Greek Science in the Long Run

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Essays on the Greek Scientific Tradition
(4th c. BCE-17th c. CE)

Edited by

Paula Olmos

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However, I would like to make special mention of three of these people, to whom, for different reasons, I should be particularly thankful. First of all, most certainly, Prof. Geoffrey Lloyd, who helped me more than anyone else with the organization of the conference. His academic prestige as well as his personal connections and generous mentorship of many of the participants made the event the best and most fruitful gathering possible, in both scholarly and personal terms. He was present at every session, took part in every discussion and was prepared and especially supportive when this publication started to take shape. In fact, he was the first to submit his contribution. Prof. Reviel Netz also played an exceptional role in the success of this volume, as he mentored and made possible my research term at Stanford University during the summer of 2011, when I was able to undertake editing of the text in a unique scholarly and collegial atmosphere that no doubt contributed to its improvement and completion. Finally, I must express my deepest gratitude and indebtedness to Prof. Antoni Malet, who made the decision to continue with his contribution to this volume despite very sad personal circumstances, doing us the honour of considering our project a means of solace, which I hope it was.

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Madrid, November 2011

INTRODUCTION: THE GREEKNESS OF SCIENCE

PAULA OLMOS

Throughout both classical and late antiquity, the European Middle Ages and most of our early modern period, systematic *technical* knowledge related to fields as varied as medicine, biology, mathematics, astronomy or even rhetoric and grammar repeatedly made reference to Greek sources, theories and discussions. In classical and post-classical Latin texts, medieval compilations or the Renaissance humanist return *ad fontes*–in part made possible through the teachings of Greek-speaking Byzantine émigré scholars–the content and legacy of Greek scientific production was esteemed and treasured. It was of course also reinterpreted, but not (not yet) discarded or much contradicted. If the dominance of the Aristotelian corpus as the basis of a broadly-construed scientific curriculum–albeit philosophically-driven–was a remarkable late medieval phenomenon with a rather prolific afterlife, more sustained indeed was the medical authority of a figure like Hippocrates, whose attributed corpus came to be mostly interpreted through the work of Galen, another Greek authority. Euclid and Archimedes were synonymous with advanced mathematics and although Cicero's textbooks were the main medieval source of information on technical rhetoric in the West, any attentive reader would note that what they really transmitted was the Latin translation of a systematic model based mainly on Hermagoras of Temnos and other Greek sources.

As is well known, the Greekness of technical literature and vocabulary was already an issue for early Latin writers such as Cicero, who tried to coin Latin terms equivalent to those used in Greek philosophy and rhetoric. However, for example, his *ratiocinatio* for *sullogismos* had very little impact, and he did not even try to come up with an alternative to *enthumema*. In the case of fields in which concept and object identification and naming were particularly pervasive, such as mathematics or rhetorical elocution–which included the study of an extensive and ever-growing list of "figures of speech" or *schemata lexeos*–Greek vocabulary remained the norm … until today.

The present volume is not intended to give a full account of the historical problem of the *longue durée* of the dominance of Greek science in itself. This is a complex topic that must be framed within a broader comprehension of the formation, development and survival of ancient literary culture as a whole, an issue to which some of the contributors to this volume are beginning to pay significant attention.[1] Our more modest aim is to offer a series of individual studies focused on different aspects, periods and fields related to the ongoing recycling of the Greek scientific legacy undertaken from the perspective of full awareness of this phenomenon.

When I began to make preparations for the international colloquium that was the starting point for this book and contacted different scholars about it, most of them seemed fascinated by the implications of the very title chosen, “Twenty centuries of Greek science”. Nevertheless, I now clearly see that it was a somewhat restrictive one, if we take into account both the legacy of the Greek archaic period and the present-day usefulness of many Greek technical concepts (especially in fields like mathematics and rhetoric). In any case, the limits I set at the time and which are still present in the subtitle of this volume–4th century BCE to 17th century CE–remain useful in focusing our attention on the long period during which the Greekness of most of science was to a certain extent widely taken for granted.

The essays in this collection thus contain the response of a significant group of scholars to this call for attention to the way Greek scientific traditions enjoyed such an incredibly long reputation, while also displaying a kind of versatility that challenges any simplistic, dogmatic or *a priori* viewpoint about the meaning and social functioning of systematic knowledge. They revisit the different processes by which such doctrinal traditions originated, were transmitted and received within diverse socio-cultural contexts and frameworks. The concepts of continuity and discontinuity, *deuteronomic* or meta-textual recycling and contextualized originality therefore inform the various approaches presented here, while some of the contributions (see especially Lloyd, Netz and King) address them in a more direct way. Although the boundaries between scientific

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[1] Reviel Netz is currently working on what he calls the *parameters* of ancient literary culture that, among other things, account for their preservation. I myself am working on the way the classical cultural legacy, and Greek science in particular, was filtered and *packed* for its future reception in the context of the crumbling world of late antiquity, a little studied period in the history of the sciences which is particularly crucial to the issues addressed in this volume.

fields and their denominations are rather problematic,[2] I have tried to structure the book in such a way that groups the different contributions into more or less coherent sections.

