

# Basic Calculations in Finance and Insurance

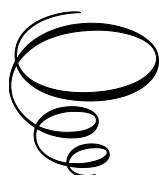


# Basic Calculations in Finance and Insurance

By

Tomas Cipra

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# CHAPTER 1

## INTRODUCTION

The seventeen chapters of this publication present basic calculation principles in finance and insurance. They are concerned with the main practical aspects of financial and insurance issues that become important parts of contemporary life. The reader of this publication should be capable of repeating and then applying all presented calculations numerically by means of simple calculation instruments (e.g., numerical calculators or Excel) unassisted by any specialized software. It may sound like an anachronism in the era of AI, but it helps substantially to make sense of the fundamentals of various problems and to make important financial or insurance decisions. Moreover, this fact predetermines the text as suitable training material in contrast to computer-oriented courses in finance.

Roughly speaking, Chapters 2–13 are devoted to finance, and 4–18 to insurance. At first, basic *interest* and *discount concepts* are described and generalized in Chapters 2–5. *Systems of cash flows*, including *investment rules*, *financial annuities* and *debt amortization* are presented in Chapters 6–8. Several chapters are devoted to securities, namely, *short-term securities* (Chapter 3), *bonds* (Chapter 9), *stocks* (Chapter 10), and *derivative securities* (Chapter 11). *Financial risk* and *portfolio analysis* are studied in Chapter 12. Chapter 13 deals with *currencies* (including *cryptocurrencies*). The remaining chapters concern insurance topics (after a general survey in Chapter 14), namely, *life insurance* including *life tables* (Chapters 15–16), *pension insurance* (Chapter 17), and *non-life insurance* (Chapter 18).

In addition to solved examples, each chapter contains a section of *unsolved examples*, which enables one to train the material of each chapter checking one's results with the ones given in the Appendix A. The Appendix B presents *financial tables* that can be useful for a quick evaluation of practical financial problems (even though such tables may again look like an old-fashioned instrument nowadays). The *recommended bibliography* in each chapter covers the concepts introduced and applied (the referenced monographs or textbooks may be suitable for further

reading). If one needs to get acquainted with any concept in more detail, nowadays, it is not difficult to find qualified reference sources for this purpose.

The formal structure of each chapter is the same, starting with keywords of particular sections with separated parts corresponding to particular keywords. For each keyword one usually starts with its basic characteristics, then basic computational formulas and relations are given with descriptions of corresponding symbols, and finally, numerical examples show possible numerical applications. Sometimes, various remarks indicate modifications that can appear in practice. The text uses standard symbols of financial and actuarial mathematics. References to examples, remarks, and formulas are organized in a natural way, e.g., Example 6.2.1 is the first example in Section 6.2 of Chapter 6. The ending of examples and remarks is always highlighted by the symbol  $\square$ .

Primarily, the publication may be helpful for people dealing professionally with problems of the quantitative economy (in banks, insurance companies and pension funds, financial institutions, investment planning, treasury, government bodies and others), or they are simply interested in financial or insurance problems they meet in practical life. However, the book can also be used as a teaching and training material for undergraduate courses in finance or actuarial sciences.

# CHAPTER 2

## SIMPLE INTEREST AND DISCOUNT

Chapter 2 deals with basic concepts of *simple interest* and *simple discount*:

2.1. Interest terminology and concepts:

*bank rate – factors affecting interest rates – interbank interest rate – interest margin – interest rate – rate of return – repo rate*

2.2. Simple interest:

*calendar conventions – interest number and divisor – simple interest*

2.3. Simple discount:

*certificate of deposit – discount principle – simple discount*

### 2.1. Interest terminology and concepts

***Interest:***

*Interest:* may be regarded as an amount paid by one person or organization (*borrower/ debtor/investor*) for the use of an asset, referred to as *capital* (or *principal* when expressed in monetary terms) belonging to another person or organization (*lender/creditor/depositor*). The pragmatic financial approach defines the interest as the amount of money that borrowers promise to pay to lenders to compensate them for the use of their money. Some aspects of interest should be mentioned:

- Interest *from the point of view of borrower*: is the price paid for borrowed money (e.g. for obtaining a credit). The debtor is willing to pay high additional charges for borrowed money because of its urgent necessity or considering expected profits (possibly higher than the paid interests). The world economy is said to be powered by credits.
- Interest *from the point of view of creditor*: is the award for postponed consumption and uncertainty of investment (e.g. for providing a credit). The creditor temporarily loses the right to dispose of lent

money and demands an adequate premium due to potential risks (defaults, inflation and others).

- The interests can be of various types, e.g.:
  - *effective*: are actual interests respecting various effects that have influence on returns (due to conversions  $m$  times per year, time shifts, charges, and the like);
  - *nominal*: are quoted interests that do not respect the actual conversion periods and other possible effects;
  - *real*: are interests adjusted for inflation;
  - *returns*: are yields (gains, profits) due to investment.

