### Energy Losses in Big Woodworking Machines

### Energy Losses in Big Woodworking Machines:

Analysis and Optimization

Ву

Boycho Marinov

Cambridge Scholars Publishing



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By Boycho Marinov

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### INTRODUCTION

Woodworking machines are a certain class of machines that are used in the woodworking industry. The study of these machines can be done with the help of different types of schemes. According to the structural schemes, each woodworking machine is made up of three main mechanisms: the motor mechanism, the transmission mechanism and the actuating mechanism. The actuating mechanisms provide the main and auxiliary motions of the machine. The main motions are performed by cutting mechanism and feeding mechanism. Auxiliary motions include starting, stopping, automatic control, etc.

Circular saw machines are widely used in packaging, furniture and plywood production. Their wide application is due to a number of their advantages such as simple construction, high productivity, easy operation, easy maintenance, relatively easy service, quality cutting and more. Due to the greater width of the cut, which they make and respectively the higher consumption of wood, circular saw machines are being replaced by band saw machines, but only in some technological processes. In most technological processes, these machines continue to find their application. General characteristic of circular saw machines is made in the technical literature, (Filipov, 1977, 167-168), (Obreshkov, 1996, 3). These machines are used for longitudinal and cross sawing, as well as for angular sawing of the wooden material. They are used for the processing of logs, prisms, boards, shutters and sheet materials such as plywood and particle board, as well as for the processing of furniture plates and furniture units.

According to the technological processes, the circular saw machines are divided into four main types:

- circular saw machines for longitudinal sawing;
- circular saw machines for cross sawing;
- circular saw machines for cutting up sheet materials;
- circular saw machines for format sawing of sheet materials and particle board.

Circular saw machines for longitudinal sawing are used to process the workpiece by sawing parallel to the wood fibers. Depending on their purpose, they are divided into machines for logs, for longitudinal sawing of boards and other purposes. According to the number of circular saw blades, they are divided into machines with one circular saw blade, two circular saw blades and three or more circular saw blades. According to the type of feeding mechanism, they are divided into machines with manual feed, with timber trolley, with chain feed, with roller feed and others.

Circular saw machines for cross sawing are used to roughly and accurately saw wooden workpieces across the wood fibers. Depending on the type of feeding motion, they are divided into:

- circular saw machines where the feeding motion is performed by the circular saw blade;
- circular saw machines where the feeding motion is performed by the treated object.

The circular saw machines of the first type depending on the trajectory of the circular disk are divided into:

- circular saw machines in which the feeding motion is performed on an arc trajectory;
- circular saw machines in which the feeding motion is performed on a rectilinear trajectory.

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Circular saw machines for cutting up sheet materials and for format sawing of sheet materials and particle board are specialized machines. They find a more limited application in the woodworking industry and mainly are used for sawing sheet materials such as plywood, chipboard and others. The purpose of this production is to obtain workpieces with a rectangular shape and a smaller size or for the production of particle board with standard dimensions.

Band saw machines are presented in the technical literature as one of the most common woodworking machines. They cut the workpiece through an endless band saw blade, drived by two leading wheels. The cutting part of the band saw blade is the rectilinear part that moves from top to bottom. The feeding motion is perpendicular to the cutting motion and in all cases it is made by the workpiece.

The band saw machines process the workpiece by cutting parallel to the wood fibers and very rarely by cutting across the fibers. These machines are mostly used for rectilinear sawing, but in some cases, curvilinear contours can also be processed. They have a number of advantages over other woodworking machines, which is why they are widely used in the woodworking industry.

The use of band saw machines in various proceedings as well as in various technological operations has led to a variety of constructions and dimensions. They can be classified into three groups based on a number of general principles (Filipov, 1977, 121), (Obreshkov, 1995, 130-133):

- band saw machines for sawing logs. They are also called log band saw. The diameters of the leading wheels are between 1100 and 3500 mm and the width of the band saw blade is from 140 to 360 mm;
- deal band saw machines. These machines have big performance. The
  diameters of the leading wheels are between 1000 and 1500 mm and
  the width of the band saw blade is from 70 to 175 mm;

 ordinary band saw machines. The diameters of the leading wheels are between 400 and 1000 mm and the width of the band saw blade is up to 40 mm.

