Four-Fermion Models in the Theory of Electro-Weak and Strong Interactions

Four-Fermion Models in the Theory of Electro-Weak and Strong Interactions

By

S. I. Kruglov

Cambridge
Scholars
Publishing

Four-Fermion Models in the Theory of Electro-Weak and Strong Interactions

By S. I. Kruglov

This book first published 2018

Cambridge Scholars Publishing

Lady Stephenson Library, Newcastle upon Tyne, NE6 2PA, UK

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

ISBN (10): 1-5275-1450-1
ISBN (13): 978-1-5275-1450-8

To Ludmila and Dzmitry

TABLE OF CONTENTS

PREFACE

There is a theoretical reason to consider a Higgs boson to be a composite particle. Thus, scalar particles such as a pion and kaon consist of quarks. In this monograph we explore the idea that a Higgs boson can be a composite particle and consists of quarks. In part I, the theoretical basis of four-fermion models is introduced. Then with the help of the path integration method, the dynamical mass generation is investigated in different four-fermion models including models with the internal symmetry groups SU(2), SU(3), SU(5), and with CP-violation. Ward-Takahashi identities and Schwinger-Dyson equations are obtained. The local SU(2)X U(1) four-fermion model with the composite Higgs boson is considered. The lepton masses and masses of W, Z bosons are formed due to the quark condensates. The Higgs boson is considered as the collective state of quarks and leptons in the model suggested. New experiments can verify the composite nature of a Higgs boson.

In part II the non-perturbative effects in strong interactions are considered. It is shown that the four-quark interaction appears naturally with the help of the gluon propagator in the infrared region. The mass formula for the sigma-meson, the Goldberger-Treiman relation and values of quark condensates are obtained. The four-quark model induced by instantons is investigated and it was proven that the current algebra is satisfied. It is shown that the four-quark models describe the region between the asymptotic freedom and quark confinement. The charge radii and electromagnetic polarizabilities of pions and nucleons are obtained within the instanton vacuum theory in good agreement with experimental data. Some quantum processes are considered in the framework of effective chiral Lagrangians. The decay of a pion into antineutrino and muon in the field of the electromagnetic wave is studied taking into account pion polarizabilities. Further developments of ideas considered may figure out the nature of quark confinement.

Toronto
March 2018
Sergey I. Kruglov

Chapter 1

1.1 Introduction

In recent years, the field theories of elementary particles with four-fermion interactions have been of great interest. This is due to the fact that in the four-fermion models the internal symmetry is violated through the self-interaction of fields. This mechanism is called dynamic symmetry breaking (DSB). The first proof of the chiral γ_5-symmetry breaking in the four-fermion models was made independently by Nambu, Jona-Lasinio [1], Vaks, Larkin [2], and Arbuzov, Tavkhelidze, Faustov [3]. They started from the Lagrangian of the form

$$\mathcal{L} = -\bar{\psi}\gamma_\mu \partial_\mu \psi + g_0 \left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2 \right], \tag{1.1}$$

where γ_μ are Dirac matrices, $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$, g_0 possesses the dimension of $(\text{mass})^{-2}$. Authors have shown that due to the restructuring of the physical vacuum in this field model, one-parameter group of γ_5-symmetry is broken. This is due to a condensate, which is connected with the appearance of a non-zero vacuum average $\langle\bar{\psi}\psi\rangle \neq 0$, and as a result, initially massless fermions acquire a mass $m = -g_0\langle\bar{\psi}\psi\rangle$. A similar phenomenon is known to occur in the theory of superconductivity, when due to the pairing of electrons, as a result of the phase transition, an energy gap appears. Such a theory in statistical physics, with degeneracy of the vacuum state, was developed by Bogolyubov in 1960 (see, e.g., [4], [5]).

