Data Envelopment Analysis and Its Applications to Management

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Edited by

Vincent Charles and Mukesh Kumar

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PREFACE

Building on the ideas of Farrell (1957), the seminal work *Measuring the Efficiency of Decision Making Units* by Charnes, Cooper and Rhodes (1978) introduced the concept of data envelopment analysis (DEA) in the literature, and since then, it has emerged as a powerful management science tool for measuring and evaluating the performance of different kinds of entities engaged in many different activities in various contexts around the globe. Recent years have seen great varieties of application of DEA in almost every field, such as agriculture, banking, benchmarking, education, environment, economy, government, health, insurance, information technology, marketing, operations, public policy, human resources, manufacturing, retail, regulation, services, and tourism. The popularity of DEA lies in the flexibility of its approach that readily incorporates the existence of multiple inputs and multiple outputs without any underlying assumption of a functional form. Given the set of inputs and outputs of different decision-making units (DMUs), it constructs its own functional form, thus avoiding the danger of misspecification of the frontier technology. Moreover, it does not assume that all DMUs are using the same technology but instead evaluates the efficiency of a DMU relative to its peer or combination of peers.

The book, entitled *Data Envelopment Analysis and Its Applications to Management,* will be useful to researchers in the field of DEA as well as to practitioners from various sectors who intend to apply DEA for their strategic and managerial decisions through efficiency evaluation. The book is well organised in sixteen chapters contributed by researchers from all around the globe. It covers theoretical development of DEA and its application in various fields, such as economy, banking, education, revenue management, branch network, sports, livestock production systems and cities. Each chapter begins with an introduction, followed by the literature review, methodology, applications, and concludes with suggestions for future scope of the study.

Chapter One provides the introduction to DEA representing the essential features of the core literature on DEA for interested readers coming from different disciplines.

Chapters Two, Three and Four offer a discussion of some interesting theoretical developments in DEA. Chapter Two shows how the geometric

DEA compares to classical DEA and demonstrates, with examples, some properties of geometric DEA models, which might be beneficial in the practice of decision making and/or efficiency measurement. Chapter Three deals with data sampling for large datasets using the DEA-neural network approach generalisation, with bootstrap methods. In order to overcome the difficulty of measuring the performance of DMUs in the presence of large datasets, the author proposes to reduce the dataset, using simple random sampling, and to apply the DEA-neural network combination in order to draw conclusions about the entire structure of the dataset, with specific error probability and accuracy of measurements. Chapter Four deals with the max-normalization-variable alteration technique as a means to avoid computational problems caused by imbalanced data in DEA efficiency assessments. The proposed data-rescaling-variable alteration technique allows for reformulation of the DEA models in a more effective manner, with the nonlinear virtual inputs and outputs developed earlier in the literature.

Chapters Five and Six deal with the application of DEA in the economies of two developing countries: India and the Czech Republic. In Chapter Five, estimates of the household-level, earning-frontier functions for two eastern states of rural India, namely, West Bengal and Orissa, are provided. Also, the relationships amongst levels of living, occupational status and the efficiency obtained from the DEA analysis are examined. Furthermore, the influence of social opportunities on frontier income is examined and the results in terms of policy implications are interpreted. In Chapter Six, the measures of the efficiency of export, with respect to foreign direct investment, are presented, using the panel data from 13 countries for EU15 and the Czech Republic.

Chapters Seven and Eight cover the application of DEA in the banking sectors. The rankings of Peruvian banks by using the super-efficiency DEA model under the assumption of variable returns to scale (VRS) for the period 2008 to 2010 are reported on in Chapter Seven. As it is well known that the super-efficiency DEA approach can be infeasible under the assumption of VRS, the one-model approach of Chen and Liang (2011) has been used to resolve the infeasibility problem in the present context. The rankings gathered from the model are compared with the ranking based on a super-efficiency DEA model under more restricted assumptions of constant returns to scale (CRS). In Chapter Eight, an integrated approach to DEA and analytic hierarchy process (AHP) methodologies are proposed, to reflect the priority weights of inputs and outputs in efficiency assessments. Additionally, it provides the application of the proposed methodology, adopted from Kim (2000), for bank branch studies, to compare the performance of 10 loan departments.

