Geometric and Wave Optics

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Theoretical and Technical Approaches

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General Introduction

The first chapter is devoted to a geometric theory of light, essentially based on the concept of light rays, which makes it possible to simply interpret the formation of images using lenses and/or mirrors. This approximate theory does not account for the wave aspect of light. However, we know from the electromagnetic theory of Maxwell and its confirmation by Hertz, that light is an electromagnetic wave. Consequently, certain optical phenomena cannot be interpreted without taking into account these wave aspects. I propose in this second chapter a wave theory of light. That being said, I recall some results of the electromagnetic theory so that the reader keeps in mind the vectorial and transverse nature of light, which makes it possible to explain certain phenomena which escape the scalar theory.

From antiquity to the incandescent lamp to the era of the laser, light mastery has constantly been a source of human progress. However, it took centuries for the greatest minds like Newton, Fresnel or Einstein to fully understand light's real, surprisingly complex nature. Physicists now know how to use its many properties to shape matter and adapt it to their needs.

Optics is the branch of physics that deals with light, its behaviour and properties, electromagnetic radiation, its relationship to vision and systems that use or emit light. Historically, optics has appeared since antiquity. Due to its wave properties, visible light spans from Ultraviolet (UV) to Infrared (IR) wavelengths. These properties then cover the field of radio waves, microwaves, X-rays, Gamma-rays and all electromagnetic radiation.

Most optical phenomena can be explained using the classical electromagnetic description. However, this description, although complete, is often difficult to apply in practice: simplified models are more often used. The most common of these is a model of geometric optics which views light as a collection of rays travelling in a straight line and curving as they pass through or reflect off surfaces. Physical optics is a more complete model, including wave effects such as diffraction and interference, which are not taken into account in the geometric model. Historically, the ray-based model was developed first, followed by the wave model. Advances in electromagnetic theory during the 19th century led to the discovery that light is electromagnetic radiation.

Some phenomena depend on the fact that light has both particle properties and wave properties. The explanation of these effects is possible thanks to quantum mechanics. When we consider light as a particle, we can model it as a collection of photons. Quantum optics deals with the application of quantum mechanics to optical systems. In this book, we are not interested in quantum optics but an introduction to will be given.

Optics finds applications and is studied in many fields, including astronomy, photography, medicine and various fields of engineering. The practical applications of optics can be found in a large number of technologies and everyday objects such as lenses, mirrors, lasers, fiber optics, microscopes or telescopes.

Historically, optics appeared in antiquity, then was developed by Muslim scholars including Persians. First of all, it is geometric. Ibn Al-Haytham (965-1039), the Persian scientist, known to westerners as Alhazen is considered the father of modern optics, experimental physics and the scientific method. A Latin translation of a chapter of his work, the Treatise on Optics, greatly influenced western science.

The classification of the fields of optics is structured in two branches: geometric optics and wave or physical optics, the first applying when the wavelengths involved in the study are much lower than the dimensions of the measuring device, the second occurring when the wavelengths are of the order of magnitude of the latter.

The development of measurement theory and various fields of knowledge investigation has also introduced other classifications, the most important of which are:

- energy optics which takes into account the light intensity, its distribution and its action on the various receivers.
- quantum optics which considers the particle aspect of light in its energy exchanges with matter.
- electronic optics which studies the wave manifestations of electron beams.
- instrumental optics based on the calculations and reasoning of geometric optics and wave optics and which takes into account the properties of materials and the various physical effects which allow the generation or measurement of an optical signal.
- nonlinear optics, the main purpose of which is the high concentration of power in a small volume, generally produced using a laser.

Geometric optics offers an analysis of light propagation based on simple principles: rectilinear propagation and inverse return. It was able to explain the reflection and refraction phenomena. It was perfected until the 18th century, when the discovery of new phenomena, such as the light deformation in the vicinity of obstacles or the splitting of light when passing through certain crystals (George Harburn, Charles Alfred Taylor, Thomas Richard Welberry T.R., “Atlas of optical transforms”), led in the 19th century to the development of physical or wave optics. This branch of optics is mainly based on the light ray model. It is a simple approach allowing for the realisation of metrical constructions of images. It is the most flexible and efficient tool for the study of dioptric and catadioptric systems. It allows, among other things, the explanation of the formation of images produced by such optical systems. It is the first optical theory formulated and validated a posteriori by wave optics, by adopting the approximation that the dimensions of all the elements used are larger than the light wavelength. Indeed, from the physical point of view, geometrical optics is an alternative approach to wave optics (also called physical optics) and quantum optics, which is also the oldest; developed since antiquity. Indeed, optics remains on the notion of the light ray proposed by Euclid, and until the sixteenth century its progression was only empirical; appearance, for example, of the first glass correctors and the discovery of the laws of Snell-Descartes by Snell in 1621 then Descartes in 1637.