Models with DSB are well-known in connection with the nature and existence of a massive scalar Higgs boson. The theory of Glashow−Weinberg−Salam (GWS) [6] - [8]), successfully predicted the gauge vector bosons $W^{\pm}$, Z, discovered by experiments [9] - [12], and does not give the exact numerical values of the mass of the Higgs particle. The known Higgs procedure [13], [14] implies the existence of the field Φ (the weak iso-doublet) of point particles with the Lagrangian of the form

$$\mathcal{L} = -\left(D_\mu \Phi\right)^+ \left(D_\mu \Phi\right) + \mu^2 \Phi^+ \Phi - \lambda \left(\Phi^+ \Phi\right)^2 , \qquad (1.2)$$

where $D_\mu = \partial_\mu - i \left(g b^a_\mu(x) t^a - (1/2) g' Y a_\mu(x) \right)$, g_0, g'_0 are constants of self-interaction of vector fields, Y is the weak hypercharge, and the fields of real vector bosons $W^{\pm}_\mu$, Z_μ, A_μ are defined through potentials of gauge fields b^a_μ ($a = 1, 2, 3$) and a_μ in the following way

$$W^{\pm}_\mu = \frac{1}{\sqrt{2}} \left(b^1_\mu \pm i b^2_\mu \right), \qquad A_\mu = \frac{g a_\mu - g' b^3_\mu}{\sqrt{g^2 + g'^2}},$$

$$Z_\mu = \frac{g' a_\mu + g b^3_\mu}{\sqrt{g^2 + g'^2}}. \qquad (1.3)$$

Scalar fields Φ also participate in the Yukawa interactions of leptons and quarks. If the parameters μ^2 and λ in Lagrangian (1.2) are positive, there is spontaneous symmetry breaking with the non-zero vacuum expectation value of the scalar iso-doublet:

$$\langle\Phi\rangle_0 = \begin{bmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{bmatrix}, \qquad v = \sqrt{\frac{\mu^2}{\lambda}}. \tag{1.4}$$

In the unitary gauge, we have

$$\Phi(x) = \begin{bmatrix} 0 \\ \frac{v+\eta(x)}{\sqrt{2}} \end{bmatrix}, \tag{1.5}$$

and the mass of the field $\eta(x)$ of the physical Higgs particle is $m = \sqrt{2}\mu$. Since the parameter μ in Lagrangian (1.2) is not fixed, then the mass of the Higgs field is uncertain, and as we know, there is the main drawback of the theory of GWS. According to current estimates the mass of the Higgs particle should be $>$ 60 GeV [15], [16]. Recent LHC results [17], [18] (see also [19], [20]) on the observation of a Higgs-like particle give the mass of 125 CeV. But the existence of an unstructured scalar particle of such a large mass can be called into a question, at least in view of the fact that all known particles of spin 0 ($\pi^{\pm}$, π^0, K, etc. mesons) have a composite quark structure. Therefore, the development of alternative theories with composite Higgs fields is of particular interest. Studying various possibilities of particle mass generation, based on

DSB without the involvement of fundamental scalar fields, remains one of the most important tasks of the field theory of electro-weak interactions. There are several approaches that lead to the spontaneous symmetry breaking of the vacuum state, other than the Higgs procedure.

In one of the first of these approaches, the Schwinger mechanism [21] is used. Schwinger showed the possibility of mass generation for the vector gauge field in two-dimension electrodynamics by shifting the poles of the transverse part of the photon propagator. This scheme has been further developed in [22] - [26] by the extension to the case of 4-dimensional space-time. In subsequent studies [27] - [31], it was attempted to use the Schwinger mechanism in electro-weak theory without the Higgs Lagrangian. It should be noted, however, that the proof of the mass generation in such schemes was based on approximate nonperturbative solutions of the Dyson−Schwinger (DS) equation. A rigorous proof of the shift of the pole of the propagators of the vector particles in such models is still lacking.