Chapters Nine and Ten present the application of DEA in the education sector in Brazil. In Chapter Ten, the efficiency of the Centre for Higher Distance Education of the State of Rio de Janeiro (CEDERJ's) centres is evaluated by using the DEA multi-objective model (MORO-D-R). As the MORO-D-R model provides multiple targets for inefficient DMUs, it offers decision makers some flexibility in choosing the most suitable target. Chapter Ten deals with a new use for cross evaluation through DEA. A self-organising map (SOM) and Kohonen's neural network are used to cluster the DMUs, using inputs as the values on the cross-evaluation matrix of each DMU and also its classic efficiency index. To avoid distortion, the efficiencies are normalised by the classical DEA efficiency index. A practical application is presented on a case study in educational evaluation.

In Chapter Eleven, the applications of a DEA-based approach in revenue management (RM) are described. RM is aimed at maximising revenue by selling the right product to the right customer at the right time for the right price and through the right channel. The common modelling approaches assume that customers are passive and they do not engage in any decision-making processes. This simplification is often unrealistic for many practical problems. Today's customers actively evaluate alternatives and make choices. The evaluation of alternatives can be done by DEA-based evaluation methods. The efficient frontier provides a systematic framework for comparing different policies and highlights the structure of the optimal controls for the problems.

Chapter Twelve presents the application of the non-radial zero-sum-gains DEA model (ZSG-DEA) with weight restrictions, for the distribution of funds transferred by the Agnelo/Piva Law in 2008 to the Brazilian Olympic Committee. This distribution or allocation of financial resources is based on the results (medals) obtained by the different Olympic sports and not on outside factors. The results obtained by applying the aforementioned model are used to analyse whether the distribution made by the Brazilian Olympic Committee (based on meritocracy) is consistent with the efficient allocation suggested by the DEA model.

Chapter Thirteen presents an efficiency analysis of the branch network of one of the mobile operators in the Czech Republic, using a two-stage DEA model. The first stage of the analysis measures external efficiency, where the main output is the number of transactions of the branch, which subsequently serves as one of the inputs in the second stage. This stage evaluates internal efficiency of the branch. Total efficiency of the branch is given by synthesising both the external and internal efficiency. The system for efficiency evaluation is illustrated on a real data set with 67 branches.

In Chapter Fourteen, evaluations of the performance of 21 beef-cattle modal production systems, in 21 municipalities of seven Brazilian states are reported. The DEA model is used under the assumption of VRS with weight restrictions. The objective is to measure the performance of each rancher's decision regarding the composition of the production system. From the 21 systems evaluated, four were efficient and most of the systems are operating under increasing returns to scale.

In Chapter Fifteen, the evaluations of the efficiency of 127 selected towns in the Republic of Croatia, using categorical models of DEA are presented. The towns, represented as DMUs, are divided into four categories, according to their respective populations. The number of employed workers and employed assets are considered as inputs and income is considered as an output in a categorical input-oriented (output-oriented) model with constant (variable) returns to scale.

Chapter Sixteen presents a method of evaluation of workflow runtime platforms using DEA. The algorithms that enable conversion of workflow data to values applicable for DEA modelling are provided, which concern workflow structure, interoperability constraints of heterogeneous systems and quality of service attributes. Then, runtime platforms are modelled as DMUs that intend to optimise their performance in terms of the final price and quality of service.

The chapters contributed to this book should be of considerable interest and provide readers with informative reading.

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Chapter One

An Introduction to Data Envelopment Analysis

Vincent Charles and Mukesh Kumar

Abstract

The introductory chapter begins with a brief introduction to data envelopment analysis (DEA) and its origin, followed by the basic assumptions. Further, the concept of efficiency is introduced, through worked examples and graphs, to researchers unfamiliar with the technique. Finally, the basic models of DEA are provided for direct reference for those interested in efficiency evaluation, ranking, and benchmarking of decision-making units (DMUs).

1.1 Introduction

Economics and operations research have common interests in several research fields, including the analysis of the production possibilities for micro units. The stochastic frontier approach (SFA) (parametric) and the data envelopment analysis (DEA) (non-parametric) models have emerged as two alternative developments of ideas that originated with Farrell (1957). Grosskopf (1986) noted that the parametric approach has been developed mainly by economists, whereas the nonparametric has been left to those in operations research. The popularity of DEA over econometric approaches lies in its flexibility to incorporate the existence of multiple inputs and multiple outputs readily without any assumption on the functional form.