Band saw machines for sawing logs are used in the production of boards, beams, prisms and other shaped and semi-shaped materials. They, in turn, according to technological and design features are divided into vertical, inclined, single or paired, with a feeding trolley, with a carriage or chain, horizontal with a carriage and mobile horizontal band saw machines.

Deal band saw machines are used for sawing shutters, quarters of logs, short logs, etc. They are also used for sawing prisms, thick boards and covers. The material to be treated usually has at least one pre-formed base surface. These machines are usually divided into vertical and horizontal. According to the feeding of the workpiece, they are divided into deal band saw machines with chain feed, roller feed and carriage feed.

Ordinary band saw machines are used for sawing wood particle boards, for sawing wood fiber boards and for curvilinear sawing of surfaces. These machines are widely used in the packaging industry for sawing packaging boards, as well as for the production of parts used in the furniture industry.

In this book the author presents his studies on certain classes of woodworking machines. The dynamic processes that arise in operating mode in two classes of woodworking machines are analysed. In particular, the author explores the dynamic processes that occur in big circular saw machines and big band saw machines.

The final purpose of this study is to obtain analytical expressions and computer calculations that describe the losses of mechanical and, respectively, electrical energy of both classes of woodworking machines. Optimization solutions to reduce energy losses should also be proposed. This purpose is achieved by solving the following main tasks:

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- analysis of static and dynamic loads that occur in operating mode;
- obtaining analytical expressions and computer solutions for calculating the full dynamic reactions in the main links of the machines;
- obtaining analytical expressions and computer solutions that describe the static deformations, dynamic deformations and transverse vibrations of the main links of these machines;
- performing deformation checks to ensure the normal and safe operation of both classes of machines;
- with the help of these expressions, the main purpose of these researches should be solved, namely, obtaining analytical expressions and computer calculations with which to determine the energy losses for both classes of woodworking machines.

### CHAPTER ONE

# FULL DYNAMIC REACTIONS IN THE BEARING SUPPORTS OF BIG WOODWORKING MACHINES

## 1. Static and Dynamic Processes in Big Circular Saw Machines

Big circular saw machines form a specific class of woodworking machines. They are used for sawing different types of wooden material. In order to saw logs, timber sleepers and beams, circular saw blades of larger diameters need to be used. The dimensions of the diameters vary within the following limits:  $1000 \le D \le 1500$  [mm]. The circular saw blades are manufactured with inaccuracies. The centre of mass of the disc is displaced from the axis of rotation at a distance of e (eccentricity) and the axis of the disk makes an angle e0 with the axis of rotation.

These machines have a high productivity but they find limited application in the woodworking industry because the losses of timber are bigger. This is due to the width of sawing being bigger in comparison to the width of sawing of the big band saw machines. For example, the width of sawing for big circular saw machines is  $5 \le b \le 8[mm]$ , the width of sawing for big band saw machines is  $2 \le b \le 3[mm]$ . Larger resistances that occur in operating mode require the use of more powerful electric motors. However, this leads to higher energy consumption, which limits the use of this class of machines.

The large diameters of the circular saw blades and the large angular velocity of the circular saw shaft determine the large cutting speeds. As a

result of these speeds, circular saw machines can lose stability of motion. There are studies in this area (Zhang et al. 2014, 32-35). To avoid this phenomenon, special rolls are used. They are made of anti-friction material and are placed on both sides of the circular saw blades. Thus, these rolls prevent the blades from deviating from the vertical plane.

