Single-Voice Transformations

Single-Voice Transformations: A Model for Parsimonious Voice Leading

By

Brandon Derfler

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CHAPTER 1
INTRODUCTION

It is likely that no music-theoretic study in recent times has had a greater impact on subsequent research than David Lewin's 1987 book *Generalized Musical Intervals and Transformations* (hereafter abbreviated *GMIT*). Among the many possible applications of Lewin's transformational theory, two have been particularly fruitful and have been developed by Lewin and others into major branches of modern music theory: transformational theories, applied to voice leading in pioneering studies by Lewin, John Roeder, Joseph Straus, and Henry Klumpenhouwer, and so-called "neo-Riemannian" theory, stemming from chapter 8 of Lewin's book and extended by Brian Hyer, Richard Cohn, Lewin himself, and others.

Transformational theories of voice leading have relied almost exclusively on the T_n/T_nI dihedral group of operations on verticalities to map pcs from one chord to another. One particular offshoot of transformational voice-leading theory of interest to many theorists (including Lewin) involves the study of network isographies ("Klumpenhouwer networks" or "K-nets"). With their emphases on chordal and network transformations and the fractal-like recursive self-similarities found in K-nets, studies of this kind have tended to relegate voice leading to a position of lesser prominence, seemingly viewed as not much more than an incidental by-product of chordal mappings. Additionally—as will be seen—the very notion of "voice leading" itself is problematic when one attempts to make a distinction between, say, *voice* and *line* on the one hand, or between *manifest* and *projected* voice leading, to borrow Lewin's terms (1998, 18), on the other hand.

Neo-Riemannian theory at times seems to suffer from the same malady that plagues set transformational theory: much more emphasis has been given to the group properties of transformations on the Riemannian triadic *Klänge* (and, more recently, seventh chords) than to the actual voice leading that results from the transformations. In both areas of study, theorists have focused much more closely on the operations that map one pc set onto another, to the detriment of exploring the contrapuntal paths used to get there. One aspect of certain types of voice leading—namely, *smoothness* or *parsimony*—has been explored in neo-Riemannian studies (particularly by Richard Cohn), and forms an important bridge between chord-transformation theories and theories of counterpoint and line.

A third, unrelated branch of musical explanation that enjoys great currency can be traced to the writings of Heinrich Schenker and his disciples.

While Schenkerian analysis is non-transformational, not being founded on a mathematical basis as Lewin's system is,[1] it offers a much more intimate look at voice leading, at counterpoint, and at the linear aspect of music than do the branches of theory described above.

While I do not propose a type of "unified field theory" that reconciles the Schenkerian and the transformational approaches to voice leading, in this study I employ a transformational approach in a manner that takes into account the voice-leading connections between chords to a greater degree than do the transformational voice-leading and neo-Riemannian models. As will be seen, there is nothing particularly Schenkerian about my approach, other than a heightened sensitivity to the horizontal element in progressions between vertical sonorities.

As some proponents of neo-Riemannian theory privilege voice-leading smoothness, in this dissertation I too will focus much of the discussion on voice-leading parsimony, which has been a concept basic to linear writing in non-monophonic music from the advent of polyphony until the present day. This principle of smooth voice leading as the "norm," with disjunct motion serving as the exception (to provide contrast), has been recognized by theorists and composers for centuries and is perhaps best described by Schoenberg (quoting Bruckner) as "the law of the shortest way" (Schoenberg 1978, 39). Key to my approach is the *single-voice transformation* (SVT), whose archetype is the *single-semitone transformation* (SST). This function on a chord transposes one pc by T_1 while the remaining pcs map onto themselves. Multiple iterations of this transformation or its inverse through the composition of functions produces larger voice-leading intervals. As a great deal of music privileges smooth voice leading over disjunct voice leading, the SST—whether iterated singly or doubly—serves as an effective model for better than half of the voice-leading connections in most music (i.e., in music in which half- and whole-step voice leading predominates). For such music in which parsimony is the predominant voice-leading paradigm, the occasional skip can be modeled by the *single-fifth transformation* (SFT) and its multiples. This function on a chord transposes one pc by T_7 while the remaining pcs map onto themselves. As with the SST, multiple iterations of this function or its inverse—as well as its combination with an SST—produce different voice-leading intervals. The SST system and the SFT system, both based on transposition intervals that generate the complete ETS 12 pitch-class universe, are isomorphic.

