The Future of Post-Human Mathematical Logic

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By

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To Those Beyond Classical and Non-Classical Logics

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BOOKS ALSO BY PETER BAOFU

• *The Future of Post-Human Knowledge* (2008) •

• *The Future of Post-Human Unconsciousness* (2008) •

• *The Future of Information Architecture* (2008) •

• *The Rise of Authoritarian Liberal Democracy* (2007) •

• *The Future of Aesthetic Experience* (2007) •

• *The Future of Complexity* (2007) •

• *Beyond the World of Titans, and the Remaking of World Order* (2007) •

• *Beyond Nature and Nurture* (2006) •

• *Beyond Civilization to Post-Civilization* (2006) •

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• Volume 1: *The Future of Human Civilization* (2000) •

• Volume 2: *The Future of Human Civilization* (2000) •

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FOREWORD

In this newest tome, Dr. Baofu tackles yet another set of sacrosanct beliefs which few thinkers would dare to question—the foundations of mathematics and logic. He examines the reasoning of forebears, points out specific shortcomings, and offers another perspective to fulfill those shortcomings.

The breadth of issues chosen by Dr. Baofu for analysis is truly astounding. In each of his prior works he has demonstrated an inquiring mind, critical perception, and tendered an innovative process to look at issues from a futurist's point of view.

He continues on the following pages to edify his readers.

Sylvan Von Burg
School of Business
George Washington University

ACKNOWLEDGMENTS

Like all previous books of mine, this one is written with the spirit to challenge conventional wisdom.

For this simple reason of political incorrectness, it receives no external funding nor help from any formal organization or institution.

The only reward, as I often acknowledge in my previous books, is the wonderful feeling of creating something new that the world has never known.

There is one person, Sylvan von Burg at George Washington University School of Business, whom I deeply appreciate for his foreword.

In any event, I bear the sole responsibility for what is written in this book.

ABBREVIATIONS

ALD = Peter Baofu. 2007. *The Rise of Authoritarian Liberal Democracy: A Preface to a New Theory of Comparative Political Systems*. Cambridge, England: Cambridge Scholars Publishing, Ltd.

BCIV = Peter Baofu. 2006. *Beyond Civilization to Post-Civilization: Conceiving a Better Model of Life Settlement to Supersede Civilization*. NY: Peter Lang Publishing, Inc.

BCPC = Peter Baofu. 2005. *Beyond Capitalism to Post-Capitalism: Conceiving a Better Model of Wealth Acquisition to Supersede Capitalism*. NY: The Edwin Mellen Press.

BDPD1 = Peter Baofu. 2004. Volume 1. *Beyond Democracy to Post-Democracy: Conceiving a Better Model of Governance to Supersede Democracy*. NY: The Edwin Mellen Press.

BDPD2 = Peter Baofu. 2004. Volume 2. *Beyond Democracy to Post-Democracy: Conceiving a Better Model of Governance to Supersede Democracy*. NY: The Edwin Mellen Press.

BNN = Peter Baofu. 2006. *Beyond Nature and Nurture: Conceivng a Better Way to Understand Genes and Memes.* Cambridge, England: Cambridge Scholars Publishing, Ltd.

BWT = Peter Baofu. 2007. *Beyond the World of Titans, and the Renaking of World Order: A Preface to a New Logic of Empire-Building.* Cambridge, England: Cambridge Scholars Publishing, Ltd.

FAE = Peter Baofu. 2007. *The Future of Aesthetic Experience: Conceiving a Better Way to Understand Beauty, Ugliness and the Rest*. Cambridge, England: Cambridge Scholars Publishing, Ltd.

FC = Peter Baofu. 2007. *The Future of Complexity: Conceiving a Better Way to Understand Order and Chaos.* London, United Kingdom: World Scientific Publishing Co.

FCD = Peter Baofu. 2002. *The Future of Capitalism and Democracy.* MD: The University Press of America.

FHC1 = Peter Baofu. 2000. Volume 1. *The Future of Human Civilization.* NY: The Edwin Mellen Press.

FHC2 = Peter Baofu. 2000. Volume 2. *The Future of Human Civilization.* NY: The Edwin Mellen Press.