The logs for sawing must be fixed rigidly to the board during the working process to obtain a quality processing of the material. For this purpose, they are fastened on the timber trolley using special hooks. Figure 1.1 shows the scheme of such circular saw machines (Filipov, 1977, 175), (Obreshkov, 1996, 12).

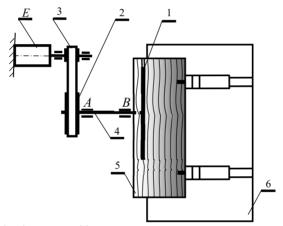


Fig. 1.1. Big circular saw machine

We define the following symbols: *E*-electric motor, 1-circular saw blade, 2 and 3-belt pulleys, 4-circular saw shaft, 5-workpiece (logs) and 6-timber trolley.

The electric motor E drives the circular saw shaft by means of a belt drive, which consists of belt pulleys 2 and 3. At the end of the shaft is mounted the circular saw blade 1, which performs a rotational motion. This is the motion of sawing or so-called main motion. The feeding motion is performed by the timber trolley 6 on which the workpiece 5 is fixed.

### 2. Static and Dynamic Reactions in the Bearing Supports of the Circular Saw Shafts

In this study, the static and dynamic reactions that load up the bearing supports of the circular saw shafts are examined. The static reactions arise as a result of the external loads. We examine the forces and moments that load up the circular saw blade and the belt pulleys separately. We also analyse the dynamic reactions that depend on the mass and kinematic characteristics of the rotating body (angular velocities and accelerations, masses, mass moments of inertia and angles of rotation). Using these correlations, we can obtain expressions for the full dynamic reactions. Taking advantage of these expressions, we can choose different machine parameter values in a way that will cause the load in the shafts and the bearings during operating mode to be minimal.

In order to solve the given problems, we use the dynamical model shown in Fig. 1.2 (Marinov, 2018, 59). The following symbols are defined: 1-circular saw blade, 2-belt pulley, 4-circular saw shaft, A and B-supports. The full dynamic reactions  $A_x$ ,  $A_y$ ,  $B_x$ , and  $B_y$  in the supports of circular saw shaft as a consequence of the external load, kinematic and mass characteristics of the rotating disk are shown in the figure. The geometric dimensions  $a_1$ ,  $b_1$ , and  $c_1$  are also shown in the figure.

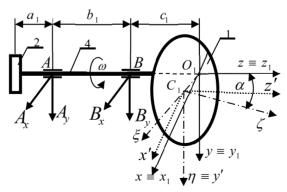


Fig. 1.2. Dynamical model

We chose the following coordinate systems: Fixed coordinate system  $O_ixyz$  and moving coordinate system  $O_ix_iy_iz_i$ , which moves along with the circular saw blade. In the initial moment  $(\varphi = 0)$ , the axes of the two coordinate systems coincide. Coordinate system  $C_ix_iy_iz_i$ , beginning at the centre of mass  $C_i$ . Its axes are parallel to the axes of the moving coordinate system. We use fourth coordinate system  $C_i\xi\eta\zeta$ . The axes of this coordinate system are principal axes of inertia of the disk. The centre of mass  $C_i$  of the circular saw blade 1 is displaced from the axis of rotation AB = z of an eccentricity  $e = O_iC_i$  and describes a circle with a radius  $\rho_{C_1} = e\cos\alpha$  around this axis. The axis  $C_i\zeta$  of the disk makes an angle  $\alpha$  with the axis of rotation. The circular saw blade 1 and belt pulley 2 perform rotations with constant angular velocity  $\omega$  about an axis AB and describe an angle  $\varphi = \omega t$ .

#### 2.1. Static Reactions Depending on the External Load

We considered the forces that load up the circular saw blade 1 and the belt pulley 2 by not taking into account weight of the belt pulley. These forces are reduced to the shaft axis.

#### 2.1.1. Circular Saw Blade

Figure 1.3 shows the circular saw blade and the corresponding load.