Some readers will recognize the single voice transformation's resemblance to Allen Forte's "unary transformation," in which one pc set mutates to another by the change of a single element of that set (Forte 1988). In order to limit the scope of this study, for the most part I will be discussing the SST, first focusing on parsimonious transformations in the abstract and subsequently examining music in which the voice leading can for the most part be modeled by SSTs. A "parsimonious transformation" may be defined as one or more SSTs acting on a chord, with a maximum voice-leading displacement of one whole step per chord member ("voice"). Thus to be considered parsimonious, a voice-leading mapping may not consist of more than two consecutive applications of the SST affecting the same voice. This limit at first appears related to Robert Morris's artificially imposed constraints on total voice leading (Morris 1998, 178). There really is no limit to the number of iterations of SSTs and SFTs that can be applied to a chord member; the whole-step artificial limit is only meant to define the term "voice-leading parsimony" and in no way compromises the integrity of the SVT system were it to be removed. Music in which the voice leading can be modeled by a combination of parsimonious and non-parsimonious transformations is the norm through all historical periods, and the works analyzed in this study are no exception to this norm.[2]

The structure of the present study is as follows:

As a preface to discussion of my model of voice leading, it is helpful to summarize some of the work of theorists in modeling transformational voice-leading space. An overview of transformational models of atonal voice leading is presented in chapter 2, examining studies by Klumpenhouwer, Straus, O'Donnell, Lewin, Cope, Jurkowski, Callahan, and Roeder. Most of these models propose variations on the theme of voice leading via T_n/T_nI operations mapping pc set to pc set of the same set-class type. While these studies are fascinating and at times quite compelling—and, in the case of Klumpenhouwer, have opened up new areas of theoretical inquiry—they all, in varying degrees, suffer from one or both of two principal shortcomings. First, and most significantly, in most of these studies, voice-leading transformations can only be mapped between pc sets of the same set-class type. This problem has recently been partially addressed in articles by Joseph Straus through his concepts of "near-transposition" and "near-inversion" (Straus 1997, 268) which he later calls "fuzzy transposition" and "fuzzy inversion" (Straus 2003, 318).[3] Second, and perhaps more relevant from a practical standpoint, is the problem of *musical voices*—as a result of pitch-class mapping via transposition and inversion operations—actually being heard (or at least musically intuited). The fact that many of the paths traced through this type of "pitch-class counterpoint"[4] correspond with neither concepts of voice-leading parsimony nor (more importantly) what most would intuit as individual voices—made audible in actual music through factors such as register, instrumentation and pitch/pitch-class interval distance—naturally causes one to question the analytical applicability of such proposed voice-leading connections. Integral to a discussion of such an important topic is an understanding of the distinction between *voice* and *line*, and part of chapter 2 emphasizes some of the finer points of this often difficult differentiation.

In Chapter 3 the single voice transformations SST and SFT are presented and formally defined as ordered *m*-tuple elements $\langle g_1,g_2,...,g_m\rangle$ of the direct product group of mod-12 transposition operations $G_1 \times G_2 \times ... \times G_m$ acting on a chord C of cardinality m, with chord members c_i also ordered as an *m*-tuple: $\langle c_1,c_2,...,c_m\rangle$.[5] The chief differences between my voice-leading model and those discussed in chapter 2 are presented in this chapter and include a number of characteristics which I will briefly outline here. Previous transformational voice-leading theories have relied on the T_n/T_nI group of operations to map pc set to pc set. My model employs only the group of transposition operations (T_n) to effect mappings between *m*-tuples. This has the net result of describing voice-leading operations in a way that is more naturally intuited by listeners and (possibly in the majority of cases) as conceived by composers. Another principal difference between methodologies is the capability of SST and SFT operations and their compositions to allow for mappings between chords belonging to different set-class types. The desirability of this result is self-evident.

The strengths and limitations of the direct-product voice-leading model are weighed in chapter 3. For example, the problem of multiple iterations of single-voice transformations is discussed: at what point does the conglomeration of "moves" become excessive? Another one of the model's shortcomings is its difficulty in convincingly explaining music in which there is a fair amount of voice leading by ic 3 or 4.

On the flip side, however, one of the strengths of my model is its use of ordered *m*-tuples—optionally indexed by a unique set-class type—as the domain and range of voice-leading functions. Through simple addition, one can apply one or more single-voice transformations to an *m*-tuple and easily calculate the resultant *m*-tuple. By extension, SST-succession classes can be created to generalize parsimonious voice-leading relations between T_n/T_nI-types on a more abstract level. The generalization to SST-succession classes builds upon recent work by Clifton Callender (2004) and Dmitri Tymoczko (2006). Through the use of ordered *m*-tuples and generalized SST-succession class types, the registral order of the chord members is unimportant, unlike some previous transformational voice-leading theories based on voice permutation. In those theories, the permutation of the voice positions drives the voice leading. If pitch-interval size between adjacent voices in a chord is a key factor in a transformational voice-leading model (and it often is), permutation of the registral order positions ("chord inversion" in the parlance of tonal harmony) completely changes the interval size; thus a different operation must be used for chords of the same set-class type appearing in different registral configurations.

Another problem which has plagued transformational theory since its inception involves mappings between sets of different cardinalities. In chapter 3 I discuss the manner in which theorists have coped with this issue, which in the pitch/pitch-class realm has involved the use of *splits* and *fuses*, terms which seem to have originated in a neo-Riemannian study by Clifton Callender (1998, 224). My concept of splits and fuses tends to differ somewhat from the "traditional" definition, and is perhaps the first to formalize these types of operations mathematically. Generalization of these concepts to split-succession classes and fuse-succession classes rounds out the chapter.

