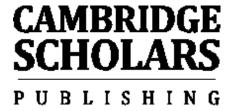# Constructive Adpositional Grammars

Constructive Adpositional Grammars:
Foundations of Constructive Linguistics

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P U B L I S H I N G

TABLE OF CONTENTS

List of Figures

LIST OF TABLES

LEGENDA

Abbreviations

adtree . . . . shortcut for 'adpositional tree'

adj . . . . generic adjectitve

adp . . . . generic adposition

dep . . . . dependent group

gc . . . . generic grammar character

gov . . . . governor group

?the Lady . . . . . example of doubtful expression (syntactically or semantically)

*good here . . . . example of ungrammatical expression (syntactically)

~~with~~ . . . . example of expression canceled by a transformation

Abs . . . . absolutive case (morphosyntactically marked)

Acc . . . . accusative case (morphosyntactically marked)

Gen . . . . genitive case (morphosyntactically marked)

Dat . . . . dative case (morphosyntactically marked)

Erg . . . . ergative case (morphosyntactically marked)

Nom . . . . nominative case (morphosyntactically marked)

Opener . . . . semantic role, case or concept in a semantic frame (example)

Symbols

← . . . . indicator of dependency

→ . . . . indicator of governement

↔ . . . . underspecified indication governement-dependency

> . . . . shortcut for transference

⇝ . . . . adpositional tree transformation

△ . . . . shortcut for a hidden adpositional tree

Letters

$\downarrow \mathcal{A}$ . . . . assertive pragmatic character
A . . . . adjunctive grammar character
$\uparrow \mathcal{C}$ . . . . commissive pragmatic character
D . . . . generic grammar character of the dependent
$\updownarrow \mathbb{D}$ . . . . Declaration (pragmatic character)
$\uparrow \mathcal{D}$ . . . . directive pragmatic character
E . . . . circumstantial grammar character
$= \mathcal{E}$ . . . . expressive pragmatic character
ϵ . . . . zero-marked adposition
F . . . . adposition grammar character imposed by the adposition
G . . . . generic grammar character of the governor
I . . . . underspecified or generic verbant grammar character
I^v_x . . . . verbant grammar character (v indicates valency, x saturation)
$\overleftarrow{I}$ . . . . unaccusative verbant (always monovalent)
$\overrightarrow{I}$ . . . . unergative verbant (always monovalent)
I^2 . . . . bivalent verbant grammar character
I^3 . . . . trivalent verbant grammar character
I^4 . . . . tetravalent verbant grammar character
I^5 . . . . pentavalent verbant grammar character
L . . . . listener (type of actant)
λ . . . . generic pragmatic character
O . . . . stative grammar character (extra-valency or generic)
O_x . . . . stative grammar character (x indicates the actant value)
O_1 . . . . stative grammar character (first valency)
O_2 . . . . stative grammar character (second valency)
O_3 . . . . stative grammar character (third valency)
O_4 . . . . stative grammar character (fourth valency)
O_x . . . . stative grammar character (extra valency)
Q . . . . extra in-valent actant in the construction (as O_x)
R . . . . receiver (a type of actant)
S . . . . speaker (a type of actant)
W . . . . fourth in-valent actant in the construction (as O_4)
X . . . . first in-valent actant in the construction (as O_1)
$\overleftarrow{X}$ . . . . unaccusative actant (always for $\overleftarrow{I}$)
$\overrightarrow{X}$ . . . . unergative actant (always for $\overrightarrow{I}$)
Y . . . . second in-valent actant in the construction (as O_2)
$\overleftarrow{Y}$ . . . . non-prominent second in-valent actant (in government)
$\overrightarrow{Y}$ . . . . prominent second in-valent actant (in dependency)
Z . . . . third in-valent actant in the construction (as O_3)

CHAPTER ONE

INTRODUCTION

This book presents a framework to understand how language is structured—in particular at a morphological and syntactic level—and how language takes its place in the general human cognition. [1]

We consider that the study of languages should be intertwined to the research in the field of cognitive sciences on one hand and with the expressive power of constructive mathematics on the other hand. In fact, constructive mathematics allows a full-blooded computational development of natural language grammar description, without invoking idealistic interpretations. In other words, every concept of our framework poses on a solid mathematical ground when interpreted in a cognitive way.

