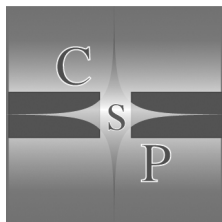


The Future of Post-Human Geometry

The Future of Post-Human Geometry

A Preface to a New Theory
of Infinity, Symmetry, and Dimensionality

By
Peter Baofu



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To Those Beyond the Conventional Wisdom on Geometry

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FOREWORD

Dr. Peter Baofu possesses a truly inquiring mind. No sooner has he analyzed the origins and characteristics of a subject than he is inspired to proceed to the next. Here he addresses the subjects of Infinity, Symmetry and Dimensionality.

In the present volume he examines Geometry in a way no known scholar has yet attempted. He deconstructs the subject, brick by brick, notes its characteristics and formulates a plan for re-assembling the subject in a more meaningful way.

Dr. Baofu presents a new perspective on a venerable subject that promises to intrigue a reader who has heretofore been enchanted with Geometry but has encountered some disturbing questions. May this additional quest provide some stimulation for further productive thought.

Sylvan Von Burg
School of Business
George Washington University

ACKNOWLEDGMENTS

Like all other books of mine, this one is written to challenge conventional wisdom—this time, however, on infinity, symmetry, and dimensionality—and then to work out a new theory to understand it better.

Because of this political incorrectness, this book receives no external funding nor help from any formal organization or institution.

My only reward is the wonderful feeling to discover something new to say in a way not proposed before.

There is one person, however, whom I deeply appreciate for his foreword, and he is Sylvan von Burg at George Washington University School of Business,.

In any event, I bear the sole responsibility for the ideas presented in this book.

ABBREVIATIONS

- ALD = Peter Baofu. 2007. *The Rise of Authoritarian Liberal Democracy: A Preface to a New Theory of Comparative Political Systems*. Cambridge, England: Cambridge Scholars Publishing, Ltd.
- BCIV = Peter Baofu. 2006. *Beyond Civilization to Post-Civilization: Conceiving a Better Model of Life Settlement to Supersede Civilization*. NY: Peter Lang Publishing, Inc.
- BCPC = Peter Baofu. 2005. *Beyond Capitalism to Post-Capitalism: Conceiving a Better Model of Wealth Acquisition to Supersede Capitalism*. NY: The Edwin Mellen Press.
- BDPD1 = Peter Baofu. 2004. Volume 1. *Beyond Democracy to Post-Democracy: Conceiving a Better Model of Governance to Supersede Democracy*. NY: The Edwin Mellen Press.
- BDPD2 = Peter Baofu. 2004. Volume 2. *Beyond Democracy to Post-Democracy: Conceiving a Better Model of Governance to Supersede Democracy*. NY: The Edwin Mellen Press.
- BNN = Peter Baofu. 2006. *Beyond Nature and Nurture: Conceiving a Better Way to Understand Genes and Memes*. Cambridge, England: Cambridge Scholars Publishing, Ltd.
- BWT = Peter Baofu. 2007. *Beyond the World of Titans, and the Renaking of World Order: A Preface to a New Logic of Empire-Building*. Cambridge, England: Cambridge Scholars Publishing, Ltd.
- FAE = Peter Baofu. 2007. *The Future of Aesthetic Experience: Conceiving a Better Way to Understand Beauty, Ugliness and the Rest*. Cambridge, England: Cambridge Scholars Publishing, Ltd.
- FC = Peter Baofu. 2007. *The Future of Complexity: Conceiving a Better Way to Understand Order and Chaos*. London, United Kingdom: World Scientific Publishing Co.
- FCD = Peter Baofu. 2002. *The Future of Capitalism and Democracy*. MD: The University Press of America.