Chapter 4 deals with network graphs of parsimonious voice-leading systems. A number of these graphs have been constructed—in two- and three-dimensional form—of voice-leading space involving triads and seventh chords of limited set-class types. For example, the familiar 2D Riemannian *Tonnetz* lattice models L, P, and R relations between major and minor triads ([037]). More recently Robert Morris (1998) has constructed a generalized "*Tonnetz* space descriptor" which adjusts the ic values on each axis of a *Tonnetz* to allow for lattice models of all trichordal set-class types, not all of which preserve parsimonious voice leading. Parsimonious voice-leading relations between all tetrachordal set-class types have been graphed by Straus in 2D (2003, 339) and by Straus (2005) and Richard Cohn (2003) in 3D.

In chapter 4 I present two distinct network graphs of parsimonious voice-leading space: the tetrachordal SST-succession-class-space graph which is a 3D version of Straus's "optimal offset" chart, and the trichordal/tetrachordal split-succession-class-space ("Christmas tree") graph which is a 3D model showing all split-succession-class relations between trichordal and tetrachordal SC types. As it is difficult to reproduce 3D renderings on the 2D printed page, links are provided to an internet URL which contains CAD files of the network graphs, along with a link to the free software used for viewing them. The purpose of this is to provide a hands-on experience in which readers can zoom in and out of the graph and pan in all three dimensions to view the structure from any angle. The website also contains short .avi video clips recorded with a virtual camera "flying" in different trajectories through the graphs. For those disinclined to use the CAD software, 2D snapshots of the graphs appear as figures in the body of the dissertation.

Both graphs are "ball-and-stick" networks, with set-class types being represented by spheres and cubes, and the associated SST- or split-relation subscripts by the connecting "sticks." In several cases, different SST-succession-class types may comprise identical pair-related set classes. In cases such as these we can speak of *duplicate paths* of distinct SST-relations between *m*-tuple elements of the ordered pair of (identical) set classes. Duplicate SST-succession-class paths are most frequently associated with symmetrical set classes. One well known—and extreme—example of this concept is the collection of SST-succession classes that relate [0369] to [0258], where there are eight distinct voice-leading path types between *m*-tuple elements of the set classes.[6]

The tetrachordal network graph, while simpler than the trichordal/tetrachordal network graph, has one clear advantage over the latter: it shows a clear correspondence between SST-succession classes affecting "order positions" of the set-class integers and the X–Y–Z axes of the Cartesian coordinate system. It will also be seen that set class [0148] occupies a unique position in the tetrachordal graph, and reasons for this are discussed in the chapter. The trichordal/tetrachordal graph comprises nested cones of set-class types sharing certain properties, and displays split-succession-class connections between trichordal and tetrachordal SC types. At first glance, this second graph appears to resemble Cohn's tetrahedral network graph (2003), but it is actually quite different; these differences are discussed in the chapter.

The study of single-voice transformations acting on *m*-tuple members of an important subset of trichordal and tetrachordal set classes—namely, the "neo-Riemannian" set classes—forms the basis for chapter 5. There is, in the literature, disagreement about exactly which set class types' members can be operated on by neo-Riemannian transformations. The original essays defining neo-Riemannian theory (originating with Riemann himself) only admitted the major and minor triads as *Klänge* which could be subjected to *Schritten* and *Wechseln* such as the *Dominantschritt* and *Leittonwechsel* operations. In the late 1990s a number of theorists began to examine parsimonious voice-leading transformations between seventh chords, particularly between dominant seventh/half-diminished seventh chords of set class [0258].[7] In this study, the "neo-Riemannian" subset is defined as set class [037] and all tetrachordal SC types that include it. This collection includes the standard major, major-minor, minor, and half-diminished seventh chords as well as several atonal tetrachords, but does not include the fully diminished seventh chord or the augmented or diminished triads (parsimonious voice-leading involving these latter chords is discussed in chapter 4).

The first part of chapter 5 provides a brief overview of the literature on neo-Riemannian theory, limited to those studies that examine 1) transformations on chords of cardinality 4 or 2) transformations that increase or decrease cardinality between chords. Work by Lewin, Cohn, Callender, Childs, Douthett and Steinbach, Gollin, Hook, and others is discussed. It will be seen that most of these studies do not directly address the relationship between neo-Riemannian operations and voice leading (the principal exception is Childs). As most of these studies regard neo-Riemannian functions such as L, P, and R as contextually-defined inversion operations ("cio"s, after Kochavi 1998), they run into the same twin problems that plague the authors discussed in chapter 2: difficulty in achieving mappings between chords belonging to different set-class types, and musically non-intuitive voice leading (if voice leading is even a consideration). By eliminating these inversion operations from the group of transformations on members of the neo-Riemannian SC collection, voice leading comes to the fore and becomes more of a determinant of chordal succession than would be suggested by cio-privileging approaches. Interestingly, only Gollin (and to a lesser extent, Shimbo) approach neo-Riemannian transformations through Riemann-like dualist systems, even if they do not explicitly and formally describe the voice-leading that results from their chord mappings. Although the present study eschews tonal and dualist tonal theories for an atonal transformational approach, by its abstracting the ordered *m*-tuples operated upon by the single-voice transformations to SST-related T_n/T_nI types, it bears some resemblance to the Riemannian dualist system. For example, the set {0,4,7} shares the same T_n/T_nI type as its Riemannian dual, {5,8,0}, namely, [037]. Applying the SST operation SST_2^{-1} to ⟨0,4,7⟩ is equivalent to the "traditional" P operation, in practical terms mapping a C-major triad to a C-minor triad. However, the SST operation SST_2 would need to be applied to ⟨5,8,0⟩ to map an F-minor triad to an F-major triad. Both of these SST-related chord successions are representatives of the SST-succession class, $\langle[037],[037]\rangle_{SST_2}$.