The second approach, which leads to DSB of the vacuum, is based on the assumption of the composite nature of the scalar fields, built of fermion fields. In this approach, the initial Lagrangian for the scalar Higgs particle is absent. Instead, we introduce an additional interaction of fermions, for example, by (1.1), through which the vacuum becomes unstable and the condensate appeases, $\langle\bar{\psi}\psi\rangle \neq 0$, leading to a violation of the original symmetry. Scalar (composite) fields arise in this case as collective excitations of fermion fields.

Fermions, forming the scalar fields, are treated as new particles - techniquarks. This approach is called technicolor (TC). Interaction between techniquarks can occur not only by self-interaction, but also through the interaction with technigluons, as new particles. Techniquarks and technigluons are unobserved particles with the confinement radius 10^{-47} cm [32]. The authors of this approach are Weinberg [33] and Susskind [34]. Their model leads to the fact that the $W^{\pm}$- and Z-bosons become massive, however, there is no natural mechanism of a fermion mass formation [35], [36]. Developing this direction, the authors of [37], [38], in order to overcome this difficulty, have introduced extended technicolor (ETC), which led to the unification of fermions and technifermions in one multiplet and yet - to the emergence of new difficulties (see [39] - [45]). Suffice to say that there is no evidence of the existence of hadrons consisting of techniquarks.

Therefore, the original line of research based on the introduction of additional four-fermion interaction of the type (1.1) for ordinary quarks or leptons (not techniquarks), has been further developed. Thus, in [46] - [49], a single theory was constructed of the strong and electro-weak interactions using the four-fermion Lagrangian which is invariant under a global, rather than local $SU(3)_c \otimes SU(2)_L \otimes U(1)$-group. In this scheme, not only the Higgs fields were built of fermions and anti-fermions but also gauge vector bosons $W^{\pm}_{\mu}$, Z_{μ}, A_{μ} and gluons. But as we know, there are no indications of the composite nature of the well-studied photonic A_{μ}, weak intermediate $W^{\pm}_{\mu}$, Z_{μ} and gluon fields. Moreover, there are additional difficulties in the way of the principal consideration of massless photons and gluons, as composite vector particles [50]. The possibility of building massive composite vector fields was investigated in a number of subsequent papers [51] - [58].

The development of schemes with composite electroweak and Higgs bosons, despite these difficulties, continues in recent years.

Thus, in Novozhilov's work [59] composite $W^{\pm}$, Z and Higgs fields are constructed from "pre-gluons" and "pre-fermions" that are subject to the dynamics such as quantum chromo-dynamics (QCD). The "pre-fermions" are massless, and they satisfy the property of confinement. This approach can be seen as a kind of variation of the TC approach.

The electro-weak gauge-invariant model was considered without the Higgs field in [60], but with an additional "Abelian" vector boson C_{μ} with a mass M. This uses the ability to generate fermion masses based on solutions of

DS equations. Neutrinos are massless in this approach, and the intermediate bosons $W_\mu^\pm$, Z_μ acquire the finite masses. With some attractive features, the model still does not seem "aesthetically better" than the standard theory of GWS, because it does not reduce the number of independent degrees of freedom (instead of the scalar Higgs field, we introduce an additional vector field C_μ), and there are the difficulties of the interpretation and the possible existence of the field C_μ. A similar approach was also proposed in [61].

In the paper [62] Higgs particles are constructed from $\bar{t}$, t - quarks. To implement the DSB mechanism, the four-fermion interaction is introduced, including t-quarks, and masses of composite Higgs scalars are obtained, $m_H = 2m_t$. It is known that the analysis of experimental data on a large $\bar{B}_d^0 - B_d^0$ - mixing and CP violation (see [63]) gives the lower bound for the t quark as follows: 78 GeV. In the work [62] t quark mass is obtained from the solution of an equation, $m_t = 84$ GeV. According to the current estimates, the mass of the t quark is $m_t = 174$ GeV, so that the mass of the composite Higgs particle is predicted to be $2m_t = 348$ GeV. In this approach, however, W-bosons are also composite. Their mass is generated by the Schwinger mechanism [21], $m_W = 80$ GeV. The realism of the model will show the further checking of the composite nature of the W-boson and the discovery of the Higgs boson of such mass.