FIA = Peter Baofu. 2008. *The Future of Information Architecture: Conceiving a Better Way to Understand Taxonomy, Network, and Intelligence.* Oxford, England: Chandos Publishing (Oxford) Limited.

FPHC = Peter Baofu. 2004. *The Future of Post-Human Consciousness.* NY: The Edwin Mellen Press.

FPHK = Peter Baofu. 2008. *The Future of Post-Human Knowledge: A Preface to a New Theory of Methodology and Ontology.* Oxford, England: Chandos Publishing (Oxford) Limited.

FPHML = Peter Baofu. 2008. *The Future of Post-Human Mathematical Logic: A Preface to a New Theory of Rationality.* Cambridge, England: Cambridge Scholars Publishing, Ltd.

FPHST = Peter Baofu. 2006. *The Future of Post-Human Space-Time: Conceivng a Better Way to Understand Space and Time.* New York: Peter Lang Publishing, Inc.

FPHU = Peter Baofu. 2008. *The Future of Post-Human Unconsciousness: A Preface to a New Theory of Anomalous Experience.* Cambridge, England: Cambridge Scholars Publishing, Ltd.

• PART ONE •

Introduction

CHAPTER 1
INTRODUCTION—THE INFLUENCE OF MATHEMATICAL LOGIC

> Mathematics is the way to understand the universe….Number is the measure of all things.
> —Pythagoras (R. Hamming 1980)

The Importance of Mathematical Logic

Why should mathematical logic be grounded on the basis of some formal requirements in the way that it has been developed since its classical emergence as a hybrid field of mathematics and logic in the 19^{th} century, if not earlier?

Contrary to conventional wisdom, the foundation of mathematic logic has been grounded on some false (or dogmatic) assumptions which have much impoverished the pursuit of knowledge.

This is not to say that mathematical logic has been useless. Quite on the contrary, it has been quite influential in shaping the way that reality is to be understood in numerous fields of knowledge—by learning from the mathematical study of logic and its reverse, the logical study of mathematics.

After all, as R. Hamming (1980) once reminded us, "[b]ecause of the…successes of mathematics there is at present a strong trend toward making each of the sciences mathematical. It is usually regarded as a goal to be achieved, if not today, then tomorrow."

The point in this book here, however, is to show an alternative (better) way to ground mathematical logic for the future advancement of knowledge (which goes beyond both classical and non-classical logics, while learning from them all).

If true, this seminal view will alter the way of how mathematical logic is to be understood, with its enormous implications for the future of knowledge.

The Varieties of Mathematical Logic

To start, the discipline of mathematical logic is diverse enough, since it consists of different subfields, with each competing for influence.

Five main subfields of mathematical logic (since its formation in the 19th century, if not earlier) can be introduced hereafter to illustrate this important point. They are, namely, (a) set theory, (b) proof theory, (c) model theory, (d) recursion theory and (e) constructive mathematics.

It is interesting to note here that (a) and (b) are more syntactic in nature, and (c) is more semantic in nature—whereas (d) and (e) are more pragmatic in nature. (WK 2008c)

With this clarification in mind, the five main subfields of mathematical logic are summarized hereafter (and also in *Table 1.1*).

Set Theory

Firstly, there is *set theory*, which is more syntactic in nature and studies sets, or "collections of objects. Although any type of objects can be collected into a set, set theory is applied most often to objects that are relevant to mathematics." (WK 2008a) For instance, in the following simple equation for the set F,

$$F = \{ n^2 - 4 : n \text{ is an integer; and } 0 \le n \le 19\}$$

In this set, "F is the set of all numbers of the form $n^2 - 4$, such that n is a whole number in the range from 0 to 19 inclusive." (WK 2008gg)

Both Georg Cantor and Richard Dedekind are often credited to initiate set theory in the 1870's—especially with Cantor's 1874 paper titled "On a Characteristic Property of All Real Algebraic Numbers." (WK 2008; P. Johnson 1972)

A well-known example of the achievements made in the history of set theory is "the axiom of choice" introduced by Ernst Zermelo (1904) to prove "that every set could be well-ordered…." (WK 2008)