Also in chapter 5, the multitude of labeling systems used by the various authors to describe the different "neo-Riemannian" functions on trichords and tetrachords are examined. These are compared with the conventions of the labeling system used in this dissertation and with each other; all are summarized in Tables 5-2 through 5-4.

Chapter 6 presents analyses of music using the single-voice-transformation model. I have deliberately chosen music composed in several distinct time periods and styles to show the near-universal applicability of the SVT model to non-monophonic works from the Western art-music tradition. The selections can be grouped in three broad categories:

1. Tonal music, specifically Romantic-era compositions more highly saturated with chromaticism than earlier styles. Pieces by Chopin (Prelude Op. 28, no. 4 in E minor), and Scriabin (Etude Op. 42, no. 5 in C-sharp minor) are analyzed;
2. Atonal music, represented here by analyses of the first movement of Webern's Second Cantata, Op. 31, and the opening of Paul Lansky's *Modal Fantasy;* and
3. Music by contemporary composers dating from the last quarter of the twentieth century which incorporates elements of atonal as well as non-functional triadic music and which, for lack of a better term, I will refer

to as “post-atonal” music. Shorter excerpts from pieces by John Adams are first examined, followed two lengthier analyses of selections from *Nixon in China*.

This last category requires some additional explanation. I have undertaken analyses of several pieces from the post-atonal repertoire for a couple of reasons. First, this literature has, until quite recently, been underrepresented in music analysis, and particularly in transformational analyses, which have instead focused on the “classic” serial literature (atonal voice-leading analyses) and on late-Romantic pieces (neo-Riemannian analyses). Another reason for the choice of this particular literature is its suitability for analysis which emphasizes parsimonious voice leading. Much of this literature fits—however tightly or loosely—under the rubric of “minimalist” music, in which smooth voice leading is, if not privileged, at least more readily apparent to the ear than in atonal music from the same time period. The dissertation concludes with a summary of topics covered in the study and posits suggestions for avenues of future research. This is followed by appendices which list tables of trichordal and tetrachordal SST- and split-succession classes.

A Brief Preliminary Remark: Conventions Used

My notational conventions for *m*-tuples, sets, and set classes in this study for the most part follow those in Rahn (1980) with one minor exception: I have eliminated the commas separating pc integers in both T_n- and T_n/T_nI-type set-class designations to follow an increasingly common practice in the literature. The absence of the commas should not create confusion if we use “t” for 10 and “e” for 11. Thus a T_n-type for the whole-tone hexachord would read (02468t) and the T_n/T_nI-type would read [02468t]. Another reason for eliminating the commas in the T_n-type is to avoid confusion with a common mathematical convention for an *m*-tuple. Following standard practice, commas will be retained between elements of *m*-tuples (enclosed in angle brackets, thus $\langle a_1,a_2,...,a_m\rangle$) and sets (enclosed in curly braces, thus $\{a_1,a_2,...,a_m\}$).

There is slight disagreement in the music-theoretic literature about the extent to which the following terms differ: transformation, operation, function. The terms are largely synonymous in the mathematical community (Hook 2002, 192; 197) but Lewin assigns subtly different shades of meaning to each in *GMIT*. Lewin’s definition of a function follows the standard mathematical convention:

> Let S and S' be sets of objects. The Cartesian product $S \times S'$ is the set of all ordered pairs $\langle s,s'\rangle$ such that s is a member of S and s' is a member of S'.

> A *function* (or mapping) from S into S' is a subset f of $S \times S'$ which has this property:
> Given any s in S, there is exactly one pair $\langle s,s' \rangle$ within the set f which has the given s as the first entry of the pair.
> We say that s', in this situation, is the value of the function f for the argument s; we shall write $f(s) = s'$ (Lewin 1987, 1).

Lewin goes on to define a *transformation* on S as a function from a set S into S itself. An *operation* on S is a bijective function (1987, 3).

It is interesting to note that by Lewin's definition, an operation is not necessarily a transformation (if the function maps set elements from S to S'). Fortunately, in most of this study the set S equals the twelve equal-tempered pitch classes, and functions on this set map elements of the set onto elements of the same set injectively. Thus Lewin's distinctions are largely a moot point here: I will use the terms "function," "mapping," and "transformation" interchangeably throughout the text. As the SVT system involves bijective functions, the term "operation" can and will also be used in subsequent chapters to describe voice-leading transformations.

