Emergent Collective Dynamics in Fault Systems

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Bridging Crustal Mechanics and Statistical Seismology

By

Davide Zaccagnino

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The belief that one's own view of reality is the only reality is the most dangerous of all delusions.

PAUL WATZLAWICK, *How Real is Real?*, 1976

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Author's preface

Earthquakes are likely the most puzzling natural phenomena on our planet. Although the basic physical mechanisms producing crustal instabilities are fundamentally well-known, our scientific approach to seismicity is still unsatisfying and the far-reaching goal of skillful forecasts has been missed till now.

Even though essential knowledge has been achieved in related fields from statistical physics to observational seismology and rock mechanics, a comprehensive view embracing key results from the parameters of the seismogenic source to the statistical features of sequences is yet to come. Therefore, the integration of different perspectives of earthquake science may greatly contribute to advance our ability to understand seismicity.

My research activity aims to improve our comprehension of the physical processes leading to the occurrence of large earthquakes. As a young earthquake physicist, I have been devoting my efforts to explain how major events emerge in previously stable crustal volumes from small magnitude ones, and their mutual relationships.

This book is a re-elaborated and harmonized report of a three years-long formative experience as a PhD student at the Department of Earth Sciences of Sapienza University and two visiting periods at the National and Kapodistrian University of Athens in Greece and Caltech, Pasadena, California.

The final outcome is itself an original piece of research in the field of seismotectonics and statistical seismology reporting a series of coherent analyses and results. Its structure is not to be intended as a concluded work, but as an ongoing travel diary towards an increasingly confident view of the physical foundations for the next generations of seismic hazard models. This book is shaped on the Ph.D. thesis “The emergence of collective behavior in fault systems” by the author himself (Zaccagnino 2025), defended on March 14, 2025; the text, figures and everything it is contained in this work is its own; part of the content has been published in research articles by the author himself and coauthors or elaborated from it (Zaccagnino et al. 2022a; Zaccagnino and Doglioni 2022a,b; Zaccagnino et al. 2022c, 2023, 2025; Zaccagnino and Doglioni 2025, 2026; Zaccagnino et al. 2025) according to the use allowed by creative commons licenses. For other papers (Zaccagnino et al. 2020, 2021, 2022b; Zaccagnino and Doglioni 2023; Zaccagnino et al. 2023, 2024; Barani et al. 2025; Zaccagnino and Doglioni 2026; Zaccagnino 2026), the author revised the content as in his thesis (Zaccagnino 2025) and re-produced all the figures employing same or updated data while using similar Matlab scripts and Adobe Illustrator sources and templates.

My investigations focus on the mechanical processes enabling the emergence of seismicity in fault systems and its statistical properties. Especially, I analyze the connection between physical conditions promoting the occurrence of major quakes, background dynamics bringing them into being and seismic forecasting. My work provides evidence that large events tend to occur under somehow favorable circumstances; moreover, they are forewarned by long-lasting, progressive preparatory phases leading to the large-scale destabilization of crustal volumes. Although often silent, they can be highlighted using appropriate techniques such as the monitoring of the seismic response to stress perturbations and clustering analysis.

In the first section, the physical origins of some key features of seismicity and their mutual relationships are discussed, while the second one is devoted to physics-based techniques for the identification of unstable crustal regions. The introduction provides a bird’s-eye view of earthquake physics and its connection to seismic hazard from the viewpoint of complex systems. I strongly emphasize the crucial role of collective behaviors of fault networks in shaping seismic activity. Furthermore, I advocate a cross-scale modeling of earthquake occurrence to grasp the essential traits determining the spatio-temporal evolution of seismicity. In the second chapter, I discuss the physical meaning of some fundamental laws of statistical seismology in the peculiar framework of optimization problems with implications for the relationships between their parameters and the tectonic environment. The following chapters show how the slip behavior of faults is mainly controlled by a few physical parameters through the action of feedback mechanisms. They do not only seem to tune the mechanics of faulting, but they also shape the key characteristics of telluric events from the composition of moment tensors to the temporal development of seismic sequences. Retracing the common thread of universal mechanisms as rulers of seismicity, the last chapters of the first part deal with the physical processes of faulting and their connection with rheology and fault styles, seismic paradoxes and the statistical properties of seismic sequences. Implications for the multi-scale modeling from laboratory experiments to natural faults are also described.