Or to put it verbally, "the axiom of choice says that given any collection of bins, each containing at least one object, it is possible to make a selection of exactly one object from each bin, even if there are infinitely many bins and there is no 'rule' for which object to pick from each. The axiom of choice is not required if the number of bins is finite or if such a selection 'rule' is available." (WK 2008k)

Zermelo then came up with a second version in 1908 to address "criticisms of the first proof," especially in relation to some paradoxes which contradicted Zermelo's claim (e.g., the Burali-Forti paradox "that the collection of all ordinal numbers cannot form a set"). (WK 2008)

Contrary to Zermelo's claims—Abraham Fraenkel in 1922 proved that "the axiom of choice cannot be proved from the remaining axioms of Zermelo's set theory with urelements," and an urelement here refers to "an object (concrete or abstract) which is not a set, but that may be an element of a set" but "is not identical with the empty set [i.e., is not zero]." (WK 2008 & 2008b; E. Weisstein 2008)

Later, Paul Cohen (1966) showed not only "that the addition of urelements is not needed" but also that "the axiom of choice is unprovable" even in set theory with the combined axioms proposed by both Zermelo and Fraenkel, or now known as "Zermelo–Fraenkel set theory (ZF)." (WK 2008)

That said—the influence of set theory is obvious enough, as it has been "used in the definitions of nearly all mathematical objects, such as functions, and concepts of set theory are integrated throughout the mathematics curriculum. Elementary facts about sets and set membership can be introduced in primary school, along with Venn diagrams, to study collections of commonplace physical objects. Elementary operations such as set union and intersection can be studied in this context." (WK 2008a)

Proof Theory

Secondly, there is *proof theory*, which, like set theory, is more syntactic in nature but seeks "formal proofs in various logical deduction systems….Several deduction systems are commonly considered, including Hilbert-style deduction systems, systems of natural deduction, and the sequent calculus developed by [Gerhard] Gentzen." (WK 2008)

For instance, in the following simple equations for a formal proof based on natural deduction (WK 2008hh),

$$A \wedge (B \wedge C) \text{ true}$$

.

.

.

B true

Verbally, it simply says that "assuming A ∧ (B ∧ C) is true,…B is true." (WK 2008hh)

Formal proofs "are represented as formal mathematical objects, facilitating their analysis by mathematical techniques" and "are typically presented as inductively-defined data structures such as plain lists, boxed lists, or trees, which are constructed according to the axioms and rules of inference of the logical system." (WK 2008 & 2008c)

Nowadays, "[f]ormal proofs are constructed with the help of computers in interactive theorem proving. Significantly, these proofs can be checked automatically, also by computer." (WK 2008c) In other words, the Information Revolution has made the process of checking formal proofs easier.

Yet, one should not mistakenly conclude that since "[c]hecking formal proofs is usually trivial" (since they can be easily checked by computers in this day and age of ours), therefore "finding proofs (automated theorem proving)" is easy. (WK 2008c)

On the contrary, unlike "checking" formal proofs—"finding" formal proofs "is typically quite hard." (WK 2008c)

Similarly, one should not be tempted to assume that, since finding formal proofs is quite hard, it is therefore better (or more advantageous) to find informal proofs instead.

Again, on the contrary, informal proofs have the main disadvantage of being unreliable, since "[a]n informal proof in the mathematics literature…[can] require…weeks of peer review to be checked, and may still contain errors." (Wk 2008c)

After all, informal proofs "are rather like high-level sketches that would allow an expert to reconstruct a formal proof at least in principle, given enough time and patience." (WK 2008c)

With this dilemma of the formalization of logic in mind—David Hilbert is considered the key figure to create Hilbert's program for the foundation of modern proof theory, with the aim of "reducing all mathematics to a finitist formal system" (just as Georg Cantor and Richard Dedekind are often credited to initiate set theory in a different context as described above). (WK 2008c)

Later and unfortunately in a way, Kurt Gödel's seminal work on "incompleteness theorems showed that this [Hilbert's ambitious aim] is unattainable," and Hilbert of course was not happy with Gödel's critique and did not recognize its validity (i.e., Gödel's work) for quite some time in his lifetime. (WK 2008c)

