dimension, the 'bulk', gravity is much more stronger. Some years ago, Randall and Sundrum (RS) proposed the existence of a 'large' extra dimension, of the order of a few millimeters. These so-called 'warped' spacetimes are very interesting variants of the super-string models. This model is a mathematical attempt to unify GRT and QM. It is an elegant mathematical model. Elementary particles behave like strings in stead of points. However, there are opponents of this model. A major problem is that the model cannot be experimentally verified, even in the distant future. In these full string models, the extra dimensions are compactified or folded into themselves, which means that almost infinite solutions are possible. Super-string theory is

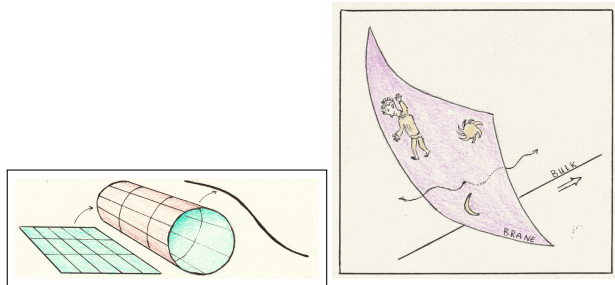


Fig. 1.10: *Left: Extra dimensions can be rolled up on a very small circle and look like a string. Right: Randall-Sundrum's brane world model. Our spacetime is embedded here in a five-dimensional bulk spacetime. All standard model fields are confined to the brane, while only gravity can propagate into the bulk.*

based on supersymmetry and operates in an 11-dimensional spacetime. It predicts super symmetric particles. However, no super symmetric particles have been discovered yet. It is possible that the future circular collider (FCC) at CERN will detect this 'new physics'. The perimeter of this accelerator will be 180 km!

The RS model is more realistic. In this model, only gravity can propagate in the bulk, while all other fields reside on the brane, i.e., our world. Einstein's gravity on the brane will change through the embedding itself and opens up a possible new way to tackle the problem of dark matter (-energy). Dark energy is an unknown form of energy that affects

the universe at the largest scales. It is needed to explain the observed accelerated expansion of our universe. Dark energy should not be confused with dark matter. See Fig. 1.12. There are many candidates for the missing dark matter, we mention Wimp's (weakly interacting massive particles), axions, neutrinos and mini black holes. To date, no evidence has been found. If one does not find these particles, an alternative explanation is welcome. Warped spacetime models could possibly also solve the problem of the gravity hierarchy, i.e., the huge discrepancy between the strength of gravity and the other forces. This is an elegant way: gravity is similar to the other forces in the bulk and is weakened in our brane. There is another strong argument in favour of Randall's five-dimensional model, Fig. (1.10). It relates to the holographic principle. We are all familiar with the hologram, an interference pattern of a three-dimensional object on a slide. When one exposes the slide in a suitable position, then one sees the three-dimensional image 'hanging' in space again. The hologram is formed by splitting a single laser into 2 beams: a reference beam and a signal beam. Both follow a separate path to the storage medium. The hologram is formed when these 2 beams come together in the storage medium, as an interference pattern is formed. The actual data is brought into the hologram using a spatial light modulator (SLM). It acts as a kind of LCD screen that displays pages of binary data as a checkerboard pattern of light and dark pixels. The amount of data in the hologram is determined by the number of pixels of the SLM. One can make a hologram with your smart phone. See Fig. (1.11). Now one can

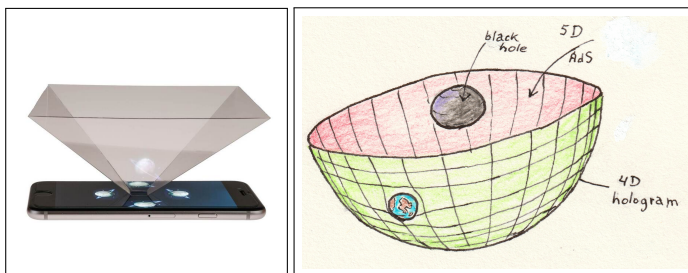


Fig. 1.11: *Left: A classic hologram where the two-dimensional information on your smart phone turns into a 3D image, which 'floats' in space. Right: The five-dimensional anti-de Sitter (AdS) universe model. Our world is the four-dimensional surface, represented as a hologram.*

also apply this principle to a black hole. The three-dimensional information in the interior of a black hole is equivalent to the information on the two-dimensional surface. Similarly, a four-dimensional space-time can be seen as a hologram on a three-dimensional surface. It was 't Hooft who first identified this principle in fundamental physics (1993). It is then a natural step to consider the information of our world (a brane) as the surface of a five-dimensional bulk spacetime. This principle also has a relation with quantum mechanical fields theory. Schrödinger already considered in 1957 a five-dimensional expanding universe model with a cosmological constant, where our universe is the surface. It was based on the work of Dutch astronomer de Sitter. Much later, a connection with conformal field theory (CFT) was found, which is obviously interesting in the development of a quantum-gravitational model. Now there are new applications of warped spacetimes. Firstly, it could



be applied to explain the observed mysterious alignment of the spin axes of quasars in large quasar groups (LQG, Slagter, 2022). Secondly, the warp factor could be used to explain the acceleration of the expansion of the universe. By default, one needs a cosmological constant to explain the phenomenon. However, there is a huge conflict between the value of this constant, when one compares the required cosmological value with the value from QM, where this constant represents the vacuum energy. In fact, the discrepancy is of the order of  $10^{-120}$ ! Ideally, one would like to get rid of the cosmological constant. Thirdly, the model