In 1665 Grimaldi carried out the first experiment indicating the limits of geometric optics with the discovery of the optical phenomenon known from then on as diffraction. Thereafter, wave optics will be highlighted with the realization of Young's double-slit experiment in the 19th century. The 20th century will be characterized by the appearance of quantum optics.

A light ray has no physical existence; it is a theoretical object that is used as a basic model of geometric optics where any light beam is represented by a set of light rays. The light ray is a method of approximating the propagation direction of the wave carrying the light or photons. Thus, and if we consider that the wave surface is plane then all rays are parallel to each other. On the other hand, if the wave surface is considered spherical, then all rays are focused at one point (a beam that converges at one point), or appear to come from one point (a beam that diverges from one point).

An energetic approach to the propagation of light is needed to link the light ray model to the wave theory of light. Indeed, the light ray represents the propagation direction of the light wave energy, which is orthogonal to the wave fronts of the wave; this is the law known as Malus's Law.

Physical optics considers light as a wave (Eugene Hecht, “Optique”); it takes into account the phenomena of interference, diffraction and polarization. At the beginning of the 20th century, Einstein's theories on the corpuscular nature of light gave birth to photon and quantum optics.

Physicists are then forced to admit that light has both the properties of a wave and a particle. From there, Louis de Broglie considers, through wave mechanics, that if the photon can behave like a corpuscle, then, conversely, corpuscles such as electrons or protons can behave like waves.

While geometric optics is purely phenomenological optics and makes no assumptions about the nature of light, except possibly that it carries energy, wave optics (called also physical optics) models the light by a wave. The scalar wave model (Huygens-Fresnel principle) makes it possible to interpret diffraction phenomena (during passage through a hole, a narrow slit, near an edge, ….) and interference. The calculations are then based on the sum of the amplitudes of sinusoidal waves which are superimposed, a sum which, depending on the phase shift, can lead to a zero result. The superposition of two beams can thus give darkness. This is what is observed at the level of the dark zones of the interference or diffraction figures. It is then necessary to consider that it is about a transverse wave, f one wants to interpret the polarization phenomenon. Finally, Maxwell will make it possible to understand that light waves are only electromagnetic waves characterized by a range of wavelengths which makes them visible to humans. Physical optics is the name of a high frequency (short wavelength) approximation commonly used in optics, applied physics, or electrical engineering. In these contexts, it is an intermediate method between geometric optics, which ignores wave effects, and wave optics, which is an exact physical theory. This approximation consists of using the rays of geometric optics to estimate the fields on a surface and then integrating these fields over the entire illuminated surface to determine the transmitted and reflected fields. In the optical and radiofrequency domains, this approximation is used to calculate interference and polarization effects and to estimate diffraction effects. Like all high frequency approximations, the physical optics approximation gains in relevance as one works with high frequencies.

This book is presented in two chapters; the first is devoted to geometric optics and the second to wave optics. The first chapter deals with the reflection and refraction phenomena, the notions of total and approximate stigmatism, conjugation relations, the description of the main fundamental optical instruments and the notions of illumination and luminance. The second chapter will deal with notions such as optical path, refractive index and propagation speed, and interference phenomena while addressing the phenomena related to the polarization of light waves and interfering devices, diffraction phenomena and diffractive devices, scattering phenomenon and absorption phenomenon. An interesting introduction to the quantum optics will be given also in the end of this second chapter.

This book is a course in optics that brings together the fundamentals of geometric and wave optics as well as many application examples and descriptions of numerous optical devices and instruments. The set of concepts covered covers the entire field of geometric and wave optics and offers a clear vision of optics. Diagrams allow a visual and concrete illustration of optical phenomena in relation to the text. Detailed technical explanations are given, with a physical approach and a precise mathematical formalism.

CHAPTER I

GEOMETRICAL OPTICS

Introduction

Geometric optics offers an analysis of light propagation based on simple principles: rectilinear propagation and inverse return. This optics branch was able to explain the reflection and refraction phenomena. It was perfected until the 18th century, when the discovery of new phenomena, such as the deformation of light in the vicinity of obstacles or the splitting of light when passing through certain crystals, led in the 19th century to the development of physical or wave optics.

Geometric optics developed based on simple observations and is based on two principles and empirical laws:

- rectilinear propagation in a homogeneous and isotropic medium.
- the principle of inverse return which expresses the reciprocity of the light path between source and destination.
- the Snell-Descartes laws for reflection and refraction.

The problems are resolved by using geometrical constructions (plots of lines materializing the rays, angles calculations), from where the name of geometrical optics originates. It gives good results as long as one does not seek to model phenomena linked to polarization or interference and that no dimension of the system is comparable to or less than the wavelength of the light used.

Geometric optics makes it possible to find almost all the results concerning mirrors, diopters and lenses or their combinations in doublets and optical systems constituting in particular optical instruments.

Moreover, within the framework of the Gaussian approximation, geometrical optics gives linear mathematical relations allowing the use of mathematical tools such as matrices and the systematization of calculations by computer.