A preliminary clarification is needed. As the main area of investigation within this book is natural language grammar, we will use the term 'language' to refer to natural languages—such as English, Chinese, or Urdu—in general terms if the context is clear enough to avoid ambiguity. Otherwise, we will specify if we are talking about natural or formal (artificial) languages. Analogously, this kind of convention will be followed for other common terms which are used both by linguists and mathematicians in very different ways, such as 'grammar', 'syntax' or 'semantics'. [2]

This clarification is needed because this book is addressed both to people belonging to the humanities—such as most linguists are, but also pedagogists involved in language learning issues—and people belonging to hard sciences, above all, mathematicians, but also logicians, computer scientists and natural language engineers. We believe that the so-called 'two cultures'—even three, as recently posed by Kagan (2009)—should at least dialogue one with the other if not put in confluence for mutual improvement.

This is the main reason why we decided to put together concepts and results of our investigation, as they are at the same time linguistic and formal in nature. In other words, there is no dedicated chapter to the formal modelling [3] or to linguistic description, simply because the insights we have on one side have an immediate and direct effect to the other side, and writing in such a separate way would lead to unnecessary intricacies in order to let the reader understand our line of reasoning.

Of course, we do not pretend to reinvent the wheel; our work is well rooted both in the linguistic and mathematical traditions of their own. The sequel of this chapter is devoted to clarify what are our start points and how they are related (or not) with other works in the fields.

1.1   The adpositional paradigm

It is worth noticing that the term 'constructive' is used in different and complementary ways within the book. In this section, we explain 'constructive' referring to constructive mathematics. The fundamental concept of this book is the *adposition*, which is linguistically a generalisation of conjunctions, prepositions, postpositions, and so on—broadly speaking, linking words. The adpositional paradigm was the main result of the PhD dissertation by Gobbo (2009).

Since then, we put our work a step forward, and this book is the presentation of the results of that effort. In fact, now adpositions—the cornerstones of the adpositional paradigm—are described in the spirit of constructive mathematics, and that's why the framework is called 'constructive adpositional grammars' (Constructive AdGrams for short).

Constructive mathematics (Bridges and Richman, 1987) is, synthetically, a way to develop the mathematical thought that strictly preserves the information content of any statement. Precisely, disjunctive and existential statements are required to indicate witnesses for their truth: for example, $\exists x.P(x)$ can be proved in some theory T if and only if we can exhibit some value v such that $P(v)$ holds, i.e., it logically descends from T. So, some forms of logical thinking are not accepted, e.g., the Law of Excluded Middle, since they introduce unjustified information: in fact, if $P \vee \neg P$ is an axiom, it does not indicate whether the first or the second disjunct holds and there is no guarantee that such an information can be recovered from the theory T, and, in general, it cannot (Troelstra and Schwichtenberg, 2000; Troelstra, 1977).

The term 'grammars' refers to the fact that the mathematical structure is one, while the instantiations are many as natural languages are. We look at different shadows (i.e., natural languages), each one with its different shape, while the sun is always the same (the structure)—see Chapters 2 and 3 for details.

Before proceeding, it is important to explain how we deal with the 'shadows', i.e., the diversity of natural languages—in other words, the problem of linguistic universals and typology. From a phylogenic point of view, while every species seems to understand their members, only humans developed different

languages—actually, more than 6,000. Furthermore, from an ontogenetic point of view, every child builds his own linguistic repertoire according to the environment he is grown up into, not his ethnic origin, and there are some regularities in the developing of language skills, from baby utterances to adult-like representations. According to Tomasello (2003), the most rigorous and plausible theory of language that characterises adult linguistic competence in child-friendly terms is rooted both in cognitive and social skills. From the cognitive side, the theory of mind has shown us that our ability to communicate is based on (a) intention-reading, i.e., the ability of inferring what the listener is expecting from us, beyond the literal meaning of what the speaker said; (b) pattern-finding, i.e., the ability to categorise our sensibilia in a mapping into the mind. From the social side, we learn the intentional actions of other humans by imitation.

These universals of learning (intention-reading, pattern-finding and imitation) applied to our innate linguistic ability are the fundamental elements of linguistic *constructions*—synthetically, patterns of usage of form-meaning correspondences that carry the messages beyond the gist of the words that took part in the construction itself. This fact leads us to another use of the term 'constructive' within this book.

