The Power of the Line

The Power of the Line: Metaphor, Number and Material Culture in European Prehistory

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Dedicated to Dobrawa

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INTRODUCTION

In ancient Greece, the rules of mathematical thinking dominated human language and thoughts to such a degree that they gave birth to the concept of eternal, constant and transcendental mathematical laws, to which we have no complete access. Mathematics became the deepest secret, a source of the purest knowledge and perfection. Pythagoras was the father of this concept, though by Plato it found its ultimate expression. He believed that numbers and geometric blocks were the expressions of a perfect world of ideas which are the cornerstones of the universe, whereas mathematics and perceiving the truth through numbers, the ruler and compass became the basis of philosophy. According to Plato, the relationship between the world of ideas and the real world is similar to that between real objects and their shadows cast on a cave wall. It would only require turning one's head away from what catches everybody's attention in order to see the true nature of things. Mathematic formulae, numbers and rules exist independently of our imperfect beings, which are unable to perceive more than their expressions alone.

When did people learn to count? How did they do it? Did arithmetic develop along with the ability to count and how did it evolve at all? Are our mathematical abilities genetically programmed or are they a product of the culture we live in? Or maybe, as Plato suggested, mathematics exists regardless of us?

In order to answer the questions mentioned above, some clarification should be provided. For example, what is counting? If we define counting as the ability to notice changes in small sets and the ability to differentiate between singularity, duality and trinity, then it is probably an ability we inherited from our animal ancestors. The observation of the behaviour of many animal species confirms that they possess an innate ability to differentiate between small sets and even understand (subconsciously) the basic arithmetic procedures, such as addition and subtraction. These observations are also confirmed by research into brain function among people with language impairment. The fact that they can perceive basic mathematical rules suggests that mathematics exists–to a certain extent–outside language and the ability to assess small sized sets is independent of using symbolic language. This still does not mean that mathematics exists outside our experiences, as Plato saw it. However, for the author of this work it means that if mathematical abilities, similarly to language abilities, are the product of the evolution of man then we must assume that the concept of numbers did not exist in the earliest periods of the development of man and society. There had to be a period in prehistory when humans could not add, subtract or even count, even though they used language to pass on symbolic contents and express the subtleties of this world. Today we may only try to answer the question of when this happened and how long this period lasted, although on the other hand we know that we are not yet able to achieve this.

Being aware of the fact that mathematics is the modern mother of all sciences, without which we cannot imagine how man could exist in a world of societies, it is justified to experience a certain intellectual discomfort when faced with the knowledge that for a very long period of time (in fact for the major part of the history of man) people functioned perfectly well without mathematics and were able to live without the ability to count and measure. At some point in history the basis for mathematical thinking had probably been created and the concepts of numbers had been established, although they still had little in common with modern mathematics. However, the matter becomes more complicated if we assume that the threshold of being able to use mathematical abilities is operating on larger sets, abstracting the concept of numbers as well as the knowledge of arithmetic operations. There is a major mental gap between the ability of concrete counting and the concept of an abstract number, which distinguishes us from animal species. How did it come to this? Research over a long period of time in the field of mankind (anthropology, archaeology, and cognitive science) clearly suggests that the development of a material culture in prehistory was a serious contribution to the mathematization of the human mind. It seems that since man started producing more and more artefacts, developing technology and exchanging objects, the role of mathematical descriptions of reality grew. Hence, from this perspective as an archaeologist in the field of material culture in prehistoric times, I would like to present this story. It is the subject of my book.

In order to tell it I have chosen several examples, which do have their own restrictions. I do not claim the right to present a story universal for the whole human species, although my ambition was also to question that universality, or in other words–to support my views on the idea of that universality. In my opinion its cornerstone is created by the following relationship: body–language–material culture. The examples presented allow us to contemplate a less universal phenomenon. The matter of similarities and differences between Near Eastern and European culture in the period of the development of farming should be mentioned in particular. These areas are the basic sources for the material discussed in this book.

In the first chapter the theoretical issues connected with the human perception of numbers and mathematical relationships are discussed. Moreover, some choices had to be made in it. I discuss these issues from the point of view of psychology, cognitive science, philosophy and linguistics, particularly the hypothesis of the importance of metaphors in perceiving reality. Two metaphors crucial for this work have been discussed with greater attention: the collection metaphor and the measuring stick metaphor. At the end of this chapter I call upon the problem of the influence of material culture on the processes of understanding mathematics. This matter is particularly important for archaeologists who study the development of material culture as well as its relationship with man and society.