The second part of this book is devoted to the physical characterization of preparatory phases preceding large earthquakes. The sixth and seventh chapters show how the responsiveness of fault systems to stress perturbations increases before the mainshocks. They support the idea that major failures may be predated by the development of long-range interactions within crustal volumes ultimately resulting in large-scale instabilities. I also demonstrate that big events tend to occur where minor and moderate seismicity shows a locally Poissonian and globally-clustered behavior (chapter 8). This phenomenon is explained in the light of the ability of crustal volumes to accommodate additional strain and fault systems to deplete it while preserving overall stability. At last, I investigate the seismogenic potential of earthquake clusters showing that a continuum exists within a wide fan of fault slip behaviors ranging from aseismic creep to cascading "foreshocks". Here a physical explanation to justify why large earthquakes can be preceded by very different seismic patterns is proposed. Due to the development of long-range interactions while approaching instability, earthquake dynamics becomes intrinsically non-local. Consequently, faults show memory effects and strong sensitivity to the stress conditions of nearby crustal volumes.

Preface

Earthquakes remain one of the most complex phenomenon confronting our understanding of the planet. Despite significant advancements in various scientific domains, from the intricacies of statistical physics to the detailed observations of seismology and the material science of rock physics, the ambitious goal of accurate seismic forecasting has thus far eluded us. A comprehensive framework that seamlessly integrates the dynamics of individual ruptures with the large-scale spatial and temporal evolution of seismic activity is still conspicuously absent. This necessitates a concerted effort to synthesize different research perspectives, a convergence that holds the promise of significantly enhancing our comprehension of seismicity.

This book tackles this grand challenge head-on, focusing on the fundamental question of how large-magnitude earthquakes emerge from a backdrop of smaller events within crustal volumes previously deemed stable. The ultimate aspiration of this research lies in its potential to contribute to more effective strategies for seismic hazard assessment.

This work represents a refined and harmonized endeavor, enriched by valuable experiences at esteemed international institutions. The resulting body of work stands as an original contribution to the fields of seismotectonics and statistical seismology, presenting a coherent suite of analyses and findings. Importantly, the author frames this work not as a definitive endpoint, but rather as an insightful and ongoing exploration towards a more robust physical foundation for understanding seismic hazard.

The central thrust of this research work lies in unraveling the intricate connections between the physical conditions that pave the way for major earthquakes, the underlying mechanical processes that drive their occurrence, and the overarching challenge of seismic forecasting. The author presents widespread evidence suggesting that large earthquakes are not simply random failures but tend to occur under specific, predisposed conditions. Moreover, the research delves into the concept of protracted, gradual preparatory phases that precede significant seismic events. While often subtle, these phases can be identified and characterized through advanced techniques such as monitoring the crust's seismic response to stress perturbations and sophisticated clustering analyses.

The work is structured into two primary sections, following an initial exposition of the scientific context of earthquake science through the lens of complex systems. The first section investigates fundamental physical properties relevant to seismicity, while the second section concentrates on developing methodologies for pinpointing regions with a higher propensity for seismic activity based on physics-informed considerations.

A key contribution of this work involves a novel interpretation of fundamental laws of statistical seismology within the framework of optimization problems without assuming criticality. This approach yields significant implications for understanding the relationships between the parameters of these laws and the ambient tectonic environment, notably offering a pathway that circumvents the common assumption of criticality prevalent in many existing models.

Subsequent chapters build upon this foundation, demonstrating how the behavior of faults is primarily governed by a limited set of physical parameters operating through feedback mechanisms. These mechanisms are shown to not only regulate the fundamental mechanics of faulting but also to intricately shape a wide spectrum of seismic event characteristics, from the individual moment tensor compositions to the collective temporal evolution of seismic sequences.

Following a unifying thread of universal mechanisms governing seismicity, the book delves into the physical processes of faulting and their intricate links with rock rheology, fault styles, and the statistical properties of seismic sequences. This discussion lays the groundwork for multi-scale modeling efforts that aim to bridge the gap between laboratory experiments and the complexities of real-world fault systems. It pivots to the critical challenge of identifying regions with elevated seismogenic potential. The author provides compelling evidence that the susceptibility of fault systems to perturbations increases as they approach a state of instability, indicating a large-scale destabilization of crustal volumes in the lead-up to major failures. Moreover, the work demonstrates that significant seismic events tend to occur in regions where small to moderate seismicity exhibits a distinctive pattern of local Poissonian behavior coupled with global clustering - a phenomenon theoretically explained through the interplay of creeping versus locked fault mechanics and seismic coupling.

Finally, this book explores the seismogenic potential of earthquake clusters, revealing a continuum that spans from aseismic creep to the abrupt occurrence of earthquakes. This continuum is associated with a diverse range of precursory behaviors, encompassing phenomena such as seismic quiescence, pre-slip events, and the more commonly observed cascading foreshock sequences.