Model Theory

Thirdly, there is *model theory*, which, unlike set theory and proof theory, is more semantic in nature and compares "(classes of) mathematical structures such as groups, fields, graphs or even models of set theory using tools from mathematical logic." (WK 2008d)

Thus, model theory is closely related to "universal [or general] algebra and algebraic geometry." (WK 2008 & 2008f)

One well-known pioneering achievement of model theory concerns the "continuum hypothesis" (or CH) by Georg Cantor, in that "two sets S and T have the same cardinality or cardinal number [the number of elements in the sets] if there exists a bijection between S and T. Intuitively, this means that it is possible to 'pair off' elements of S with elements of T in such a fashion that every element of S is paired off with exactly one element of T and vice versa. Hence, the set {banana, apple, pear} has the same cardinality as {yellow, red, green}." (WK 2008e)

In other words, as an illustration, in the following very simplistic graph, the set {banana, apple, pear} can be paired off with {yellow, red, green} for each of the elements in the set: (WK 2008e)

Banana = yellow
Apple = red
Pear = green

In many other cases, however, model theory is not so simplistic and is often highly mathematical, beyond the understandability of lay people with little mathematical background.

That qualified—the debate on whether or not CH is true of false has been hotly debated without general agreement, since "[h]istorically, mathematicians who favored a 'rich' and 'large' universe of sets were against CH, while those favoring a 'neat' and 'controllable' universe favored CH. Parallel arguments were made for and against the axiom of constructibility, which implies CH." (2008e)

For instance, "[Kurt] Gödel believed that CH is false….[Paul] Cohen…also tended towards rejecting CH….[But] recently, Matthew Foreman has pointed out that ontological maximalism can actually be used to argue in favor of CH, because among models that have the same reals, models with 'more' sets of reals have a better chance of satisfying CH." (WK 2008e; P. Maddy 1988)

A second seminal illustration of model theory concerns Kurt Gödel's 1929 proof of the "completeness theorem," which "establishes a correspondence between semantic truth and syntactic provability in first-order logic," in that "a set of sentences is satisfiable if and only if no contradiction can be proven from it. " (WK 2008g & 2008h)

In other words, the theorem proves that "if a formula is logically valid then there is a finite deduction (a formal proof) of the formula. The deduction is a finite object that can be verified by hand or computer. This relationship between truth and provability establishes a close link between model theory and proof theory in mathematical logic. An important consequence of the completeness theorem is that it is possible to enumerate the logical consequences of any effective first-order theory, by enumerating all the correct deductions using axioms from the theory." (WK 2008g)

A more general version of completeness theorem argues that "for any first-order theory T and any sentence S in the language of the theory, there is a formal deduction of S from T if and only if S is satisfied by every model of T. This more general theorem is used implicitly, for example, when a sentence is shown to be provable from the axioms of group theory by considering an arbitrary group and showing that the sentence is satisfied by that group." (WK 2008g)

Like the continuum hypothesis, the completeness theorem has yet to be totally proven. In fact, it has been shown that the completeness theorem is logically related to another theorem known as the "compactness theorem"; while "neither of these theorems can be proven in a completely effective manner, each one can be effectively obtained from the other." (WK 2008g)

For instance, the compactness theorem can be obtained from the completeness theorem in that, for the compactness theorem, "if a formula φ is a logical consequence of a (possible infinite) set of formulas Γ then it is a logical consequence of a finite subset of Γ,…because only a finite number of axioms from Γ can be mentioned in a formal deduction of φ, and the soundness of the deduction system then implies φ is a logical consequence of this finite set." (WK 2008g)

A different way to explicate the compactness theorem is that "a (possibly infinite) set of first-order sentences has a model, iff every finite subset of it has a model." (WK 2008h)

Consequenty, "the compactness theorem is equivalent to Gödel's completeness theorem." (WK 2008h)

However, like many other theorems, what is true for simple first-order logics may not hold for complicated higher-order logics.