From a physical point of view, geometric optics is an alternative approach to wave optics (called also physical optics) and quantum optics. On the other hand, it is older, having been developed since antiquity. The notion of light rays was thus introduced by Euclid in the 4th century BC. Until the 16th century, optics remained on this notion of light ray and only progressed empirically, allowing however the appearance of the first corrective lenses in 1285. The laws of Snell-Descartes were found by Snell in 1621 and then by Descartes in 1637. The first experiment showing the limits of geometrical optics was carried out by Grimaldi in 1665, who gave his name to diffraction. Wave optics will only be highlighted in the 19th century with the experience of Young's slits and quantum optics will only emerge during the 20th century.

The question of the light nature goes back to antiquity. Pythagoras, Democritus, Aristotle and others had already built a light theory and the t propagation in a straight line was already known to Euclid (300 years BC). The fall of the Roman Empire (475 A.D.) then severely hampered scientific development and it was not until the end of the 16th century that physics was reborn. It was during this period that great progress was made in optics, both experimentally and theoretically. At the theoretical level, two conceptions clash. Newton, the father of the f classical mechanics, defends a corpuscular description of light. For him, the phenomenon of diffraction of light reported in the work of Father Grimaldi (published posthumously in 1665) is explained by an inflexion of light by matter: we see here a purely mechanistic vision. At the same time, Huygens defended a wave description of light. But the success of the principles of mathematics and the influence of Newton in the scientific world certainly slowed down the development of the wave theory. It will take more than a century for the ideas of Huygens to be recognized. Let us cite a few historical landmarks relating to progress in geometric optics:

1590: Zacharias Janssen invents the microscope.

1609-1610: Galileo Galilei builds an astronomical telescope with which he will discover the sunspots and 3 satellites of Jupiter (Callisto, Europe, Ganymede).

1611: Johannes Kepler discovers total internal reflection, the law of refraction for small angles and the laws of thin lenses.

1613: Galileo Galilei demonstrates the rotation of the sun through the observation of sunspots.

1621: Dutch astronomer Willebrord Snell discovers the laws of refraction.

1637: René Descartes demonstrates mathematically that the angles of primary and secondary rainbows depend on the elevation angle of the sun.

1657: Pierre de Fermat introduces the principle of minimal time in optics (Pierre et Jean-Pierre Provost, "Optique géométrique et principe de Fermat (vol. 1) et optique ondulatoire et cohérente (vol. 2)").
1665: Father Grimaldi observes that around obstacles or at the edge of a hole, light undergoes a scattering, and calls this phenomenon diffraction.

I. Concept of light

The nature of light has long been a subject of wide discussion. It was considered a wave at the end of the 18th century by Augustin Fresnel (1788-1827), who was thus able to interpret interference and diffraction experiments. Albert Einstein showed in 1905 that certain still unexplained observations, such as the photoelectric effect, found a simple interpretation if one considered that the light was made up of corpuscles "quanta of energy", called photons. Light can thus behave like particles or like a wave.

I.1 Particle theory

According to the corpuscular theory, light is considered as a set of particles (or corpuscles) called photons which have an energy **E=hν**: where h is Planck's constant (**$h=6{,}63\ .10^{-34}$ J.s**) and ν the frequency of the light wave in Hz, and which move at a very high speed (**$c=3.10^{8}$ m.s^{-1}**; **c** is the speed of light in vacuum).

I.2 Wave theory

According to the wave theory, light is an electromagnetic wave. This wave is characterized by the propagation at speed c of an electric field and a magnetic field coupled which vibrate at a frequency ν.

Maxwell's electromagnetic theory places no limit on the frequency of electromagnetic waves. The spectrum of electromagnetic radiation (Figure below) extends from "radio waves" to "γ rays", with visible light (**$0.4\mu m<\lambda<0.8\ \mu m$**) occupying only a very small part of this spectrum.

I.3 Approximation of geometrical optics

We have seen that light has a double nature: particle and wave.

Geometric optics free itself from this duality and considers light in terms of light rays: within the framework of this theoretical approximation, it is assumed that in media transparent and homogeneous, the light propagates along straight lines called light rays.

A set of rays forms what is called a light beam. A light pencil is a narrow beam of light.

Light rays that deviate from each other are called diverging light rays. Light rays that approach each other are called converging light rays.

All the rays come from the same point called the top of the beam. If the vertex is located at infinity, a conical beam tends towards a parallel beam.

It must be kept in mind that geometric optics is only valid if all the dimensions of the problem, in particular the dimension of the diaphragms which limit the beams, are much greater than the wavelength. If not, diffraction phenomena occur, and the very notion of ray no longer makes sense.

II. Optical media

Light can propagate in a vacuum and media other than a vacuum.

A medium is said:

- homogeneous when its composition is the same at all its points; it is said to be inhomogeneous in the opposite case,
- isotropic when its properties are the same in all directions; otherwise, it is anisotropic,
- transparent if it allows light to pass without attenuation (water, glass, etc.),
- absorbent if it only allows part of the light to pass (smoked glasses, etc.),
- opaque if it does not let light through.