This work represents a significant contribution to our ongoing quest to decipher the complex and often perplexing nature of earthquakes. By thoughtfully integrating diverse scientific perspectives and employing rigorous analytical techniques, the author provides valuable insights that will undoubtedly stimulate further research and contribute to the development of more robust, physics-based approaches to seismic hazard assessment. This work will be of considerable interest to graduate students and researchers across the fields of seismology, geophysics, and statistical physics, offering fresh perspectives on the fundamental processes that govern the dynamic behavior of our planets.

Carlo Doglioni

Chapter 1

Introduction

> The limits of my language mean the limits of my world.
>
> ---
>
> Ludwig Wittgenstein, *Tractatus logico-philosophicus*, 1922

1.1 Complex systems: different worlds, same laws

On June 23, 1998 Christof Koch, former professor of neurophysiology at Caltech and president of the Allen Institute for Brain Science, bet the famous Australian philosopher David J. Chalmers that the mechanism of emergence of consciousness from neuronal activity would be well understood by 2023 (Chalmers 1998). After twenty-five years, Chalmers was appointed as winner of the adversarial collaboration by the Association for the Scientific Study of Consciousness (ASSC) (Finkel 2023). Nowadays, we still do not know how the interaction between neurons gives birth to our ability to perceive the surrounding reality, allowing us to behave as human beings. Taste, sense of smell, sight and apperception, love and hatred, sorrow and happiness. For each of such abilities, neuroscientists tracked down some key neuronal features allowing large scale information exchange with the environment even though a detailed map of how perception and human behavior originate is still missing. Yet, during the last decades computational power and diagnostic accuracy have been improving dramatically so that physicians can identify the regions of the cerebral cortex involved in visual and motor functions, memory and attention. Why are consciousness and related behaviors so elusive? The book *Consciousness: Confessions of a Romantic Reductionist* (Koch 2012) has a revealing title: most of the models in current neurophysiology of consciousness are devoted to deriving the overall features of being alive from neural physical mechanisms, hopefully localized in a well-defined brain region.

But what does consciousness have to do with seismic hazard? In order to answer this question, it may be useful to broaden a little bit my discussion to other challenging research topics.

Stock market crashes are sudden drops of financial bonds ultimately produced by traders' panic attacks (Sornette 2003). Such instabilities result from the long-term tendency to increase the value of shares and capitals and from the expansion of exchange possibilities and, on the other side, from the mounting fear of future crashes due to source of uncertainties and contingent socio-political reality (Brunnermeier and Oehmke 2013). Even though financial crises are usually triggered by large credit shocks produced by bankruptcy of major companies, they are incredibly tricky to predict (Laloux et al. 1999). The matter is that financial markets are extremely complicated systems with a lot of stakeholders and variables strongly interacting with each other. There is no unique stable configuration of parameters able to satisfy all the players, while we definitely have a huge number of marginally stable states of the system (Schmitt-Grohé and Uribe 2021). Each condition can be accessed in different ways starting from various initial conditions, but it is never permanent. As the system becomes more and more unstable because of the progressive increase of values, i.e., larger possible loss, or global conjunctures, i.e., perceived higher failure possibilities, smaller and smaller perturbations are able to dislodge it from its previous state towards a more stable one (Sornette 2003). Concurrently, the actions of stakeholders tend to become more and more coordinated, i.e., correlated. This effect produces long-range correlations to appear in the stock market with the emergence of collective behaviors. In this sense, in physical terms, we can say that financial markets are featured by critical transitions (Johansen et al. 2000). Each transition is marked by the divergence of peculiar quantities which ultimately rule a strongly nonlinear dynamics which occasionally appears as violent swift adaptations of bonds prices.