The facts that make the DEA a relatively superior tool for the evaluation of efficiency are as follow: Firstly, a typical statistical approach is characterised as a central tendency approach and it evaluates producers relative to an average producer. In contrast, DEA is an extreme-point method and compares each producer with only the *best* producer(s). Secondly, DEA does not require any underlying assumption of a functional form relating to inputs and outputs. Given the set of inputs and outputs of different firms, it constructs its own functional form. Thus, it avoids the danger of misspecification of frontier technology. In contrast, the econometric approach assumes a functional form, such as Cobb-Douglas or Translog, relating to inputs and output. Thirdly, the parametric approach estimates the efficiency of firms producing a single output with a set of multiple inputs, whereas the DEA readily incorporates the existence of multiple outputs. Fourthly, in the parametric approach, the decomposition of the error term into two parts, one representing stochastic error, and the other representing inefficiency is not useful for the datasets of fewer than 100 observations (Aigner, Lovell, & Smith, 1997). DEA, on the other hand, works well with a small sample size. As a rule of thumb, the minimum sample required for DEA analysis is just three times larger than the sum of the number of inputs and outputs (Nunamaker, 1985; Raab & Lichty, 2002).

Recent years have seen great varieties of application of DEA in almost every field, including agriculture, banking[1], benchmarking, education, energy and environment[2], economy, government, health, insurance, information technology, marketing, operations, public policy, human resources, manufacturing, retailing, regulation, services and tourism, and others. Several authors have surveyed the general DEA literature[3] and provided scenarios for DEA methodology[4] development in different time periods for a range of issues. The strong growth of DEA research in recent years has increased the DEA literature to a very large scale. The rapid increase in its popularity can be inferred from the fact that Seiford (1994) in his DEA bibliography has listed not less than 472 published articles and accepted Ph.D. dissertations, some dated even as early as 1992. Emrouznejad et al. (2008) provided a detailed bibliography of DEA literature published in various journals/book chapters/proceedings since 1978, which clearly shows an exponential growth in the literature. In one of the very recent survey papers, Liu et al. (2012) reported that up to the year 2009, the field has accumulated approximately 4,500 papers in the ISI Web of Science Database.

1.2 The Origin of Data Envelopment Analysis

The specific research stand of efficiency measurement for production units in the field of operations research took off with *Measuring the Efficiency of Decision Making Units* by Charnes, Cooper, and Rhodes

(1978)[5] as the seminal paper (Førsund & Sarafoglou, 2002). However, the intellectual root of DEA in economics can be traced all the way back to the early 1950s. In the aftermath of World War II, linear programming (LP) came to be recognised as a powerful tool for economic analysis. The papers in the Cowles commission monograph, *Activity Analysis of Production and Resource Allocation*, edited by Koopmans (1951), recognised the communality between existence of nonnegative prices and quantities in a Walras-Cassel economy and the mathematical programming problem of optimising the objective function subject to a set of linear inequality constraints. Koopmans (1951) defined a point in the commodity space as efficient whenever an increase in the net output of one product can be achieved only at the cost of a decrease in the net output of another product. In view of its obvious similarity with the condition for Pareto optimality, this definition is known as the Pareto-Koopmans condition of technical efficiency. In the same year, Debreu (1951) defined the coefficient of resource utilisation as a measure of technical efficiency for the economy as a whole, and any deviation of this measure from unity was interpreted as deadweight loss suffered by society due to inefficient utilisation of resources.

Farrell (1957) made a path-breaking contribution through the seminal work *The Measurement of Productive Efficiency* by constructing an LP model using actual input-output data for a sample of firms, the solutions of which yield a numerical measure of technical efficiency of an individual firm in the sample. He demonstrated that the economic efficiency could be decomposed into *allocative efficiency* and *technical efficiency*. The technical efficiency reflects the ability of a firm to obtain the maximal output from a given set of inputs, whereas, the allocative efficiency reflects the ability of a firm to use the inputs in optimal proportion, given the prices of the resources. The idea of Farrell (1957) can be illustrated with a simple example involving firms, using two inputs (X_1 and X_2) to produce a single output (Y) under the assumption of constant returns to scale (CRS). The assumption of CRS implies that a radial increase in input vector causes the same proportion of increase in the output vector (doubling of all inputs leads to doubling of all outputs).