The permittivity $\boldsymbol{\varepsilon}$ of a medium is different from that $\boldsymbol{\varepsilon_0}$ of vacuum whereas, in the case of a non-magnetic medium, its permeability $\boldsymbol{\mu}$ is very little different from that $\boldsymbol{\mu_0}$ of vacuum. In a transparent, homogeneous and isotropic medium, $\boldsymbol{\varepsilon}$ is a function of the frequency $\boldsymbol{\nu}$. The speed $\mathbf{v}$ of propagation of light in the medium is then written: $\mathbf{v}\ (\boldsymbol{\nu}) = \frac{\mathbf{1}}{\sqrt{\mu_0 \varepsilon_0}}$.

II.1. Index of the media

We call the index of the medium noted n, the ratio between the speed of light in vacuum (speed **c=3.10⁸ m/s**) to that in the considered medium: $\mathbf{n}(\boldsymbol{\nu}) = \frac{\mathbf{c}}{\mathbf{v}(\nu)}$.

By their definition, the indices of the usual transparent media, for the optical frequencies considered, are greater than 1; but some media (e.g. plasmas) may have an index lower than 1 in some frequency domains.

The media that we will consider in this course will be transparent, homogeneous, isotropic and with an index greater than or equal to 1.

III. Basic concepts of geometrical optics

Geometrical optics is by definition a branch of the cs that studies light rays in transparent media (*Max Born, Emil Wolf, "Principles of optics")*.

The first optical instruments for astronomy appeared in the 17th century with the use of the telescope by Galileo in 1609 and then the improvement of the telescope by Isaac Newton in 1671. The 17th century, therefore, saw the development of geometric optics as well as a corpuscular description of light, carried in particular by Newton. The light sources emit particles of light which are reflected by the mirrors and pass through the transparent media at different speeds. From this description, Fermat will draw the principle of least time to explain the phenomena of refraction (José Philippe Perez, "Optique géométrique et ondulatoire"). Light takes the fastest path to get from the emitter to the receiver. At this same time, Newton carried out the first experiments on the decomposition of light and deduced that white light was composed of the superposition of coloured lights.

III.1 Light Beams

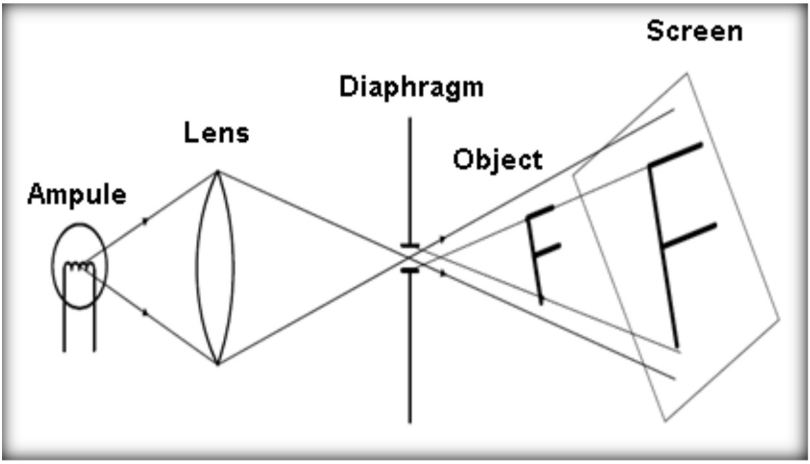

Figure No. 1: the shadow of an object

The shadow observed on the screen is homothetic to the object (the shadow is proportional in its dimensions to the object): This indicates that the light propagates in rectilinear bundles. If a light ray travels an optical path from point A to point B, then another light ray can cross the same optical path in the opposite direction; which proves the inexistence of a privileged sense of the path of light (the path followed by the light is independent of the direction of its propagation): This is the principle of the inverse return of light. The light rays are considered in geometric optics independent of each other so that the consideration of the absence of interference between the light rays is valid: Independence of light rays.

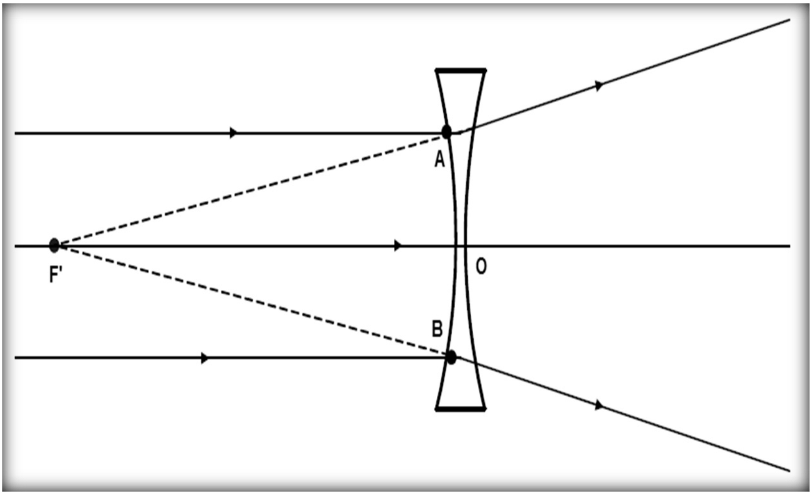

Figure No. 2: Virtual light rays

In the figure above, two incident rays parallel to the optical axis are deflected by a divergent lens and seem to come from a source point F'. Indeed, the paths F'A and F'B are not actually followed by light; they are virtual rays and point F' is a virtual source.