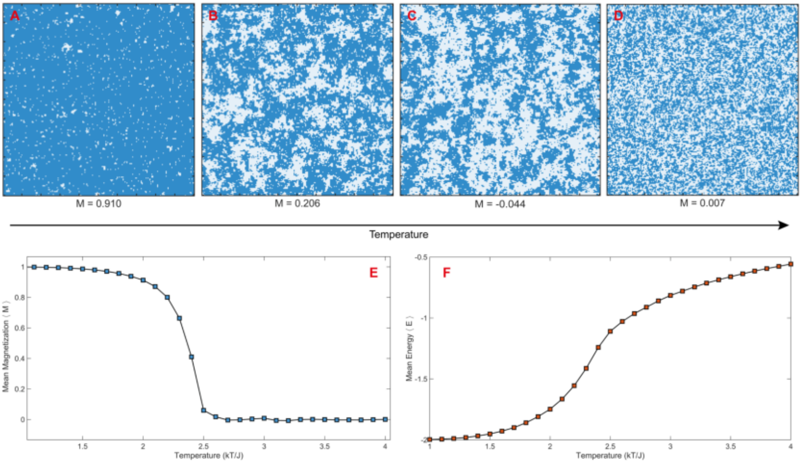

Fig. 1.1. Macroscopic realization of a square grid with 200x200 spins with two possible states (-1,1) represented by white and blue dots. At low temperature (A), spins are almost all aligned along the trend according to the external force producing a ferromagnetic behavior and large magnetization (E, left side). As the temperature kT/J approaches 2.2-2.3, the magnetization drops and the mean energy, associated with each spin, increases following an hyperbolic tangent function (F) and clusters of negative spins appear (B,C). At large temperature, a random pattern is found with no macroscopic magnetization (D).

The simplest prototype of systems going through phase transitions is the so-called 2D Ising model (Onsager 1944), describing the collective behavior of spin variables uniformly distributed over a two dimensional grid interacting each other with their nearest neighbors via a coupling factor, J_{ij}. i and j represent the location of the spin σ_i on the grid. According to the thermal state of the system and the strength of coupling, i.e., the competition between stochastic fluctuations and deterministic gradients, the general properties of the system change radically. See Figure 1.1 for a visual representation. The hamiltonian of the system, H, can be written, in a general form taking into account of a local field h_i and an external one $\mathcal{H}$, as (Young 1998)

$$H(\sigma) = -\sum_{(i,j)} J_{ij}\sigma_i\sigma_j - \sum_i (h_i + \mathcal{H})\sigma_i. \tag{1.1}$$

where (i,j) stands for the set of nearest neighbors. If random contributions are negligible, i.e., in the low temperature regime, the system tends to be ordered according to deterministic forces; conversely, at high temperature, the effect of fluctuations is dominant, which destroys geometric clusters with correlated features. In the first situation, spins tend to be spatially correlated producing macroscopic effects such as magnetization and optimizing the energy of the system. According to the initial state of the system, it evolves towards equilibrium (Figure 1.2). In the middle, a critical temperature exists

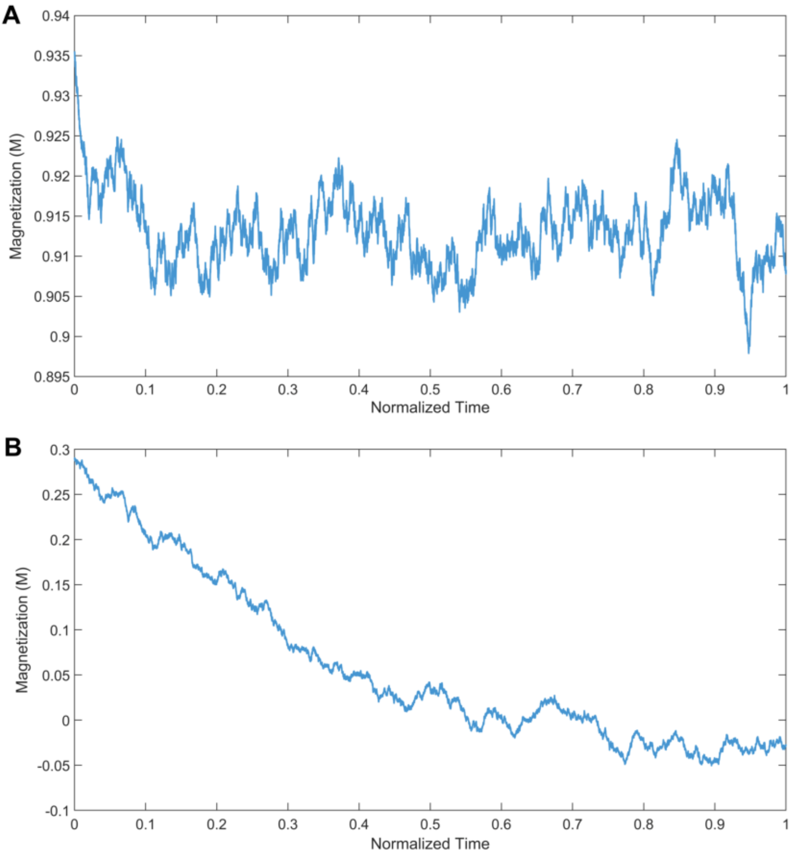

Fig. 1.2. Fluctuations around equilibrium conditions are observed at T = 2.0 in the Ising model at equilibrium (A), while a slow drift in magnetization is observed at T = 2.5 if the initial condition contains clusters of aligned spins (B).

