

Stability Principle and the Nature of Quantization

Stability Principle and the Nature of Quantization:

*Non-Inertial Dynamics
and Quantum Mechanics*

By

Timur F. Kamalov

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Preface

The present book is devoted to the foundations of quantum mechanics from the perspective of non-inertial dynamics and Stability Principles. Its main objective is to demonstrate that the emergence of quantization can be understood as a consequence of classical variational principles supplemented by physical stability requirements, rather than as an independent postulate.

The motivation for this work originates from a long-standing problem in the axiomatization of physics. While classical mechanics is based on a transparent set of principles, the axiomatic structure of quantum mechanics remains conceptually opaque. As famously remarked by R. Feynman, quantum mechanics is a theory that works extraordinarily well, yet resists intuitive understanding.

In this book, we pursue a unified viewpoint in which classical and quantum descriptions are not separated by an ontological boundary. Instead, quantum phenomena are interpreted as manifestations of non-inertial and stochastic features inherent in real physical systems. The key role is played by an extended variational formalism, including higher-order time derivatives, and by a Stability Principle selecting physically realizable dynamical regimes.

Special attention is given to the role of phase coherence, action increments, and statistical averaging over non-inertial trajectories.

Within this framework, quantization appears as a condition of stable phase evolution, while operator methods of quantum mechanics emerge as an effective representation of an underlying phase-based dynamics.

Locality is strictly defined as dependence on the spatial coordinates of an individual system. Within this definition, the acceleration of a reference frame and its higher-order time derivatives do not depend on coordinates and are therefore non-local in a strict kinematic sense. These higher-order derivatives are identified with hidden parameters that shape the statistical measure and phase correlations of physical observables. Estimates for the hydrogen atom show that characteristic scales of action and energy naturally cluster around the Planck constant, while classical radiative losses remain negligible compared to phase-sensitive effects. Within this model, correlation phenomena and Bell-type experiments are analyzed. It is shown that in non-inertial reference frames the assumption of a fixed, factorizable probability measure is violated. Consequently, violations of Bell inequalities are interpreted as manifestations of a non-inertial statistical structure, rather than evidence of fundamental dynamical non-locality.

The material presented here is based on the author's original research and on previously published results, which are synthesized into a coherent conceptual framework suitable for a monographic presentation. The book is intended for researchers and graduate students in theoretical physics, particularly those working in the foundations of quantum mechanics, non-inertial reference frames, and classical-quantum correspondence. No prior commitment to a specific interpretation of quantum mechanics is assumed.

Chapter 1

Introduction

1.1 Motivation and Conceptual Background

One of the central challenges of modern theoretical physics lies in the foundations of quantum mechanics. Despite its unparalleled empirical success, the axiomatic structure of quantum theory relies on postulates—such as the wave function, operator formalism, and probabilistic interpretation of measurements—that lack a direct classical analogue and whose physical meaning remains a subject of debate [1]. This work proceeds from the position that the key role in shaping observable physical regimes is played not by probability as such, but by the stability of dynamics with respect to small perturbations. This view naturally stems from variational principles, which throughout the history of physics have served as the basis for constructing fundamental theories. The Principle of stationary action, supplemented by the requirement of stability, makes it possible to formulate a criterion for the physical realizability of dynamic regimes without resorting to the a priori introduction of quantum postulates. A central element of the approach is the consideration of