Limits of geometrical optics: (see also Chapter II):

1. The diffraction is a phenomenon due to the wave nature of the light. We consider the wavelet model of Huygens shown in the diagram of **Figure No. 3** below.

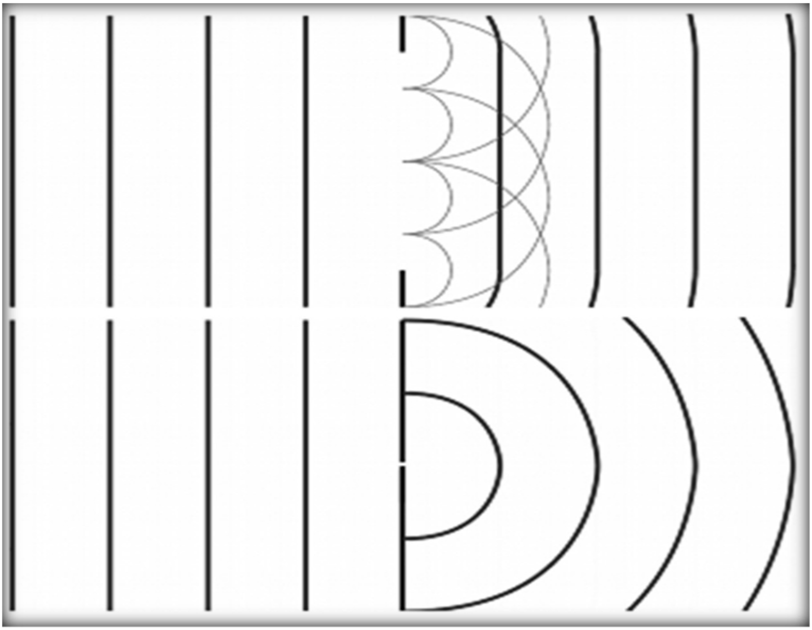

Figure No. 3: Wavelets of Huygens

When the opening is large, the incident wave, which is a plane, remains plane after the obstacle therefore the phenomenon of diffraction is negligible. Conversely, when the opening is of low dimensions, the wave transmitted almost spherical wave, and more the opening is small the more the diffraction phenomenon is important. In effect, the angle of divergence of the beam is inversely proportional to the size of the opening. The phenomenon of diffraction appears as soon as a light ray crosses his path of an object whose dimensions are comparable to the wavelength of the ray (more generally changes in opacity on the scales of the order of the wavelength, such as the edges francs per example).

In the figure below an incident ray with a wavelength $\boldsymbol{\lambda}$ on a diaphragm of diameter **d**, then two cases of figures exist:

$\boldsymbol{\lambda} < \mathbf{d}$: The emerging beam remains aligned with the incident beam.

$\boldsymbol{\lambda} > \mathbf{d}$: The emerging beam is widening. This is the phenomenon of diffraction (interference phenomenon; intimately related and necessary to the wave nature of the light).

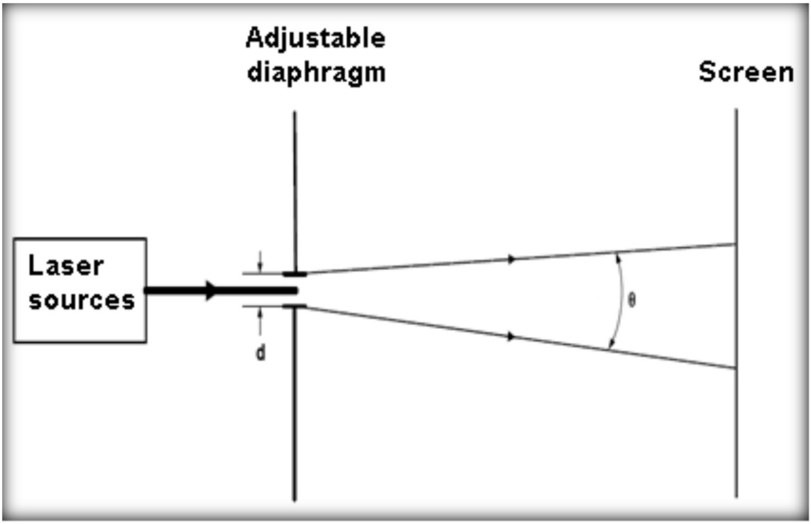

Figure No. 4: Passing a beam of light rays through a diaphragm

2. The light is composed of photons that are massless particles, giving a corpuscular appearance to light. By this corpuscular aspect, the light exerts a pressure force (with a value much less than the atmospheric pressure) designated by the radiation pressure. The radiation pressure exerted by a beam of light on a surface is thus attributed to a flow of photons on the surface (An analogy with the kinetic theory of gases).