In the second chapter I present the history of the development of basic mathematical concepts in the Near East. It is exceptionally well prepared and widely discussed in the world of archaeology and is perfectly suited for illustrating the thesis concerning the leading role of material culture in forming mathematical perception. It is also useful for outlining important cultural differences between the Near Eastern and European areas in the Neolithic period as well as for presenting the question of whether it was in some way a determining factor for societies in those areas. In other words, were clay tokens–an invention originated from Near–eastern societies–also responsible for the development of mathematical abilities in Europe?

The third chapter is focused solely on Europe. In it, our area of interest is the organization of space in Neolithic and Eneolithic settlements as well as other types of structures. I elaborate on the symbolic meanings of all constructions from those periods as well as discuss the universal rules of their design. Many researchers emphasize their symbolic and metaphoric meaning. Settlements, houses and the massive constructions of that time are interpreted as a manifestation and embodiment of the cosmic order, which bears many analogies with ethnological studies. As some archaeologists believe, this "domestication" of space had to bear with it a great amount of symbolic meaning among farming communities. There are two basic elements of these structures: straight lines and circles. However, it is the line which deserves special attention. Linearity requires more thought, from its simple manifestations in the monumental form to its complex use in later megalithic structures. Afterwards, I show that linearity was not only an ephemeral symbol and a metaphor but also a practical tool in building anthropogenic spaces–the linear measure, which relates to properties of the human body. In my opinion all of this together requires a new approach. Only when we see a metaphor in the omnipresent linearity can we understand it properly in combination with the cosmologic aspects of architecture, the role of the human body as well as the concept of number. In connection with this I also discuss the problem of the so called megalithic yard and other derived Neolithic measures while proposing a new, more holistic way of understanding this issue.

The aim of the fourth chapter is to focus on the characteristic features of tool production of the European Neolithic and Eneolithic periods and its role in shaping human relations. I mostly concentrate on axes and flint blades and their relation with the fragmentation process, the *signum temporis* of that era, as well as the phenomenon which is less discussed in literature, namely the enlargement of tools. These observations show that both the fragmentation and macrolithization of tools could be two sides of the same coin, as we are faced here with a manipulation of the length of these socially important tools in different social communication contexts. I try to emphasize their symbolical role in shaping the new concept of value among the early farming societies. In my opinion this role is not without a connection with the contents of chapter three, that is with some concepts and idealizations concerning space assessment in the Neolithic period. Furthermore, if we realize that macrolithic technology was directly connected with the development of early metallurgy then we require a new descriptive language, which will place it in a field of a rationalized communication medium. However, we must be aware that in the initial stages of metallurgy, before an abstract concept of measure had been perfected, macroliths could be the only type of link between two categories of perception: they were physically measurable in the times when the weight of metal was still unknown. Many factors point to the fact that the measuring stick metaphor found a fruitful path of development in the case of macrolithic industries, introducing numerical and mathematical messages between people directly.

In the fifth chapter the further transformations of the measuring stick metaphor are discussed. These are best illustrated by means of small copper objects, e.g. beads, three examples of which are presented. The beads from Cortaillod, Geröllfingen and Colmar present clear metrological structures, which illustrate the complex manipulations of the measuring stick far surpassing the simple understanding of proportion, which was present in the case of macrolithic industries. I shall try to show that the clarity of these structures allows us to provide them in a certain "numerical grammar". Apart from the beads, I also discuss copper axes, which also seem to fall under the process of rationalization of the value of metal. Taking into consideration the latter events in the Bronze Age, the fragmentation of metal axes can be understood as a part of this process. I also discuss ceramic vessels, which complete the presented thesis in an intriguing way.

Concluding these arguments I suggest distinguishing two development paths of mathematization and numerosity in Europe and the Near East–the birthplace of farming. These two categories may be perceived as being opposites: the measuring stick metaphor *versus* the object collection metaphor. As it is known, the wealth finance economy is underlined in the European path which stands in opposition to the Near–eastern staple finance economy–a dichotomy which had been described by archaeologists. In the final chapter I have tried my best to outline the theoretical implications which result from this perspective for European prehistory.