*Interest rate*: is the interest expressed relatively (mostly in per cent) of the value of principal capital. For instance, the annual interest rate 10 % (denoted as 10 % p.a.) means that the creditor obtains ten cents of each euro for one year of the loan. The spectrum of interest rates valid currently in a given region belongs to important economic indicators. We shall also switch freely between expressing interest rate  $i$  as a percent or decimal. It is possible to have  $i > 1$  (interest rate of over 100% per year) or  $i < 0$ , which can be interpreted as a bank charging you for holding your principal.

*Rate of return (RoR, return, yield to maturity)*: is the interest rate in the context of investment. From the calculation viewpoint it is equivalent to the classical concept of interest rates.

### ***Factors affecting interest rates:***

The interest rates depend on various factors:

- *Discount rate (or bank rate)*: is the interest rate which is charged by a central bank while lending to a commercial bank (e.g., to meet its liquidity needs).
- *Repo rate*: relates to repurchase agreements (*repos*) that are in general short term, well collateralized loans of liquid securities used to provide short-term financing to borrowers and short-term investing to lenders. In a repo, a financial institution that owns a security agrees to sell that security for a price  $P$  and buy the security back in the near future for a slightly higher price  $P + \text{repo rate interest}$  (a repo is like a well-collateralized loan except that the lender takes actual ownership of the security until it has been repurchased to settle the loan). Mostly in practice, the repo rate is the interest rate for which commercial banks lend short-term money (e.g., for two weeks) to the central bank against government securities (mainly T-bills, see Section 3.2). In many

countries the repo rate plays the key role when setting the level of important interest rates including the *mortgage interest rates*.

- *Interbank interest rates*: are the interest rates for short-term loans among commercial banks. For instance, EURIBOR (Euro Interbank Offered Rate) and EURIBID (Euro Interbank Bid Rate) are published daily for various maturities from 1 week to 12 months as an average of nearly thirty leading interest rates on the interbank market in Eurozone (including several banks outside Eurozone). Similarly, one applies FIBOR in Germany (Frankfurt), PIBOR in France (Paris), PRIBOR in Czech Republic (Prague) and TIBOR in Japan (Tokyo). Recently, LIBOR (London) previously widely applied in international finance was replaced by a new set of rates SOFR (*Secured Overnight Financing Rates*) based on observations of transactions in various money markets.
- *Bank strategy factor*: may consist in the demanded *interest margin* which is in classical banks the difference between the interest rates of credits on one side and deposits on other side.
- *Risk factor*: influences the interest rates significantly. Obviously the interest rates increase with the increasing risk of the given financial instrument. Due to the risk factor the return of government financial instruments (e.g. government bonds) is lower than the return of commercial instruments. The extremes are (nearly) *riskless* (or *risk-free*) instruments on one hand (e.g., U.S. Treasury rates corresponding to U.S. dollars T-bills from Section 3.2 are often viewed as proxies of the riskless interest rates) and *high-risk* instruments on the other hand (e.g., junk bonds). In the case of corporate bonds, there appears a risk that the corporation may default on its scheduled coupon and principal payments, and potential investors demand that the risk-free interest rate must be increased by a *risk premium* (*bond yield spread*) compensating the bond holder for the default risk.
- *Inflation factor*: another important factor that may influence the interest rate of any transaction is given by the inflation which is a general increase of the prices of goods and services in an economy. This factor is obviously very important in the times of high inflation (see Section 5.4).
- *Tax factor*: is important due to the fact that usually the *net returns* (in contrast to the *gross returns*) are crucial in finance. A simplifying role plays the *withholding tax* (e.g., bank depositors receive directly the net interests after taxation).

- *Term factor*: means that interest rates depend on the terms of financial instruments. Usually the annual returns of long-term financial instruments (e.g., deposits) are higher than for the short-term ones. One could give further factors and aspects influencing the interest rates which are not mentioned here.

## 2.2. Simple interest

The simple interest calculation is based in any time on the original principal capital without including interests from previous times so that the amount due increases *linearly*. One applies this principle typically for short-term financial instruments (see Chapter 3).

**Simple interest:**

$$u = P \cdot i \cdot t = P \cdot \frac{p}{100} \cdot \frac{k}{360} \quad (2.2.1)$$

- $u$  ...simple interest
- $P$  ...*principal (principal capital, present value)*
- $i$  ...annual (p.a.) interest rate given as a decimal value (e.g.,  $i = 0.035$ )
- $p$  ...annual (p.a.) interest rate given as a per cent value (e.g.,  $p = 3.5\%$ )
- $t$  ...time measured as a fraction of year (mostly  $0 < t \leq 1$ , e.g.,  $t = 0.5$  means the time of half year)
- $k$  ...time measured in days: e.g.,  $t = k/360$  (see Remark 2.2.1)

*Simple interest rate* (analogously for the principal or for the time of loan):

$$i = \frac{u}{P \cdot t} \quad \left( P = \frac{u}{i \cdot t}, \quad t = \frac{u}{P \cdot i} \right) \quad (2.2.2)$$

**Example 2.2.1.** The preferred stock with annual dividend rate 4.5 % and the face value 1 000 EUR apiece was purchased for the market price 900 EUR apiece. What is the annual rate of return for the investor (preferred stocks are described in Section 10.1)?