- FHC1 = Peter Baofu. 2000. Volume 1. *The Future of Human Civilization*. NY: The Edwin Mellen Press.
- FHC2 = Peter Baofu. 2000. Volume 2. *The Future of Human Civilization*. NY: The Edwin Mellen Press.
- FIA = Peter Baofu. 2008. *The Future of Information Architecture: Conceiving a Better Way to Understand Taxonomy, Network, and Intelligence*. Oxford, England: Chandos Publishing (Oxford) Limited.
- FPHC = Peter Baofu. 2004. *The Future of Post-Human Consciousness*. NY: The Edwin Mellen Press.
- FPHCT = Peter Baofu. 2009. *The Future of Post-Human Creative Thinking: A Preface to a New Theory of Invention and Innovation*. Cambridge, England: Cambridge Scholars Publishing, Ltd.
- FPHE = Peter Baofu. 2009. *The Future of Post-Human Engineering: A Preface to a New Theory of Technology*. Cambridge, England: Cambridge Scholars Publishing, Ltd.
- FPHG = Peter Baofu. 2009. *The Future of Post-Human Geometry: A Preface to a New Theory of Infinity, Symmetry, and Dimensionality*. Cambridge, England: Cambridge Scholars Publishing, Ltd.
- FPHK = Peter Baofu. 2008. *The Future of Post-Human Knowledge: A Preface to a New Theory of Methodology and Ontology*. Oxford, England: Chandos Publishing (Oxford) Limited.
- FPHML = Peter Baofu. 2008. *The Future of Post-Human Mathematical Logic: A Preface to a New Theory of Rationality*. Cambridge, England: Cambridge Scholars Publishing, Ltd.
- FPHMM = Peter Baofu. 2009. *The Future of Post-Human Mass Media: A Preface to a New Theory of Technology*. Cambridge, England: Cambridge Scholars Publishing, Ltd.
- FPHST = Peter Baofu. 2006. *The Future of Post-Human Space-Time: Conceiving a Better Way to Understand Space and Time*. New York: Peter Lang Publishing, Inc.
- FPHU = Peter Baofu. 2008. *The Future of Post-Human Unconsciousness: A Preface to a New Theory of Anomalous Experience*. Cambridge, England: Cambridge Scholars Publishing, Ltd.

• PART ONE •

Introduction

CHAPTER 1

INTRODUCTION—THE IMPORTANCE OF GEOMETRY

God created the natural numbers, all else
is the work of man.
—Leopold Kronecker (J. Cabillon 2000)

The Millennial Concern with Geometry

Why should some essential properties of geometry (i.e., infinity, symmetry, and dimensionality) be both necessary and desirable in the way that they have been constructed—albeit with different modifications over time—since time immemorial?

Contrary to the conventional wisdom in all history hitherto existing, the essential properties of geometry do not have to be both necessary and desirable.

This is not to suggest, of course, that one has nothing to learn from geometry. On the contrary, geometry has contributed to the advancement of knowledge in many ways since its inception as a field of knowledge some millennia ago.

The point in this book, however, is to show an alternative (better) way to understand the nature of geometry, which goes beyond human conception, intuition, and imagination, together with worldly experience of course, as its foundation, while learning from them all.

If true, this seminal view will fundamentally change the way that the nature of abstraction in the thinking process is to be understood, with its enormous implications for the future advancement of knowledge, in a small sense, and what I originally called its “post-human” fate, in a large one.

Mathematics and Geometry

A good starting point for this inquiry in question concerns the place of geometry in the field of mathematics in the first place.

The term “mathematics” has its etymological origin in the Greek *μάθημα* (or *máthēma*), to refer to “learning,” “study,” or “science”—and can be classified in terms of two separate broad categories for analysis. (WK 2008a)

In the absence of better words, let me call them, namely, (a) *pure mathematics* and (b) *interdisciplinary mathematics*, to be summarized hereafter—with the qualification, however, that the word “pure” here means “unmixed,” only in an appromixate sense, as opposed to something which is “interdisciplinary” (to mix with non-mathematical disciplines). (MWD 2008c)

Pure Mathematics

Pure mathematics is concerned with the study of four major areas, that is, (i) “quantity,” (ii) “structure,” (iii) “space,” and (iv) “change”—with the qualification, however, that, although there can be other areas besides these four, the case studies here are deemed sufficient for the current purpose of illustration. (WK 2008a)

And these four areas correspond to the four sub-fields of pure mathematics, that is, (i) “arithmetic,” (ii) “algebra,” (iii) “geometry,” and (iv) “analysis.” (WK 2008a)

In this sense, geometry is related to mathematics as one of its sub-fields.