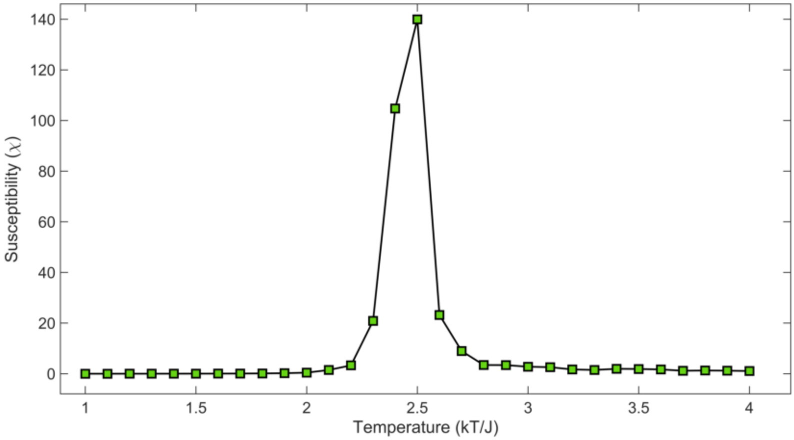

Fig. 1.3. Power-law divergence of magnetic susceptibility χ in the same system of Figure 1.1 and Figure 1.2 near the critical temperature.

marking the transition from the first state to the second one. For instance, compare with the behavior of magnetic susceptibility in the Ising model represented in Figure 1.3. Some features such as magnetization, susceptibility and correlation lengths follow power laws diverging at the critical temperature with universal scaling exponents depending on few characteristics of the system (Wilson 1971) (such as the number of dimensions it is embedded and the type and strength of internal disorder). This kind of prototypes can be applied to several different physical systems where spins can be replaced by vectors or scalar values of interest with suitable mathematical reformulation; global properties such as magnetization can be replaced by analog quantities and the temperature can be interpreted in terms of random fluctuations due to noise (Sornette 2006b). Nevertheless, the physics is the same (Kadanoff et al. 1989). In the case of financial systems, marginal stability is one of the features that has to be faced routinely, e.g., (Diamond and Dybvig 1983; Chan and Thakor 1987; Biais et al. 2015). Able economists and public affairs ministers should collaborate to find the best path within a fan of possible temporal trends towards an optimal economic configuration. The choice is not unique and depends on the political choices which constrain the horizons of social and inner affairs dynamics. Figure 1.4 represents the idea of "multiple equilibria" associated with saddle points and minima of the energy potential of complex systems (Parisi 2023). This concept is well known, for instance, in the field of bank deposits contracts which show multiple possible configurations, one of which is bank run. Therefore, a major goal consists in arranging contracts that can prevent failures and understanding how government provisions can help yield superior contracts. “Multiple equilibria” was indeed the title of the 2022 Nobel Prize for Economy Lecture given by Philip H. Dybvig (Dybvig 2023). The concept of multiple equilibria is also well known in sociology and mathematics, for instance in Game Theory, where reaching one of the Nash equilibria is the final goal of games with complex elements and several players, usually after trials and errors. This scenario of varying choices over time before getting equilibrium is one of the most common in the world of business. Analogous patterns are found in the temporal and spatial organization of large ecosystems and species within (Higgins et al. 2002). The interaction of living beings, historically modeled starting from the pioneering works by Lotka and Volterra, displays non-unique marginal stability conditions (Biroli et al. 2018). This is the physical base of adaptive processes leading to the evolution of species and allowing the selection of the best characteristics for the survival of Life, which unavoidably implies that less adaptable species become extinct. Even in this case, the raise of new species and their extinguishment generally occur suddenly and clustered in time (large mass extinctions and blooming) because of rapid variations in the environmental conditions. Therefore, long periods of stability are interspersed with dramatic changes which quickly lead ecosystems towards new equilibria. The evidence of gaps in fossil records was well documented since the late years of the XIX century (e.g., compare with *Elements de Paleontologie* by Felix Bernard, 1895). They were firstly modeled by N. Eldredge and S.J. Gould in 1972, when they introduced the theory of punctuated equilibria of speciation (Gould and Eldredge 1972). The concept of dynamic equilibrium is also important in other fields of biology and medical sciences. For instance, the ability of tumor tissues to differentiate themselves thanks to numerous genetic mutations is one of the reasons why some kinds of cancers are still so challenging to be treated (Loeb et al. 2003).