non-inertial reference frames. Real physical experiments are always carried out under conditions far from ideal inertiality. In this work, it is shown that non-inertiality can play a fundamental role, forming additional dynamic degrees of freedom and influencing the phase structure of motion. Extension of the configuration space by including higher derivatives of coordinates leads to a natural generalization of the variational formalism. In such an extended space, dynamics acquires a richer structure, and the requirement of stability of phase evolution becomes a key criterion for selecting physical trajectories. It is in this context that a statistical and phase description is introduced, which arises as a consequence of summation over families of close trajectories. An important feature of the developed approach is that quantization of action is considered not as an initial rule, but as a condition of phase coherence and stability. Discreteness of spectra, wave properties of states, and interference effects arise here as dynamic consequences, conditioned by the structure of the functional integral and the phase selection of stable contributions. A special place in the book is occupied by the analysis of correlation effects and Bell inequalities. In the traditional interpretation, violation of Bell inequalities is regarded as evidence of the fundamental non-locality of quantum theory. In the present work, a view is proposed, based on the consideration of non-inertial characteristics of the reference frame. It is shown that higher derivatives of coordinates, common to different subsystems, can play the role of non-local hidden variables in the coordinate sense, without requiring the introduction of superluminal interactions or abandonment of causality.

1.2 Mathematical formulation of Stability Principle as a tool

The mathematical core of classical mechanics is the variational principle of stationary action. In its standard form, this principle leads to second-order differential equations and uniquely determined trajectories. However, when higher-order time derivatives are included in the action functional, the space of admissible trajectories expands substantially.

Such extended variational formalisms have been studied since the work of Ostrogradsky [4, 5], and are known to introduce additional degrees of freedom as well as potential instabilities. In most treatments, these features are regarded as pathological. In the present approach, however, they are interpreted differently.

We propose that physical realizability requires not only stationarity of the action, but also dynamical stability. This leads to the introduction of a *Stability Principle*, formulated as the non-negativity of the second variation of the action [6, 7]. The Stability Principle acts as a selection rule, distinguishing physically observable regimes from mathematically admissible but unstable ones.

1.3 Basis of Non-Inertial Dynamics

Ideal inertial reference frames play a fundamental role in classical mechanics. However, strictly inertial frames do not exist in nature. Any real physical system is subject to external perturbations, accelerations, and environmental influences, which render its reference frame effectively non-inertial.

In non-inertial systems, additional dynamical contributions arise that are absent in idealized inertial descriptions. These contributions

may manifest as effective forces, higher-order time derivatives in the equations of motion, or stochastic fluctuations associated with uncontrolled degrees of freedom. While such effects are often neglected in macroscopic classical mechanics, their cumulative influence becomes significant at microscopic scales.

This observation suggests that certain features commonly attributed to the “quantum nature” of physical systems may, in fact, reflect the non-inertial and stochastic character of realistic dynamics. The present book explores this possibility in a systematic and mathematically controlled manner.

1.4 Introduction to Non-Inertial Dynamics

This book is devoted to an attempt to reinterpret the origin of quantum properties of physical systems from the standpoint of extended classical dynamics. The main motivation of the work was the desire to minimize the number of initial postulates of quantum mechanics and to trace to what extent its formalism can arise as a consequence of more general physical principles.

The starting point of the investigation is the Stability Principle, which supplements the standard variational principle of classical mechanics. The book advocates the viewpoint that physically realizable are not all stationary trajectories, but only those that preserve stability of phase dynamics under the inevitable non-inertiality of real reference frames. Such an approach allows a new look at discreteness of action, energy spectra, and correlation effects.

Special attention is paid to the role of non-inertial reference frames. In most fundamental theoretical constructions they are regarded as a technical complication or a source of small corrections. In this work, on the contrary, non-inertiality is treated as a physi-

cally inevitable property of real experimental conditions, exerting a fundamental influence on the structure of statistical description and phase coherence.

The material of the book has been formed over a long period and relies both on classical results of analytical mechanics and on the author's own research devoted to non-inertial dynamics, stochastic models, and analysis of correlation experiments. A significant part of the presented ideas was previously published in the form of separate articles and presentations, but in this book they are for the first time collected into a single, logically consistent conception.

The book is addressed to researchers in theoretical physics, foundations of quantum mechanics, and non-inertial dynamics. The author also hopes that it will be useful to readers interested in physical interpretations of quantum theory and in searching for alternative ways of understanding its fundamental principles.