Quantity

Firstly, the study of “quantity” has to do with the nature of “numbers,” and the “arithmetical operations [i.e., division, addition, multiplication, and subtraction] on them, which are characterized in arithmetic.” (WK 2008a)

For instance, numbers are classified in different ways, starting with the smaller units and ending with “infinite” ones (or sometimes known as “relative infinity” as coined by Georg Cantor, who wanted to separate “relative infinity” in “the mind of Man” from “absolute infinity” in “the mind of God”). (WK 2008a & 2008e)

To start, there are “natural” (or “counting”) numbers (e.g., 0, 1, 2, 3, 4,...). (P. Suber 1998; WK 2008a)

Then, there are “whole” numbers or “integers” (e.g., ..., -3, -2, -1, 0, 1, 2, 3, ...), with “nonnegative integers” (e.g., 0, 1, 2, 3, ...) and “positive integers” (e.g., 1, 2, 3, ...) (WK 2008b)

Then, there are “rational” numbers or “fractions” (e.g., -3, $\frac{1}{2}$, 1.45, ...). (WK 2008a)

Then, there are “real” numbers to include “continuous” quantities (e.g., $-e$, $\sqrt{3}$, π , etc.), so that “real” numbers can be divided into two sets, namely, “rational” numbers (e.g., 42, $-23/129$, etc.) and “irrational” numbers (e.g., e , $-e$, $\sqrt{3}$, π , etc.). (WK 2008c)

Then, there are “complex” numbers, which combine “real” numbers with “an imaginary unit, denoted i ,” such that $i^2 = -1$. (WK 2008d; K. Joshi 1989) Each complex number is normally written “in the form $a + bi$, where a and b are real numbers called the *real part* and the *imaginary part* of the complex number, respectively.” (WK 2008d)

“Number theory” is to study more deeply the nature of numbers like this in arithmetic, and, therefore, “number theory” is sometimes treated like “the higher arithmetic.” (WK 2008j)

Structure

But the study of quantity is only a starting point, since quantity also has its own structure to be further studied.

For instance, “[m]any mathematical objects, such as sets of numbers and functions, exhibit internal structure. The structural properties of these objects are investigated in the study of groups, rings, fields and other abstract systems, which are themselves such objects.” (WK 2008a)

For illustration, the study of a “group” in mathematics (also known as “group theory”) refers to “an algebraic structure consisting of a set together with an operation [e.g., addition] that combines any two of its elements to form a third element.” (WK 2008g)

Thus, as an example, suppose $a + b = c$. So, if $a = 1$, $b = 2$, then $c = 3$. This mathematical operation looks very trivial and obvious, but mathematicians like to find out about something else in relation to the structure of equations like this.

For instance, in this example, since, 1, 2, and 3 are integers, which means that there is an internal structure in this operation, so that “[f]or any two integers a and b , the sum $a + b$ is also an integer. In other words, the process of adding integers two at a time can never yield a result that is not an integer. This property is known as *closure* under addition.” (WK 2008g)

Some other good examples of the internal structures for simple operations like addition and multiplication are shown below, including the one of “closure” (as indicated above): (WK 2008b)

	Addition	Multiplication
Closure	$a + b$ is an integer	$a \times b$ is an integer
Associativity	$a + (b + c) = (a + b) + c$	$a \times (b \times c) = (a \times b) \times c$
Commutativity	$a + b = b + a$	$a \times b = b \times a$
Identity	$a + 0 = a$	$a \times 1 = a$
Inversion	$a + (-a) = 0$	

In the case of a “ring” (in “ring theory”), the set usually has two operations (not just one), like addition and multiplication. (WK 2008h) And in the case of a “field” (in “field theory”), all four arithmetic operations (e.g., addition, subtraction, division, and multiplication) can be performed. (WK 2008i)

The study of the internal structures like this in numbers falls into the field of “abstract algebra.” (WK 2008f)

Space

Things start more interesting, however, when, on top of “quantity” and “structure”—“space” is also considered, which is where geometry comes into play (and is the focus of this book, albeit in relation to other sub-fields of mathematics and non-mathematical disciplines too).

The reason is that “[t]he study of space originates with geometry—in particular, Euclidean geometry,” in relation to, say, the “size, shape, and relative position of figures and with properties of space.” (WK 2008a & WK 2008k)

Space, however, is seldom studied just alone, but often in relation to other areas of mathematics (and other fields of knowledge, for that matter).