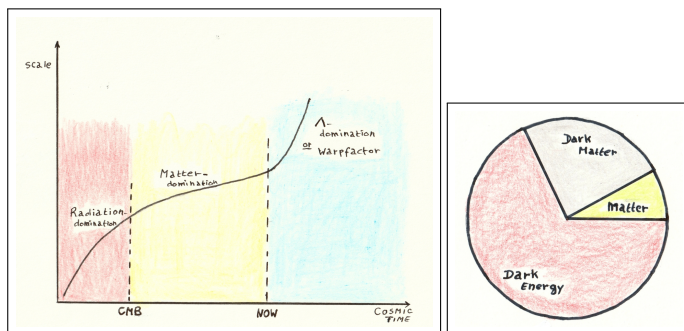


Fig. 1.12: *Left: The expansion epochs of our universe. We now live in the era of accelerated expansion. This could be caused by a cosmological constant. However, it could also be explained by a warp factor in a 5-dimensional spacetime. Right: The distribution of mass/energy in our universe. Only a minor part consists of ordinary baryonic matter.*

could be used to solve the paradoxes that occur near the horizon of black holes. One can avoid the exotic solutions, such as white holes or even other universes. Moreover, in the our new model under consideration, gravitons become hard in the extra dimensional bulk, which can be included in the new boundary condition of the antipodal map. There is also a conceptual reason behind the vision of the warped spacetime. It is quite possible that the two theories, QM and GRT, could become closer together if you assume an extra dimension. The basis of QM will not change drastically. We will see that the entangled Hawking particles remain pure states as they travel through bulk space and that no infinite velocity is needed to measure on any of them (the non-locality issue). In any case, the question is who to blame for the struggle for unification of QM and GRT. Should we modify gravitational theory or quantum mechanics? However, undermining the foundations of QM is like 'swearing in church'. One of the great mysteries in quantum mechanics, which one would like to solve, is this 'spooky action at a distance' according to Einstein's famous words. If entities move faster than the speed of light, then we get problems with locality and thus with the special theory of relativity (SRT). However, it seems that one measures infinite velocities, when one performs measurements on entangled quantum systems of particles. Quantum mechanics seems to be incomplete. Perhaps QM at its base, is also deterministic. People call this 'super-determinism'. Many scientists have tried to solve this locality problem. John Bell, for instance, used hidden variables in the measurement of entanglement to preserve the non-deterministic principles of quantum mechanics. Bell's hypothesis is

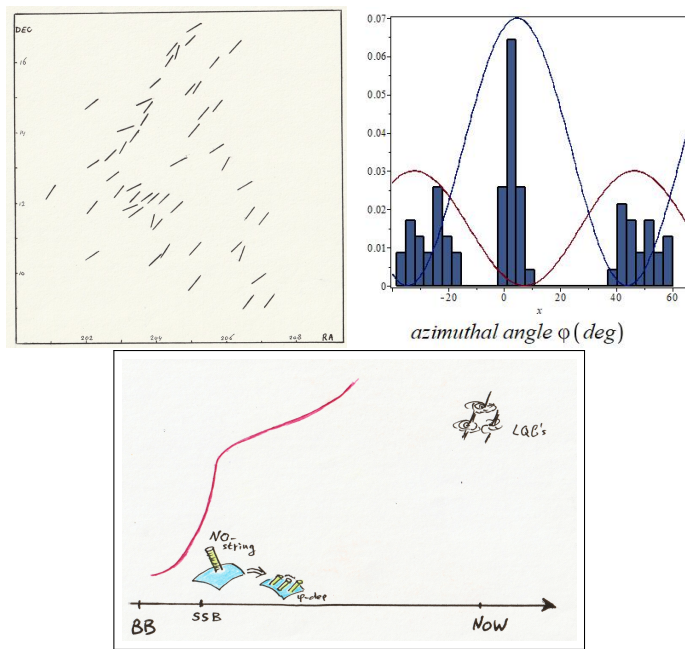


Fig. 1.13: There is a ghostly alignment of the spin axes of quasars in large quasar groups (LQG. Slagter, 2022). A possible explanation could lie in the primordial decay of excited vortices with a high spin number just after a spontaneous symmetry breaking (SSB) in the early universe in a warped spacetime (Slagter 2022). The formation of vortices in relativistic field theory has lived up to its reputation. The famous superconductivity is explained by a complex scalar field, i.e., the order parameter, which is the same field as the Higgs field in the standard model. Here we have plotted the azimuthal angles ( $\phi$ ) of the quasars in the LQG U1.27 from the NASA/IPAC databases. These results can be explained using the warped brane world model.