Lewin's definitions for semigroups and groups follow the standard mathematical definitions: A *semigroup* is an ordered pair $\langle X,\bullet \rangle$ comprising a set X and an associative binary composition • on X (Lewin 1987, 5). A *group* is a semigroup with identity in which every element has an inverse (6). In this study for the most part I will be limiting the discussion to the group G of transpositions modulo 12.

Finally, chapter 4 presents a number of graphical representations of pitches and relations between them (as in, for example, the Riemannian *Tonnetz*), of sets of pitches (chords) and transformations that relate them, and of SC types and parsimonious voice-leading paths between them. These representations have been called anything from models to maps, graphs, and networks. While in a general sense these terms are fairly interchangeable (and I will ask the reader's forgiveness if I occasionally slip into this practice), each term carries its own subtle shade of meaning. The term "model" is perhaps best reserved for theoretical systems devised to explain aspects of pitch and pitch-class relations in compositional space, and will not be used here to describe transformational diagrams. Straus (2003) prefers to use the term "map" for his diagrams, but to me this term is too close to the mathematical term "mapping" for comfort. This may explain his choice to use the term "graph" (of voice-leading space) for the same type of diagram in 2005. However, "graph" itself is a loaded word, which after Lewin's usage implies a node-and-arrow diagram without content assigned to the nodes.[8] Lewin himself uses the terms "transformation network" or "network graph" for his node-and-arrow diagrams; the term *network graph* seems perfectly reasonable to describe transformational “ball-and-stick” representations of voice leadings and SST-succession classes between *m*-tuples and paired set classes, respectively.

CHAPTER 2

SOME TRANSFORMATIONAL MODELS OF VOICE-LEADING SPACE

While the study of voice leading in tonal music has a long and venerable historical pedigree (with Schenker as the most visible proponent), the study of voice leading in non-tonal music has only recently been an area of serious theoretical inquiry. Although a rudimentary model of voice leading existed as early as the 1940s in the writings of Joseph Schillinger, it was not really until the 1990s that scholars turned their attention to the possibilities of transformational theory applied to voice leading in atonal music.

The term "transformational" is key here, describing a model based on concepts of David Lewin as set forth in *GMIT* and earlier works. Transformational theories are one of three types of models for atonal voice leading, as recognized by Joseph Straus (1997, 237), the other two being "prolongational" and "associational" models. Prolongational voice-leading theories owe much to Schenkerian analysis and are usually considered to be modifications to his theory to allow for the analysis of non-tonal music. Associational models attempt to explain the voice leading of specific musical lines as "projections" of vertical harmonies or set-class types that are prominent at the musical surface (241). As my approach is transformational, in this chapter I will give an overview of some of the transformational models of atonal voice leading beginning with Henry Klumpenhouwer's 1991 dissertation and continuing to recent studies.

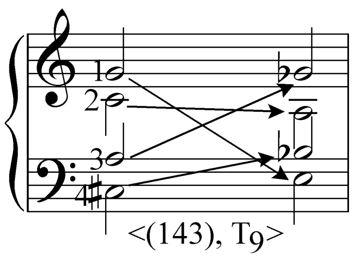

Fig. 2-1: Voice leading of a [0146]-type tetrachord, as a result of a permutation/transposition function (after Klumpenhouwer 1991).

Inspired by Lewin's *GMIT,* Klumpenhouwer's dissertation, "A Generalized Model of Voice-Leading for Atonal Music" is perhaps the *locus classicus* of subsequent approaches to atonal voice leading in that it posits transposition and inversion operations on pc sets as the driving forces behind its transformational method. The model is basically a permutational one, where stacked pcs, ordered by register, "lead" to verticalities through permutation cycles affecting the ordered registers. For example, the permutation cycle (sbt) maps the pc in the "soprano" register position to that in the "bass" position in the subsequent verticality; the "bass" pc maps to the pc occupying the "tenor" position, and the "tenor" pc maps to the pc occupying the "soprano" position. Given a four-register texture, the "alto" pc maps to its own register position (not necessarily to the same pc). These register positions can also be (and are later) re-labeled 1, 2, 3, or 4, from highest to lowest. When a register position is associated with specific pcs, a registral permutation, paired with a transformation belonging to the T_n/T_nI group of TTOs, acts upon the pcs and maps them to a new verticality of the same set class. The operation is expressed as an ordered pair $\langle A,X \rangle$ where A = a registral permutation and X = a TTO. For example, $\langle (143),T_9 \rangle$ acting on the collection {2,C}, {3,A}, {4,C♯}, {1,G} produces {2,A}, {1,G♭}, {3,B♭}, {4,E} (see Figure 2-1). Klumpenhouwer later fashions a quotient GIS which makes all transformations with the same permutational cycle congruent, regardless of TTO employed.