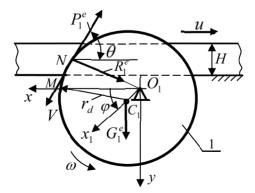


Fig. 1.3. Circular saw blade 1- external load

There are various studies to determine the cutting forces. (Porankiewicz, Goli. 2014, 204), (Krenke, Frybort, Müller. 2017, 437-448), (Marchal, Mothe, Denaud, Thibaut, Bleron. 2009, 158-165), (Kopecký, Hlaskova, Orlowski, 2014, 828-830), (Orlowski, Ochrymiuk, Atkins, Chuchala, 2013, 951-954), (Orlowski, Ochrymiuk, 2017, 150-152).

The tangential force  $P_1^e$  and the radial force  $R_1^e$  in the literature are usually calculated using the following expressions (Filipov, 1977, 184), (Porankiewicz, Bermudez, Tanaka, 2007, 671-672), (Porankiewicz, Axelsson, Grônlund, Marklund. 2011. 3687-3688), (Obreshkov, 1996, 17).

$$P_1^e = \frac{K_{\Delta(\lambda)}bHu}{V} , \quad R_1^e = mP_1^e ,$$
 (1.1)

where  $K_{\Delta(\lambda)}$  is the specific work of the cutting. It can be calculated by different expressions according to different factors. The most used expressions are listed below (Gochev. 2005. 106). The index ( $\Delta$ ) is used for swage-set teeth and the index ( $\lambda$ ) is used for part-set teeth.

$$K_{\Delta} = k + \frac{a_{\rho}p}{u_z \sin \Theta_a} + \frac{\alpha_{\Delta}H}{b}, \qquad K_{\lambda} = k + \frac{a_{\rho}ps}{bu_z \sin \Theta_a} + \frac{\alpha_{\lambda}H}{b},$$
 (1.2)

where k is a fictitious pressure on the front side of the teeth, p is a fictitious specific force on the back side of the teeth. They can be calculated using different formulas (Gochev, 2005, 106).

- coniferous wood  $k = 10^{6} \left[ (11.24 + 12.9\Theta_{a})\delta + (0.0687 + 0.0843\Theta_{a})V (5.4 + 8.43\Theta_{a}) \right],$   $p = 3920 + 2020\Theta_{a},$
- deciduous wood  $k = 10^6 [(15.75 + 19.3\Theta_a)\delta + (0.0883 + 0.1124\Theta_a)V (7.46 + 11.24\Theta_a)],$   $p = 4900 + 2580 \Theta_a,$

where  $\delta$  is the cutting angle. Values for this angle can be obtained from the technical literature.

The coefficient of blunt teeth is denoted by  $a_{\rho}$ . The values of this coefficient are changed in the following range:  $1.4 \le a_{\rho} \le 2.8$ . The thickness of the circular saw blade is marked with s. The friction intensity of the shavings on the sawing walls is marked with  $\alpha_{\Delta}$  and  $\alpha_{\lambda}$ ,  $\alpha_{\Delta} = 589.10^3$  [Pa] and  $\alpha_{\lambda} = 736.10^3$  [Pa].  $\Theta_{\alpha}$  is the average kinematic angle. This angle can be calculated from the following relation (Gochev, 2005, 105):

$$\Theta_a = \arccos \frac{2a + H}{D},\tag{1.3}$$

where  $D=2r_d$  is the diameter of the circular saw blade, a is the distance from the centre of the shaft to the broad board of the machine or to the upper surface of the workpiece.