The first section includes six papers which address relatively general, method-focused and trans-disciplinary issues. They can be read in pairs as dealing first with method-centred caveats regarding our initial approach to Greek scientific traditions (Lloyd, Andò); then with the characteristically Greek–and specifically Aristotelian–concern with reasoning and argumentation in and of themselves (Vega, di Piazza) and finally, with the historically subsequent, *deuteronomic* compilation and reception (in different periods) of a comprehensive encyclopaedia of knowledge based on Greek sources (Olmos, Raschieri). The essays in the second section of the volume are devoted to arts and sciences based on numbers or quantification: mathematical or exact sciences in very general terms, with two original contributions to the study of Greek musical theory and its concepts, focused on very different authors of different periods and with very different aims (Tomasello, Tolsa), and two reflections on the historical processes of continuity, discontinuity, tradition and reception of Greek mathematics (Netz, Malet). The third section includes essays dealing with the arts and sciences relating to life and health. Here it is possible to distinguish two distinct blocks, the first exploring the broad social and intellectual spectrum of medical concerns (theoretical, philosophical) and practices (healing disciplines and professions) within ancient Greek culture (Cano, Macías), and the second looking at different aspects of the historical shaping (particularly undertaken by Galen) of a comprehensive, and to a certain extent forcefully coherent, idea of Greek medicine worthy of being passed along (Vegetti, King, Vélez).

Geoffrey Lloyd's paper, "Categorical Anachronisms and their Consequences for the History of Science", warns us about the perils of an anachronistic approach to the evolving fields and genres of Greek science that could create false expectations as to what we may find, for example, in Greek *mathēmatikē* or *harmonikē*. Following on from the fruitful comparative approach of his recent publications, he turns once more to his extensive knowledge of Chinese culture in order to explore significant cross-cultural differences in the classification of scientific fields. They thus appear as characteristic cultural products in themselves, rather than being dictated by the nature of their object.

[2] See Lloyd in this volume, although Netz has a somewhat different take on this issue, at least regarding the compactness of the field of the "exact sciences".

For its part, the exegetic review offered by Valeria Andò in her contribution, “Ancient Greece and Gender Studies”, reminds us of the necessity to take gender issues into account–as the authors referenced by her have successfully done–in our exploration of the Greek literary and cultural realms. It will serve as a guide for anyone interested in this aspect of classical studies, which is necessarily intertwined with certain scientific practices and concerns, especially within the field of medicine, as the paper by Helen King in this same volume so keenly reveals.

The paper by Luis Vega, “The Field of Argumentation: From Aristotle to the Present Day”, consciously exceeds the chronological limits initially set for this volume in presenting the particular case of Aristotle’s broad and comprehensive approach to argumentation and its contemporary relevance. Aristotle’s argumentative theories–contained in works like the *Topics*, the *Analytics* and the *Rhetoric*–although preserved and studied piecemeal for many centuries, seem to have required today’s involvement with such issues to reveal all their potentialities. However, Vega shows how Aristotle’s *approaches* essentially differ from the contemporary perspectives that have inherited the denominations of his triple legacy: logic, dialectic, rhetoric.

In “Stochastic Knowledge: *For the Most Part* and Conjecture in Aristotle”, Salvatore di Piazza tries to reconstruct the meaning and philosophical import of the Aristotelian expression “ὡς ἐπὶ τὸ πολὺ” (“for the most part”), used with characteristic determinations (predications) applied to objects or states of the world, and opposed to both what is *necessary* and what is merely *by chance*. Di Piazza demonstrates the pervasive nature of this expression within the Aristotelian corpus–which is so abundant in self-referent methodological considerations–and seeks to establish its relationship with a kind of *conjectural* cognition that would be proper for wise (φρόνιμος) and reasonable people, capable of dealing with “truths *for the most part*” in a flexible, adaptable way.

My own contribution to this volume, entitled “*Euge, Graeculi nostri!* Greek Scholars among Latin Connoisseurs in Macrobius’ *Saturnalia*”, focuses on this significant text, a literary piece with encyclopaedic ambitions dating from the first third of the 5th century–and therefore belonging to the still poorly understood literary and scientific culture of late antiquity. The *Saturnalia* in fact reveals to us one of the main contentions of this volume, the Greek character attributed to systematic knowledge even within the context of a truly advanced and somewhat multicultural state within Latin culture. The text, in a Ciceronian mood, portrays a three-day gathering with highly social and partially *sympotic* or convivial erudite dialogues between members of the Roman elite and some

Greek scholars. It presents to us–mainly through its clever choice of characters–a series of confrontations and oppositions in which the tensions, as well as a certain sought-after harmony, between different approaches to advanced knowledge are brought to the fore. Among these approaches some are considered more Greek, which usually means more technical and systematic, than others.

To conclude the first, method-centred and metadisciplinary, section of the book, we have Amedeo Raschieri's contribution, "Giorgio Valla, Editor and Translator of Ancient Scientific Texts", focusing again on an author with encyclopaedic ambitions based on the Greek legacy, although this time situated at the latter extreme of our chronological range. Raschieri offers a very precise description of the significance of Giorgio Valla's publishing agenda, undertaken during the late 15th century in Northern Italy. This included a summary text aiming at compiling, in Raschieri's words, "the entire scientific and philosophical knowledge of the time" (*De expetendis et fugiendis rebus*), but also a vast programme of Latin translations of Greek scientific texts by, among others, Aristotle, Galen, Alexander of Aphrodisias, Aristarchus of Samos, John Philoponus, Nicephorus and Hypsicles. The works addressed such fields as rhetoric, mathematical and astronomical science, medicine and Aristotelian philosophy. Their presentations and extremely revealing preliminary material are the main focus of the paper.