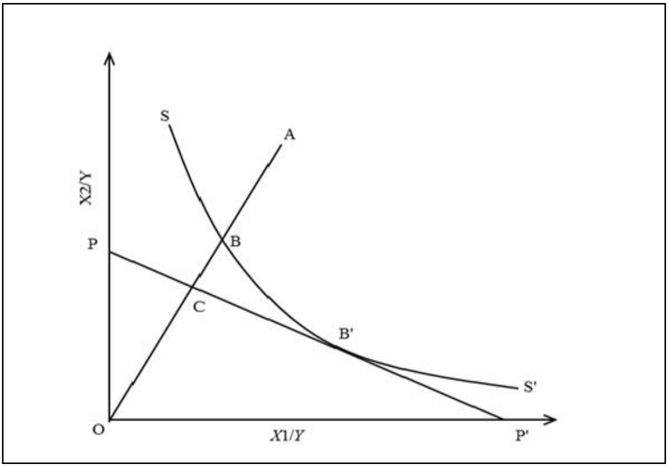
A further assumption is that the efficient production function is known. The curve SS' in Figure 1-1 represents the unit isoquant of an efficient producer. To produce a unit of output, let a firm uses the quantities of inputs denoted by the point A. If we draw a line from the origin to the point A, it will intersect the efficient isoquant at the point B. This means that if inputs can be reduced equiproportionately, the efficient point will be at point B, which must lie on the efficient isoquant SS'. Thus, point B

represents the combination of inputs in the same proportion as in point *A* but with a lesser amount of both inputs to produce a unit level of output. An *OB/OA* fraction of inputs is now needed to produce the same level of output or, in other words, *OA/OB* times of output can be produced from the given level of both the inputs. The *technical efficiency* of the firm can be measured by the ratio

$$TE = OA/OB$$

where $TE \leq 1$. $TE = 1$ indicates that the firm is technically efficient, whereas, $TE < 1$ indicates that the firm is technically inefficient.

Figure 1–1 Technical, allocative and economic efficiency



In the above definition of efficiency, the role of input price in measuring the efficiency is not considered. In order to access the efficient allocation of inputs in terms of input price, let us introduce the price line or isocost line PP′ in Figure 1-1. It represents the minimum cost required, given the price of inputs for the use of same proportion of inputs as is used at point *B*. Thus, the ratio *OC/OB* gives the measure of price efficiency or *allocative efficiency*.

$$AE = OC/OB$$

where the distance *BC* represents the reduction in production costs that would occur if the production were to occur at the allocatively (and technically) efficient point B' instead of at the technically efficient, but allocatively inefficient, point B.

If the firm is efficient, both technically as well as allocatively, then the ratio OC/OA will be the measure of overall efficiency or *economic efficiency* (EE).

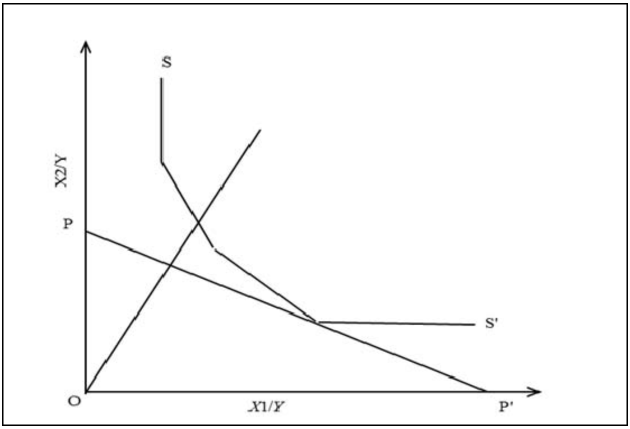
$$EE = OC/OA$$

where the distance AC can also be interpreted in terms of cost reduction. It should be noted that the product of technical and allocative efficiency provides the economic efficiency and all three measures are bounded by 0 and 1.

$$TE \times AE = \frac{OB}{OA} \times \frac{OC}{OB} = \frac{OC}{OA} = EE$$

There are two empirical approaches to the measurement of efficiency based on the above concepts of technical and allocative efficiency. The first, favoured by most economists, is parametric (either stochastic or deterministic), where the form of production function (or isoquant) is either assumed to be known or is estimated statistically. However, in many cases, the functional form of production function (or isoquant) is unknown. In the nonparametric approach, no functional form is assumed *a priori* and the piecewise linear convex isoquant is constructed empirically from observed inputs and outputs, as is shown in Figure 1-2.

Figure 1–2 Piecewise linear convex isoquant



If Farrell's (1957) article is taken as the seminal work, the fundamental research reported in 1978 is undoubtedly the basis for subsequent developments in the nonparametric approach to evaluating technical efficiency. In the subsequent work, Charnes and Cooper (1985) provided the formal definition of efficiency as follows:

100% efficiency is attained for a production unit only when

a) None of the outputs can be increased without either (i) increasing one or more of the inputs, or (ii) decreasing some of its other outputs.
b) None of its outputs can be decreased without either (i) decreasing some of its outputs, or (ii) increasing some of its other inputs.