A similar approach is also developed in the works [64], [65], where the masses of W and Z bosons are generated by condensation of t-quarks. This uses the four-fermion interaction in the quark sector and it is shown (see [64]), that the theory is asymptotically free, and the composite operator $\bar{\psi}\psi$ has a large anomalous dimension $\gamma_m = 2$. The importance was specified [64] of checking the phase diagrams for the four-fermion schemes in the lattice approximation using computers.

There are other papers [66] - [71], which also develop an approach that is associated with the composite nature of the Higgs particle.

Note that in the earlier work [72] it is indicated that the mass of the weak bosons W, Z can be obtained by the dynamic Higgs mechanism (without fundamental scalars) if there are heavy quarks or leptons with masses of the order of $30 - 100$ GeV. A similar result was obtained in [23].

Four-fermion field models, along with their use in the theory of electro-weak interactions, in recent years, are also used in the study of QCD at its low energy limit. We can say that a new level revives Heisenberg's ideas, but instead of "fore-matter," the quark fields are introduced.

Heisenberg (see [73]) made the first attempt to use a four-fermion model for a unified theory of elementary particles. The underlying approach to the concept of "fore-matter" is described by the nonlinear spinor equation. The excited states of a nonlinear spinor field were treated with strongly interacting particles - hadrons. This approach, as we know, has been studied initially in the development of the nonlinear meson theory and the theory of nuclear forces [74].

Recently, the attention of physicists was attracted by various nonlinear equations admitting particle-type solutions - solitons. This has still not diminished interest in obtaining exact solutions of nonlinear spinor equations [75], [76]. Baryons were considered as non-topological chiral solitons in the framework of the NJL model in [77].

The approach of Volkov with colleagues [78] - [91] (see also [92]) is based on a four-quark interaction Lagrangian of the type (1.1), but with given properties of the internal symmetry. On the basis of that postulated and used to describe the low-energy hadron physics Lagrangians, many characteristics were calculated: meson masses, decay widths, the scattering cross-sections and so on, which turned out to be in good agreement with the experimental values. In addition, this approach produces the vector dominance model, well-proven in practice.

A similar approach is used in [93] - [96], which also postulates a four-fermion Lagrangian, from which the effective chiral Lagrangian is derived. The latter, in particular, contains a topological Wess–Zumino [97] and Skyrme's terms [98], [99]. The corresponding coefficients of the Lagrangian calculated lead to the observed values, which are in good agreement with the phenomenology of [100] - [105]. The resulting effective Lagrangian reproduces the soft-pion theorem, the Goldberger–Treiman relation, PCAC and other well-tested relations [106] - [108]. The properties of dense and hot baryonic matter within the NJL model were investigated in [109] - [118]. We note the works [119], [120], which dealt with four-fermion Lagrangians to describe non-leptonic kaon decays $K \to 2\pi$, $K \to 3\pi$ strangeness-changing $|\Delta S| = 1$ and the change in isospin $|\Delta T| = 1/2, 3/2$. Along the way, however, we note that in the theory there are still some difficulties in explaining the increase of transitions with $|\Delta T| = 1/2$ [121] - [125]. An approach based on the Wilson lattice action for gauge fields and fermions in QCD is considered in [126] - [128]. When some assumptions in the low-energy physics were made, the authors come to the contact interaction of the four-fermion interaction with the effective Lagrangian containing Wess–Zumino's and generalized Skyrme's terms.