The second section of the book, devoted to Greek traditions in the exact sciences, begins with Reviel Netz's useful and extensive panorama. Under the title "The More it Changes … Reflections on the World Historical Role of Greek Mathematics", this contribution covers the variations–continuities and discontinuities–endured by what Netz sees as the relatively compact and identifiable field of the Greek exact sciences, from pre-Socratic times up to the "late ancient synthesis" (4th to 6th centuries CE). This paper–along with Helen King's on Greek medicine–is a very serious response to the challenge posed by our initial call for reflections upon "twenty centuries of Greek science", providing a thought-provoking and insightful overview. After reviewing the different periods of creativity in Greek mathematics and exploring its genres and ways of expression, Netz concludes that, whereas Greek medicine and philosophy base their continuity on a canon of authors and schools, it is "formal style" that defines Greek mathematics, which can, once and again, avoid the mention of masters and individuals.

According to Netz's periodization, Marianna Tomasello's contribution can be seen as a case study on the *philosophical mathematics* of the 4th century BCE and the way authors like Plato or Aristotle engage in methodological reflections that make use of mathematical terminology and concepts–the terminology of music or harmonics in this case–albeit with significant cosmological repercussions. Thus, “Musical Terminology in Plato's Dialogues: The Image of Concord in the *Republic* and in the *Timaeus*” reviews Plato’s understanding of the concept of σύμφωνος (what is concordant) and its crucial role in the theoretical comprehension of a kind of cosmology that, as is to be expected from this author, has ethical and political consequences. The mathematical basis of a cosmos which is initially perceived through its sensible phenomena–an allegedly Pythagorean idea–and its similarity to the also mathematical structure of the human soul, is conceived by Plato as suitable justification for his “teleological discourse” involving the harmony between the subject and the object of her knowledge.

Cristian Tolsa engages with a rather different aspect of Greek mathematical practice, so brilliantly explored in other publications by Reviel Netz: the use and meaning of diagrams. Medieval manuscripts have transmitted to us both the words and the graphical schemas of the original Greek mathematical treatises, but until recently, most attention has been focused on the words, while the diagrams have been deemed mere dispensable *representations* of the theorems and propositions allegedly developed and fully proved within the text. After the work of Netz and others, this is no longer acceptable and a whole world of new research on Greek mathematical diagrams has been opened up. In “On Ptolemy’s *Harmonics* 2.4: Does the Text Refer to the Diagram?”, Cristian Tolsa analyses a particular diagram belonging, according to part of the manuscript tradition–and thus assumed by modern editions–to the aforementioned treatise, but whose relationship to the text is rather problematic. His conclusion is that this particular diagram does not belong within the original treatise, but must rather be part of the commentary on it by Porphyry, a conclusion that in itself reveals the rather fascinating possibility of a visual commentary. The paper brings to the fore interesting aspects of the inner story of the texts within the long-standing tradition that is the main focus of this volume.

Antoni Malet’s “Euclid’s Swan Song: Euclid’s *Elements* in Early Modern Europe” closes this section with a detailed review of the early modern reception of Euclid’s *Elements*, ranging from the philologically-driven concerns of 16th-century editions of the text, to the conceptual and pedagogical criticism of its 17th-century editors and commentators, who tried to “amend” what they saw as the text’s geometrical bias with clarifications and modifications that would make it more appropriate to the theoretical challenges of a general science of magnitude, especially in regards to the theory of ratios or proportions. His account ends with the changed context in which 18th-century authors such as Euler reinterpreted the content of the *Elements*, further confirmation of the lasting significance of Euclid's contribution.

After the section devoted to sciences based on numbers and quantification comes the vast field of sciences which examine and give an account of the diverse physiological aspects of living creatures. These are inevitably intertwined–something especially present in some of the papers (Macías, King)–with their most obvious practical side, concerning the health and physical condition of human beings.

As was the case with Tomasello's contribution on harmonics and cosmography, there is also room in this section for a paper on Plato's philosophical approach to and exploitation of scientific ideas and models in several of his dialogues, but most especially in the *Timaeus*, a work which because of its particular story, transmission and reception could in itself be considered one of the pillars of the long-standing tradition of Greek science. Jorge Cano's paper "Philosophy and Teleology: The Creation of the Marrow and the Head in Plato's *Timaeus*" addresses Plato's rationalistic treatment of the somewhat inferred anatomy and physiology of the organs allegedly associated with the body-soul relationship, i.e. the marrow and the brain. Cano explores the way Plato's contentions regarding the *encephalo-myeloid* system relate to the theories and speculations of other authors and schools, most especially Empedocles, Philolaus of Croton and the Hippocratic tradition, and concludes by restating the Platonic commitment to a general teleological scheme of divine creation and order: "As happens with arithmetic and geometry, other *mathemata*, medical physiology serves as proof and evidence of the mark of the demiurge underlying the entire universe."

With Sara Macías' "*Pharmaka*: Medicine, Magic and Folk Medicine in the Work of Euripides", we concentrate on matters relating to popular magic and folk medicine that have had a prominent place in recent literature. In this particular case, what is explored is the evidence of certain widespread ideas and popular prejudices involving these issues–and particularly the use of remedies with various properties (*pharmaka*)–in the literary work of Athens's most successful drama writer of his day, the enduring canonical author, Euripides. Reflecting the vastly *performative* culture of the Greek *polis*, with its high standard of citizen social participation, Euripides' tragedies bring to us expressions and images that were purportedly shared and understood by the laymen of the city, even as they also reflected certain controversies and conflicts within the technical and specialized realm of medical practice and literature.