This definition is in accordance with the economist's concept of Pareto (Pareto-Koopmans) optimality. If there is no way of establishing a *true* or theoretical model of efficiency, that is, the absolute standard, then the definition needs to be adapted so that it refers to levels of efficiency relative to known levels attained elsewhere in similar circumstances. Charnes and Cooper (1985) thus provided the further definition:

> 100% relative efficiency is attained by any (unit) only when comparisons with other relevant (units) do not provide the evidence of inefficiency in the use of any input or output.

1.3 Evaluating Efficiency: Numerical Examples and Graphic Presentation

The heart of DEA analysis lies in creating the *best* virtual producer for each real producer. If a given producer, *A*, is capable of producing *Y*(*A*) units of output with *X*(*A*) units of inputs, then other producers should also be able to do the same if they were to operate efficiently. Similarly, if producer *B* is capable of producing *Y*(*B*) units of output with *X*(*B*) units of inputs, then other producers should also be capable of doing the same. Producers *A*, *B*, and others can then be combined to form a composite producer with composite inputs and composite outputs. Since this composite producer does not necessarily exist, it is called a *virtual producer*.

The producer is considered as inefficient if virtual output > actual output for a given input level or virtual input < actual input for a given output level. The former is known as *output-oriented technical efficiency*, whereas, the latter is known as *input-oriented technical efficiency*. These two orientations of efficiency measurement address two different questions:

- The input-oriented measures of technical efficiency address the question, *By how much can input be reduced while attaining the current level of output?*
- The output-oriented measures of technical efficiency address the question, *By how much can output be increased while keeping the level of current input fixed?*

1.3.1 Technical efficiency: one input-one output case

To understand the concept of efficiency, we begin with a simple example of a one-input and one-output case. Consider four branches of Interbank in Peru for the evaluation of efficiency. For each branch, we have a single output measure, personal transactions (measured in thousands) and a single input measure, managerial staff (measured in numbers), as shown, respectively, in columns 2 and 3 of Table 1-1a. For example, for the Chacarilla branch in one year, there were 50,000 personal transactions and 17 staff members were employed.

Table 1–1a Input (#1) and Output (#1) of Bank Branches and their Relative Efficiency (Input-Oriented)

Code	Branch	PT (Y_1)	Staff (X_1)	PT/Staff (Y_1/X_1)	Relative Efficiency
A	Santa Anita	130	20	130/20 = 6.50	6.50/6.50 = 1.00
B	Chacarilla	50	17	50/17 = 2.94	2.94/6.50 = 0.45
C	El Polo	85	18	85/18 = 4.72	4.72/6.50 = 0.73
D	Jockey Plaza	25	11	25/11 = 2.27	2.27/6.50 = 0.35

Here, branches can be seen as taking inputs and converting them (with varying degrees of efficiency) into outputs. The commonly used method to measure efficiency is ratio, which requires a straightforward calculation based on data on a single output and input. In this case, the single ratio can be obtained by dividing number of personal transactions (Y_1) by number of staff (X_1), as shown in column 4 of Table 1-1a. One can observe that Santa Anita (A) has the highest ratio of personal transactions per staff member (6.50), whereas Jockey Plaza (D) has the lowest ratio of personal transactions per staff member (2.27).

As Santa Anita (A) has the highest ratio of 6.50, all other branches can be compared to it and their relative efficiency calculated with respect to the Santa Anita branch. To do this, the ratio of each branch is divided by 6.50 (the value for Santa Anita), as shown in the last column of Table 1-1a, to attain the relative efficiency of each branch. One can observe that the value of efficiency varies between 0 and 1. The only branch that is efficient is Santa Anita (A), with an efficiency score of 1. The most inefficient branch is Jockey Plaza (D), with its efficiency score of 0.35.

Similarly, one can obtain the efficiency of bank branches by using the ratio of number of staff (X_1) to number of personal transactions (Y_1), as shown in the fifth column of Table 1-1b. It can be seen that Santa Anita (A) incurs the lowest cost per personal transaction, whereas Jockey Plaza (D) incurs the highest cost per personal transaction.