**Solution:** One can solve this example formally in two steps:

Step 1: According to (2.2.1) for  $P = 1\,000$ ,  $i = 0.045$ ,  $t = 1$  one has

$$u = P \cdot i \cdot t = 1\,000 \cdot 0.045 \cdot 1 = 45 \text{ EUR.}$$

Step 2: According to (2.2.2) for  $P = 900$ ,  $u = 45$ ,  $t = 1$  one has

$$i = \frac{u}{P \cdot t} = \frac{45}{900 \cdot 1} = 0.05, \quad \text{i.e. } p = 5 \%$$

Hence the annual rate of return of the given preferred stock is 5 %.

□

*Amount due in simple interest model:*

$$S = P + u = P(1 + i \cdot t) = P \left( 1 + \frac{p}{100} \cdot \frac{k}{360} \right) \quad (2.2.3)$$

$S$  ...amount due

*Principal  $P$  of amount due  $S$  in simple interest model:*

$$P = \frac{S}{1 + i \cdot t} = \frac{S}{1 + \frac{p}{100} \cdot \frac{k}{360}} \quad (2.2.4)$$

**Calendar conventions:**

*Calendar conventions:* are rules how to count the difference between two dates, i.e., the time  $t$  or fractions of the type  $k/360$  in the formulas of the type (2.2.1). One applies them in the formulas of simple interest model (see Section 2.2) and simple discount model (see Section 2.3). In their description the usual symbol for date is *DMY* (e.g. the date November 6, 2020 corresponds to  $D = 6$ ,  $M = 11$ ,  $Y = 2020$ ). For a given time interval one needs the initial date  $T_1 = D_1M_1Y_1$  and the final date  $T_2 = D_2M_2Y_2$ :

- *Standard 30E/360:* means that all months have 30 days and all years have 360 days:

$$\begin{aligned} t = \frac{k}{360} &= \frac{360(Y_2 - Y_1) + 30(M_2 - M_1) + (D_2 - D_1)}{360} = \quad (2.2.5) \\ &= \frac{360(Y_2 - Y_1) + 30(M_2 - M_1 - 1) + (30 - D_1) + D_2}{360}, \end{aligned}$$

where  $D_1 = 31$  or  $D_2 = 31$  must be changed to  $D_1 = 30$  or  $D_2 = 30$  before substituting them to (2.2.5). One can see from the numerator of the last fraction in (2.2.5) that the first day  $D_1.M_1.Y_1$  is not included (see the summand  $(30 - D_1)$ ), whilst the last day  $D_2.M_2.Y_2$  is included in  $k$  (see the summand  $D_2$ ). If it is e.g.  $D_1.M_1.Y_1 = 15. 3. 2023$  and  $D_2.M_2.Y_2 = 6. 11. 2023$ , then according to (2.2.5) one obtains  $k = 360 \cdot (2023 - 2023) + 30 \cdot (11 - 3 - 1) + (30 - 15) + 6 = 231$  days so that  $t = 231/360$ . In practice the standard 30E/360 is usually used for the operations with securities.

- *Standard ACT/360*: applies months with the calendar number of days (ACT means the abbreviation for *actual*) but the years with 360 days in the denominator:

$$t = \frac{k}{360} = \frac{\text{calendar number of days}}{360} \quad (2.2.6)$$

(some technical details are omitted). In practice the standard ACT/360 is usually used for operations with deposits.

There are other calendar conventions, e.g. the standard *ACT/365* popular in the UK. Moreover, there are conventions that are applied when the maturity date is not the bank day:

- *following day*: the maturity date is taken as the following bank day;
- *modified following day*: the maturity date is taken as the following bank day if it still lies in the same month: otherwise one takes the preceding bank day;
- *preceding day*: the maturity date is taken as the preceding bank day;
- *second-day-after*: the maturity date is taken as the following second bank day.

**Example 2.2.2.** A client deposited to a bank the amount 15 000 EUR on March 15 and collected it including interests on November 6 of the same year. What amount did the client receive if the bank account guaranteed the annual interest rate 3 %?

**Solution:** Using the standard ACT/360 (usual for bank deposits) one has according to Remark 2.2.1

$$t = \frac{k}{360} = \frac{(31-15) + 30 + 31 + 30 + 31 + 31 + 30 + 31 + 6}{360} = \frac{236}{360}.$$

Hence the amount due according to (2.2.3) is

$$S = P(1 + i \cdot t) = 15\,000[1 + 0.03 \cdot (236/360)] = 15\,295 \text{ EUR.}$$

As a matter of fact, after applying the withholding tax of 15% the net simple interest is only  $0.85 \cdot 295 = 250.75$  EUR, so that the net balance is 15 250.75 EUR.