Linguistic communication is symbolic, and symbols are "social conventions by means of which one individual attempts to share attention or mental state to something in the outside world" (Tomasello, 2003, 8). Symbols are used in patterned ways, and these patterns give form to linguistic *constructions*. Hence, constructions arise by two different forces: by the meaning of their parts, and by the frequency of use of the pattern itself. This species-unique process occurs over time and is called *grammaticalization*. [4]

The results of this cultural and historical process are natural language grammars, and that is why we have more than one. Therefore, grammars should be investigated as a result of the process of grammaticalization, not as static, monolithic entities. Most puzzling linguistic phenomena can be enlightened in terms of the grammaticalization process, as we will explain throughout this book. In fact, the frequency of pattern-usage leads to loose (hide) unnecessary information: that's why highly frequent words are usually short in terms of syllables.

Each grammar is made of constructions, providing us the linguistic data to be analysed. Constructions being central, our work can be put into relation with the early stages of transformational grammar (Chomsky, 1957, 1965) and to the works of cognitive linguists proposing constructional analyses, such as Tomasello (2003), Goldberg (1995, 2006), and Croft (2001).

Most of our linguistic observations derive from the works by cognitive lin-

guists. The main limit of these analyses lies in the fact that they are *constructional* instead of constructive. That is, they take constructions as primitives, rejecting *in toto* any formalism, which is seen as inevitably 'Chomskyan' in nature. Matthews (1993) already observed:

> […] for more of the past fifteen years, despite occasional disparagement from one side or another, each school has in practice had little reason to refer to the other. It is worth noting, for example, that Croft's recent introduction to *Typology and Universals* (1990) cites no work by Chomsky. (Matthews, 1993, 45)

The situation is even getting worse, as exemplified by the most radical constructional approach, which states that there is no linguistic category that is both formal and universal, as asserted by (Croft, 2001, 4):

> Of course, abandoning universal categories and relations leads to a very different view of Universal Grammar. Under the alternative view, Universal Grammar does not consist of an inventory of universal categories and relations available to all speakers. […] The formal structures in grammars are language-particular, and universals of language must be sought elsewhere. […] In principle, that appears to be the direction that Chomskyan generative grammar has headed: general constraints on syntactic structure but a proliferation of syntactic categories. In practice, however, the syntactic categories are assumed to be cross-linguistically valid, and the same categories (or a subset thereof) are posited of every language. This practice also holds for other formal syntactic theories.

We claim that a formalism is possible without neglecting the usage-based results of cognitive linguists. The problem is in the general perspective. Most cognitive linguists are 'maximalists', i.e., they consider semantics and syntax indissociable, and therefore semantics is put at the centre of analysis, while syntax is put at the periphery—traditionally, most results in cognitive linguistics are devoted to the semantic level, in particular the analysis of metaphors (Lakoff, 1997, for example). Conversely, in whatever (linguistic) formalism, the algebraic rules are insensitive to the meanings of the element they algorithmically combine, and hence linguistic meaning is put at the periphery of the system, the (morphosyntactic) rules being the core.

Nonetheless, we take into account the critique of later Chomskyan development (Chomsky, 1981, 1992) that constructions cannot be disregarded as epiphenomena, while the earlier Chomskyan models showed that the formalisation of

constructions is not only possible but also feasible (Goldberg, 1995, 1). After all, Chomsky is a leading figure in the Cognitive Revolution, along with George Miller, Marvin Minsky, Allen Newell and Herbert Simon [5]: we claim that at least part of Chomsky's work can be put in the stream of cognitive linguistics, if we do not reject formalisation as a whole. At the same time, we believe that another way—not based on constituents—to formalise grammars is possible and worth attention, particularly for cognitive linguistics. [6]

In sum, it's no more the time of the "linguistic wars" (Harris, 1995): we claim that both approaches are valid, as they talk about different aspects of the same phenomenon, i.e., natural language grammars: the Chomskyan approach is top-down, deductive, because it looks for regularities beyond the variety of languages, while cognitive linguists try to explain variety on a usage-based perspective, following a data-driven, inductive approach. Our aim is to take the best practices from both approaches, without adhering to any 'Church'. Here we offer a strong, general, explicative formalism with a lot of examples taken from different languages of the world, with a special regard to English.