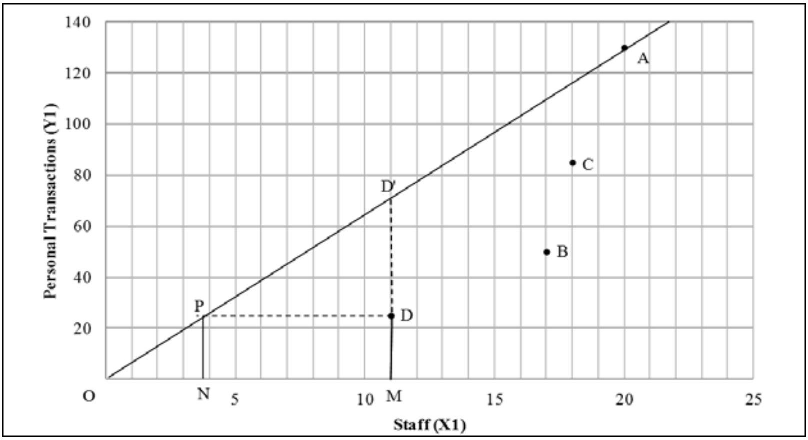
Table 1–1b Input (#1) and Output (#1) of Bank Branches and their Relative Efficiency (Output-Oriented)

Code	Branch	PT (Y_1)	Staff (X_1)	Staff/PT (X_1/Y_1)	(Staff/PT)/ Min(Staff/PT)	Relative Efficiency
A	Santa Anita	130	20	20/130 = 0.15	0.15/0.15 = 1.00	1/1.00 = 1.00
B	Chacarilla	50	17	17/50 = 0.34	0.34/0.15 = 2.27	1/2.27 = 0.44
C	El Polo	85	18	18/85 = 0.21	0.21/0.15 = 1.40	1/1.40 = 0.71
D	Jockey Plaza	25	11	11/25 = 0.44	0.44/0.15 = 2.93	1/2.93 = 0.34

As Santa Anita (*A*) has the lowest per unit cost of personal transactions (i.e., 0.15), we can compare all other branches to it and calculate their relative efficiency with respect to the Santa Anita branch. To do this, we can divide the ratio of each branch by 0.15 (the value for Santa Anita), as shown in column 6. However, in this case, the efficiency of branch (shown in column 7) is the reciprocal of values presented in column 6.

The concept of input and output-oriented technical efficiency is well demonstrated in Figure 1-3. The output, number of personal transactions (Y_1), is represented in the *Y*-axis, whereas, the input, number of staff (X_1), is shown in the *X*-axis. The input-output combinations of bank branches are represented by branch codes, *A*, *B*, *C,* and *D*. The straight line from the origin through point *A* measures the frontier technology, which shows the maximum attainable level of output (number of personal transactions), given the level of input (number of staff). Given the frontier technology *OA*, all other branches are inefficient except the Santa Anita (*A*) branch.

Figure 1–3 Technical efficiency: one-input and one-output case



Let the reference branch be Jockey Plaza (*D*), which is using *OM* amounts of input to achieve the current level of output, *MD*. If Jockey Plaza (*D*) were technically efficient, the input that is required to achieve the current level of output would have been *ON*. The technical efficiency under input-orientation is defined as the maximum proportional reduction in input for a given level of output.

$$TE_I = ON/OM = 3.84/11 = 0.35$$

The subscript *I* indicates that the efficiency is *input-oriented*.

The efficiency of Jockey Plaza (*D*) can also be evaluated from the output-orientation. Given the current level of input *OM*, the maximum potential level of output that could be achieved by branch *D* is *MD′*. Under the output-orientation, the technical efficiency is the reciprocal of the maximum proportional expansion of output (i.e., the reciprocal of the ratio of distances *MD′* to *MD*).

$$TE_O = MD/MD' = 25/71.43 = 0.35$$

The subscript *O* indicates that the efficiency is *output-oriented*.

It should be noted that under CRS, the efficiency under output orientation is exactly the same as the efficiency under input orientation. This can be easily inferred from Figure 1-3 through the property of triangle as given below:

$$\Delta OPN \cong \Delta OD'M$$

$$\Rightarrow \frac{NP}{ON} = \frac{MD'}{OM} \Rightarrow \frac{NP}{MD'} = \frac{ON}{OM} \Rightarrow \frac{MD}{MD'} = \frac{ON}{OM} \Rightarrow TE_O = TE_I$$

1.3.2 Technical efficiency: one input-two outputs case

Suppose now that we have two output measures (personal transactions completed and number of business transactions, *BT*, completed) and the same single input measure (number of staff), as shown in Table 1-2a. For example, for the branch, Chacarilla (*B*), in one year, 50,000 transactions relating to personal accounts and 25,000 transactions relating to business accounts were conducted, and 17 staff members were employed. Here, the performance of the branches needs to be assessed on how efficiently they use their single input (number of staff) to produce the two distinct categories of transaction outputs.