The author is aware that the proposed approach does not remove all questions and does not claim to be a final solution to the problems of the foundations of quantum mechanics. Nevertheless, it demonstrates that a number of key quantum properties can be consistently explained without introducing additional non-physical assumptions, which, in the author's opinion, is of independent interest.

In this book we use an extended dynamic and statistical model intended for describing physical systems in non-inertial reference frames. Since many of the employed concepts differ from their standard interpretations in classical and quantum mechanics, this section provides a brief exposition of the notation and conventions adopted in the text.

The purpose of this section is twofold. First, it establishes a unified and unambiguous notation for dynamic variables, actions, phases, and statistical measures. Second, it defines the precise mean-

ing of such terms as locality, non-locality, hidden variables, and stability, which play a central role in the subsequent analysis.

1.4.1 Coordinates, Time, and Reference Frames

Space-time coordinates are denoted as $x^\mu = (ct, x)$, where t is coordinate time and x denotes spatial coordinates. Unless explicitly stated otherwise, all physical quantities are described relative to a given reference frame, which is not assumed to be inertial.

A reference frame is characterized by the trajectory of its origin $q(t)$ relative to an underlying inertial frame. In realistic physical situations this trajectory is considered smooth but otherwise arbitrary.

1.4.2 Dynamic Variables and Higher-Order Derivatives

The generalized coordinate of a physical system is denoted as $q(t)$. Its time derivatives are written as:

Velocity

$$v = \dot{q} = \frac{dq}{dt}$$

Acceleration

$$a = \ddot{q} = \frac{d^2q}{dt^2}$$

Jerk

$$j = \dddot{q} = \frac{d^3q}{dt^3}$$

Snap

$$s = \dots\ddot{q} = \frac{d^4q}{dt^4}$$

N-order

$$q^{(n)} = \frac{d^n q}{dt^n}$$

etc. The extended configuration of the system is defined as:

$$Q = (q, \dot{q}, \ddot{q}, \dots, q^{(n)}, \dots) = (q, v, a, j, s, \dots).$$

Throughout the book special attention is paid to the role of higher-order time derivatives. While the coordinate q and the velocity \dot{q} characterize the local state of an individual body, higher-order derivatives encode properties of the reference frame itself.

1.4.3 Local and Non-Local Quantities

In this work a basic distinction is made between local and non-local quantities.

A physical quantity is called local if it explicitly depends on the coordinates of an individual body,

$$A_{\text{loc}} = A(x, t).$$

In this sense the coordinate q and the velocity \dot{q} are local variables, because their values generally differ for spatially separated systems.

Conversely, quantities that are common to all bodies in a given reference frame and do not depend on their spatial coordinates are called non-local in the coordinate sense. In particular, the acceleration of the reference frame and its higher time derivatives, $\ddot{q}_{\text{ref}}(t), \dots$, are identical for all bodies and therefore cannot be represented as local functions of position.

This notion of non-locality is purely kinematic and should not be confused with dynamic non-locality or superluminal signal transmission.

1.4.4 Action Principle and Variations

The action functional is denoted $S[q]$ and in the extended formulation may depend on higher-order derivatives of the generalized coordinate,

$$S[q] = \int_{t_1}^{t_2} L(q, \dot{q}, \ddot{q}, \dots, t) dt.$$

Stationary trajectories satisfy the generalized Euler–Lagrange equations, and physical realizability is further constrained by the Stability Principle [6, 7], expressed as

$$\delta S = 0, \quad \delta^2 S \geq 0.$$

The action plays a dual role: it governs classical dynamics and defines the phase of the statistical description.

1.4.5 Phase, Wave Function, and Statistical Description

The phase associated with a trajectory is defined as

$$\phi = \frac{S}{Y},$$

where Y is a parameter with the dimension of action. At intermediate stages of the analysis Y is treated as a formal scale and is not a priori identified with Planck’s constant.