Our formalism takes into account the results of linguistic typology. Are there any language universals? The cross-analyses of grammars made by typologists showed us that grammar categories, arisen in Greek and Roman context for educational purposes—i.e., teaching and learning of Greek and Latin as written languages—cannot be forced as such into native languages of most part of Africa, Southeast Asia, the Americas and Australia. Should we look through the Procrustean lens of Standard Average European to those languages? Certainly not. Tomasello (2003) takes a radical conclusion, claiming that no universals of form exist, in particular no linguistic symbols, no syntactic constructions and no grammatical categories. If linguistic symbols highly depend on the socio-cultural context within they emerge (but what about the word *mama*?) and typological studies have shown that the variety of syntactic constructions is really impressive, nevertheless this variety *should* have a common, cognitive basis—and Greenberg's results in syntactic typology cannot be disregarded so easily. Tomasello claims that the only language universals that exist are the ones of communication and cognition, being in particular the presence of expressions of reference and the presence of predication. Moreover, they have no counterpart in terms of linguistic universals of form. We claim exactly the opposite: the mechanisms underlying the constructions are the linguistic counterparts of the cognitive ability of pattern-finding and intention-reading, which let the speaker and the listener—in the simpler, default case—to build up the linguistic representation within their joint attentional frame. The best account of the attention focus as a cognitive linguistic ability we have found in literature is the trajectory/landmark asymmetry

by Langacker (1987, 1990, 1991), of which we will give a formal interpretation in terms of information prominence—see Chapter 2.

Attention should be directed somewhere in order to function properly: we claim that four grammar characters are universal, being the expressions of reference, predication, and their respective modifiers—respectively, adjuncts and circumstantials. Whorf (1945) and Tesnière (1959) did come to the same conclusion, even if starting from different bases. In particular, the work by Tesnière (1959) was the start of the so-called Dependency Grammars, which was rarely put into relation with cognitive linguistics. [7] We deeply analysed that classic work through the lens of modern formalisms and we have found that most, if not all, derived works have taken the concepts of dependency and verbal valency from Tèsniere, but, at the same time, completely disregarding his use of grammar characters. [8]

Moreover, we retain the concept of 'prepositional system', borrowed from Pennacchietti's analysis of Brøndal (1940): each natural language grammar has a relatively small set of prepositions (or other kind of adpositions) whose function is determined by the result of the opposition with the other prepositions in the grammar itself. In fact, even genetically close languages—such as French and Italian—show a considerable difference in the use of their prepositions because they belong to different prepositional systems (Pennacchietti, 2009, 2006).

The central role given to adpositions—broadly, prepositions, postpositions or in-positions, depending on the specific language—lead to another fundamental grammar character: adpositions. This grammar character collects the 'structural morphemes' of a language. Because of its somewhat technical nature, it has been neglected for a long time. The name 'adpositional paradigm' followed naturally.

The adpositional paradigm was first developed in Gobbo (2009), which extends Pennacchietti's work in an original way. Adpositions become a more abstract, general element in order to understand grammar structures. In particular, zero-marked adpositions—signed through an epsilon (ϵ)—put word order phenomena in the same realm of morphology. Thus morphology and syntax are clarified through a unified model and a unique mechanism—with some special features of their own. Semantics and pragmatics also have their place in the model, while phonetics and phonology have not. [9]

A remark made by an anonymous reviewer on the linguistic foundations of the adpositional paradigm was that the model was tested only into nominative-accusative grammar cases. We have taken this remark very seriously, and the present book fills this gap, providing a new constructive adpositional grammar model which takes into account both nominative-accusative and ergative-

absolutive grammars. Our reference for ergativity under a theoretical point of view is the work by Dixon (1994), who notes in the appendix that, surprisingly, there are few theoretical models which gives an explanation of ergativity, because the theory "would have to recognise that there are three basic syntactic-semantic primitives (A, S and O) rather than just two ('subject' and 'object') however these are defined", see (Dixon, 1994, 236).

Constructive adpositional grammars solve this problem in terms of information prominence, as explained in the next chapters. Furthermore, we adhere to the unaccusative hypothesis, originally proposed by Perlmutter (1978), which gives a clear and convincing linguistic account of the problem, even if the Relational Grammar (pseudo)formalism is not convincing. [10]

Finally, no agreement is possible on a finite, fixed list of the universal types of semantic roles—called in a Chomskyan perspective 'theta-roles'—simply because they depend on the constructions they belong to. In other words, semantic roles are construction-dependent and hence language-defined, and so their list is open and undefined, at least for the purposes of the present book. [11]

For example, if a construction deals with food, such as *X eat Y* it has much more sense to call *X* Eater instead of Agent, following the tenets of cognitive linguistics, as the semantic roles belonging to the semantic frame of food activated by a constructions such as *X eat Y* show distinctive characteristics in the distributional analysis of the corpus in different languages—e.g., English, German, and Bengali (Croft, 2009)— which cannot be explained with a suitable degree of precision by the more used dichotomy Agent *vs.* Patient.