III.2 Laws of reflection and refraction (Laws of Snell-Descartes)

In 1637, Descartes published his famous treatise "Discourse on the method to well lead his reason and seek the truth in the sciences, the more the dioptric, meteors and the geometry". There are demonstrated laws of reflection and refraction (see below) using a mechanical analogy; based therefore on a corpuscular design of the light.

III.2.1 Laws of reflection

Consider a surface separating two distinct environments (a diopter) which is supposed to be a plane (**Figure No. 4**). Let **i** be the incidence angle and let **i'** be the reflection angle.

- ✓ **1st Law of reflection:** The reflected beam is in the incidence plan (Plan formed by the normal **N** to the diopter and the incident beam).
- ✓ **2ième law of reflection:** The incidence angle **i** = the reflection angle **i '**.

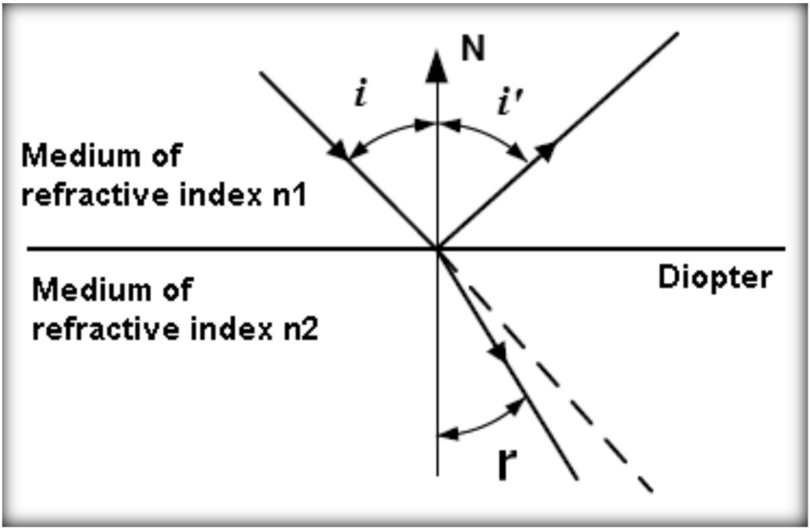

Figure No. 5: Reflection and refraction of a light ray

The case of a tennis ball which is projected on a wall is a mechanical analogy of the reflection.

III.2.2. Laws of refraction

They were established in 1621 experimentally by the Dutch astronomer and mathematician **Willebrord Snell** or **Snellius** (1580-1626).

Natural manifestation and characteristics: Examples of objects immersed in water: A stick plunged into the water appears as broken at the level of the surface separating two environments. Similarly, the bird in the sky seems to the fish higher than it is in reality, and the fish appears to the bird closest to the surface of separation air-water than it is in reality (**Figure No. 6**).