The feeding of one tooth is marked with  $u_z$ . This feeding can be calculated depending on various factors. We use formulas that take into account the feeding of one tooth according to the power of the electric motor to sawing  $N_e$  (Gochev. 2005. 105))

$$u_{z\Delta} = \frac{\frac{N_e \eta_p}{zHn} - \frac{a_\rho pb}{\sin \Theta_a}}{kb + \alpha_\Delta H}, \quad u_{z\lambda} = \frac{\frac{N_e \eta_p}{zHn} - \frac{a_\rho ps}{\sin \Theta_a}}{kb + \alpha_\Delta H}, \quad (1.4)$$

where n is the rotational frequency of the circular saw blade. The number of teeth of the circular saw blade is designated with z. The thickness of the workpiece or sawing height is marked with H, b is width of the sawing, V is the cutting speed and u is the feeding speed. m is a coefficient, which depends on the state of the circular saw blades. The values for this coefficient that we use for practical calculations change within the following limits  $0 \le m \le 1$ . These values can be expanded for more accurate calculations, but we need to determine the circular saw

condition more accurately.  $\eta_p$  is the efficiency of the transmission. The cutting speed V is determined from the next dependence  $V = \omega O_1 N$ . Taking into account the fact that the eccentricity  $e << r_d$ , we may assume  $O_1 N \approx r_d$ . Thus, the cutting speed can be calculated from the following expression:

$$V = \omega r_d . ag{1.5}$$

The tangential force  $P_1^e$  generates the moment  $M_1^e = P_1^e O_1 N$ . Taking into account the fact that  $O_1 N \approx r_d$ , we can record

$$M_1^e = P_1^e r_d \,. \tag{1.6}$$

By projecting expression (1.1), we receive the components of the forces along the axes  $\mathcal X$  and  $\mathcal Y$ .

$$\begin{split} P_{1x}^{e} &= P_{1}^{e} \cos \theta \,, \quad P_{1y}^{e} = P_{1}^{e} \sin \theta \,, \\ R_{1x}^{e} &= m P_{1}^{e} \sin \theta \,, \quad R_{1y}^{e} = m P_{1}^{e} \cos \theta \,. \end{split} \tag{1.7}$$

The angle  $\theta$  is marked in Fig.1.3. This angle is the angle between the tangential axis through point N and a horizontal axis parallel to the x-axis. We can assume  $\Theta_a = \theta$ , where  $\Theta_a$  is the average kinematic angle. This angle can be calculated from the expression (1.3).

We accept that  $e\sin\alpha\approx 0$  because the eccentricity e and the angle  $\alpha$  have very small values. Due to this fact, we assume that point  $C_1$  lies on the axis  $x_1$ . In fact, this point is displaced at a distance of  $e\sin\alpha$ 

#### 2.1.2. Belt Pulley

Figure 1.4 shows the belt pulley 2 and the corresponding external load.

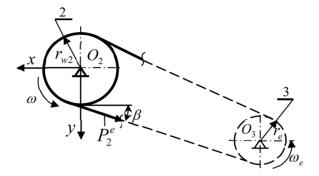


Fig. 1.4. Belt pulley 2 - external load

The tangential force  $P_2^e$  generates the moment  $M_2^e$ , i.e.

$$M_2^e = P_2^e r_{w2}. (1.8)$$

This moment is equal to the moment  $M_1^e$ , defined in the expression (1.6). We can calculate the force  $P_2^e$  from the equality of these moments.

$$P_2^e = P_1^e \frac{r_d}{r_{w2}}. (1.9)$$

The projections of this force along the x and y-axes are presented below.

$$P_{2x}^{e} = P_{2}^{e} \cos \beta = P_{1}^{e} \frac{r_{d}}{r_{w2}} \cos \beta,$$

$$P_{2y}^{e} = P_{2}^{e} \sin \beta = P_{1}^{e} \frac{r_{d}}{r_{w2}} \sin \beta.$$
(1.10)

#### 2.1.3. Forces and Moments Loading the Circular Saw Shaft

The external forces that load up the circular saw blade 1 and the belt pulley 2 as well as the moments generated by these forces are reduced to the

axis of rotation  $AB \equiv z$  . They also generate the static reactions shown on Fig.1.5.