For example, the English construction *Y-eating X*, exemplified by *oil-eating microbes*, states that *Y* is the Food and *X* is the Non-human eater, prototypically having the sememe animal, so *oil-eating robot* is also acceptable, while *fish-eating child* is not, since *child* is not a Non-human eater. [12]

What it is important to underline here is that the *productivity* of constructions—i.e, how much *X*s and *Y*s can vary within the scope of that particular construction—is bound to the construction itself.

The aim of semantic roles is exactly to represent this degree of freedom, which can be zero at the limit, as in the case of completely grammaticalized idiomatic expressions—such as *kick the bucket*, where each lexical item cannot be moved paradigmatically—or very high, such as *Let's X!*, where the only constraint applied to *X* is its grammar character, in particular the need to belong to the class of English verbs.

1.2   The formal model

In the fields of mathematical and computational linguistics there are many natural language grammar formalisms currently under investigation. In particular, our formalism can be put into the realm of the so-called 'categorial grammars'—i.e., representations of natural language grammars in terms of categories. This line of research is far from being new, being rooted in the works by Ajdukiewicz (1935), Church (1940), Bar-Hillel (1953), and Lambek (1958). [13]

Our formal model is intended as a guiding reference for the development of linguistic concepts. In this sense, it should be understood as a 'weak' model which provides enough insight to understand the complexities of natural languages but not yet enough power to treat them in a computational fashion. The full details of the formal model are presented in mathematical terms in Appendix B: the reader is referred to that part for the formal details.

When the formal model is applied to any natural language, a parser, i.e., a computer program which allows to recognise the expressions belonging to the grammar, can be naturally derived, due to the constructive nature of the formal representation. [14]

The starting point is to consider any linguistic expression as a formal object, represented as an adpositional tree (adtree, for short) on morphemes. The precise structure of such trees is explained in the following chapters; for the moment being, we just need to know that each of them represent a unique piece of text in some natural language. Moreover, a tree represents a 'unique interpretation' of the corresponding text, i.e., a unique way to understand the grammar characters of each element in the text (the concept of 'grammar character' will be discussed and explained in detail in Chapter 3). For example, let's take the Italian sentence:

(1-1.)   La vecchia porta la sbarra.

Example (1-1) can be understood in two completely different ways: *the old lady brings the bar* or *the old door bars her*. In the first interpretation, *la vecchia* (the old lady) counts as a noun and plays the role of the first actant, *porta* (brings) counts as the bivalent verb, *la sbarra* (the bar) again counts as a noun, playing the role of the second actant. In the second interpretation, *la vecchia porta* (the old door) is grouped as the first actant, while *sbarra* (bars) acts as the bivalent verb. Finally, the pronoun *la* (her) is a still unresolved anaphoric place-marker of a noun, playing the role of the second actant.

The two interpretations correspond to two similar but distinct adtrees [15], as shown in Figure 1.1. Both interpretations are fully acceptable as the ambiguity

Figure 1.1: The two interpretations of *la vecchia porta la sbarra*

stands in the parser of the Italian language. In fact, two chains of constructions equally valid and semantically plausible can generate the respective adpositional trees. Also in English there are some known examples of sentence where there is more than one valid syntactic relationship, for example:

(1-2.) Time flies like an arrow and fruit flies like a banana.

where 'flies' counts as a verb in the first occurrence and as part of the noun in the second one.

By contrast, there are cases where there is only one adpositional tree but there is more than one semantic interpretation.

(1-3.) È la rossa di Maranello.

Example (1-3) can be interpreted as *She's the red-haired girl coming from Maranello*, an Italian town in the Emilia-Romagna region, as well as *it's the Ferrari car*, meaning the famous sport car produced in Maranello.

In this case, we have two different meanings for the same text, under the same syntactic interpretation: the adtree will be the same (Figure 1.2).