The effective Lagrangian directly derived from the fundamental QCD Lagrangian is very important, as it is now generally recognized that the QCD is a theory of the strong interactions of quarks and gluons. However, as you know, a reformulation of QCD in terms of hadrons as bound states of quarks possesses serious mathematical difficulties. One reason is the impossibility of functional integration over the gluon fields in the generating function for the Green functions as the corresponding path integral is not Gaussian because of the self-interaction of gluons.

Currently, intensive development of this field is running. In the papers [129], [130] (see also [131], [132]), based on simplifying assumptions of QCD at low energies, we obtain the effective Lagrangian for pseudoscalar meson nonet, comprising Wess−Zumino's interaction. The central point of this work is the consideration of the axial anomaly [133] - [148]. The integration of the chiral anomaly using methods of differential geometry was considered in [140] - [143].

Accounting anomalies lead to violation of U_1-symmetry [144], [145], which can help to solve the "old" U_1-problem formulated by Weinberg in 1974, the great mass difference of $\eta(549)$ and $\eta'(958)$ mesons (see reviews [146] - [149]).

In some other way, the chiral Lagrangian was obtained by Andrianov and Novozhilov [150] - [154]. This Lagrangian describes correctly the $\pi\pi$-scattering [130], [150]. Important components of the chiral Lagrangian are the term Wess−Zumino [97] (see also [155]), which describes the decay of $\pi^0 \to 2\gamma$ and is associated with the Adler anomaly.

In the Karchev and Slavnov work [156] the nonlinear chiral Lagrangian is obtained as a low-energy approximation of QCD at large N_c (N_c is the number of quark colors) under the assumption of chiral symmetry breaking. One of the first studies where a computational scheme has been developed for large N_c is the work of 't Hooft [157].

Note also the works of [158] - [163], where using a variety of mathematical techniques, the authors derive the effective chiral Lagrangians directly from the fundamental QCD Lagrangian.

Thus, the effective chiral Lagrangians describe well the low-energy hadron physics and the ability to dynamically implement the current algebra [164] - [168]. Under this approach, the nonlinear chiral Lagrangian containing the pion field, leads to the existence of stable soliton solutions, which describe the nucleon. By the way, the chiral Lagrangian, which leads to a stable soliton, interpreted as a nucleon, was proposed long ago by Skyrme [98], [99]. This ideology was then developed in [169], [170] (see also reviews [171] - [173]). Today, we know that the quantum numbers of the chiral soliton determine the above Wess−Zumino term. In this case, the baryon charge of the nucleon is treated as a topological charge of the soliton, and Δ-resonances arise here as rotational states of the quantum soliton.

It should be noted that the Skyrme model was postulated independently of the QCD, and is therefore an approximation to the true theory. Chiral Lagrangians, obtained from QCD besides Skyrme members also contain higher derivatives of the chiral pion field. Therefore, the observed values, calculated from a consistent theory, will, in general, differ from the values obtained from the Skyrme model.

In the papers [100] - [102], [174] - [185] the chiral Lagrangians of the general form are constructed having a large number of parameters, which are then determined from a comparison with the experimental values of the decay widths and the scattering cross-sections of mesons. This approach should also recognize the model, which does not follow directly from QCD.

Instantons play a very important role in the nonperturbative QCD, i.e., at low energies, when not to use perturbation theory [186]. Instantons are associated with an infinite number of sub-barrier transition oscillators from one state to another and can be obtained in pure gluodynamics as solutions of equations of motion in Euclidean space-time. It was shown in papers of Shuryak [187] - [189], Diakonov and Petrov [190] - [192] that the true vacuum of QCD can be considered as instantons and anti-instantons gas. This gas is in equilibrium with the average size of instantons $\bar{\rho} \simeq 0.3$ fm and the average distance between the centers of instantons is $\bar{R} \simeq 1$ fm.