Table 1–2a Input (#1) and Outputs (#2) of Bank Branches and their Output-Input Ratios

Code	Branch	PT (Y_1)	BT (Y_2)	Staff (X_1)	PT/Staff (Y_1/X_1)	BT/Staff (Y_2/X_1)
A	Santa Anita	130	55	20	6.50	2.75
B	Chacarilla	50	25	17	2.94	1.47
C	El Polo	85	60	18	4.72	3.33
D	Jockey Plaza	25	15	11	2.27	1.36

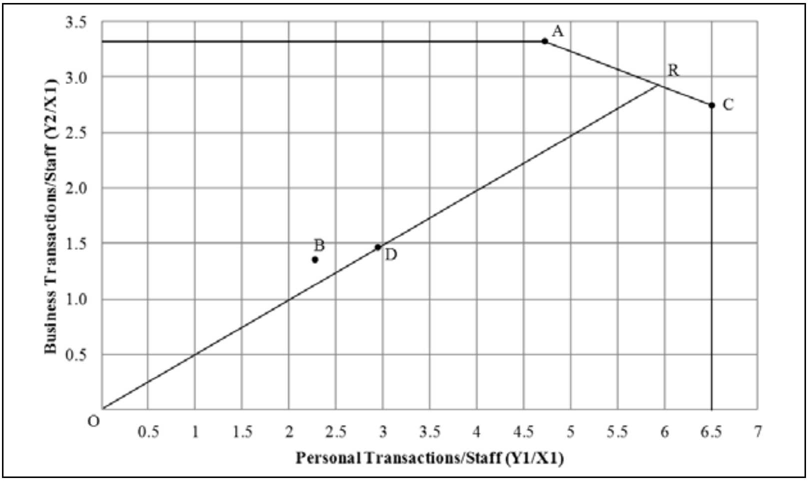
The output-oriented technical efficiency of each branch in producing two outputs can be found by dividing each of their outputs by their input. As one can observe, Santa Anita (*A*) has the highest ratio of personal transactions per staff member, whereas El Polo (*C*) has the highest ratio of business transactions per staff member. Chacarilla (*B*) and Jockey Plaza (*D*) do not compare so well with Santa Anita (*A*) and El Polo (*C*), so are presumably underperforming. Thus, Chacarilla (*B*) and Jockey Plaza (*D*) are relatively less efficient at using their given input resource (number of staff) to produce outputs, number of personal transactions, and number of business transactions.

One problem with comparison via ratios in this case is that different ratios can give a different picture and it is difficult to combine the entire set of ratios into a single numeric judgment. For example, consider

Chacarilla (B) and Jockey Plaza (D): Chacarilla is (2.94/2.27) = 1.29 times as efficient as Jockey Plaza at personal transactions but only $(1.47/1.36) = 1.08$ times as efficient at business transactions. Thus, the challenge here is to combine these numbers into a single judgment.

Figure 1-4a shows the X-axis representing the personal transactions per staff member (Y_1/X_1) and the Y-axis representing the business transactions per staff member (Y_2/X_1). The observed level of two outputs of a bank branch is represented by its branch code. The frontier technology is formed by connecting the observations A and C and further extending the line horizontally from point A to the Y-axis and extending the line perpendicularly from point C to the X-axis.

Figure 1–4a Output technical efficiency: one input – two outputs case



Any branches on the frontier are technically efficient $(TE_o = 1)$. Hence, for our example, Santa Anita (A) and El Polo (C) are efficient. However, it does not mean that the performance of Santa Anita (A) and El Polo (C) could not be improved. In fact, what we can say is that on the evidence (data) available, we have no idea of the extent to which their performance can be improved. Clearly, Chacarilla (B) and Jockey Plaza (D) are inefficient $(TE_o \leq 1)$.