In the case of the completeness theorem, "[s]econd-order logic, for example, does not have a completeness theorem for its standard semantics…, and the same is true of all higher-order logics. It is possible to produce sound deductive systems for higher-order logics, but no such system can be complete. The set of logically-valid formulas in second-order logic is not enumerable.” (WK 2008g)

Recursion Theory

Fourthly, there is also *recursion theory* (or “computability theory”), which, unlike set theory, proof theory, and model theory, is more pragmatic in nature and “studies the properties of computable functions and the Turing degrees, which divide the uncomputable functions into sets which have the same level of uncomputability. Recursion theory also includes the study of generalized computability and definability.” (WK 2008i)

Like set theory, proof theory, and model theory—recursion theory also has its own founders, especially “from the work of Alonzo Church and Alan Turing in the 1930s, which was greatly extended later by [Stephen] Kleene and [Emil] Post in the 1940s.” (WK 2008i)

An important illustration of recursion theory in action involves the “Church-Turing thesis.” It all started from “Turing computability as the correct formalization of the informal idea of effective calculation. These results led Stephen Kleene (1952) to coin the two names 'Church's thesis'…and 'Turing's Thesis.' Nowadays these are often considered as a single hypothesis, the Church-Turing thesis, which states that any function that is computable by an algorithm is a computable function.” (WK 2008i)

More technically speaking, in a computable function with a set of natural numbers, for instance, the “set of natural numbers is said to be a computable set (also called a decidable, recursive, or Turing computable set) if there is a Turing machine that, given a number *n*, halts with output *1* if *n* is in the set and halts with output *0* if *n* is not in the set. A function *f* from the natural numbers to themselves is a recursive or (Turing) computable function if there is a Turing machine that, on input *n*, halts and returns output *f(n)*.” (WK 2008i)

An interesting outcome of recursion theory is the understanding that many mathematical problems are not effectively decidable: “With a definition of effective calculation came the first proofs that there are problems in mathematics that cannot be effectively decided. Church [1936 & 1936a] and Turing [1937], inspired by techniques used in by Gödel [1931] to prove his incompleteness theorems, independently demonstrated that the Entscheidungsproblem is not effectively decidable. This result showed that there is no algorithmic procedure that can correctly decide whether arbitrary mathematical propositions are true or false.” (WK 2008i)

In fact, “[m]any problems of mathematics have been shown to be undecidable after these initial examples were established….[For example], [Andrey] Markov and [Emil] Post [1947] published independent papers showing that the word problem for semigroups cannot be effectively decided. Extending this result, Pyotr Sergeyevich Novikov and William Boone showed independently in the 1950s that the word problem for groups is not effectively solvable: there is no effective procedure that, given a word in a finitely presented group, will decide whether the element represented by the word is the identity element of the group.” (WK 2008i & 2008jj)

Constructive Mathematics

And finally, there is *constructive mathematics*, which, like recursion theory, is more pragmatic in nature and proposes a different way to prove the existence of an object.

But unlike the other four subfields (as described above), constructive mathematics has not been quite accepted in the mainstream of mathematical logic.

For instance, constructivism “asserts that it is necessary to find (or 'construct') a mathematical object to prove that it exists,” which differs from the traditional approach, in which “one assumes that an object does not exist and derives a contradiction from that assumption.” (WK 2008j)

But, for constructive mathematics, this proof by contradiction “still has not found the object and therefore not proved its existence.” (WK 2008j)

As a major school of thought within constructive mathematics, L. E. J. Brouwer has contributed to the development of constructive mathematics, with his “intuitivist” theory of mathematical logic, which makes use of “intuitionistic logic and is essentially classical logic without the law of the excluded middle. This is not to say that the law of the excluded middle is denied entirely; special cases of the law will be provable. It is just that the general law is not assumed as an axiom….” (WK 2008j)

Brouwer considered “the law of the excluded middle as abstracted from finite experience,…[which is] then applied to the infinite without justification. For instance, [Christian] Goldbach's conjecture is the assertion that every even number (greater than 2) is the sum of two prime numbers. It is possible to test for any particular even number whether or not it is the sum of two primes (for instance by exhaustive search), so any