□

### ***Interest number and divisor:***

Interest numbers and divisors are a useful instrument of systematic approach to simple interest calculations in financial practice, particularly in

situations when the principal varies frequently during year, e.g. in the current accounts (checking account):

*Interest number*: is defined as

$$IN = \frac{P \cdot k}{100}. \quad (2.2.7)$$

*Interest divisor*: is defined as

$$ID = \frac{360}{p}. \quad (2.2.8)$$

*Simple interest by means of IN and ID*:

$$u = P \cdot \frac{p}{100} \cdot \frac{k}{360} = \frac{P \cdot k}{100} / \frac{360}{p} = \frac{IN}{ID}. \quad (2.2.9)$$

*Simple interest by means of IN and ID under varying principal*:

$$\begin{aligned} u &= P_1 \cdot \frac{p}{100} \cdot \frac{k_1}{360} + \dots + P_n \cdot \frac{p}{100} \cdot \frac{k_n}{360} = \frac{P_1 \cdot k_1}{100} / \frac{360}{p} + \dots + \frac{P_n \cdot k_n}{100} / \frac{360}{p} = \\ &= \frac{IN_1 + \dots + IN_n}{ID}, \end{aligned} \quad (2.2.10)$$

where

$P_i$  ...principal during  $k_i$  days ( $i = 1, \dots, n$ ), e.g. the account balance between previous and next turnovers

$IN_i = \frac{P_i \cdot k_i}{100}$  ...interest number corresponding to capital  $P_i$  during  $k_i$  days ( $i = 1, \dots, n$ )

$ID$  ...interest divisor according to (2.2.8)

**Example 2.2.3.** Table 2.2.1 describes the development of a current account which was open on January 12 with the interest rate 2 % p.a. and standard 30E/360. What is the simple interest settlement at the end of the given calendar year?

<i>Date of turnover</i>	<i>Balance (in EUR)</i>	<i>Number of days</i>	<i>Interest number</i>
12. 1.	160 000	133	212 800
25. 5.	90 000	129	116 100
4. 10.	240 000	86	206 400
31. 12.	240 000		
$\Sigma$		348	535 300

**Table 2.2.1.** Development of current account in Example 2.2.3

**Solution:** In Table 2.2.1 one has completed numbers of days for particular balance values (see the third column) and then calculated corresponding interest numbers (see the fourth column). E.g. for the first account balance of 160 000 EUR one has

$$IN_1 = \frac{P_1 \cdot k_1}{100} = \frac{160000 \cdot 133}{100} = 212800.$$

As the interest divisor is permanently

$$ID = \frac{360}{p} = \frac{360}{2} = 180,$$

one obtains the interest settlement according to (2.2.10) as

$$u = \frac{IN_1 + IN_2 + IN_3}{ID} = \frac{535300}{180} = 2\,973.89 \text{ EUR.}$$

□

### 2.3. Simple discount

*Simple discount:* is an interest transaction common mainly for short-term financial instruments, i.e., with maturity up to one year (bills of exchange, certificates of deposits (CD's), Treasury bills (T-bills), etc., see Chap. 3):

- the price of the corresponding loan is set down by subtracting the so-called *discount* from the amount due;
- such a loan makes use of the *discount principle*, i.e., the corresponding interest is credited at the beginning of the discount period (*interest in advance*, while in the simple interest model the interest is credited *in arrears* at the end of the interest period). E.g., if the discount rate in a one-year loan is 10%, then the debtor receives from each borrowed euro only 90 cents even though one must return one euro at the end of year.

*Discount:*

$$D = S \cdot d \cdot t = S \cdot d \cdot \frac{k}{360} \quad (2.3.1)$$

$D$	...discount
$S$	...amount due
$d$	...annual (p.a.) <i>discount rate</i> given as a decimal value
$t$	...time measured as a fraction of year ( $0 < t \leq 1$ )
$k$	...time measured in days

*Principal  $P$  of amount due  $S$  in simple discount model:*

$$P = S - D = S \cdot (1 - d \cdot t) = S \cdot \left(1 - d \cdot \frac{k}{360}\right) \quad (2.3.2)$$

**Example 2.3.1.** What is the price of a 9-months certificate of deposit with the face value (amount due) 10 000 EUR and the discount rate 5%? *Certificates of deposit* are short-term securities representing savings deposits in commercial banks or savings institutions which may be traded on the discount principle (in contrast to the classical bank deposits).

**Solution:** The price of this certificate is the principal  $P$  of loan  $S = 10\,000$  EUR which the bank in the role of debtor obtains immediately when it sells the certificate to the client in the role of creditor. Hence according to (2.3.2) for  $S = 10\,000$ ,  $d = 0.05$ ,  $t = 270/360 = 0.75$  and standard 30E/360

$$P = S \cdot (1 - d \cdot t) = 10\,000(1 - 0.05 \cdot 0.75) = 9\,625 \text{ EUR} .$$

It means that the client buys the certificate for 9 625 CZK and in 9 months should receive back the amount due of 10 000 EUR. However, the return  $10\,000 - 9\,625 = 375$  EUR is taxed by the withholding tax 15 % so that the client will obtain the net amount calculated as  $10\,000 - 0.15 \cdot 375 = 10\,000 - 56.25 = 9\,943.75$  EUR.