The statistical description is introduced through a complex amplitude of the form

$$\psi = \sqrt{P} e^{iS/Y},$$

where P denotes the probability density. Observable quantities depend on phase differences and ensemble averages, not on individual trajectories.

1.4.6 Hidden Variables and Extended Configuration Space

In the context of correlation phenomena, hidden variables are identified with higher-order derivatives of the reference-frame motion. In particular, the hidden variable λ is defined as:

$$\lambda \equiv Q^{(n>1)} = (\ddot{q}(t), \dots, q^{(n)}(t), \dots), \quad n > 1.$$

Coordinates and velocities are excluded from the space of hidden variables because they are local variables tied to individual systems. In contrast, higher-order derivatives are global properties of the reference frame and act as common, unobservable variables affecting all subsystems.

1.4.7 Probability Measure and Ensemble Averages

Ensemble averages are denoted by angle brackets $\langle \dots \rangle$. The probability measure on the space of hidden variables is written as $\mu(\lambda)$.

In non-inertial reference frames this measure is not assumed to be fixed or factorizable. Instead it is dynamically determined by the statistical properties of the higher-order derivatives of the reference-frame motion.

This point is crucial for the interpretation of correlation experiments [8] and the analysis of Bell-type inequalities.

1.4.8 Units and Constants

Unless otherwise stated, physical constants are kept explicit. In particular, Planck's constant appears as the minimal scale of action selected by stability considerations, not as a fundamental postulate. Natural units are not used in order to maintain transparency of dimensional estimates and physical scales.

1.5 Scope and Structure of the Book

The aim of this book is to demonstrate that the fundamental structures of quantum mechanics can be understood as consequences of classical dynamics extended to non-inertial and statistically stable regimes. The operator formalism of quantum mechanics is shown to arise as an effective representation of an underlying phase-based dynamics, rather than as a primary postulate.

The book is organized as follows. Chapter 1 introduces the Stability Principle within an extended variational framework. Subsequent chapters develop the role of higher-order dynamics, statistical averaging, and phase coherence. Special attention is devoted to the emergence of quantization conditions, the appearance of wave functions, and the connection with the Schrödinger equation. The later chapters address non-inertial interpretations of quantum correlations and the status of Bell-type inequalities.

Throughout the book, the emphasis is placed on conceptual clarity and logical continuity, rather than on interpretational speculation. The results presented here are intended to provide a unified perspective on classical and quantum physics, grounded in well-defined dynamical principles.

Chapter 2

The Stability Principle

2.1 Variational Principle as the Foundation of Inertial Dynamics

In classical mechanics the dynamics of physical systems is formulated in terms of a variational principle [9]. The state of the system is described by generalized coordinates $q(t)$, and its evolution is determined by the extremality of the action functional

$$S[q] = \int_{t_1}^{t_2} L(q, \dot{q}, t) dt,$$

where L is the Lagrangian of the system. The stationarity condition

$$\delta S = 0$$

leads to the Euler–Lagrange equations, which are equivalent to Newton’s second law for a wide class of systems.

Within this approach all trajectories for which the action is stationary are considered physically realizable. However, the station-

arity condition by itself does not guarantee stability of motion. A stationary trajectory may turn out to be unstable with respect to small perturbations, which makes its physical realization doubtful.

A key consequence of the extended variational framework is the enhanced role of phase. When higher-order derivatives and non-inertial effects are present, small variations in trajectories can lead to significant phase differences. As a result, a single classical trajectory no longer provides a complete physical description.

Instead, the observable state of a system is naturally described in terms of an ensemble of nearby trajectories and their associated phases. This motivates a statistical description based on phase averaging and leads, in a natural way, to expressions formally analogous to path-integral representations.

Within this framework, quantization does not appear as an imposed rule, but as a condition for stable phase coherence. Discrete action increments emerge from the requirement that phase fluctuations do not destroy constructive interference.