The introduction of instantons leads to the challenging problem of the origin of the gluon condensate, the formation of which is associated with the finiteness of the action that is functional for the instanton (anti-instanton) and short-wavelength fluctuations of the gluon field. Instantons at large distances attract and repel each other at short distances, so that the gas-phase environment is stable [190]. When the quarks are present in the instanton medium, the chiral pairs $\bar{\psi}_R\psi_L$ and $\bar{\psi}_L\psi_R$ ($\psi_L = \frac{1}{2}(1+\gamma_5)\psi$, $\psi_R = \frac{1}{2}(1-\gamma_5)\psi$) are created. As a result, at the finite density of the instanton gas, the chiral condensate $\langle\bar{\psi}\psi\rangle \neq 0$ will occur, which leads to the DSB.

Thus, instantons lead to the axial anomaly needed to solve the U_1-problem, and break chiral symmetry.

Note that the concept of instantons still does not provide an explanation of quark confinement, as the latter is not associated with short- and long-wavelength fluctuations. These types of fluctuations are usually taken into account, attracting the bag model [193]. However, it is well known that the confinement problem of the quarks is still not solved in the framework of nonperturbative QCD.

The complicated structure of the QCD vacuum and the vacuum fluctuations are pointed out in the Novikov, Shifman, Vainshtein and Zakharov works [194] - [196].

In the papers [197], [198] it is shown that the inclusion of the small size instantons leads to the quark interaction which is described by the Lagrangian of the form

$$\mathcal{L}_{det} = \lambda \det\left(\bar{\psi}_R^i \psi_L^j\right) + c.c., \tag{1.6}$$

where ψ^i is the quark field of flavor i ($i = 1, 2, ...N$), c.c. means the complex conjugate of the expression, λ is a constant, which is related to the density of instantons.

The role of interactions (1.6) for the low-energy hadron physics is noted in [199] - [207].

Considering only the u, d-quarks ($\psi^1 = u$, $\psi^2 = d$), the Lagrangian (1.6) can be written as [208], [209]

$$\mathcal{L}_{det} = \frac{\lambda}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}\gamma_5\psi\right)^2 - \left(\bar{\psi}\tau^a\psi\right)^2 - \left(\bar{\psi}\gamma_5\tau^a\psi\right)^2\right], \tag{1.7}$$

where τ^a are Pauli matrices. In (1.6), (1.7), a summation over the color degrees of freedom of quarks ($N_c = 3$) is implied.

Thus, we arrive again at the four-fermion interaction. It is important to note that the Lagrangian $\mathcal{L}_{det}$ in the general case is invariant under the group $SU(N_f) \otimes SU(N_f)$, but breaks the $U_A(1)$ symmetry, in contrast to the NJL-type of Lagrangians (1.1). This is a consequence of the axial anomaly.

It is supposed in [210] - [217] that quark flavor dynamics are described at medium energies by the interaction

Lagrangian

$$\mathcal{L}_{int} = \mathcal{L}_{NJL} + \mathcal{L}_{det}, \tag{1.8}$$

where

$$\begin{aligned}\mathcal{L}_{NJL} = G_1 \sum_{\alpha=0}^{N_f-1} \left[\left(\bar{\psi} \frac{1}{2} \lambda^\alpha \psi \right)^2 + \left(\bar{\psi} \frac{1}{2} \lambda^\alpha i \gamma_5 \psi \right)^2 \right] \\ + G_2 \sum_{\alpha=0}^{N_f-1} \left[\left(\bar{\psi} \frac{1}{2} \lambda^\alpha \gamma_\mu \psi \right)^2 + \left(\bar{\psi} \frac{1}{2} \lambda^\alpha i \gamma_5 \gamma_\mu \psi \right)^2 \right].\end{aligned} \tag{1.9}$$

Here λ^a are the Gell-Mann matrices acting in the color space.

Authors of papers [210], [211] suggest that the introduction of the interaction with the Lagrangian of NJL type (1.9) is necessary to account for the exchange of heavy gluons, and this interaction is expected to be dominant. In these works, bosonization of the Lagrangian (1.8) is carried in the general case of an arbitrary number of flavors.