□

**Remark 2.3.1.** The annual rate of return for the buyer of the certificate of deposit  $i$  in Example 2.3.1 must fulfill the equation

$$9\,625(1 + i \cdot 0.75) = 10\,000$$

with the solution  $i = 0.052$ . The rate of return of 5.2% is higher than the corresponding discount rate of 5% (this relation between  $i$  and  $d$  holds generally). If one calculates the net rate of return, then 10 000 on the right-hand side of the equation above must be replaced by 9 625.

□

## 2.4. Examples

**Example 2.4.1.** What should the market price of the preferred stock from Example 2.2.1 be to provide the annual rate of return 6 %?

**Example 2.4.2.** The client's short-term credit of 50 000 USD for 6 months with annual credit interest rate 6.5 % provided by a security broker includes a condition that the balance of client's margin account

must be permanently kept at least at 30 % of the credit. What is the annual effective credit rate for the client?

*Hint: The calculation is similar as in Example 2.2.1 but the principle capital in the second Step 2 is only 35 000 USD.*

**Example 2.4.3.** A client deposited to a bank the amount 90 000 USD on January 8, 2024 and withdrew it including interests on December 10 of the same year. What amount did the client receive if the bank account guaranteed the annual interest rate 2.5 % (notice that 2024 is the leap year using standard ACT/360, the withholding tax is 15 %)?

**Example 2.4.4.** Table 2.4.1 describes the development of a current account which was open on January 14 with the interest rate 1.5 % p.a. and standard 30E/360. What is the simple interest settlement at the end of the given calendar year?

<i>Date of turnover</i>	<i>Balance (in EUR)</i>
14. 1.	30 000
12. 5.	20 000
27. 5.	15 000
22. 8.	40 000
20. 9.	60 000
31.12.	60 000

**Table 2.4.1.** Development of current account in Example 2.4.4

**Example 2.4.5.** What is the price of a 12-months certificate of deposit with the face value (amount due) 50 000 EUR and the discount rate 7.2 %? What is the net amount due (the withholding tax is 15 %)?

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## CHAPTER 3

### SHORT-TERM FINANCIAL INSTRUMENTS

Chapter 3 applies the simple interest and discount principles for short-term financial instruments. *Short-term financial instruments* are usually securities with a maturity not exceeding one year. They are frequently priced (quoted) using the *discount principle*, i.e., their price is set by subtracting the discount from the face value (amount due) using the corresponding discount rate and a suitable calendar convention. The short-term instruments are mostly *money market instruments* and usually *fixed-income instruments* (i.e., guaranteeing investors that they will receive specified cash flows at specified times in the future). In this chapter, some examples of such instruments are presented, including their computational aspects:

- 3.1. Bill of exchange:  
*bill of exchange – commercial paper – discount credit*
- 3.2. Treasury bill:  
*T-bills*
- 3.3. Current account:  
*current account – overdraft account – payment card*
- 3.4. Cash discount:  
*cash discount*

#### **3.1. Bill of exchange**

*Bill of exchange (draft)*: is an unconditional order which is

- issued (drawn) by a first party (*drawer*)
- and directing a second party (*drawee*, mainly a bank)
- to make a specific payment to a third party (*payee*) at a future date (sometimes the bill of exchange may not be an “order” but a “promise”).

*Types of bills of exchange:*

- *demand bills (sight bills)*: are payable at any time (no fixed date is enjoined);
- *term bills of exchange*: are payable after a specific period or on a fixed date;
- *trade bills*: are typically drawn by the seller of goods and accepted by the buyer;
- *order bills*: are payable to a specific person whose name is written on the bill;
- *bearer bills*: are payable to any person that has the bill on the date of maturity;
- bills of exchange can be in the form of *cheques* or tradable securities (*commercial papers, banker's acceptances* and others).

***Discount credit:***

*Discount credit*: is a short-term credit by a bank that discounts a bill exchange before its maturity. The bank reduces the original amount due by the discount corresponding to the period from the discount date to the maturity date.

**Example 3.1.1.** A commercial bank discounted a bill of exchange with a face value of 500 000 EUR due in 3 months (the bank applies the annual discount rate of 10 % and the discount provision of 0.05 % of the face value). What amount was obtained by the bearer of this bill of exchange from the bank on the date of the discount?

**Solution:** According to (2.3.2) for  $S = 500\,000$ ,  $d = 0.10$ ,  $t = 90/360 = 0.25$  one has

$$P = 500\,000 \cdot (1 - 0.10 \cdot 0.25) = 487\,500 \text{ EUR} ,$$

so that after subtracting the discount provision, the resulting amount is

$$487\,500 - 0.0005 \cdot 500\,000 = 487\,250 \text{ EUR} .$$

□

**Example 3.1.2.** A building firm issued commercial papers to cover the payroll with the total face amount 25 000 000 USD due on December 29. An investment company purchased this bill on January 18, applying the annual discount rate 9 %, but sold it again on November 30, applying the annual discount rate of 7.2 %. What annual rate of return did the investment company achieve in this transaction? *Commercial paper* is a money-market security issued by a large corporation to obtain funds to meet short-term debt obligations. It is backed only by an issuing bank or

company promise to pay the face amount on the maturity date specified on the bill.