Strong correlations in many-body systems with cooperative phenomena plagued by avalanches are not exclusive of the realms of biology and economy. The internal structure of glasses is the final outcome of slow cooling and heterogeneity which frosts the systems in a frustrated disordered configuration (Charbonneau et al. 2017). Cracks in heterogeneous materials develop along paths by optimizing the balance of free surface energy and elastic potential

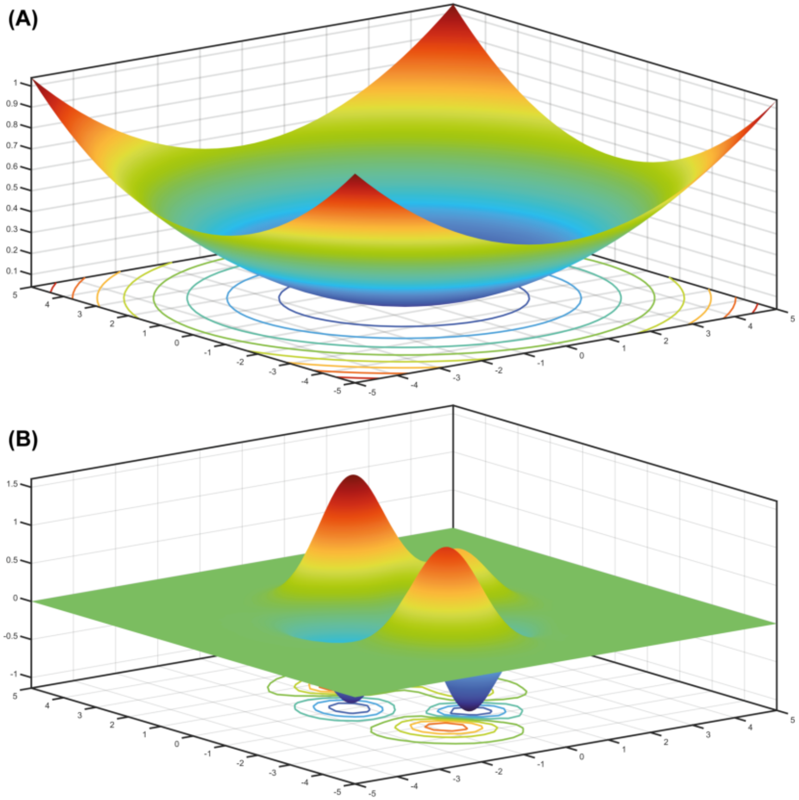

Fig. 1.4. Simple physical systems tend to explore the space of phases around their equilibrium point, corresponding to a minimum of their potential energy. Close to equilibrium, each potential profile can be reasonably approximated by a parable and the system oscillates around such equilibrium until an external perturbation will be able to dislodge it (A). In several complex systems, there is a huge number of equilibrium points and only marginally stable, so that the system may easily move from one to another.

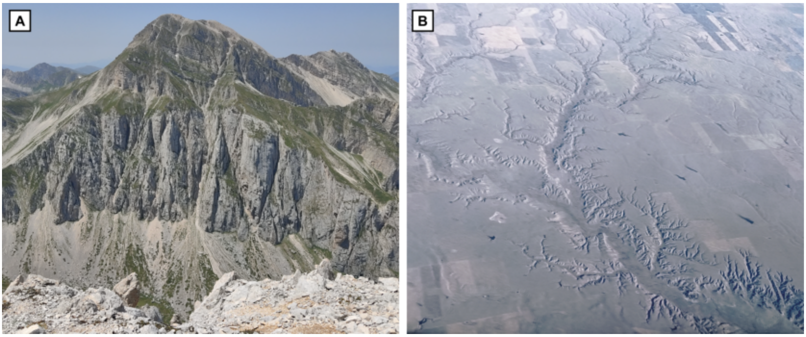

Fig. 1.5. Examples of fractal structures in geosystems. (A) Erosional slopes with self-similar patterns in the Gran Sasso Massif, Central Apennines, Italy. (B) Aerial view of a fluvial basin in Montana, USA.

budget depicting nice fractal ruptures analogous to the spatial patterns of lightnings and dielectric breaking (Niemeyer et al. 1984). The most general equation describing the development of interfaces $h(\mathbf{x}, t)$ in heterogeneous and disordered media affected by thermal (uncorrelated) noise and quenched (correlated) disorder is the celebrated Kardar-Parisi-Zhang equation (Kardar et al. 1986):

$$\frac{\partial h(\mathbf{x}, t)}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2} (\nabla h)^2 + \eta(\mathbf{x}, t) + \zeta(\mathbf{x}, h), \tag{1.2}$$