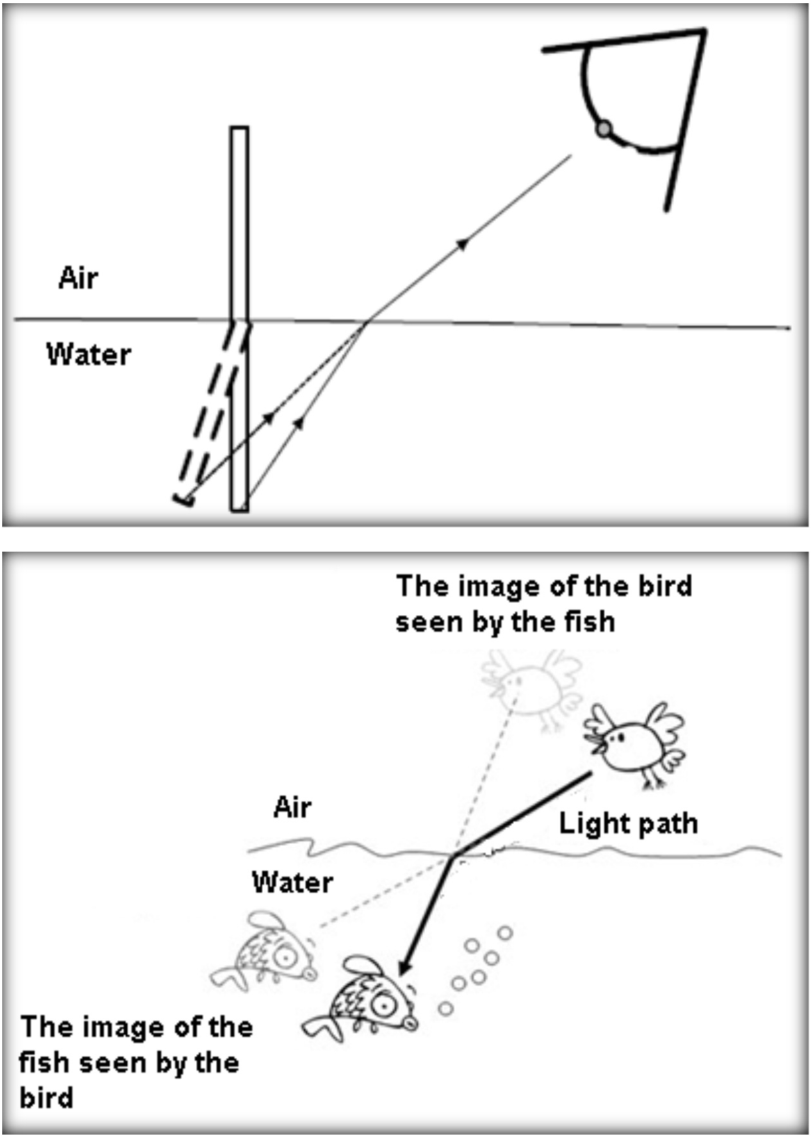

Figure No. 6: Example of the natural observations of the phenomenon of refraction

Example of the mirage (an optical phenomenon that is due to the deviation of the light beams by superimpositions of air layers that are at different temperatures) (**Figure No. 7**): Parameters can vary the refractive index of the air which is not a constant. Among these parameters are s, the temperature and the atmospheric pressure, as well as humidity and more generally the composition of the air. Thus, the layers of cold air are denser so that their index is greater because the refractive index increases proportionally to the pressure and decreases inversely proportional to the temperature. As a result, the density ρ of the air varies with the temperature, which implies the variation of the refractive index n as a function of the density. Thus, for a gas: $\frac{\mathbf{n-1}}{\boldsymbol{\rho}} = \mathbf{cte}$.

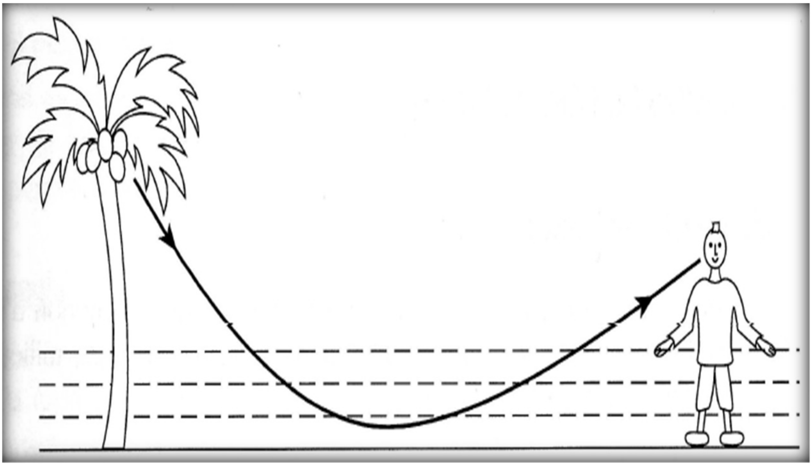

Figure No. 7: Deviation of the light beams by superposition of layers of air

The table below gives the refractive index of a few materials:

Medium	Refractive index (At λ = 589 nm)
Empty	1
Water	1.33
Crown glass	1.52
Flint glass	1.67
Diamond	2.42
Fluorine	1.43

Properties checked by the refracted ray (Figure No. 5):

- ✓ The refracted ray is in the plane of incidence.
- ✓ Angles of incidence **i** and refractive **r** are bound by the relation:

$$\mathbf{n_1 sin(i) = n_2 sin(r)}$$

With $\mathbf{n_1}$ is the index of the medium **1** and $\mathbf{n_2}$ the index of the medium **2**.

The total reflection angle: In the case where the light passes from a medium 1 of index n_1 to a medium 2 of index n_2, then for any value of the incidence angle i the inequality i <r is verified if $\mathbf{n_1 > n_2}$.

Assume that **n1>n2**, so **i<r** whatever the value of **i**. When **i** increases from the value of **0** to the value of $\frac{\pi}{2}$, **r** will also increase until reaching the angle $\frac{\pi}{2}$ for a certain angle $\mathbf{i = \lambda}$ defined as:

$$\mathbf{n_1 sin(\lambda) = n_2 sin(\tfrac{\pi}{2})} \Longrightarrow \mathbf{sin(\lambda) = \frac{n_2}{n_1}}$$

This phenomenon is manifested experimentally when **i> λ** by the fact that all the light is reflected. In this case, the diopter, therefore, behaves like a mirror; that is the phenomenon of total reflection and the lambda is the angle of total reflection.