The construction of the quotient GIS is the first of several steps which gradually lead the topic of inquiry away from voice leading *per se*, toward the more abstract and generalized topic of network isomorphisms. These later chapters of the dissertation, in fact, develop Klumpenhouwer's best-known contribution to transformational theory: not a permutational model of voice leading, but rather the Klumpenhouwer network (or K-net). Klumpenhouwer posits the K-net as a way of relating chords belonging to different SCs by comparing their internal structure, noting where there is a "skew" in moving from one set type to another. He also spends considerable time studying networks of networks, noting the "possibility of recursive structuring" between the internal structure of networks and network isomorphisms (see his chapter 6).[1]

So much time, in fact, is spent on K-nets that one starts to wonder whether the dissertation is accurately titled. Certainly the author begins by constructing a system for atonal voice leading, but the model becomes so generalized and remains at such a macro-level for so long that voice leading as a topic is largely abandoned by the fifth of nine chapters. In the end, his dissertation becomes more a celebration (or fetishization?) of network isographies than an exploration of voice leading in atonal music, and as such it should be regarded as an important contribution to theories of isomorphisms in music. Its value as a theory of voice leading lies chiefly with its proposal of a transformational T_n- and T_nI-based model for voice mapping between pc sets that became the prototype for several subsequent studies. These later studies inherit from the dissertation a number of issues that raise questions about the efficacy of the T_n- and T_nI-based transformational model in convincingly portraying voice leading in a way that is commensurate with one's logical and audiological intuitions about the music in question.

One of the principal issues or problems attached to *any* T_n- and T_nI-based analytical approach (not just limited to models of transformational voice leading) is that of relating objects of different class-types, in this case chords of different set-class types. Klumpenhouwer provides a roundabout way of relating chords belonging to different SCs through his K-nets, but the network isographies are stronger between certain pairs of chords than others, and in any case the voice leading between any two chords is never directly shown. In fact, in the early chapters of the dissertation he is compelled to skip certain chords in his analyses, chords that do not belong to the SC type of the particular verticality undergoing a voice-leading transformation.

It should also be pointed out that the registral (vertical) arrangement of chord members is of paramount importance in Klumpenhouwer's model. This is an *absolute* registral ordering and does not allow for voice crossing as frequently occurs in the more abstract concept of a musical "voice." Once a registral permutation has occurred, a transformed "line" is assigned a new order number based on its new registral position; thus the concept of a quasi-independent voice cannot be maintained, since it would be destroyed with each mapping. Without taking this absolute ordering into account, Klumpenhouwer's model does not work. Again, it will be seen that this issue continues to arise in later voice-leading studies.

A recent paper by Michael Callahan (2006) reviews some previously noted parallels between Klumpenhouwer networks and George Perle's cyclic sets and then investigates sum-and-difference motion along a network graph comprised of two separate K-net cycles. The first of these is the "strong-isography class cycle" in which the K-net interval and one of the K-net sums increase or decrease in tandem by one while the remaining sum decreases or increases equally in opposition to them. Any three-pc K-net strong-isography cycle will contain twelve steps. The second K-net cycle is the "semi-transpositional cycle" (after Perle) which maintains the same K-net interval and one of the sums while increasing or deceasing the remaining sum by one. Like the strong-isography cycle, the semi-transpositional cycle contains twelve steps. Callahan later constructs a 3D network graph of three-member K-nets that includes "axial-isography cycles," which are used to relate K-nets which share no transposition- or inversion-related interval-classes.

Callahan's contention is that one can reinterpret K-nets as pc sets and show the semitonal voice leading between the sets as a combination of "moves" along the network graph. His claim that "the treatment of these relationships in terms of semitonal motion allows us to avoid the potential objection that sums-of-sums and differences-of-sums may be abstract and too far removed from the musical surface" seems to fall short simply because K-nets themselves, upon which his network graphs are constructed, are already a significant theoretical abstraction. Additionally, Callahan does not express motion along any of the axes of his network graphs in terms of a mathematical transformation, instead choosing to show parsimonious relations in his graphs without identifying specific transformations—either musically intuitive (such as an L, P, or R operation) or purely abstract—leading from one K-net/pc set to another. Thus constituted, Callahan's study perhaps does not precisely belong in a chapter devoted to transformational models of voice-leading space, but it does show a heightened sensitivity to independent voice-leading paths and does not rely upon *in toto* set-class transposition and inversion as Klumpenhouwer's and many others' approaches do (as summarized later in this chapter). It is also notable for its being the only study that I am aware of that combines the privileging of voice-leading parsimony (à la Richard Cohn and the neo-Riemannians) with Lewin's research on K-nets.

Callahan's study is not without precedent, however. Edward Jurkowski's dissertation, "A Theory of Harmonic Structure and Voice Leading in Atonal Music" (1998), also relates non-TTO-related sets through special K-nets which he calls "Interval-Difference Networks," or I-DIFF Networks. Operations within these networks are on intervals and ics rather than on pitches and pcs; this gives Jurkowski the capability of discarding inversion operations entirely and using only transpositions. The I-DIFF Networks are composed of dyadic subsets of verticalities, and like Klumpenhouwer, rely upon reorderings (permutations) of registral positions in order to create the pitch-class counterpoint of transformational lines. Though an intriguing investigation of a new direction for the study of K-nets, Jurkowski's method is somewhat hampered in the same way Klumpenhouwer's and Callahan's are: all rely heavily on a very abstract concept, the K-net, which—though elegant theoretically—is so far removed from the musical surface and from an actual listener's experience that there is little, if any, correlation between the intra-chordal K-net transformations and the perceived voice leading. Even the models which propose set transpositions/inversions as the driving force behind the mappings of the individual voices (see the discussion of Straus, O'Donnell, and Lewin, below) seem to conform more closely to one's listening experience than do those which first require a K-net transformation *before* proceeding to a TTO on a set.