where $\eta(\mathbf{x}, t)$ represents thermal noise with $\langle \eta(\mathbf{x}, t) \eta(\mathbf{x}', t') \rangle = 2D\delta^d(\mathbf{x} - \mathbf{x}')\delta(t - t')$, and $\zeta(\mathbf{x}, h)$ is quenched noise with correlations $\langle \zeta(\mathbf{x}, h) \zeta(\mathbf{x}', h') \rangle = \Delta \delta^d(\mathbf{x} - \mathbf{x}')\delta(h - h')$. The first term is a deterministic force acting on the interface, the second one represents the tendency to surface relaxation, i.e., smoothing, and the third term stands for the simplest contribution to describe growth. Strongly connected with this mathematical framework, self-similarity is another ubiquitous trait of such phenomena also shared by several geomorfological settings due to hydrological drainage from mountains to alluvial basins (Rodriguez-Iturbe and Rinaldo 1997), the wind shaping of dunes (Day and Kocurek 2018), wildfires (Morvan 2011), snow avalanches and landslides development (Chiaia and Frigo 2009) (Figure 1.5). Self-similar patterns have been proven to enable optimal transportation schemes; for instance, in the case of fluvial basins, the fractal organization of tributaries allows efficient flows of precipitations from mountain peaks to the sea. Among the consequences of the growth of fractal structures, as well as of deep interactions between small-scale and collective dynamics, power-law distributions take the lion's share. Several complex systems can be mostly characterized using power-laws: the number of tectonic plates as a function of their angular surface extension is well fitted by a power-law (Sornette et al. 1990b; Zaccagnino et al. 2025)

$$N(\geq A) \approx cA^{-1/3} \tag{1.3}$$

with $c \sim 7$ and $N(\geq A)$ represents the cumulative number of plates with an area A expressed in steradians.
Earthquakes magnitudes, M, follow the Gutenberg-Richter law in a wide range of sizes, i.e., a power law with scaling exponent b, henceforth b-value, which can be written as (Gutenberg and Richter 1944)

$$\log(N(\geq M)) = a - b(M - M_c) \tag{1.4}$$

where N is the number of events with magnitude larger than M and M_c is the completeness magnitude, i.e., the size above which the number of undetected events does not affect statistical analyses. The modified Omori's law (Utsu et al. 1995)

$$n(t) = \frac{k}{(c+t)^p} \tag{1.5}$$

describes the attenuation over time, t, of the seismic rate of earthquakes, $n(t)$, induced by stress perturbations produced by a mainshock on unbroken fault segments. Analogously, the number of citizens of the largest cities all over the world, the strength of hurricanes, the number of animals and plants in different kinds, species, orders and classes, the visualization rates of web pages on the Internet and wealth in different countries are power-law distributed. So, we can write a general scaling law for each one of such observables, x, as follows:

$$\log(N(\geq x)) = \alpha - \beta(x - x_{min}), \tag{1.6}$$

where x_{min} is the detection threshold. However, the most amazing property of such systems is likely their strong non-linearity. By a mathematical viewpoint, it means that the superposition of solutions of their associated deterministic equations does not apply, producing complicated future evolution usually impossible to be found analytically. Moreover, apparently negligible perturbations can be quickly amplified by the nonlinear terms resulting in highly unpredictable outcomes, i.e., a chaotic behavior. Then, the gap between theoretical prediction and actual measurements diverges exponentially as

$$\Delta(t) \sim \Delta(0)e^{\lambda t} \tag{1.7}$$

from the initial error $\Delta(0)$. In mathematical terms, if we assume that our system can be described using a set of state variables, $\mathbf{x}$, and its temporal evolution can be modeled using the deterministic equation

$$\frac{d\mathbf{x}}{dt} = f(\mathbf{x}) \tag{1.8}$$

where $f(\mathbf{x})$ is a nonlinear function of the state variable; then, the prediction horizon, which defines the longest theoretical time interval for reliable forecasts, is given by

$$\mathcal{T}^{event} \sim \frac{1}{\lambda_{max}} \ln\left(\frac{\delta}{\Delta(0)}\right), \quad \delta, \Delta(0) \longrightarrow 0 \tag{1.9}$$

where δ is the accuracy required for our prediction. $\Delta(0)$ is the initial error in measuring our state variable $\mathbf{x}$, while λ_{max} is the largest Lyapunov exponent given by

$$\lambda_{max} = \lim_{t\to\infty} \lim_{\Delta\to 0} \frac{1}{t} \ln\left(\frac{\Delta(t)}{\Delta(0)}\right). \tag{1.10}$$