If the branch, Jockey Plaza (D), is placed under evaluation, the ratio personal transactions/business transactions = (25/15) = 1.67, that is, there are 1.67 personal transactions for every business transaction. Mathematically, the value 1.67 is also the ratio of personal transactions per staff member/business transactions per staff member, that is, 2.27/1.36 = 1.67. Numerically, we can measure the (relative) efficiency of Jockey Plaza (*D*) by the following ratio: length of line from origin to Jockey Plaza (*OD*)/length of line from origin through Jockey Plaza to efficient frontier (*OR*) = 2.646/6.126 = 0.432.

The input-oriented technical efficiency of each branch can be found by dividing their input by each of their outputs, as shown in Table 1-2b. Here we can see that Santa Anita (*A*) incurs the highest number of personal transactions in terms of output Y_1, whereas El Polo (*C*) incurs the highest number of business transactions in terms of output Y_2. On the other hand, Jockey Plaza (*D*) incurs the least number of transactions in terms of both the outputs.

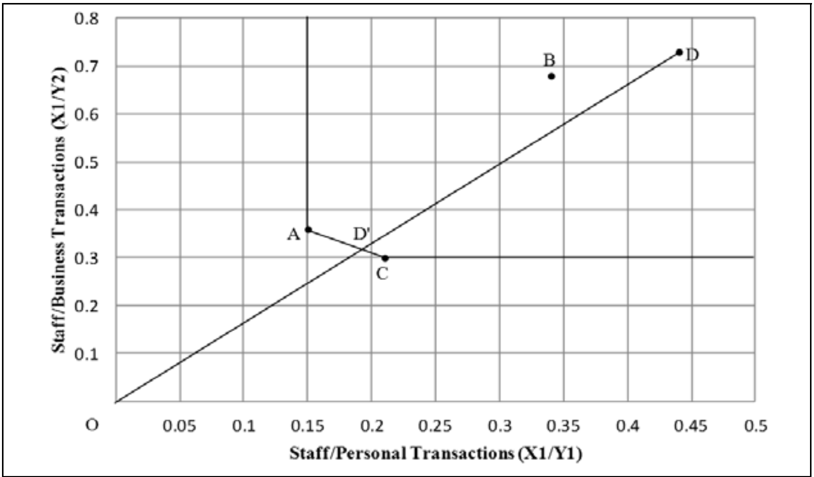
Table 1–2b Input (#1) and Outputs (#2) of Bank Branches and their Input-Output Ratios

Code	Branch	PT (Y_1)	BT (Y_2)	Staff (X_1)	Staff/PT (X_1/Y_1)	Staff/BT (X_1/Y_2)
A	Santa Anita	130	55	20	0.15	0.36
B	Chacarilla	50	25	17	0.34	0.68
C	El Polo	85	60	18	0.21	0.30
D	Jockey Plaza	25	15	11	0.44	0.73

Let the *X*-axis represent the number of staff per number of personal transactions (X_1/Y_1) and the *Y*-axis represent the number of staff per number of business transactions (X_1/Y_2). The frontier is formed by joining *A* to *C* and extending the line from point *A* (parallel to the *Y*-axis) and point *C* (parallel to the *X*-axis). Clearly, the branches *A* and *C* are efficient $(TE_I = 1)$ as they lie on the frontier of the isoquant, whereas the other two branches, *B* and *D*, are technically inefficient $(TE_I \leq 1)$ as they lie above the isoquant. The further away a branch is from the isoquant, the more inefficient it is.

The input-oriented technically efficiency of branch *D* as shown in Figure 1-4b can be obtained by taking the following ratio: length of line from origin to efficient frontier (*OD′*)/length of line from origin to Jockey Plaza (*OD*) = 0.368/0.852 = 0.432.

Figure 1–4b Input technical efficiency: one input – two outputs case



1.3.3 Technical efficiency: two inputs-one output case

Now let us assume that each branch is producing the single output Y_1 (number of personal transactions) with the help of two inputs, namely X_1 (number of staff) and X_2 (size of the branch), as shown in Table 1-3a. For example, Chacarilla (*B*) completes 50,000 personal transactions with the help of 17 staff members in a branch of 80 meter square.

Table 1–3a Inputs (#2) and Output (#1) of Bank Branches and their Output-Input Ratios

Code	Branch	PT (Y_1)	Staff (X_1)	Size (X_2)	PT/Staff (Y_1/X_1)	PT/Size (Y_1/X_2)
A	Santa Anita	130	20	150	6.50	0.87
B	Chacarilla	50	17	80	2.94	0.63
C	El Polo	85	18	40	4.72	2.13
D	Jockey Plaza	25	11	45	2.27	0.56