**Solution:** The price of the commercial paper on January 18 can be calculated according to (2.3.2) as follows (there are  $(31 - 18) + 28 + 31 + 30 + 31 + 30 + 31 + 31 + 30 + 31 + 29 = 315$  days between January 18 and November 29):

$$P = 25\,000\,000 \cdot (1 - 0.09 \cdot 315/360) = 23\,031\,250 \text{ USD} .$$

Analogously, the price on November 30 can be calculated as follows (there are 29 days between November 30 and December 29):

$$P = 25\,000\,000 \cdot (1 - 0.072 \cdot 29/360) = 24\,855\,000 \text{ USD} .$$

Hence, the achieved annual rate of return can be calculated according to (2.2.3) as follows (there are  $315 - 29 = 286$  days between January 18 and December 29):

$$i = \frac{S - P}{P \cdot t} = \frac{24\,855\,000 - 23\,031\,255}{23\,031\,255 \cdot (286/360)} = 0.0997, \text{ i.e., } 9.97\% .$$

One concludes that the commercial company achieved an annual rate of return of 9.97 % p.a.

□

### 3.2. Treasury bill

*Treasury bills (T-bills):* are government securities issued by the central bank (the Ministry of Finance, the Fed in the United States) to finance the government in short-terms, i.e., to fund the short-term state deficit. They back the repo agreements (see Section 2.1). Moreover, the government can realize monetary regulation by means of T-bills, e.g., to call in the money when the inflation increases.

- T-bills are usually priced using the discount principle with maturities varying from weeks to one year. They are widely regarded as low-risk (or even riskless) instruments and secure investments with high liquidity. On the other hand, their rate of return usually lies on the lower limit of the spectrum of interest rates.
- The most countries do not redeem state debts and rely on the so-called *debt rolling*. T-bills and government bonds (see Section 9.1) present new debt issues to redeem the maturing debts.

**Example 3.2.1.** The following information was published on the auction of T-bills by the Czech National Bank:

Face value (mil. CZK)	Date of auction	Date of maturity	Time to maturity (days)	Auction	Price of surrender (CZK)	Net YTM (% p.a.)
1	6. 5.	3. 6.	28	Dutch	998 560	10.59

- (a) Find the primary auction price of this T-bill.  
 (b) Check the net YTM value.

The *Dutch auction* is a public offering auction in which the price of the offering is set after taking in all bids to determine the highest price at which the total offering can be sold (the investors place their bid for the amount they are willing to buy in terms of quantity and price). The *net YTM* is obtained after taxation by the withholding tax of 15% in the Czech Republic.

**Solution:** (a) Obviously, the difference of the face value and the price of surrender

$$1000\,000 - 998\,560 = 1440 \text{ CZK}$$

corresponds to the 15% tax of the yield (which is the discount in this case). Hence, the discount can be calculated as

$$1440 \cdot (100/15) = 9600 \text{ CZK ,}$$

so that the resulting primary price of the T-bill in the Dutch auction was

$$1000\,000 - 9600 = 990\,400 \text{ CZK .}$$

- (b) The net *YTM* is according to the (2.2.3)

$$i = \frac{S - P}{P \cdot \frac{k}{360}} = \frac{998\,560 - 990\,400}{990\,400 \cdot (28/360)} = 0.1059, \text{ i.e., really } 10.59\%.$$

□

**Example 3.2.2.** A commercial bank decided to invest a part of its financial reserves to T-bills with a total nominal value of 10 000 000 EUR and a maturity of 84 days (i.e., 12 weeks) offered for 9 870 000 EUR. After 35 days (i.e., 5 weeks), the bank made use of the opportunity to sell these bills conveniently to an investment firm that wanted to deposit the prepared investment amount for a short time, and purchased the bills for 9 940 000 EUR. Was the premature sale of T-bills convenient for the bank? (The convenience of this transaction for the investment firm is obvious: the firm earned in 7 weeks the amount of  $10\,000\,000 - 9\,940\,000 = 60\,000$  EUR exploring the capital that would otherwise lie idle and, moreover, avoiding the risk that the capital is unavailable in the required term.)

**Solution:** According to (2.3.2) one can calculate the annual discount rates corresponding to the alternatives:

- without the sale of T-bills:

$$d = \frac{S - P}{S \cdot \frac{k}{360}} = \frac{10\,000\,000 - 9\,870\,000}{10\,000\,000 \cdot (84/360)} = 0.0557, \text{ i.e., } 5.57\%;$$

- with the sale of T-bills:

$$d = \frac{9\,940\,000 - 9\,870\,000}{9\,940\,000 \cdot (35/360)} = 0.0724, \text{ i.e., } 7.24\%.$$

Comparing the discount rates, the second alternative (i.e., the premature sale of the bills) is more convenient for the bank (obviously under the condition that the bank will find immediately a new investment opportunity to allocate the money received by the premature sale of the bills). □

### 3.3. Current account

Even though the current accounts are not securities, they serve as typical examples of simple interest calculations in practice. Let us remember that typical calculation instruments for current accounts are interest numbers and divisors (see Section 2.2).