CHAPTER ONE

THEORETICAL FOUNDATIONS

Piaget and the childhood of mankind

It has been noted in the past centuries that in many aspects phylogenesis bears a resemblance to ontogenesis. Following this path, the early stages of cultural development could be treated as humanity's childhood[1]. This idea turned out to have great consequences. However, if this is the case then can observations and studies concerning the development of the growth of children lead us to discovering new facts concerning the prehistory of mankind? This idea has indeed been introduced. On the basis of observing children, psychologists of the 20th century began building models of the development of mental abilities, which were derived from basic perceptions. The birth of mathematics was seen in the gradual process of comprehending logical operations. The discussion which began between psychologists and philosophers, such as Peano, Russel and Frege, also led to the involvement of one of the best known cognitive psychologists of that time–J. Piaget, who used it to explain the psychological basis of mathematics as a derivative of logical operations' development[2].

Let us observe that children use their bodies as calculators in their early stage of learning to count. This method, as we shall see below, is the basic rule of the quasi–mathematical register of events. Most often they stop at their fingers using them to show the amount of specific objects without mentioning the number, which they do not know. However, children are often subjected to the rigors of modern education as well as cultural and social pressure from their very early stages of development. These were also matters which interested Piaget. He was a versatile researcher. In 1918 he completed his PhD degree in biology and then focused on the subject of the minds of little children. His studies revolutionized our way of perceiving the development of human thinking.

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[1] Morgan 1877; Engels 1884.
[2] Miller 1992, 4.

According to Piaget, children travel a path from egocentrism towards sociocentrism during their personal development[3]. In his studies, both in psychological (interviews) as well as clinical (experiments) methods in interviews he often used the mixed questions strategy, interchangeably asking several standard questions and one surprising question while observing the reactions of his small interlocutors. In their answers he searched for something he called "spontaneous convictions". In the course of these studies he concluded that a process exists which is based on a gradual passing from wider and intuitional reactions to such which were expected on the level of social norms adequate to the given situations. When in the presence of older children, younger children adapted their reactions to the expectations of the older ones, while the behaviour of adults were treated as the highest authority[4].

Piaget believed that everything we know about the world, as well as how we reach this knowledge, is the result of how we act in it, how we interact with objects and how we manipulate them. He observed that children under the age of two perceive things which surround them in a literal "objectified" way. At first they behave in an exceptionally egocentric way and cannot yet see the world from another person's point of view. Only in time do they learn to do this as they live in a society. They also learn to pay attention to objects/things and develop a characteristic addiction to them. They learn to recognize things anew after they lose contact with them for some time.

At this stage children cannot make mathematical references between objects and they do not know that, for example, an apple and pear are two objects. They make quantitative evaluations between terms such as "a lot" and "not many". Direct experience dominates over logical thinking[5]. Only after their 18th month are children able to recognize an object which was absent for some time or has been placed in another context as the same one. In other words, they learn to recognize that some things, no matter what happens to them, are still the same things, thanks to which they can be defined as one of a kind and compared with other ones. Children gradually acquire abilities to organize small objects into collections which allows them to compare and understand all tables, chairs, cars, although sometimes very different from each other, as a single collection of objects similar in their nature. Hence, the ability to group items into collections is acquired culturally through learning. Piaget called this the preoperational stage. Magical and egocentric thinking dominates throughout it, which

[3] Gruber & Voneche 1977, 91–117.
[4] Gruber & Voneche 1977, 182.
[5] Miller 1992, 8.

gradually begins to fade in time, while logical thinking does not exist at all[6].

When having the ability to group objects into collections we can proceed to the next stage, which is comparing in order to evaluate which of them is smaller or bigger. These are the first steps on the path to mathematization, even if those are quite unsure. As the mathematician John Barrow reminds us, children are quite easy to fool at this stage. For example, collections which we seemingly enlarge by spreading candies appear larger in the eyes of a child[7]. Piaget also had conducted such research[8]. Determining what is larger and what is smaller is in fact quite a complicated cognitive process. Only when having greater abilities, which is the ability to count, can this problem be dealt with definitely. Only at the age of four–five does a young person achieve the ability to efficiently connect collections of different sizes with numbers, which is the numeral expression of collections. This stage has been defined by Piaget as the operational stage.