Nonlinearity can also arise in a quite simple system through the collective behavior of its components. A straightforward addition of its parts does not allow to recover its large-scale temporal evolution. Instead, the interactions between its fundamental elements lead to emergent phenomena such as fractal patterns, avalanche dynamics and extreme sensitivity to perturbations. It is the case of global climate, resulting from a complex radiative equilibrium between the Sun and our planet, thermal balance of oceans and continents, aerial and sea currents, hydrological circulation and atmosphere composition (Ferreira et al. 2011). Notwithstanding the combination of so many contributions, small random perturbations can be remarkably amplified by the interaction between non-linearity of atmospheric dynamics with external orbital forcing. This explanation has been utilized to understand several macroscopic effects such as glaciations and hot periods during the whole history of Earth (Benzi et al. 1981). Even in this case, multiple equilibria play a key role allowing long-lasting stationary periods alternated with swift changes. Their stability is enhanced by the coupling between oceans, atmosphere and glacial masses which determine feedback mechanisms. They, in turn, allow to compensate local unbalance keeping the whole system around its initial condition.

Feedback processes, i.e., coupling terms in which the evolution of a variable depends on itself and on other variables,

$$\frac{dy}{dt} = \sum_{i=1}^{N} A_i x^{\alpha_i} + C y^{\gamma}, \tag{1.11}$$

are responsible for an extremely wide range of features in complex systems, such as stability, effect amplification, and dynamic transitions. Here $x(t)$ and $y(t)$ are time-dependent variables, while α_i and γ are suitable exponents (typically real numbers). Rainfall time series are well-known to show alternations of droughts and wet periods as well as rivers' seasonal flooding. This phenomenon is also referred with the name of "Joseph Effect". It is inspired to the Old Testament story of Joseph, recounting the Pharaoh's dream about seven fat cows devoured by a herd of seven lean bovines, while seven ripe spikes were destroyed by seven desiccated heads. Joseph foresaw the future of Egypt in his dreams and suggested the interpretation that seven years of plentiful crop harvesting would have been followed by seven years of drought. According to Genesis, Joseph saved the population from famine. The Joseph effect, i.e., long-term trend of persistence, has been investigated since the pioneering studies by H.E. Hurst in the Fifties about the long-term trends in the fluctuations of water levels in the Nile River (Hurst 1951). The Hurst exponent was formerly found using empirical scaling relationships in time series recordings and it is nowadays applied in several different fields of science to study long-range dependency (Beran 2017; Mielniczuk and Wojdyłło 2007). Persistence arises whenever such connection of the system with its past configurations is strong.

Feedback processes are crucial to guarantee long-lasting memory and their role has been recognized in several contexts: the rate and intensity of future precipitation is found to be negatively correlated with both recent and prolonged drought (Rosenfeld et al. 2008), wildfire extension with abundant rain precipitations; analogously, antigens allow a prompt immunological response against viruses, while previous slope instabilities trigger further landslides (Yang et al. 2019).

The mechanism of memory in Life Sciences, discovered in its physiological foundations by the Austrian-american neuro-psychiatrist Eric R. Kandel (Nobel Prize for Physiology in 2000), is a clear example of how feedback mechanisms and persistence on complex networks can cooperate to generate emerging properties and adaptation (Kandel and Schwartz 1982). In his works, Kandel chose to investigate memory in *Aplysia Californica*, a giant marine snail whose brain only counts about 20'000 large cells organized in a few clusters. Their size allowed to locate easily small electrodes to capture electric signals. Thanks to his ingenious choice, Kandel and his colleagues were able to ascertain the occurrence of plastic changes in neural connections after repeated stimulation, resulting in learned fear, habituation or sensitization (Hawkins et al. 1983).

1.2 Earthquakes: emergent phenomena in fault systems

Seismic activity shares several common features with other systems mentioned above. It is indeed renowned that earthquakes tend to occur in sequences, grouped in space and time, so that long quiescent periods are alternated with instability. Moreover, large seismic events are much less frequent than smaller ones, being their frequency-magnitude distribution a power law. Their hypocenters have been proven to showcase multi-fractal patterns occurring along intricate interfaces called faults, densely clustered in heterogeneous crustal volumes. In addition, seismicity is extraordinary sensitive to tiny perturbations such as the level of aquifers, e.g., (Christiansen et al. 2007), tides (Métivier et al. 2009), other far seismic events and anthropic activity (Velasco et al. 2008). The brittle crust is itself a disordered complex system subject to background remote tectonic forces and several local effects. While its chemical and structural properties have been studied extensively and its long-term dynamic evolution is well understood, little is known about its behavior when it approaches the breaking point provoking earthquakes. Even by this viewpoint, our current state of knowledge is limited and a physical theory for seismic hazard is missing. In the absence of this, most of the attempts are focused on immediate applications of knowledge acquired during friction