Adtrees are built from some basic bricks via a few standard constructions. Correctly identifying the bricks and the basic constructions is the main objective of this book. But, let's suppose to have them: then, we can generate the whole language and, vice versa, we can check if a piece of text is syntactically correct by asking if there is a corresponding adtree. Moreover, we would say that a piece of text is syntactically ambiguous if it admits more than one correct adtree. [16]

Figure 1.2: The adpositional tree of *È la rossa di Maranello*

In mathematical terms, adtrees and constructions between them form a structure **AdTree** which is a *category* [17], see Mac Lane (1998) and Borceux (1994). A mathematical category is an algebraic structure composed by two classes, the *objects* and the *arrows*; arrows lie between two objects, the *source* or *domain*, and the *target* or *codomain*. Also, a category states that there are distinct arrows, the *identities*, one for every object A and such that the source and the target are A. Moreover, a category is equipped with a partial operation allowing to compose two arrows whenever one has the domain which is the target of the other one. Composition is required to be associative and identities act as one expects with respect to composition.

Intuitively, there is an arrow f from A to B whenever we can construct the B tree starting from the A tree applying the construction f. We do allow complex constructions obtained by sequentially composing simpler ones; if f and g are constructions such that $f(A) = B$ and $g(B) = C$, that is, if f maps A into B, and

g constructs *C* from *B*, then *g* ∘ *f* is the construction which maps *A* into *C* by doing *g* after *f*. [18]

We observe that, calling *M* the free monoid over the alphabet of morphemes of some natural language, i.e., the set of all possible (finite) sequences of morphemes obtained by juxtaposition, the functions mapping the trees in **Adtree** into the sequences of *M* comprehend the textual renderings of adpositional trees. If we restrict our attention to *contravariant functors*, i.e., the functions preserving the identical transformation and the reverse composition of adpositional trees, we get a class of functions which is called *presheaves over M*. Requiring that a presheaf maps morphemes in the adtree into themselves in the monoid, we get exactly the lexicalizations of adtrees. In other words, there is a subclass of presheaves which directly corresponds to the texts the adtrees represent and which encodes the transformations that constitute the grammar. It is this space of presheaves which is generally understood as the subject of linguistics.

It is possible and fruitful to interpret the basic constructions of Category Theory, e.g., natural transformations, limits and colimits, etc., in the framework we have just introduced. But it requires a better understanding of the basic blocks of the **AdTree** category. So, we stop here in our analysis, at least until the following chapters will introduce the required elements.

As a side effect of this intended model of interpretation, it follows that whatever construction over adtrees which is built by combinatorially composing the fundamental constructions, is an arrow. Lifting the structure of the **AdTree** category into the spaces of presheaves, which is a category, we can reason in a larger and richer environment, where the full power of mathematical methods can be applied: in fact, the presheaves space is a *Grothendieck topos* (Mac Lane and Moerdijk, 1992; Johnstone, 2002), one of the richest mathematical structures we can deal with. [19]

As we have suggested, categorial mathematics give a set of elegant tools to build natural language grammars because it provides a transparent account of the correspondence between the syntactic and the semantic combinatorics—in other words, how meaning is reflected into collocation and word order phenomena, hypothesis put at first by Montague (1973) in Hintikka et al. (1974).

From a mathematical point of view, formalisms based on categories, [20] usually employ some variant of the Lambek calculus, because it corresponds to a non-commutative, substructural variant of linear logic. So, Lambek calculus, as well as derived formalisms, is based on a logical system where we have limited *resources*, allowing to model, e.g., that a phrase may contain at most one verb, and connectives are not commutative, modelling the ordering of elements in a

phrase. An important result about pure categorial grammars is (Pentus, 1993) that shows how their generative power is that of context-free grammars and thus inadequate for theories of natural language syntax; this result applies only to categorial grammars which have a natural computational counterpart defined as an extension of the λ-calculus, i.e., where composition rules are functional applications. Unfortunately, leaving the simple and safe world of λ-abstraction and application in favour of more sophisticated rules allows for an explanation of more complex linguistic phenomena, but at the price of having models whose behaviour is far from being well-understood. In fact, they have been nearly abandoned for the automatic processing of natural languages.