At the age of 6–7 further, more complicated abilities are acquired. Two values can be precisely evaluated and compared mentally. It is a new quality which shows that some mental objects appear in a child's mind, which it can manipulate in the same way as it manipulated objects/things. They do not have to be perceived directly (they are seen as through the eyes of the soul). This step opens the path to true mathematics. Children at the age of 9–10 are able to replace objects, and actions performed on them, with symbols as well as using them freely in their minds. Dividing and multiplying becomes possible as purely symbolic actions, detached from their object past. Algebra becomes possible.

At the end of this short introduction it should be said that Piaget was also interested in prehistory. Inspired by Lèvy–Bruhl he formed a concept according to which primitive (prehistoric) societies reached only a pre–operational level. It is said that sometimes he expressed his idea suggesting that the reconstruction of mental stages in the phylogenetical sense, which is the cognitive evolution of man, was his main research target. However, being aware that the reconstruction of this type of development in the case of researching prehistoric societies is very difficult, he devoted himself to ontogenesis[9]. Hence, Piaget's study is a good starting point for this discussion in more than one way.

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[6] Damerow 1996, 1–37.
[7] Barrow 1999, 260.
[8] Gruber & Voneche 1977, 299.
[9] Damerow 1996.

Criticism of Piaget's ideas

Piaget's development scheme, which is universal for all people, suggests that basic mathematical abilities are based on the experience of a person who manipulates objects. In other words, logical–mathematical thinking cannot be acquired *a priori* from objects in the surrounding world but from the way we manipulate them by the use of the body and how we relate to them by means of our bodies. In time, the process of manipulation underwent such an advanced process of internationalization, even dismissal, which we are not able to comprehend how we are even able to count. Hence, we learned this similarly to speaking, which does not mean, however, that the world has a verbal nature.

Nonetheless, being researchers of the past, we are obliged to assume something more. We must assume that the basic logical–mathematical mental structures, apart from individual development ontogenesis process, also develop in confrontation with the widely understood culture. At this point it is worth emphasizing the culture context, which does not occur for Piaget, or does not find any significant use. However, cognitive abilities, in connection to which mathematical abilities develop, are not independent from the influence of the social surroundings and specific space–time in which they are resolved. Following this thesis we may be tempted to connect the psychological concept with its culture–historical correlate. On the other hand, however, we may still count psychological concepts in our discussion of the past.

In time the researchers divided into two groups: those who acknowledged Piaget's theory and methodology and those who tried to deny it through experimentation. Some of the experiments showed gaps in the Pythagorean model as rudiments of mathematical thinking have been found even among infants. The research showed that infants become accustomed with small collections and perceive any manipulations performed on them from outside, as if they had some sort of mathematical precognition. The widely understood criticism made by later researchers slightly shook the theory of the numerical perception development presented by Piaget, although its assumptions generally remained unchanged[10].

The view that the concept of numbers is in some way dependant on the development of logical thinking, as Piaget suggested, was also questioned by Wittgenstein, who claimed that mathematics does not require any logical basis. It can be defined as a "motley collection of language games".

[10] Miller 1992, 12.

Wittgenstein promoted the view that the concept of numbers does not necessarily have one cognitive source and that there is no one source of mathematics[11]. He expressed it in typical fashion for him: "*teach it to us, and then you have laid its foundations.*"[12]

In such a general form this view might be accepted from the perspective of this publication under the condition that this study would last a millennia. As a matter of fact, nothing stands before understanding a philosopher's words. However, Wittgenstein's problem is very useful for outlining one of the main levels of our research. We may say that it is included in evolution itself, which occurred in his mind. The early Wittgenstein, present on the pages of the *Tractatus Logico–Philosophicus,* aimed at proving the direct link between language and reality. The main idea of this thesis connected the meaning of an affirmative sentence with its correctness so as to correlate the sentence with the facts in a sensible way. It seems that at times Wittgenstein saw gaps in his early understanding as in the later *Philosophical Investigations* he replaced this concept of meaning with the idea of the "conditions of affirmability": a sentence is sensible if certain circumstances occur which justify its utterance. And, as Kripke pointed out, one of these conditions is inter alia a place in practice of social communication, in the "stream of life"[13].