This eminently technical and, by Hellenistic and Roman times, clearly professionalized medical field is the main focus of Mario Vegetti's contribution, "Galen on Body, Temperaments and Personalities". Galen's only somewhat successful efforts in building a coherent and unified theory of temperaments and humours that would also be the basis of a theory of personality, becomes Vegetti's object of study in what is also an account of the synthetic reworking of an earlier Greek legacy at the hands of the great Greek physician of a then Roman world.

The role of Galen in passing on his own synthesis of Greek medicine to the Western scientific tradition is also emphasized in Helen King's "Knowing the Body: Renaissance Medicine and the Classics". Before focusing on the very revealing and complicated reception of renowned classical case studies–involving moreover gender issues–in Renaissance medical and exemplary literature (that which compiles didactic stories, anecdotes and *exempla*), this paper offers us an insightful historical panorama of what would traditionally be considered Greek medicine. According to King's account, the concept of Greek medicine itself was an early construct whose canonical origins can be found in Hippocrates and which rather successfully resisted any temptation of fragmentation into different specialized practices–diet, surgery, drugs or gynaecology–by presenting itself as a unified field for an all-around practitioner. The Renaissance continued to revere the classical authors of Greek ancient medicine, while sometimes offering confused amalgamations of their case studies.

The final contribution to the volume is by Andrés Vélez: "The *Forum Vulcani* in the Work of Juan Huarte: Geographical Argument and Renaissance Medicine". The paper focuses on how renowned Spanish Renaissance physician Juan Huarte, author of the extremely well-received *Examen de Ingenios*, made use of the classical imagery associated with Pozzuoli. Located on the Gulf of Naples, this area of sulphuric emissions was traditionally linked, through mythological accounts, with the gates and halls of hell. Vélez's avowed aim is to explore the function of this and other geographical references as a basis for arguments based on location (*argumentum a loco*) in Renaissance medicine and science. Thus for Vélez, the preservation of the long tradition of classical, and particularly Greek, medicine involving itself with geographical knowledge in texts like *Airs, Waters and Places* (within the Hippocratic Corpus), as well as in Galen's writings, several Platonic dialogues, especially the *Timaeus*, and Aristotle's *Problemata*, is something that must be taken into account in the analysis of Juan Huarte's work and his way of employing erudite, typically humanist, scavenging of an ample encyclopaedic tradition for his medical goals.

This last paper, concerning the theories and writings of a renowned professional physician, while showing evident meta-disciplinary concerns, like those addressed in the first section of the volume, rounds off our journey through different aspects of the long tradition of the Greek sciences. It completes a book that has been put together with the aim of quite consciously attempting to convey what Greek science has meant in the long run.

PART I

METHOD, ARGUMENTATION AND TRANSDISCIPLINARY ISSUES

CATEGORICAL ANACHRONISMS AND THEIR CONSEQUENCES FOR THE HISTORY OF SCIENCE

GEOFFREY LLOYD

Most historians of science are well aware that the Greek term *mathēmatikē* is far from being the exact equivalent of our "mathematics", that the Greek term *phusikē* (the study of nature) is radically different from the modern understanding of "physics" and that "biology" as such is nowhere to be found in Greek antiquity.[1] If we want to grasp how Aristotle set about *peri zōōn historia* (the study of animals) we have to pay careful attention to his account of the different types of causes that are his principal target. Spreading our net wider, a slight acquaintance with classical Chinese texts reveals that their conceptual map of the major intellectual disciplines differs widely from that of the ancient Greeks and from our own. I shall be discussing one of their overarching categories, *shu shu*, 數 術 , "calculations and methods", in some detail in a minute. But for now it is enough to note that it spans both what we should think of as mathematical subjects (the study of numbers) and what we would class as physical ones, that is the explorations of the interactions between things that the Chinese discussed under the rubric of *wuxing*, 五行 , the five phases. Medicine provides another instance, where Greek *iatrikē* and Chinese *yi xue*, 醫 學 , represent very different ideas of the causes of diseases and what needs to be done to treat them, and widely different, in each case, from modern biomedicine. And many other examples could be given.

But this bewildering heterogeneity may lead us into two profound mistakes, each of which needs to be avoided like the plague. The first is to think that the differences between Greece and China (for instance) are such that no comparison is possible between them, that we are dealing with incommensurable conceptual systems. The second is to hold that in neither ancient civilisation is there anything at all close to (modern)

[1] See especially Cunningham (1988); Cunningham and Williams (1993).

science, which must therefore be considered a uniquely modern, specifically Western, phenomenon. So there is a second incommensurability in our way, this time between ancient and modern thought.

Let me first discuss, then, some of the complications of a cross-cultural study of "mathematics".[2] Our term "mathematics" is, of course, derived from the Greek *mathēmatikē*, but that word comes from the verb *manthanein*, which has the quite general meaning of "to learn". A *mathēma* can be any branch of learning, anything we have learnt, as when in Herodotus, 1 207, Croesus refers to what he has learnt, his *mathēmata*, from the bitter experiences in his life. So the *mathēmatikos* is, strictly speaking, the person who is fond of learning in general, and it is so used by Plato, for instance, at *Timaeus* 88c, where the point at issue is the need to strike a balance between the cultivation of the intellect (in general) and that of the body–the principle that later became encapsulated in the dictum "mens sana in corpore sano". But from the fifth century BCE certain branches of study came to occupy a privileged position as the *mathēmata* par excellence. The terms mostly look familiar enough, *arithmētikē*, *geōmetrikē*, *harmonikē*, *astronomia*, and so on, but that is deceptive. Let me spend a little time explaining first the differences between the ancient Greeks' ideas and our own, and secondly some of the disagreements among Greek authors themselves about the proper subject-matter and methods of certain disciplines.