Drawing on Klumpenhouwer's transformational approach and on earlier, related work by John Roeder (1984 and 1994, discussed later in this chapter), Joseph Straus (1997) also uses a T_n- and T_nI-based model to describe voice leading between pc sets. In Straus's system, voice-leading connections are still limited to pc sets of the same SC type. Like Klumpenhouwer's model, the destination tones of individual voices comprising a chord are determined by transposition or inversion operations on the set. Straus, however, abandons the permutational aspects of the earlier study, instead tracing individual voice paths by connective lines in diagrams placed under the score (see for example Figure 2-2). In addition to showing the chord-to-chord voice-leading connections effected by set transposition and inversion operations, Straus shows the overall T_n or T_nI level traced by summing all the chord-to-chord transformations, from the first to the last chord of a musical passage. By examining the total sum of voice-leading operations on small-, medium-, and large-scale levels, Straus is able to show transformational patterns that otherwise would not be readily apparent. These patterns suggest intriguing but inexact parallels to foreground, middleground, and background structures in Schenkerian analyses.

However, Straus's most important contribution lies in his proposal of (as yet unformalized) concepts of "near-transposition" and "near-inversion."[2] Despite this tantalizing suggestion, unfortunately the model still suffers from the frustrating by-product of pc-set transposition- and inversion-based transformational systems: the resultant "voice leading" usually runs counter to what is musically perceived/intuited. Reconciling audible musical lines with the more abstract concept of voices is not a problem unique to Straus's study; on the contrary, it has been an ongoing challenge for most theorists attempting to construct logical models of voice leading. I will have more to say on this later in this chapter.

An important contribution to the theory of transformational voice leading is found in Shaugn O'Donnell's dissertation, "Transformational Voice Leading in Atonal Music" (1997). O'Donnell's principal contribution lies not so much in his voice-leading model or system, which is cast in the Klumpenhouwer and Straus set transposition- and inversion-driven mold, but rather in his insightful criticism of the very limitations of this type of model. Among the most severe deficiencies he cites are those mentioned earlier. For one thing, voice leading may occur only between chords of the same SC type, and these are often widely separated in the actual music:

> The most significant limitation of a transformational voice-leading model based solely on transposition and inversion is the self-contained nature of the set-class equivalence classes. . . . The transformational interpretation of the traditional operations, that is, transposition and inversion as dynamic processes rather than static relationships, is severely weakened in such

> circumstances. . . . It is absolutely essential that a successful voice-leading model be capable of describing complete passages, because there must be some aural continuity in these musical gestures or we would not reify them with the term "passages." (16–7)

O'Donnell offers more perceptive criticism of the T_n- and T_nI-based voice-leading model than any of his predecessors, and promises to expand "the transformational machinery" in his dissertation to compensate for the system's deficiencies. He is only somewhat successful in achieving this promise, however. His solutions for the most part seem to be relatively minor fixes, subtle "tweaks" to the system that patch up only the most egregious deficiencies.

O'Donnell recognizes—perhaps more than any other author on this subject—the rôle that our auditory faculties and mental pre-conditionings play in shaping our perception of voice leading. "I want voices that most richly interact with my experience of the musical surface. In other words, I prefer voices that are reinforced by one or more surface lines, such as registral or instrumental lines" (64). Much of the dissertation focuses on the attempt to choose, from multiple possible analyses of a piece's voice leading, a reading that most closely approximates a hypothetical listener's experience. This is not always successful, as a purely perceptual interpretation of the voice leading frequently comes in conflict with more abstract constructs, as it does, for example, with voice-leading paths determined by T_n and T_nI operations on pc sets or with Klumpenhouwer network isography.[3] In trying to "have it both ways"—tracing the voice leading on a perceptual level while at the same time justifying voice leading as a result of strict transformational processes, O'Donnell runs into a problematic result familiar to many: a jarring disjunction between a perfectly logical theory and a musical surface that refuses to pigeonhole itself by readily conforming to that theory.

As a way of coping with the problem of transposition and inversion between members of different set classes, O'Donnell endorses the idea of the "singleton transformation" (26–9). In this type of transformation, all chordal elements but one map by the same operation. The remaining element (voice), in mapping by some other operation, violates the integrity of the composite mapping, keeping it from being an exact transposition or inversion. There are several precedents for this type of transformation. Forte's "unary transformation" (Forte 1988) maps pc sets of different set types by the transposition of only one of its members. In "Transformational Techniques in Atonal and Other Music Theories" (1982–3) David Lewin proposes an "if-only transformation," which is basically the reverse of the unary transformation in that all members of a pc set but one participate in a TTO. (The remaining singleton maps to itself.) Straus's "near-transposition" and "near-inversion" (1997, 268–70) are similarly conceived. This "new" type of transformation allows for the modeling of a greater number of voice-leading connections between proximate pc sets than would simple, straightforward set transposition and inversion.