In time even *Philosophical Investigations* became the subject of a certain correlation (although, according to some interpreters sometime later[14]). Kripke, who dealt in detail with this problem, formed a conclusion which is interesting for our discussion in two ways. One of them I shall present now, the other in chapter 3. Kripke stated that Wittgenstein's view concerning the way names gain their correlation is incorrect as there is no sense of either speaking of the meaning or following the rule in an individual perspective[15]. The correlation for the names (e.g. number) which employ rules are not defined by some definite identifying marks, some specific properties of the world, to which the form relates. This problem is most graphically illustrated by the mental model of the twin Earth presented by Putnam, who wants to show that even when two people speak of the same thing it does not have to mean the same thing[16]. In such a case, this correlation indeed seems only to describe the fact that the speaker is a member of a society which uses a certain term or rule in a

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[11] Wittgenstein 1974, 297.
[12] Miller 1992, 13.
[13] Soin 2001, 166.
[14] Ibidem.
[15] Soin 2001, 167.
[16] Putnam 1973, 699–711.

specific way. In other words, the norms and rules are understood not only by the use of cognitive categories in the Cartesian or Kantian style but make sense only on the basis of some external system of meanings, passed on by means of tradition [17]. However, as Maciej Soin points out, Wittgenstein had been misunderstood by Kripke, as inside relations, relations between the rule and its use, should not oppose socialization. The rule is not something separate from use, since to understand a rule is to be able to use it. Wittgenstein was aware that it is not an individual mental experience but behaviour regularity, in time transforming into a norm, which forms the basis of language[18].

From this short overview of the voices of criticism we may thus far conclude that the situation is slightly more complicated than it would logically seem from Piaget's experiments. Perhaps children learn from representative symbolic systems before they are completely familiarized with the concept of what a number is and what are its various uses? Maybe they possess something similar to mathematical precognition? On the other hand, the tools used to represent numbers and the ways of using them in a particular society clearly influence the mathematical abilities of individuals. Moreover, we cannot forget that manipulating objects creates the basis for creating rules for speaking about them, as a result of which, at the same time, creates an initial base for socialization resulting from the criticism of Wittgenstein and Kripke. In other words, a model based on one source and one path of development does not explain much, although its main assumptions could be generally correct.

Communication and culture as sources of mathematical thinking

The transition from psychology to linguistic philosophy initiated by Wittgenstein leads us to the sphere of human communication issues. We cannot allow ourselves to present the enormous increase of knowledge concerning this subject which has occurred since Wittgenstein's time. The achievements made by Jürgen Habermas and Michael Tomasello are in my opinion particularly important for the idea presented here. Let us begin with the latter.

Tomasello's theoretical proposition is a link between evolutionism and the humanities [19]. According to him, language and the mathematical

[17] Kripke 2001, 147.
[18] Soin 2001, 174.
[19] Tomasello 1999; 2002.

abilities of man are a product of historical and ontogenetic development, which make use of the currently “available” cognitive abilities of the human mind. Some of these abilities are common for all primates and some only for humans. For example, the abilities we share with all primates are “scenes of joint attention” which could have been the initial potential for the first “verb islands”, that is concepts and words describing basic types of human activity, such as moving, picking up, passing objects, etc. Hence, Tomasello states that, from the cognitive perspective, language is more primal than man’s mathematical abilities. Because of this it is such a symbolizing display of man’s abilities, which originated directly from actions that both require attention and communication actions[20]. These in turn, as Tomasello states, originate from understanding other beings as intentional entities. However, a certain paradox is inhered in it, as language, although it is an exceptional ability, is not as exceptional as it is accredited. Jane Aitchison, who researched the prehistory of language, expresses a similar view[21].

Tomasello sees mathematical cognition as an activity which requires gaining, changing and coordinating perspectives originated from social interaction and discourse in order to develop. In the model situation which he presented, this clash of two perspectives results in finding an appropriate solution to the problem. According to him it is also by no means a coincidence that the concepts of abundance and relation characterize language structure. Creating categories and classes of objects (paradigmology) as well as relating them to each other as elements of a sequence (syntagmatics) is specific for language[22].

Habermas[23] presented a similar view and, despite the fact that he was not dealing with the process of creating abilities of mathematical comprehension of reality in the past, on the basis of the theory of communicative action and Tomasello’s research we have grounds to presume that the abilities to count and measure are strongly based in an environment of social cooperation and discourse, an environment of language communication. The tendency to avoid the misunderstandings outlined by Habermas could have played an important role in this process. Such a way of communication indeed seems typical for humans and it should not raise doubts that the ability to determine quantity and number precisely is an important element of avoiding misunderstanding in some situations. The philosopher imagined this process as a development from

[20] Tomasello 2002, 277.
[21] Aitchison 2000; 2000a.
[22] Tomasello 2002, 250.
[23] Habermas 1981; 1999.

strongly ritualized mimetical communication (with the use of gestures and mimics) to verbal communication, when the basic characteristic is the tendency to rationally reach an understanding[24].