Arithmētikē is the study of *arithmos*, but that is usually defined in terms of what we would call positive integers greater than one. Although Diophantus, who lived at some time in late antiquity, possibly in the third century CE, is a partial exception, the Greeks did not normally think of the number series as an infinitely divisible continuum, but rather as a set of discrete entities. They dealt with what we call fractions as ratios between integers. Negative numbers are not *arithmoi* and do not enter into consideration. Nor is the monad an *arithmos*: it is thought of as neither odd nor even. Plato draws a distinction, in the *Gorgias* 451bc, between *arithmētikē* and *logistikē*, calculation, derived from the verb *logizesthai*, which is often used of reasoning in general. Both studies focus on the odd and the even, but *logistikē* deals with the pluralities they form, while *arithmētikē* considers them–so Socrates is made to claim–in themselves. That at least is the view Socrates expresses in the course of probing what the sophist Gorgias was prepared to include in what he called the art of rhetoric, though in other contexts the two terms that Socrates thus

[2] This subject is the topic of a comprehensive collective volume, Robson and Stedall (2009).

distinguished were used more or less interchangeably. Meanwhile a different way of contrasting the more abstract and the more practical aspects of the study of *arithmoi* is to be found in Plato's *Philebus* 56d, where Socrates distinguishes the way the many, *hoi polloi*, use them from the way philosophers do. Ordinary people use units that are unequal, speaking of two armies, for instance, or two oxen, while the philosophers deal with units that do not differ from one another in any respect, abstract ones in other words. A gap thus opens up between the mathematical concepts that anyone might be expected to have, and a deeper, more sophisticated, approach, though that is here represented as depending not so much on advanced mathematical skills as on a philosophical grasp of the subject. We shall find other examples, in our ancient Greece, where it is the philosophers, as much as mathematicians themselves, who claim superior knowledge in the domain.

At the same time the study of *arithmoi* encompassed much more than we would include under the rubric of arithmetic. Like some other peoples the Greeks represented numbers by letters, where α represents the number 1, β the number 2, γ 3, ι 10 and so on. This means that any proper name could be associated with a number. While some held that such connections were purely fortuitous, others saw them as deeply significant. When in the third century CE the neo-Pythagorean Iamblichus claimed that "mathematics" is the key to understanding the whole of nature and all its parts (*On the Common Mathematical Science*), he illustrated that with the symbolic associations of numbers, the patterns they form in magic squares and the like, as well as with more widely accepted examples such as the identification of the main musical concords, the octave, fifth and fourth, with the ratios 2:1, 3:2 and 4:3. The beginnings of such associations, both symbolic and otherwise, go back to the pre-Platonic Pythagoreans who are said by Aristotle (*Metaphysics* 987b11, 1036b12) to have held that in some sense "all things" *are* or *imitate* numbers. Yet this is quite unclear, first because we cannot be sure what "all things" covers, and secondly because of the evident discrepancy between the claim that they *are* numbers and the much weaker one that they merely *imitate* them.

What about "geometry"? The literal meaning of the components of the Greek word *geōmetria* is the measurement of land. According to a well-known passage in Herodotus, 2 109, the study was supposed to have originated in Egypt in relation, precisely, to land measurement after the flooding of the Nile. Measurement, *metrētikē*, still figures in the account Plato gives in the *Laws* 817e when his spokesman, the Athenian Stranger, specifies the branches of the *mathēmata* that are appropriate for free citizens, though now this is measurement of "lengths, breadths and depths", not of land. Similarly in the *Philebus* 56e we again find a contrast between the exact *geōmetria* that is useful for philosophy and the branch of the art of measurement that is appropriate for carpentry or architecture.

Those remarks of Plato already drive a wedge between practical utility–mathematics as securing the needs of everyday life–and a very different mode of usefulness, namely in training the intellect. One classical text that articulates that contrast is a speech that Xenophon puts in the mouth of Socrates in the *Memorabilia*, 4 7 2-5. While Plato's Socrates is adamant that mathematics is useful primarily because it turns the mind away from perceptible things to the study of intelligible entities, in Xenophon Socrates is made to lay stress on the usefulness of geometry for land measurement and on that of the study of the heavens for the calendar and for navigation, and to dismiss as irrelevant the more theoretical aspects of those studies. Similarly Isocrates too (11 22-3, 12 26-8, 15 261-5) distinguishes the practical and the theoretical sides of mathematical studies and in certain circumstances has critical remarks to make about the latter.

The clearest early extant statements of the view privileging the theoretical come not from the mathematicians but from philosophers commenting on mathematics from their own distinctive perspective. What mathematics can achieve that sets it apart from most other modes of reasoning is that it is exact and that it can demonstrate its conclusions. Plato repeatedly contrasts that with the merely persuasive arguments used in the law-courts and assemblies where what the audience can be brought to believe may or may not be true, and may or may not be in their best interests. Philosophy, the claim is, is not interested in persuasion but in the truth.[3] Mathematics is repeatedly used as the prime example of a mode of reasoning that can produce certainty: and yet mathematics, in the view Plato develops in the *Republic*, is subordinate to dialectic, the pure study of the intelligible world that represents the highest form of philosophy. Mathematical studies are valued as a propaedeutic, or training, in abstract thought: but they rely on perceptible diagrams and they give no account of their hypotheses, but rather take them to be clear.[4] Philosophy by contrast moves from its hypotheses up to a supreme principle that is said to be "unhypothetical".