O'Donnell then ups the ante by allowing two distinct T_n or T_nI (or even wedge—"W") operations to occur simultaneously. This "split transformation"[4] likewise adds another option for explaining voice leading between chords of different SCs, and serves fairly well in analyses of passages from Scriabin (98ff) and Babbitt (154–5). However, even this broader palette of transformations produces analyses faintly tinged with rose-colored methodological hues, as the voice-leading paths traced by the mappings are often still musically non-intuitive or inaudible, especially when resulting from an inversion operation. Partly to defend his split transformations, O'Donnell writes, "Recent transformational models tend to parse pc sets into subsets to explain transformations among sets belonging to different SCs" (48). I believe that O'Donnell is on to something important here. An even greater number of simultaneous transformations on chords will be necessary if one is to have the freedom to connect a series of chords from *any and all* of the different SCs, let alone to chords of differing cardinalities. As the number of concurrent transformations acting on a pc set increases to encompass more than half the cardinality of the set, it would seem to be wise to re-evaluate what exactly these specialized transformations are acting *upon,* and whether it would be propitious to begin thinking less about transformations on sets or even subsets, and more about transformations on *individual pitch classes*.

In summary, O'Donnell fulfils his promise to expand "the transformational machinery" originally conceived by Klumpenhouwer and Straus, but even with this expanded toolbox he achieves only mixed results in terms of offering insight into the voice leading of any given musical passage. Ultimately his most important contribution to the field of atonal voice leading studies consists of some important criticisms of T_n/T_nI pc-set transformational models in general.

Lewin's important *JMT* article, "Some Ideas about Voice-Leading Between Pcsets" (1998), also touches upon transformations of the type discussed earlier, in which all but one member of a pc set participate in a straight transposition. If the nonparticipating set member's transposition differs as little as possible from the transposition of the other set members (in ETS 12 systems by a semitone) the voice leading is called *maximally uniform.*[5] Lewin assigns a "pseudo transposition number" to the transposition which is equal to T_n where n = the transposition number of the majority of the set members, and also designates an "offset number" which equals the deviation from true transposition by the total (absolute) number of nonconforming semitones (33).

In his article, Lewin also raises an important issue touching on the "retrogradability" of voice-leading functions, namely that only bijective functions have an inverse function. To allow for "inverses" (retrogradable voice leading) for non-bijective voice-leading functions between pc sets, Lewin dispenses with the label "function" altogether in favor of the more general term *relation*. A relation *R* between two sets *X* and *Y* is a collection of ordered pairs $\langle x,y \rangle$ where the *x* of each $\langle x,y \rangle$ is a member of *X* and the *y* of $\langle x,y \rangle$ is a member of *Y*. *R* is a subset of the Cartesian product $X \times Y$. Functions are simply relations *R* in which every member of *X* appears once on the left, within some member-pair of *R,* and only once (Lewin 1998, 64).[6]

Lewin's choice to "settle for" the mathematically weaker relations to describe voice leading transformations in the last section of his article, instead of the functions that he had so carefully prepared in the previous sections of his paper, is very curious.[7] Even the non-bijective voice leadings can still be described in terms of functions; they simply do not have unique inverses. Although functions can be injective, surjective, or both, they can also be neither. As an example, consider Lewin's example of voice leading between the sets *X* = {D,F,A} and *Y* = {E,G♯,B}. The *closest* voice leading between pitch classes would result from a function *V* where *V*(D) = E, *V*(F) = E, and *V*(A) = G♯. Although the voice-leading transformation *V* is neither surjective (the element B of *Y* did not occur as a result of the function *V*) nor injective (elements D and F of *X* both mapped to the same element of *Y*) it still satisfies the definition of a function as described at the end of the previous paragraph. If all functions were unidirectional this would not be a problem, but what if we wanted to "undo" this voice leading by moving set element E of *Y* "back" to both D and F of set *X*? A move like this would defy the criteria for a function, as the image of the argument E under this inverse transformation V^{-1} would not be uniquely determined. The simpler term "relation" must be used instead to describe such a transformation.

It is for this reason—to mathematically justify retrogradable voice leading—that Lewin discards voice-leading functions in the last section of his paper, even though he had very carefully constructed a number of specific types of voice-leading functions in the previous section.[8] I assume—although it is nowhere explicitly stated—that Lewin is suggesting that functions should be used to model voice leading except in cases that require retrogradable mappings, or to put it in musical terms, in cases of voice doubling or of increasing/decreasing the number of voices in a homophonic texture. In these cases, the best one can do, mathematically, is to revert to the simpler relations. Why Lewin does not simply use relations to describe *all* voice-leading moves, ditching the more narrowly restrictive functions to allow for more voice-leading flexibility, is open to conjecture. There is another alternative, wherein Lewin could have used bijective functions for *all* voice-leading moves, dispensing with relations entirely. This