Habermas found that human communication is most effective when seeking an agreement verbally. Language itself bears the rationality potential specific only to the human species, whereby development and actual progress becomes possible. It is suffice to say that the language model of reaching agreements would also play a significant role in forming mathematical and metrological truths. Their origin would involve mostly communicating through gestures and rituals, while further development was based on more abstract reasoning, such as the one present in modern mathematics. It is difficult to say what the end of this process looks like, as this subject is a different story, although Habermas emphasizes that man remains man as long as he retains a "primal" language model of reaching agreements in culture, along with the emotional experiences of the world[25].

The second factor for developing numerical concepts in human communication was defined by Habermas as steering media. Power, and its complex relations, is the first of them. However, in time other measurable steering media appeared, money being the best example here[26]. It is obvious that before its invention, societies had to struggle to reach mathematical claims which were necessary to use money.

Numbers and language

The concepts of the origins of mathematics presented above clearly suggest that its early stages were much more based in language. Perhaps they rather resembled the grammatical properties of language in their structure than their modern, highly abstract, equivalent. The concepts were being executed by creating specific expressions on the same basis as producing sentences in any language with an appropriately complex grammar. Research concerning children's abilities to perform these tasks seem to acknowledge that learning to count is based greatly on the development of lexical abilities[27]. While discussing the idea of man acquiring numerical systems, which had a set counting base[28], Pollmann

[24] Habermas 1999.
[25] Habermas 2003.
[26] Habermas 2002, 474.
[27] Tomasello 2002; Polmann 2003, 1–31.
[28] The base in the decimal system is the number 10.

reached the conclusion that the following elements make up the basis of this process:

1. The ability to form rhythmic utterances which had a certain hierarchic structure based on the rhythmic recursiveness of grammar.
2. The ability of words to mark specific beats.
3. The properties of the base itself, which enable the handling of the specific hierarchy of the rhythmic structure as well as the introduction of specific recursive[29] grammar for new numerical words.
4. The ability to reduce the hierarchization of the rhythmic structure.

Moreover, Pollmann pointed out that the process of learning to count/measure is also an extension of vocabulary and grammar. If we were to learn numbers by heart we would never learn to count as the sequence of symbols is infinite. During the process of learning to count we are faced with the problem of generating infinite numbers of utterances by means of a finite vocabulary. In other words, counting is identical to discovering a specific numerical grammar–a lexical structure which enables us to effectively manipulate symbolic expressions. We are not only able to produce the numerical expressions of an infinite length but also a certain order which defines the meaning. This order results from possessing a counting base. The sequence of concepts of numbers which our numerical grammar can produce are nothing more than lexical units. Hence, numerical grammar is also morphological grammar[30].

The possibilities presented in points 1 and 4 are biologically possessed by every human being. However, it is beyond doubt that point 3 mentions a certain cultural invention created in a specific time under specific life circumstances. In short, the abilities connected with counting and measuring originate from specific human capabilities of using language which has a recursive grammar, characterized by the hierarchic subordination of reality. The first rational invention in this field was the invention of the base, which became a constant element of rational numerical grammar. A certain paradox, based on the fact that the ability of such numerical grammar and vocabulary to develop was impossible during a large part of human history, is interpreted by Chomsky as a lack of ability to make use of the recursive nature of language. Following

[29] Grammar which is comprised of a finite collection of rules which make use of a finite supply of words is recursive as long as one may use it to create an infinite number of sentences. For in this case at least some of these rules have to be able to be used more than once in generating the same sentence. Hence, these rules are named recursive.

[30] Pollmann 2003, 28.

Chomsky's train of thought we can imagine that language recursiveness in the Palaeolithic was strongly attached to the narrative abilities of language communication, which means it was strongly anchored in group identity by storytelling[31]. When observing modern development from the historical perspective we can even say that the invention of the counting base, the development of mathematical concepts in the later periods may be understood as the beginning of the end of great narratives[32], or maybe even the beginning of the dehumanization of language, exemplified by computer language, which count only by means of the binary code.