[3] This is a recurrent motif in Plato's *Gorgias* especially.

[4] The interpretation of the shortcomings of mathematics is controversial. See, for example, the articles in the collective volume edited by Anton (1980); Vlastos (1988); Lloyd (1991: ch. 14) and Burnyeat (2000). The *actual* role of the lettered diagram in Greek mathematical reasoning has been brilliantly explored by Netz (1999).

The exact status of that principle, which is identified with the Form of the Good, is highly obscure and much disputed.[5] Likening it to a mathematical axiom immediately runs into difficulties, for what sense does it make to call an axiom "unaxiomatic"? But Plato was clear that both dialectic and the mathematical sciences deal with independent intelligible entities.

Aristotle contradicted Plato on the philosophical point: mathematics does not study independently existing realities. Rather it studies the mathematical properties of physical objects (Lear, 1982). But he was more explicit than Plato in offering a clear definition of demonstration itself and in classifying the various indemonstrable primary premisses on which it depends. Demonstration, in the strict sense, proceeds by valid deductive argument (Aristotle thought of that in terms of his theory of the syllogism) from premisses that must be true, primary, necessary, prior to and explanatory of the conclusions. They must, too, be indemonstrable, to avoid the twin flaws of circular reasoning or an infinite regress. Any premiss that can be demonstrated should be. But there have to be *ultimate* primary premisses that are evident in themselves. One of Aristotle's examples is the equality axiom, namely if you take equals from equals, equals remain. That cannot be shown other than by circular arguments, which yield no proof at all, but it is clear in itself.

It is obvious what this model of axiomatic-deductive demonstration owes to mathematics. I have just mentioned Aristotle's citation of the equality axiom, which figures also among Euclid's "common opinions" (sometimes called "common notions") and most of the examples of demonstrations that Aristotle gives, in the first book of the *Posterior Analytics*, are mathematical. Yet in the absence of substantial extant texts before Euclid's *Elements* itself (conventionally dated to around 300 BCE) it is difficult, or rather impossible, to say how far mathematicians before Aristotle had progressed towards an explicit notion of an indemonstrable axiom (Knorr, 1975).

Of course the principles set out in Euclid's *Elements* themselves do not tally exactly with the concepts that Aristotle had proposed in his discussion of strict demonstration. Euclid's three types of starting-points include definitions (as in Aristotle) and common opinions (which, as noted, include what Aristotle called the equality axiom) but also postulates (very different from Aristotle's hypotheses). The last included especially the parallel postulate that sets out the fundamental assumption on which

[5] Modern discussion was opened up especially by Robinson (1953 [1941]: ch. 10). For a highly original study of Plato's ideal of the Good, see Burnyeat (2000).

Euclidean geometry is based, namely that non-parallel straight lines meet at a point. However, where the philosophers had demanded arguments that could claim to be incontrovertible, Euclid's *Elements* came to be recognised as providing the most impressive sustained exemplification of such a project (Mueller, 1981). It systematically demonstrates most of the known mathematics of the day using especially reductio arguments and the misnamed method of exhaustion. Used to determine a curvilinear area such as a circle by inscribing successively larger regular polygons, that method precisely did *not* assume that the circle was "exhausted", only that the difference between the inscribed rectilinear figure and the circumference of the circle could be made as small as you like. Thereafter the results that the *Elements* set out could be and were, treated as secure by later mathematicians in their endeavours to expand the subject. Mastery of the *Elements* came to be an essential prerequisite for those seeking to join the ranks of what became an increasingly self-conscious elite.

The impact of this development first on mathematics itself, then further afield, was immense. In statics and hydrostatics, in music theory, in astronomy, the hunt was on to produce axiomatic-deductive demonstrations that basically followed the Euclidean model. But we even find the second century CE medical writer Galen attempting to set up mathematics as a model for reasoning in medicine–to yield conclusions in certain areas of pathology and physiology that could claim to be incontrovertible (Lloyd, 2006: chh. 4 and 5). Similarly Proclus attempted an *Elements of Theology* in the fifth century CE, again with the idea of producing results that could be represented as certain.

This development is often held up as one of the glories of Greek rationality. Yet three points must be emphasised to put it into perspective. First, for ordinary purposes, axiomatics was quite unnecessary. Not just in practical contexts, but in many more theoretical ones, mathematicians and others got on with the business of calculation and of measurement without wondering whether their reasoning needed to be given ultimate axiomatic foundations (Cuomo, 2001).

Secondly, it was far from being the case that all Greek work in arithmetic and geometry, let alone in other fields such as harmonics or astronomy, adopted the Euclidean pattern. The three "traditional" problems, of squaring the circle, the duplication of the cube and the trisection of an angle were tackled already in the fifth century BCE without any explicit concern for axiomatics (Knorr, 1986). Much of the work of a mathematician such as Hero of Alexandria (first century CE) focuses directly on problems of mensuration using methods similar to those in the traditions of Egyptian and Babylonian mathematics by which, indeed, he may have been influenced (Cuomo, 2001; Tybjerg, 2004).

Thirdly, the recurrent problem for the model of axiomatic-deductive demonstration that the *Elements* supplied was always that of securing axioms that would be both self-evident and non-trivial. Moreover it was not enough that an axiom set should be internally consistent: it was generally assumed that they should be true in the sense of a correct representation of reality.