For instance, in “trigonometry,” both “space” and “quantity” are considered, and a beautiful achievement is the well-known “Pythagorean theorem,” which is mathematically expressed as “ $a^2 + b^2 = c^2$ ” (or verbally means that “[t]he sum of the areas of the two squares on the legs [a and b] equals the area of the square on the hypotenuse [c].” (WK 2008a & 2008l; T. Heath 1921)

And in “linear algebra,” three of the fundamental areas of mathematics (i.e., “quantity,” “structure,” and “space”) are combined, for the study of “vectors.” (WK 2008a)

A “vector” (sometimes known as a “geometric or spatial vector”), especially in elementary mathematics, physics, and engineering, is defined as “a geometric object that has both a magnitude (or length), direction and sense that is orientation along the given direction. A vector is frequently represented by a line segment with a definite direction, or graphically as an arrow, connecting an initial point *A* with a terminal point *B*.” (WK 2008m; A. Ivanov 2001; J. Heinbockel 2001; B. Muvdi 1997; K. Ito 1993)

A set of vectors is called a “vector space” in Euclidean geometry and “may be scaled and added....As much of the theory around vector spaces is of linear nature, these objects are a keystone of linear algebra. From this point of view vector spaces are well-understood, since vector spaces are completely described by a single number called dimension.” (WK 2008n)

Change

Yet, “quantity,” “structure,” and “space” are not the end of the story. There is also “change,” or a more dynamic understanding of the other three areas in mathematics.

After all, “[u]nderstanding and describing change is a common theme in the natural sciences, and calculus was developed as a powerful tool to investigate it. Functions arise here, as a central concept describing a changing quantity.” (WK 2008a)

For instance, in relation to “quantity” and “change,” there is now “real analysis” (for “[t]he rigorous study of real numbers and real-valued functions”) and “complex analysis” (for “the equivalent field for the complex numbers”). (WK 2008a) In fact, “differential equations” are to study problems in regard to the “relationships between a quantity and its rate of change.” (WK 2008a)

In relation to “quantity,” “structure,” “space,” and “change”—there is now “vector calculus,” which, unlike the study of vectors (with only “quantity,” “structure,” and “space”), “expands the field into a fourth fundamental area, that of change.” (WK 2008a)

The analysis of functions so understood, or more generally, “functional analysis,” is also applied for the natural sciences. For instance, “[o]ne of many applications of functional analysis is quantum mechanics. Many phenomena in nature can be described by dynamical systems; chaos theory makes precise the ways in which many of these systems exhibit unpredictable yet still deterministic behavior” over time. (WK 2008a)

Interdisciplinary Mathematics

Interdisciplinary mathematics, unlike pure mathematics, combines different mathematical and non-mathematical disciplines, and two major ones to be considered here are, namely, (a) mathematical logic (and foundations) and (b) applied mathematics—to be summarized below.

Mathematical Logic (and Foundations)

Unlike pure mathematics—mathematical logic is a hybrid field of mathematics and logic (as the name suggests) and has different sub-fields like “set theory,” “proof theory,” “model theory,” “recursion theory,” and “constructive mathematics,” as already analyzed in one of my previous books titled *The Future of Post-Human Mathematical Logic* (2008).

It searches for the foundations of mathematics, on whether or not mathematics can be grounded on some proper formal axioms. In other words, it is “concerned with setting mathematics on a rigid axiomatic framework, and studying the results of such a framework.” (WK 2008a)

A good illustration of this “foundational controversy” is none other than the debate “in twentieth century history of mathematics” between David Hilbert, the founder of “formalism” and the main proponent of mathematical foundationalism, on the one hand, and his opponent, L. E. J. Brouwer, a supporter of intuitionism, on the other hand. (WK 2008o)

But it is Kurt Gödel, who showed, in his “incompleteness theorem, perhaps the most widely celebrated result in logic,” that “any formal system that contains basic arithmetic, if sound (meaning that all theorems that can be proven are true), is necessarily incomplete (meaning that there are true theorems which cannot be proved in that system).” (WK 2008o) His conclusion is that Hilbert’s formalist program is not possible.

Applied Mathematics

Mathematics has also been used in relation to other non-mathematical disciplines—not just logic alone.

A good case in point concerns “applied mathematics,” which “considers the use of abstract mathematical tools in solving concrete problems in the sciences, business, and other areas,” and illustrative examples include, say, “mathematical physics,” “financial mathematics,” “game theory,” “optimization,” “numerical analysis,” “statistics,” and “probability theory.” (WK 2008a)

As an illustration, “an important field in applied mathematics is statistics, which uses probability theory as a tool and allows the description, analysis, and prediction of phenomena where chance plays a