*Current account:* is a deposit account held at a financial institution (mostly a bank) that allows (frequent) withdrawals and deposits. The current accounts are very liquid and can be accessed using automated teller (AT) machines and electronic funds transfers.

*Overdraft account (overdraft):* is a special type of current account that allows the account holder to continue withdrawing money even when the account has insufficient funds to cover the amount of the withdrawal. In other words, the bank admits a negative balance (*debit*) of the account and provides a short-term credit to the client up to a contracted *credit limit*.

*Interest operations* in the framework of *overdraft account*:

- interests for the account holder (bank's client):
  - *credit interests:* from positive balances (denoted as *credit balances*);

- interests and charges for the bank:
  - *debit interests*: from negative balances (denoted as *debit balances*);
  - *provision for exceeding credit limit*;
  - *liquidity provision*: due to the possibility that the client's debit balance may be raised to the credit limit, the bank must hold liquid funds for this potentiality;
  - *administrative charges*: depend on the bank's strategy.

<i>Date of turnover</i>	<i>Inflows</i> (in EUR)	<i>Outflows</i> (in EUR)	<i>Balance</i> (in EUR)
24. 3.	10 000	–	10 000
10. 6.	–	11 000	–1 000
15. 9.	6 000	–	5 000
15. 10.	–	11 000	–6 000
5. 12.	19 000	–	13 000
31. 12.	–	–	13 000

**Table 3.3.1.** Development of overdraft account in Example 3.3.1

**Example 3.3.1.** An overdraft account developed during a calendar year, according to Table 3.3.1. What is the simple interest settlement at the end of the given calendar year if the credit limit is 4 000 EUR, the credit interest rate is 3 % p.a., the debit interest rate is 14 % p.a., the provision for exceeding debit limit is 4 % p.a., the liquidity provision is 0.7 % p.a. and the annual flat charge for the account administration is 50 EUR (the calendar convention is the standard ACT/360)?

**Solution:** In Table 3.3.2, one has completed the number of days for the balance values of particular interest items and then calculated corresponding interest numbers. E.g., the interest number for the credit balance 10 000 CZK is

$$IN = \frac{10\,000 \cdot 78}{100} = 7\,800$$

and the interest divisor for the interest rate of 3 % is

$$ID = \frac{360}{3} = 120.$$

All balances are taken as without signs. In particular, the balance of 2 000 EUR corresponding to exceeding credit limit is calculated as 6 000 – 4 000, and the balance of 3 000 EUR corresponding to liquidity provision is calculated as 4 000 – 1 000.

<i>Number of days</i>	<i>Credit balance (in EUR)</i>	<i>IN</i>	<i>Debit balance (in EUR)</i>	<i>IN</i>	<i>Exceeding credit limit (in EUR)</i>	<i>IN</i>	<i>Liquidity provision (in EUR)</i>	<i>IN</i>
78	10 000	7 800	–	–	–	–	–	–
97	–	–	1 000	970	–	–	3 000	2 910
30	5 000	1 500	–	–	–	–	–	–
51	–	–	6 000	3 060	2 000	1 020	–	–
26	13 000	3 380	–	–	–	–	–	–
$\Sigma$		12 680		4 030		1 020		2 910

**Table 3.3.2.** Calculation of interest numbers for particular interest items of the overdraft account in Example 3.3.1

Hence according to (2.2.10) we obtain

- credit interests  $\frac{12\,680}{360/3} = 105.67$  EUR
- debit interests  $\frac{4\,030}{360/14} = 156.72$  EUR
- provision for exceeding credit limit  $\frac{1\,020}{360/4} = 11.33$  EUR
- liquidity provision  $\frac{2\,910}{360/0.7} = 5.66$  EUR

Therefore, the final balance of the overdraft account after the interest settlement is

$$13\,000 + 105.67 - (156.72 + 11.33 + 5.66 + 50) = 12\,881.95 \text{ EUR.}$$

□

**Remark 3.3.1.** The *eurodollars* are time deposits denominated in U.S. dollars held at banks outside the United States (mainly in European banks), i.e., they can be classified as short-term (e.g., 3-months) certificates of deposit. Eurodollars are not under the jurisdiction of the U.S. Federal Reserve Board and, consequently, are subject to much less regulation than similar deposits within the U.S.

□

### **Payment cards:**

*Payment cards* are payment instruments issued mainly by banks to customers. They enable the cardholders to access the funds in bank accounts and make payments by electronic transfer with payment terminals and access automated teller machines (ATMs). Sometimes, they are designed exclusively for use on the Internet (the so-called *internet banking* or *electronic banking*) without any physical form.