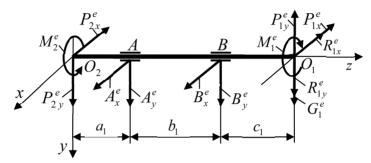


Fig. 1.5. Static reactions caused by the external load

The components of the external forces  $P_1^e$ ,  $R_1^e$  and  $P_2^e$  as well as the moments  $M_1^e$  and  $M_2^e$  are calculated from the expressions (1.7), (1.10), (1.6) and (1.8).  $G_1^e = m_d g$  is the weight of the circular saw blade 1,  $m_d$  is its mass and g is the acceleration of gravity.

After using the corresponding conditions for equilibrium, we get the following expressions for the reactions  $A_x^e$ ,  $A_y^e$ ,  $B_x^e$  and  $B_y^e$ :

$$A_{x}^{e} = \frac{-\left(R_{1x}^{e} + P_{1x}^{e}\right)c_{1} + P_{2x}^{e}\left(a_{1} + b_{1}\right)}{b_{1}}, \quad A_{y}^{e} = \frac{\left(R_{1y}^{e} + G_{1}^{e} - P_{1y}^{e}\right)c_{1} - P_{2y}^{e}\left(a_{1} + b_{1}\right)}{b_{1}},$$

$$B_{x}^{e} = \frac{\left(R_{1x}^{e} + P_{1x}^{e}\right)\left(b_{1} + c_{1}\right) - P_{2x}^{e}a_{1}}{b_{1}}, \qquad B_{y}^{e} = \frac{\left(P_{1y}^{e} - G_{1}^{e} - R_{1y}^{e}\right)\left(b_{1} + c_{1}\right) + P_{2y}^{e}a_{1}}{b_{1}}.$$
(1.11)

### 2.2. Dynamic Reactions Dependent on the Kinematic and Mass Characteristics of the Rotating Body

In order to express these reactions, we use the scheme shown on Fig.1.6.

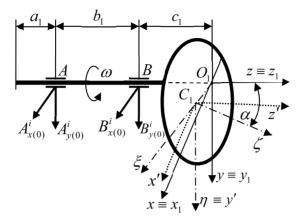


Fig. 1.6. Dynamic reactions caused by the kinematic and mass characteristics of the rotating body

We assume that  $\omega=const$ , whence it follows that the angular acceleration is zero, i.e.  $\varepsilon=0$ . In this case, the dynamic reactions are constant. What varies is their orientation according to the change of the angle  $\varphi=\omega t$ . The magnitudes of these forces can be found using well-known expressions (Pisarev, Paraskov, Bachvarov, 1988, 318-322), (Kisyov, 1979, 245-247).

$$\begin{split} & m_{d}(-\varepsilon y_{C_{1}} - \omega^{2} x_{C_{1}}) = A_{x(0)}^{i} + B_{x(0)}^{i}, \\ & m_{d}(\varepsilon x_{C_{1}} - \omega^{2} y_{C_{1}}) = A_{y(0)}^{i} + B_{y(0)}^{i}, \\ & - \varepsilon J_{xz}^{(0)} + \omega^{2} J_{yz}^{(0)} = A_{y(0)}^{i} (b_{1} + c_{1}) + B_{y(0)}^{i} c_{1}, \\ & - \varepsilon J_{yz}^{(0)} - \omega^{2} J_{xz}^{(0)} = -A_{x(0)}^{i} (b_{1} + c_{1}) - B_{x(0)}^{i} c_{1}. \end{split}$$

$$(1.12)$$

We choose the initial position when  $\varphi=0$ . For this position, the axes of the fixed coordinate system  $O_1x_1y_2$  and the moving coordinate system  $O_1x_1y_1z_1$  coincide. In this case, the coordinates of the centre of mass  $C_1$  are calculated by the following expressions:

$$x_{C_1}^{(0)} = e\cos\alpha, \ y_{C_1}^{(0)} = 0, \ z_{C_1}^{(0)} = -e\sin\alpha.$$
 (1.13)