Ethnological examples

In order to show the problem discussed here it is best to refer to ethnological examples. Ivanov presents some examples of Brazilian tribes, among which the concept of number can be expressed only by the use of special verbal constructions. For example, the Arara Indians, who belong to the same language group as the Pirahã, would say the following: *ma'wit ip #iy matet iagarokum–nem*, which means: "yesterday a man [in double number] caught [two] fish". Among the Kwaza, who come from the same group as the Arara, the numerals are mixed into nouns: *ka'nwã aky–'kai e'mã ele'le–tse*, which means "the car has more–than–two–wheels" (classifier)–three (=four wheels). This is of course very strange from our modern point of view, but it lets us assume that these examples illustrate the early method of expressing numbers. Moreover, users of languages described by Ivanov in fact have some problems with performing calculations and mathematical operations of an above–basic level, aiding this process with gestures. He even suggests that the development of mathematics has been much easier in languages in which numbers could be expressed as nouns just like objects, not as predicates as in the case of the Amazon languages he researched[33].

However, it is Benjamin Lee Whorf who became a classical figure in linguistic studies concerning native languages. This self–taught linguist, who attained a science degree in the field of chemistry, a researcher of the Hebrew and Aztec languages became renowned thanks to his theory concerning the influence of language on thinking. Two of its variations exist. The stronger version is based on the assumption that the language we use everyday determines our way of perceiving and understanding the

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[31] Kordys 2006.
[32] Lyotard 1979.
[33] Ivanow 2007, 190.

world, which has been described by Whorf in detail in his studies of the Hopi language, which was published after his death[34]. In it he dealt with inter alia by means of expressing time and space in the Indian language. At present, the strong version does not find so many supporters as its weak version, which simply states that the structure of a given language influences the way of thinking about things and phenomena does not determine it completely. Nevertheless, the strong version has a couple of supporters.

One of them is Peter Gordon, who stated that language may influence the way people perceive spatial relations and the properties of objects. However, as Gordon points out, none of this research proves that the structure of our language is an impassable barrier; that it does not allow perceiving that which is seen by the use of other languages[35].

The Amazon tribes remain the key object of research concerning language determinism as not only do they mix concepts of number into their utterances in an odd way but also have a very limited numerical vocabulary. Hence, the Pirahã use the following vocabulary which concerns numbers: "hói" (dropping tone = "one") and "hoi' (rising tone = "two"). Greater numbers are referred to as "baagi' or "aibai", which simply means "a lot". This system may simply be called: one–two–a lot and, as researchers presume, a similar system could have been an initial understanding of number/measure in prehistory.

Referring to Whorf's theory, Gordon wondered if the Pirahã could bend their humble mathematical abilities in order to come closer to a real counting system even by making use of language's recursive feature. After performing the appropriate research he reached a conclusion that the Pirahã did not use recursiveness in creating new complex numerical expressions. They never used expressions such as "hoi–hoi" to describe greater numbers. Instead they used gesture language, which is the so called counting on fingers, as a technique which aids verbal counting. Gordon noticed that counting on fingers was often quite inaccurate even in the case of small numbers (up to 5). Moreover, the words "hói" and "hoi", which mean "one" and "two", were not used only to describe those numbers in particular. While the word "two" referred to a collection greater than the word "one", "one" was sometimes used to describe small numbers such as 2 or 3. In connection with this fact it seemed that the Pirahã language is unable to express precise mathematical numbers (table 1.1).

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[34] Whorf 1956; 2002. The idea of linguistic determinism is called also the Sapir–Whorf hypothesis.
[35] Gordon 2004, 496–499.

Gordon claimed that societies which do not possess at least the simplest numerical system cannot differentiate sets comprised of more than 4 or 5 elements, which means they do not surpass the *subitizing* barrier which has been built in our brains through evolution. The language used by the Pirahã may be used to present and communicate in detail amounts that are less than 3 in number. Discussing this example of linguistic determinism, it is safe to say that the Pirahã's language is disproportionate to languages which have numerals, which allow the representation of precise mathematical numbers. Moreover, an interesting fact is that the Pirahã do not even possess a standard term for describing unity. Instead, the word "hói" bears the meaning of "roughly one" or "not many", which depends on the context. So does the strong version of Whorf's theory take place in this case? In fact, Gordon leans towards accounting this instance to the strong version of linguistic determinism. However, he adds that it is an exceptional case and it may even be the only one among ethnological languages[36].