Calculated on the basis of (1.8) the mass spectrum of light mesons and some structural properties of mesons received [210] - [218] are in good agreement with the experiment. This suggests, that both, the instanton effect (taking into account the axial anomaly of QCD) and the four-fermion interaction (due to gluon exchange, which has a larger symmetry group $U(N_f) \otimes U(N_f)$) contribute to the low-energy physics of mesons.

The properties of hadrons were discussed in the literature, based only on the NJL Lagrangian (see [78] - [237]), which gave good description of the low-energy physics of mesons. Solitons arising in the NJL model (1.9) were studied by Alkofer and Reinhardt [238] - [243]. It has been shown that this approach approximately mimics the Skyrme model.

There are also investigations [244] - [255], to study models of type (1.6), (1.9) at the finite density of matter and temperatures. The chiral symmetry restoration at a critical temperature, the density of the matter and the external electromagnetic field were found. The results obtained can be used in the study of the theory of nuclear matter.

An important role in physics of electro-weak interactions is played by weak instantons (relevant fields are $W^{\pm}$ bosons). As shown by ‘t Hooft (see [256] and references therein), in GWS theory, there are processes with non-conservation of baryon and lepton charges. At energies of 15 TeV, cross-sections (due to many-fermion interactions (lepton-quark)), due to weak instantons, can reach the observed values. Now there is intensive work on the calculation of the function $F(\varepsilon)$, which determines the dependence of the total cross-sections in violation of the baryon number (VBN) on the energy. The knowledge of this function would appreciate the value of the energy at which processes can be observed with the VBN on the accelerator SSC ($\sqrt{s} = 40$ TeV).

Various aspects of the models with four-fermion interactions are considered also in [257] - [297]. In [298] Fukushima included the Polyakov loop in the NJL model. In this model (PNJL) the confinement of quarks holds in the NJL model via the Polyakov loop. The phase diagrams of strongly interacting matter were obtained [299]-[301]. Recent studies within the PNJL model are [302]-[303].

Thus, the four-fermion interactions play an important role in the theory of electro-weak and strong interactions of elementary particles. Hence, in particular, the further development of models of the NJL type is important. In the way of learning and using the NJL model, despite the progress and above-mentioned success, there are still many unsolved problems.

In the area of electro-weak interactions, first of all, the construction of such a model, while maintaining the fundamental property of local gauge invariance, would disclose a mechanism of mass generation of leptons and quarks, and intermediate vector bosons. This scheme, in contrast to the theory of GWS should not contain as source fields, fundamental scalar fields. This is due to natural reasons of simplicity and a minimal number of inputs to the theory of the fundamental fields. But here the scalar Higgs field can also occur, but as a collective excitation of the initial fields of quarks and leptons.

The above brief analysis of the situation in the field of non-perturbative QCD shows that there is an actual derivation of the effective low-energy Lagrangian, which would be based on the use of known solutions of the DS equations for the propagator of the gluon fields. The proof of the chiral symmetry breaking is very important. Of great interest is the study of the pseudoscalar bosons in the framework of this approach.

In addition to solving these global problems, it is obviously necessary, along with the study and calculation of the nonperturbative properties of hadrons in the well-known model of the instanton vacuum, to perform calculations for specific processes on the basis of phenomenological chiral Lagrangians by the known limitations of the application of the concept of the instanton vacuum in the range of energy above 600 MeV. Only within the framework of nonperturbative QCD, can we calculate the following characteristics of hadrons: form factors and polarizabilities.

The main objectives of this work are as follows:

1) to study the general properties of quantum field theory, taking into account the four-fermion interaction of different groups of internal symmetry;

2) the construction and study of the local $SU(2)_L \otimes U(1)$-invariant model of the electro-weak interaction with the four-fermion link without fundamental Higgs bosons;