A lot more could be said about "mathematics" and the mathematical sciences in Greece, but it is time to introduce some of the complications that arise when we attempt to compare another great civilisation, China, and what they had to say on the subject. We can recognise their interest in numbers and shapes, and that looks familiar enough. But their maps of the relevant intellectual disciplines turn out to be very different from those of the Greeks and from our own. One of the two general terms for number or counting, *shu*, 數 , has meanings that include "scolding", "fate" or "destiny", "art" as in "the art of", and "deliberations" (Ho, 1991). The second general term, *suan*, 算 , is used of "planning", "scheming" and "inferring" as well as "reckoning" or "counting". The latter term figures in the titles of the two main canonical works on "mathematics" that date from a hundred years either side of the millennium, namely the *Zhoubi suanjing* (the mathematical canon of the Zhou Gnomon, Cullen, 1996) and the *Jiuzhang suanshu* (the Nine Chapters of Mathematical Procedures, Chemla and Guo, 2004).

When around the turn of the millennium the Han bibliographers, Liu Xiang, 劉 向 , and Liu Xin, 劉 歆 , catalogued all the books in the imperial library under six generic headings, one of these was *shu shu*, 數 術 , that is, on one view, "calculations and methods". Its six sub-species comprise two that deal with the study of the heavens, namely *tianwen*, 天 文 , (the patterns in the heavens) and *lipu*, 曆 譜, (calendars and tables), as well as *wuxing*, 五 行 (the five phases) and a variety of types of divinatory studies. The five phases provided the main framework within which change was discussed. They are named fire, earth, metal, water and wood, but these are not elements in the sense of the basic physical constituents of things, so much as processes. Fire, one text says,[6] is flaming upwards: water is soaking downwards. (Heraclitus might well have understood, but those like Aristotle who thought of fire as an elemental substance would have been lost).

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[6] *Hong Fan* from the *Shang Shu*, see Karlgren (1950: 28 and 30); cf. Lloyd and Sivin (2002: 259-60).

So, on the one hand these Chinese inquiries deal with aspects of what we call astronomy and astrology and physics, the constitution of things, their correlations and associations, and their changes. But on the other the Chinese subsume all of these under a general category that straddles our "mathematics" and "physics". Moreover the aims of their more purely mathematical studies are strikingly different from those of the Euclidean tradition in Greece. We have direct evidence on the point in the work of a third century CE Chinese mathematician, Liu Hui, who wrote the first extant commentary on the *Nine Chapters*.

The key point, for our purposes, is that the notion of an axiom is absent from traditional Chinese mathematics, right down until the time of the arrival of the Jesuits in the sixteenth century. Rather the chief aim of Chinese mathematics, as Liu Hui describes it, was to identify the procedures that unify the subject and to extend its range. It is the *same* procedures, Liu Hui says, that provide the solutions to problems in different subject-areas. What he looks for, and finds, in such procedures as those he calls "homogenising", *qi*, 齊, and "equalising", *tong*, 同, is what he calls the guiding principles, *gangji*, 綱紀, of mathematics (*suan*, 算). In his account of how, from childhood, he studied the *Nine Chapters*, (91.6ff.) he speaks of the different branches of the study, but insists that they all have the same trunk, *ben*, 本. They have a single source or principle, *duan*, 端. The realisations and their categories, *lei*, 類, are elaborated mutually. Over and over again the aim is to find and show the connections between the different parts of *suanshu*, extending procedures across different categories, making the whole "simple but precise, open to communication but not obscure". Describing how he identified the technique of double difference, he says (92.2) he looked for the essential points or characteristics, *zhi qu*, 指趣, to be able to extend it to other problems.

While Liu Hui is more explicit in all of this than the *Nine Chapters*, the other great Han classic, the *Zhoubi*, represents the goal in very similar terms. That text contains an interesting exchange between the Master and one of his Pupils on how "mathematics" holds the key to the understanding of all sorts of obscure phenomena (shades of Iamblichus, one might think), including especially astronomical ones, the height and distance and magnitude of the sun, for instance. But in the Chinese text the Master does not overwhelm the student with the claim to have demonstrated his results (quod erat demonstrandum, *hoper edei apodeixai*). Rather the Master insists that the student must internalise the procedures, to be so competent in them that he can get the results on his own (Lloyd, 2002: 51-2).

This parallelism between the *Zhou Bi* and Liu Hui shows that we are not dealing with some isolated, maybe idiosyncratic, point of view on the latter's part, but with one that represents an important, maybe even the dominant, tradition–at least among those in the forefront of Chinese mathematical speculation. "It is the ability to distinguish categories in order to unite categories" which is the key according to the *Zhoubi* (25.5). Again, among the methods that comprise the *Dao*, the Way, it is "those which are concisely worded but of broad application which are the most illuminating of the categories of understanding. If one asks about one category and applies [this knowledge] to a myriad affairs, one is said to know the Way" (24.12ff.).

I may now attempt to sum up what my rapid and selective survey suggests so far. The study of just two ancient mathematical traditions already brings to light certain generic similarities, but also some suggestive differences (Lloyd, 2009: ch. 2; Robson and Stedall, 2009). Axiomatization was a distinctive preoccupation of one Greek tradition. Faced with arguments as to the inadequacy of mere persuasion some mathematicians set about giving the subject a basis that would secure incontrovertibility. But if that was an ambition of some–certainly not all–Greek mathematicians, it is quite absent from classical Chinese mathematics, which is more concerned with exploring the connections and the unity between different studies, including those we consider to belong to physics and cosmology as well as those we classify as mathematics. Their aim was not to establish the subject on a self-evident axiomatic basis but to expand it by extrapolation and analogy.