Other research concerning the Pirahã language seem to only partly confirm Gordon's conclusions. This time the group of researchers focused on answering the question of whether the use of language which lacks numerals changes the way the speaker perceives the collections? Referring to Gordon's work, experiments have been conducted during which it occurred that, despite the lack of numerals, the studied natives were able to differentiate the precise sizes of the collections which contained even more than five elements, albeit with one exception. They were not able to memorize the numbers. The amount of wrong answers to the questions asked increased as time passed after the end of the experiment.

The results of the research clearly suggest that mathematical vocabulary is a cultural invention and not something universal for man. However, if words which describe numbers do not change the "external" representations of numbers and are only a type of cognitive technology which serves the ability to memorize, then perhaps something similar to mathematical precognition does exist?[37] Transforming it into a rational mathematical vocabulary would only be a matter of "extraction", as the mathematician Barrow proposes[38]. Nevertheless, the development of metrological–mathematical systems had to depend on social–economical circumstances. Societies which made intensive exchange contacts required new technologies of memorizing complex liabilities made during the exchange of goods. As the Pirahã did not perform intensive exchange, they

[36] Ibidem.
[37] Frank et al. 2008, 819–824.
[38] Barrow 1996.

did not develop the techniques required to do so and that is the reason why counting has such a marginal meaning for them.

The results of the second series of research did not confirm the Whorfian strong thesis in full, which as Gordon admitted could occur. According to them, language plays a comprehensive role allowing a sufficient codification of information concerning size, colour, spatial orientation, etc. However, in a situation when an appropriate code is underestimated or unnecessary, speakers communicate in a more descriptive, narrative, and gesticulative fashion–a way used by speakers of a language that does not possess appropriate vocabulary. Hence, colours, numbers and the vocabulary used to describe cardinal directions do not seem to change the cognitive processes which are their basis, contrary to what some researchers suggest, or if they do this is not made in a direct way. Instead, similarly as in the case of other technologies such as alphabetic scripture, the appropriate vocabulary grants its users the potential of a new–and more efficient–means of coding experiences[39].

Table. 1.1. The use of fingers and number words by the Pirahã (after Gordon 2004, reworked by the author)

Number of objects counted	Number words used	Number of fingers
1	hói (= 1)	?
2	hoí (= 2) aibaagi (= many)	2
3	hoí (= 2)	3
4	hoí (= 2) aibai (= many)	5 – 3
5	aibaagi (= many)	5
6	aibaagi (= many)	6 – 7
7	aibaagi (= many)	5 – 8
8	?	5 – 8 – 10
9	aibaagi (= many)	5 – 10
10		5

[39] Frank et al. 2008.

The problem of language determinism will probably engage researchers in the future. This matter will not be resolved in this book. Here we may only remark that the most recent studies concerning language and the development of mathematical cognition among the Pirahã tribes again seem to lean towards the initial thesis of Lee Whorf and Gordon. Caleb Everett's & Keren Madora's opinion is that the phenomena on a world scale upholds the idea that the ability to recognize precisely amounts that are greater than 3 is determined by a culturally developed concept, a mathematical vocabulary, which is not universal to all human societies[40].

Measuring or counting?

What is the difference between counting and measuring? From the cognitive point of view these are two different processes. According to Gregory Bateson, the number is not the same as quantity, as numbers are a product of counting, whereas quantity is a product of measuring. In the cognitive sense the concept of number must be manifested by the existence of a formula while the concept of quantity does not. Formulas are basic elements of the world, hence all living organisms are able to recognize them. The most obvious manifestations of formulas are simple geometric forms which can be expressed even by means of small numbers. That is why, according to Bateson, some animal species possess the ability to count small collections. The little time required by humans to recognize small numbers without counting is probably realized in this way. Bateson writes that numbers originate from the world of formulas, figures and numerical calculations and quantity belongs to the world of analogue and probabilistic calculations[41].

Let us employ a historical point of view. Is our perception of the passing of time a process of measuring or counting? Or maybe both?

The perception of astronomical changes, such as the change of daytime and seasons, the shape of the moon, etc. was probably the oldest method of measuring/counting. Natural cycles automatically become concepts which are used to describe changes in time. Also other phenomena, such as the growth of plants and animals, the change of the seasons as well as ageing and life in society (the chronology of one's experiences) were the earliest motive not to use number concepts to measure time. Archaeological data also suggests that the observation and adequate record

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[40] Everett & Madora 2012, 130–141.
[41] Bateson 1996, 72.