In classical theory one distinguishes minima, maxima, and saddle points of the action functional. The condition

$$\delta S = 0$$

does not allow one to discriminate between these cases. Yet in real physical systems trajectories corresponding to a maximum or a saddle point of the action are generally not observed.

This circumstance indicates the need for an additional physical criterion that would separate mathematically admissible solutions from physically realizable ones. In the present work such a criterion is proposed to be the Stability Principle.

2.2 Stability Principle as formulation of Non-Inertial Dynamics

The Stability Principle is formulated as follows:

Physically realizable are only those trajectories for which the action is not only stationary, but also stable with respect to small variations.

Mathematically this is expressed by the conditions [6, 7]

$$\delta S = 0, \quad \delta^2 S \geq 0.$$

In other words, the Principle of Stability is a boundedness from below, meaning the minimality of the action function.

The second variation of the action

$$\delta^2 S$$

characterizes the response of the system to small deviations from the stationary trajectory. The condition of non-negativity of the second variation means that small fluctuations do not lead to unlimited growth of deviations.

Thus the Stability Principle refines the standard variational principle by adding to it the physical requirement of stability.

2.3 Physical Meaning of Stability of the Action

The physical meaning of the Stability Principle becomes especially clear when non-inertial effects are taken into account. In real reference frames there are always small perturbations associated with accelerations, vibrations, and fluctuations of external conditions. Un-

der such conditions trajectories that lack stability are destroyed even by arbitrarily small perturbations.

From this point of view the Stability Principle reflects the requirement of reproducibility of physical states. Only stable regimes can be reproduced upon repetition of an experiment and therefore possess physical meaning.

For describing non-inertial reference frames it is necessary to account for the contribution of the acceleration of the reference frame and its higher derivatives. This leads to considering Lagrangians that depend on higher derivatives of the coordinate [4]:

$$L = L(q, \dot{q}, \ddot{q}, \dots, t).$$

The corresponding action has the form

$$S[q] = \int_{t_1}^{t_2} L(q, \dot{q}, \ddot{q}, \dots, t) dt.$$

The equations of motion in this case are determined by the generalized Euler–Lagrange equations (Ostrogradsky formalism). Despite known problems related to instability of such systems, within the present work higher derivatives are regarded not as independent dynamic degrees of freedom, but as variables describing properties of the reference frame.

2.4 Stability and Selection of Physical Regimes

In extended dynamics the Stability Principle acquires special significance. Higher derivatives of the motion of the reference frame introduce additional contributions to the action, which may lead to phase sensitivity and amplification of fluctuations.

The requirement

$$\delta^2 S \geq 0$$

in this case performs the function of selecting admissible motion regimes. Only those trajectories for which the contribution of non-inertial effects does not destroy the stability of phase dynamics turn out to be physically realizable.

The action plays a dual role: it determines both the dynamics of the system and the phase of the statistical description. The phase associated with a trajectory is defined by the expression

$$\phi = \frac{S}{Y},$$

where Y is the scale of action.

Fluctuations of the action lead to fluctuations of the phase. If phase fluctuations become large, phase coherence is destroyed, and the corresponding state cannot be stably observed. Thus stability of the action is equivalent to stability of the phase.

2.5 Stability Principle and Discreteness

Under conditions of stochastic non-inertial fluctuations the stability requirement leads to the selection of such regimes for which phase shifts turn out to be multiples of the minimal scale of action. This results in the natural appearance of discreteness in allowed values of action and energy.

Quantization in this approach arises not as an external postulate, but as a consequence of the requirement of stability of phase dynamics under non-inertial conditions.

2.6 Role of the Stability Principle in Subsequent Chapters

The Stability Principle is a fundamental element of the whole subsequent construction. In the following chapters it is used for:

- introducing the statistical description and phase summation;
- analyzing atomic spectra;
- investigating correlation effects;
- interpreting Bell inequalities in non-inertial reference frames.

Thus the Stability Principle serves as a link between classical dynamics, statistical description, and quantum properties of physical systems.