also called the free will theorem. Does the experimenter have free will to perform an independent measurement? In chapter 2 and 7 we will return to these important issues. You can argue that if the experimenter has free will, even an elementary particle must have free will, which of course is absurd. Einstein was not in favour of the basic principles of quantum mechanics. But even Schrödinger was not entirely convinced. Famous is his cat paradox. On the other hand, the abandonment of free will evokes a lot of resistance from many scientists. Bell's theorem is a kind of projection of the obvious hypothesis that humans have free will. Recently, famous scientists like Penrose, Lloyd and 't Hooft have also tended to give up free will, albeit in a different context. One can conclude that a kind of monism ('all is one') has emerged in QM. This philosophical concept goes back in time to Plato.





Fig. 1.14: *Quantum mechanics is concerned with probabilities for microscopic objects. Rather controversial problems arise when trying to extend quantum properties to the macroscopic world we live in.*

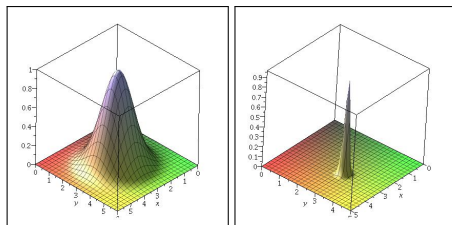


Fig. 1.15: *Left: The wave function spread out. Right: Sharp location of the wave function, similar to a particle.*

Platonism became the dominant viewpoint in the Roman empire. In modern times, this idea of 'the One' was pushed to an otherworldly afterlife. This is nature and is opposed to the vague metaphysical world. Anyone who dared to confuse the monastic One and nature, was persecuted. However, monism and science do not belong together. Monism is not real science. Yet monism resurfaced from time to time. In Chapter 8 we will come back to metaphysics. QM, however, reintroduced in some way, this controversy. The duality of particles and waves was the birth of the foundation of QM. Bohr, Pauli and Heisenberg formulated the Copenhagen interpretation of QM. You can look at nature with two different eyes: particles or waves, but if you open both eyes, the duality disappears. They believed that there is no underlying, hidden reality behind QM. It describes precisely the incompleteness of our knowledge of nature. This 'smells' like metaphysics. As mentioned earlier, the forms of quantum objects are not in a well-defined state. They do not really exist until we capture them with experiments. The whole One is destroyed. In quantum language, this is called

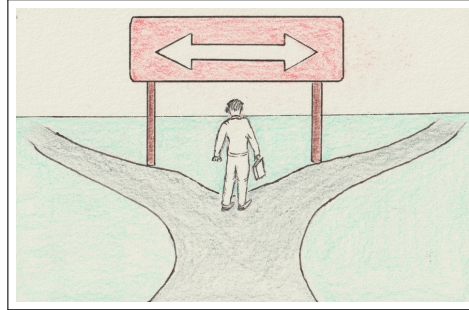


Fig. 1.16: *Do we have free will?*

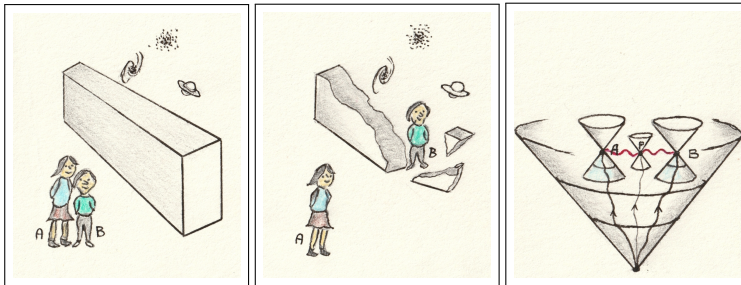


Fig. 1.17: *Left: The deterministic world. Observers Alice and Bob look at the universe that sits out there. Center: Quantum mechanics. It replaces the pure observer as participant. The outcome of observation can only be described statistically, taking its strangeness for granted. Right: The super-deterministic, or ontological viewpoint. To bring QM closer to GRT, one could change the basis of QM. Alice, Bob and the entangled pair of photons originating from a certain point in the past, are all three correlated, because they share variables in their past light cones.*

decoherence. The Copenhagen interpretation is still adhered to by the majority of physicists. Schrödinger and Einstein were the first to doubt this interpretation. In recent decades, more physicists followed. Is there a hidden reality? Should we take monism in fundamental physics seriously to make progress in solving the crisis? During observation, the observer interacts with the object and simultaneously with the whole environment. The observer cannot be isolated from his or her environment. While the universe still remains an entangled quantum object, the observer imagines that it is divided into many things. But the whole is still the One. It is destroyed from the specific perspective of the observer. Another peculiar issue is the arrow of time, which does not seem to follow from underlying physical laws. The physical laws are invariant under time reversal. A fundamental measure of time is the thermodynamic time, as we will encounter. The gradual diffusion of energy towards a ther-