Each of those two aims has its strengths and its weaknesses. The advantages of axiomatization are that it makes explicit what assumptions are needed to get to which results. But the chief problem was that of identifying self-evident axioms that are not trivial. The advantage of the Chinese focus on guiding principles and connections was to encourage extrapolation and analogy, but the corresponding weakness was that everything depended on perceiving the analogies, since no attempt is made to give them axiomatic foundations. It is apparent that there is no one route that the development of mathematics had to take, or should have taken. We find good evidence in these two ancient civilisations for a variety of views of its unity and its diversity, its usefulness for practical purposes and for understanding.

So what can we learn about our two basic questions from this foray into the "mathematics" of early civilisations? While there are considerable differences to which I have drawn attention, it is not that comparison is impossible. We can recognise interests in number theory in both civilisations. Despite a common view that the Chinese were no good at geometry, they investigated the circle-circumference ratio (π) just as the Greeks did. There are sophisticated investigations of the tricky problem of the volume of the pyramid that again involve the decomposition of the figures in question and recursive techniques that lead to better and better approximations to the desired result. In this instance both the conception of the problem (how to determine a particular volume) and the result are strictly comparable in Greece and China. There is this difference, however, that whereas the Greek (misnamed) "method of exhaustion" proceeds by an indirect proof (the volume cannot be greater, nor less, than the given formula, and so it must be equal to it) the Chinese use a direct method, getting closer and closer approximations and inferring that the series converges on the desired result.[7] But that difference in the methods of getting a result is far from legitimating any conclusion to the effect that the mathematics in the two civilisations are totally incommensurable.[8]

Once we take into account the whole gamut of Greek and Chinese investigations, the idea of dismissing Chinese work as somehow not proper mathematics at all can be seen to be mere Eurocentric prejudice. Of course many areas of the work of today's mathematicians are distinctly modern developments, and axiomatization itself underwent radical redefinition with Hilbert. But it is worth reminding ourselves that issues concerning the proper subject-matter and aims of mathematics continue to be hotly disputed in the twenty-first century. What mathematics studies and what knowledge it produces are questions that continue to be answered very differently by Platonists, constructivists, intuitionists, logicists and formalists (just to mention some of the principal divergent views). Those disagreements and the divergences in different historical mathematical traditions do not mean that there is no such subject as mathematics, after all: rather they suggest that what we can recognise as elements of the subject have been and continue to be pursued very

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[7] See Lloyd (1996: 152-4, on the *Nine Chapters*, ch. 5: 167-8).

[8] It is important to distinguish between strong and weak versions of the incommensurability thesis. When Kuhn (1970 [1962]) introduced the notion, that was to draw attention to the undeniable point that radical shifts in the meaning of key terms took place as between Ptolemy, say, and Copernicus, or Aristotle and Galileo, or Newton and Einstein. But incommensurability has sometimes been used to suggest that there is a mutual unintelligibility across paradigms. Yet Galileo certainly thought he could understand Aristotle's views on weight and force, and modern interpreters proceed on the assumption that neither is totally beyond the reach of our understanding.

differently and with very different ambitions by different students of numbers and shapes.

But mathematics, some may think, is a comparatively easy subject in which to find answers to my two principal questions, of the possibilities of comparison between divergent ancient traditions and between them and modern work. So I turn now to the apparently much more difficult case of "science" itself.[9] Here there are no "actors" categories in ancient languages that provide a possible starting-point for comparisons across them and with our conception of science. Yet let me begin by reminding you of ongoing divergences in the latter. In the UK in one common usage "science" without qualification is taken to refer first and foremost to the "natural sciences". Of course psychology, anthropology, economics and the "social sciences" in general also stake a claim to be (proper) science, but the yardstick by which they are judged is provided by those natural sciences. Yet the other side of the English Channel, French "science", Spanish "ciencia", Italian "scienza" are not so restrictive, and in Germany "Wissenschaft" is certainly not limited to the "Naturwissenschaften".

How to define science, whether in terms of results, or methods, or aims, continues to be hotly contested. Any attempt to use results–the delivery of the truth–as the test is open to the objection that they, the results, are always subject to revision, never definitive. Some parts of what is accepted in 21st century chemistry, for instance, may look robust enough. But many of the fundamental problems in particle physics, in cosmology, even in parts of genetics, are, as is well known, still very far from resolved.

So rather than use results, most would focus on either methods or aims as the key criterion–and yet they yield very different views on our question. The invocation of the, or at least a, scientific method tends to favour the thesis of a Great Divide, which asserts that science has only been practised in the last 150 years or so (see, for example, Goody, 1977; Gellner, 1985). Science, on that view, may not require the entire gamut of the sophisticated gadgetry of modern laboratories, but it depends on such concepts as the mathematisation of physics, and on quantitative analyses generally, on the use of hypotheses and postulates, and especially on the experimental method. It is certainly not, then, a world-wide phenomenon to be found, to some extent at least, in any human society at any period.

But notions of correct methods have also been subject to fluctuation and the goal of being able to specify *the* scientific method, good for all

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[9] See Cunningham (1988); Cunningham and Williams (1993), and cf. Lloyd (2009: ch. 8).