A modern formalism currently under development is the Combinatory Categorial Grammar (CCG) by Steedman and Baldridge (2007), a lexicalized grammar formalism which can be parsed in non-deterministic polynomial time. It is especially used in the field of statistical parsing. Linguistic categories are assigned to words by the lexicon, e.g., an intransitive verb has the category $S \backslash NP$ while a transitive verb has the category $(S \backslash NP)/NP$. Under a theoretical linguistic point of view, a CCG is constituent-driven, i.e., it retains the advantages and limits of the Chomskyan syntactic perspective we put into question here, so we can't use CCGs as such. Under a mathematical point of view, a grammar is a set of inference rules controlled by the linguistic categories, interpreted as functional spaces, see van Benthem (1995); such rules can be inferred by suitable techniques from machine learning (Manning and Schütze, 1999), essentially using statistical measures. Also, the probabilistic approach is used to cope with the fact that non-deterministic polynomial problems cannot be efficiently solved, as far as modern mathematics knows. [21]

Ranta (2004) proposes a formal framework for writing grammars and linguistic theories—following the motto *sentences-as-proofs*. From a mathematical point of view, it is based on Martin Löf Type Theory, see (Martin Löf, 1984), using the Coquand's algorithm for type construction/partial proof derivation (Coquand, 1996). So, it is ultimately based on a dependent-type, predicative variant of the λ-calculus.

There are two advantages in this line of research: first, the Grammatical Framework is a piece of software which can be used freely and it has a growing up community (Ranta, 2009); second, it might be used for implementation regardless of the linguistic theory behind. However, software implementation is out of the scope of the present book, which is dedicated to linguistic analysis.

Dynamic Syntax is based on the program of Labelled Deductive Systems, which aims to bring semantics back into syntax (Kempson et al., 2001). La-

belled formulae and deductive systems are connected with algebraic frames and type logical grammars. In fact, Labelled Deductive Systems have been first introduced by Gabbay (1996) as an algorithmic way to control proof development using labels from some simple algebra whose operations are used to model the potential application of inference rules.

Unlike Chomskyan grammars, in Dynamic Syntax the trees represent the structure of the sentences first as the result of a growth obtained by a parsing strings uttered in contexts—this is the sentence's 'dynamics'. As it will be clear in the sequel of this book, we follow the hypothesis posed by Dynamic Syntax that (morpho)syntax is the mechanism for constructing representations of content. However, there are important differences, namely the use of linguistic constructions—based on cognitive linguistics results—is formally resolved in a totally different way. Moreover, while Dynamic Syntax makes use of Hilbert's epsilon calculus in order to solve quantified noun phrases, our formal bases are completely different.

1.3   What is in this book?

The aim of the present work is foundational. In particular, we want to give the right hints to understand natural language grammars within the adpositional paradigm. For us, 'understanding' means both having insights about the functioning of real-world usage of a given natural language—in terms that are comparable with other languages as well—and having insights about what is structural in a given natural language, in constructive mathematical terms. In practice, there are no distinct "linguistic *vs*. mathematical" insights; rather, the joint linguistic and mathematical apparatus should be regarded as the two sides of the same coin.

Under a different perspective, this book is foundational as here the reader can acquire all the instruments necessary for the founding of Constructive Linguistics, i.e., an application of constructive mathematics to the realm of linguistics—broadly, the study of natural languages. When foundations are posed, many building details are still to be defined. In this book, we indicate which details, instantiations, specific phenomena are not covered, in order to help the reader see possible further directions of this work, standing on the solidity of the foundations posed here.

A crucial part of the foundations are the instruments from Category Theory, in particular, the intuitions about sheaves and Grothendieck topologies—the im-

patient reader is invited to read Appendix B at first.

The choice of the mathematical instruments apt to represent the results obtained within the adpositional paradigm was lead by two simple criteria: expressiveness and naturalness. From the point of view of expressiveness, it is easier to obtain more general result with an elegant formalism than narrower results with a less powerful formalism, and topos-theory is the strongest, general, elegant and broad formalism belonging to constructive mathematics we have nowadays.

On the other hand, the representation of linguistic phenomena in constructive mathematical terms should be natural. In other words, the mathematical constructs we use should have a direct and immediate counterpart in linguistic terms: in that way, readers interested only in the linguistic side can still follow the intuition of the model, without being forced to use abstract tools whose linguistic nature is not clear. Nevertheless, we invite even readers not used to mathematical formalisms to give a chance to our formal model, in reading the whole book in the order of presentation, even Appendix B. We are confident that—after the reading and understanding of the main text—the Appendix will be readable with profit and interest to all readers.