*Types of payment cards:*

- *debit cards*: are linked to the user's bank account and limited by the amount money in it. Funds are withdrawn directly either from the cardholder's bank account, or from the remaining balance on the card. Debit cards are also known as *bank cards* or *check cards*.
- *credit cards*: provide the user with a line of credit that they can borrow against as needed and pay back later. Credit cards can be used to build credit, while debit cards cannot. Particularly, credit cards charge interest on the money the cardholder borrows (unless it's paid back within the grace period).
- *stored-value cards*: enable monetary values to be stored on the card, and not in externally recorded accounts (unlike the debit cards, which are usually anonymous). The *term stored-value cards* specifically mean that all data are physically stored on the card.
- *fleet cards*: are payment cards, most commonly used for gasoline, diesel, and electricity at stations or for vehicle maintenance. Using fleet cards reduces the need to carry cash, thus increasing security for fleet drivers and preventing fraudulent transactions.

### 3.4. Cash discount

*Cash discount*: A cash discount is a deduction allowed by some sellers of goods or by some providers of services in order to motivate customers to pay within a specified time. The cash discount is also referred to as an *early payment discount*:

- An example of a typical cash discount is a seller who offers a 5 % deduction on an invoice due in 30 days if the buyer pays within the first 5 days of receiving the invoice. Giving the buyer a small cash deduction would benefit the seller in accessing the cash sooner. The sooner a seller gets the cash, the sooner this seller can put the money back into business to purchase more supplies or support the company in other ways. The amount of the cash discount is usually a percentage of the total amount of the invoice, but it is sometimes stated as a fixed amount.
- If the buyer takes advantage of this offer, then the buyer lends money to the seller and receives the offered deduction instead of interest in advance. From the calculation point of view, the convenience of the cash discount can be evaluated by comparing it with the interest that the seller can realize when receiving the money ahead of schedule.

**Example 3.4.1.** Goods worth 4 000 000 EUR are due in 30 days. If payment is realized in 3 days, then the seller provides a price deduction of 0.5 %. Is it convenient to exploit this cash discount from the point of view of the buyer applying the interest rate of 6 %?

**Solution:** The cash discount is

$$4\,000\,000 \cdot 0.005 = 20\,000 \text{ EUR.}$$

The interest that can be achieved by the seller when the cash discount is accepted may be calculated according to (2.2.1) as

$$(4\,000\,000 - 20\,000) \cdot 0.06 \cdot (27/360) = 17\,910 \text{ EUR.}$$

As the price deduction is higher than the corresponding interest, the possibility of a cash discount is convenient for the buyer.

### 3.5. Examples

**Example 3.5.1.** A commercial bank discounted a bill of exchange with a face value 60 000 EUR due in 2 months (the bank applies the annual discount rate of 12.5 % and the discount provision of 0.1 % of the face value). What amount did the bearer of this bill of exchange obtain on the discount date?

**Example 3.5.2.** A speculative investor bought on April 5 a banker's acceptance with the amount due 10 mil. EUR due on December 1, applying the annual discount rate of 7.5%, but sold it again on September 11, applying the annual discount rate of 6%. What annual rate of return did the investor obtain in this transaction? A *banker's acceptance* is a short-term security issued by a bank that guarantees payment later. It is often used in importing and exporting, with the importer's bank guaranteeing payment to the exporter. Similar to buying another short-term security, an investor in the secondary market might buy the acceptance at a discounted price but still get the full value at maturity.

**Example 3.5.3.** The following information was published on the auction of U.S. T-bills:

<i>CUSIP</i>	<i>Type</i>	<i>Issue Date</i>	<i>Maturity Date</i>	<i>Security Term</i>	<i>Price per \$100</i>
912797FW2	Bill	Oct10 2023	Jan4 2024	13-Week	98.648903

- Find the discount rate applied for pricing this T-bill.
- What is the corresponding YTM value?

**Example 3.5.4.** An overdraft account developed during a calendar year, according to Table 3.5.1.

<i>Date of turnover</i>	<i>Inflows (in CZK)</i>	<i>Outflows (in CZK)</i>	<i>Balance (in CZK)</i>
15. 3.	29 000	–	29 000
13. 4.	–	104 000	–75 000
28. 4.	64 000	–	–11 000
24. 7.	–	80 000	–91 000
20. 9.	85 000	–	–6 000
31. 12.	–	–	–6 000

**Table 3.5.1.** Development of overdraft account in Example 3.5.4

What is the simple interest settlement at the end of the given calendar year if the credit limit is 60 000 CZK, the credit interest rate 3 % p.a., the debit interest rate 13 % p.a., provision for exceeding debit limit 5 % p.a., liquidity provision 0.4 % p.a. and annual flat charge for the account administration is 1 200 CZK (the calendar convention is the standard ACT/360)?

**Example 3.5.5.** If a commercial claim for 9 000 USD due at most in six months is realized during 10 initial days, then the payer obtains a deduction of 1 %. Is it convenient to pay early if the savings accounts offer 5 % p.a. for the corresponding amount?

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