The centrifugal moments  $J_{xz}^{(0)}$  and  $J_{yz}^{(0)}$  for the initial position- $\varphi = 0$  can be calculated using the Huygens-Steiner theorem (Pisarev, Paraskov, Bachvarov, 1988, 193-198).

$$J_{xz}^{(0)} = J_{x'z'}^{(0)} + m_d x_{C_1}^{(0)} z_{C_1}^{(0)},$$

$$J_{yz}^{(0)} = J_{y'z'}^{(0)} + m_d y_{C_1}^{(0)} z_{C_1}^{(0)},$$
(1.14)

where  $J_{xz}^{(0)}$  and  $J_{yz}^{(0)}$  are the centrifugal moments relative to the respective axes of the coordinate system  $C_1x^{'}y^{'}z^{'}$ . These moments are calculated from the following dependencies:

$$J_{xz}^{(0)} = \int_{(V)} x'z'dm, \quad J_{yz}^{(0)} = \int_{(V)} y'z'dm. \tag{1.15}$$

We need to express the coordinates x', z' and y' through the coordinates  $\xi$ ,  $\zeta$ ,  $\eta$ . The two coordinate systems  $C_1\xi\zeta\eta$  and  $C_1x'y'z'$  have a common origin and are rotated relative to one other at an angle  $\alpha$ , which is shown in Fig. 1.6.

The transition from the first rectangular coordinates x', z', y' to the other rectangular coordinates  $\xi, \zeta, \eta$  takes place according to known formulas, which are written below (Kisyov, 1979, 301).

$$x' = \xi \cos \alpha + \zeta \sin \alpha,$$

$$z' = -\xi \sin \alpha + \zeta \cos \alpha,$$

$$y' = \eta.$$
(1.16)

We replace these expressions in expressions (1.15) and obtain the following dependences for calculating the centrifugal moments  $J_{xz}^{(0)}$  and  $J_{yz}^{(0)}$ .

$$J_{\overrightarrow{xz}}^{(0)} = \int_{(V)} (\xi \cos \alpha + \zeta \sin \alpha)(-\xi \sin \alpha + \zeta \cos \alpha)dm,$$

$$J_{\overrightarrow{yz}}^{(0)} = \int_{(V)} \eta(-\xi \sin \alpha + \zeta \cos \alpha)dm.$$
(1.17)

We transform the above expressions and obtain the expressions for the centrifugal moments, keeping in mind the fact that the axes  $\xi$ ,  $\eta$  and  $\zeta$  are the principal axes of inertia.

$$J_{x'z'}^{(0)} = \frac{1}{2} (J_{\xi} - J_{\zeta}) \sin 2\alpha, \quad J_{y'z'}^{(0)} = 0. \tag{1.18}$$

We substitute the above expressions in the expressions (1.14) and receive the dependencies (1.19) for calculating the centrifugal moments  $J_{\nu z}^{(0)}$  and  $J_{\nu z}^{(0)}$ .

$$J_{xz}^{(0)} = \frac{1}{2} \left( J_{\xi} - J_{\zeta} - m_d e^2 \right) \sin 2\alpha \,, \quad J_{yz}^{(0)} = 0 \,, \tag{1.19}$$

where  $J_{\xi}$  and  $J_{\zeta}$  are the mass moments of inertia of the rotating disk toward axes  $\xi$  and  $\zeta$  .

Using the well-known expressions for the mass moment of inertia  $J_{\xi}$  and  $J_{\zeta}$  and after corresponding transformations, we get the expression for the centrifugal moments  $J_{xz}^{(0)}$  and  $J_{yz}^{(0)}$  at the initial moment when  $\varphi=0$ . These expressions are written below.

$$J_{xz}^{(0)} = -\frac{m_d \sin 2\alpha}{2} \left(\frac{r_d^2}{4} + e^2\right), \quad J_{yz}^{(0)} = 0. \tag{1.20}$$