The limit refraction angle: Assume that **n1<n2**, it was then that **i>r** whatever the value of **i** between **0** and $\frac{\pi}{2}$. For $\mathbf{i} = \frac{\pi}{2}$, **r** will reach a limit angle **β** said the limit refraction angle which is defined as:

$$\mathbf{n_1 sin\left(\frac{\pi}{2}\right) = n_2 sin(\beta)} \Longrightarrow \mathbf{sin(\beta) = \frac{n_1}{n_2}}$$

The law of Kepler: For small enough angles, **i** and **r**, Descartes's refraction law (the second law of refraction) takes the form: $\mathbf{n_1 . i = n_2 . r}$

The refractive index of a material: It is a number noted **n** which characterizes the power that has this matter, to deviate the light and slow down.

By definition, the index of refraction of a material **is** the ratio of the speed of light in vacuum **c = 299792 km/s** and the speed of light **v** in this material:

$$\mathbf{n = \frac{c}{v}}$$

The refractive index is therefore a parameter without dimension.

For the empty **n=1** and for the air **n= 1.0008** but it is often taken as equal to **1** also. But this index varies as a function of temperature, pressure and humidity (which cause, for example, poor sharpness of images of telescopes (which had led to install the Hubble telescope in Orbit)).

Below, is a table which gives the refractive index as well as the speed of light in a few materials:

Field	Index of Refraction	Speed of Light
Empty	1	299 792 km/s
Air	1,0008 (it may vary with the temperature and the pressure as well as the moisture)	299 552 km/s
Water	1,3300	225 407 km/s
Organic glasses	1,500 - 1,740	199 861 - 172 294 km/s
Mineral glasses	1.525 - 1900	196 584 - 158 452 km/s
Diamond	2,420	123 881 km/s

Fermat principle: (**Pierre de Fermat** is a French magistrate, polymath and especially mathematician (1601-1665), he was also interested in the physical sciences):

"The light propagates from one point to another on trajectories such that the duration of the course is locally minimal (Locally signifying: For a "small" trajectory)" (René Descartes, "« Œuvre de Descartes Tome 10» published by Tom Cousin). The Law of Refraction is related to the principle of Fermat (José Philippe Perez, "Optique géométrique et ondulatoire"). The figure below illustrates by an analogy this principle:

Just like the light, the rescuer will choose the shortest route. Path (1) which is in a straight line, is longer than path (2) because the movement is slower in the water than on the sand. By replacing sand with air (refractive index ~ 1), this reasoning applies well to light. This principle also makes it possible to establish Descartes' second law of refraction and thus explain the formula that links the refractive index n of a medium to its velocity in this medium and that vacuum.

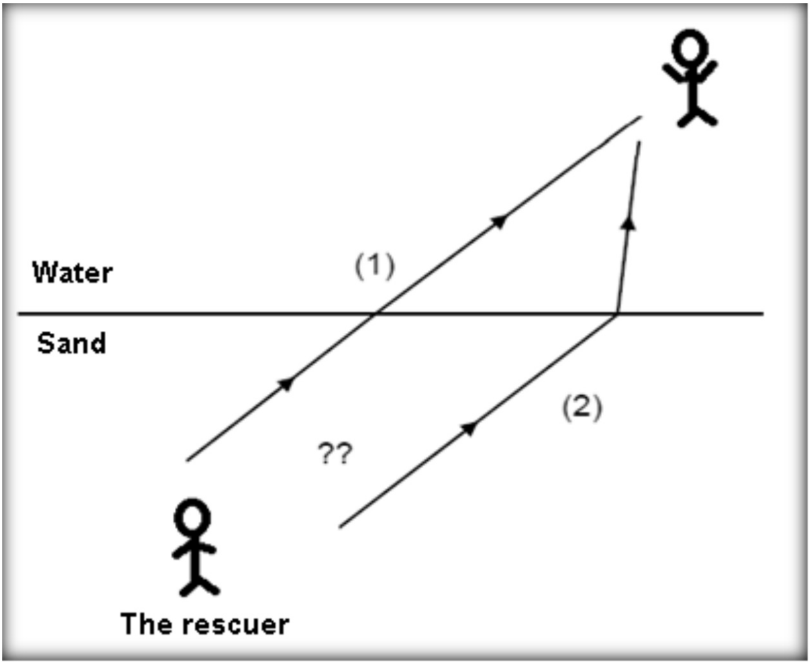

Figure No. 8: An analogy with Fermat's principle of Fermat (just like the light of the rescuer will choose the shortest route)

Wavelets Huygens and refraction of light: Either an incident plane wave that moves at the speed c in the air and the velocity v <c in the glass. To determine the shape of the wave in the glass, Huygens assumes that each point on the separation surface re-emits a spherical wavelet that propagates at the velocity v in the glass. The representation of some of these spherical wavelets allows visualization of the surface of the wavelet envelope.

III.3. Rigorous and approximate stigmatism

III.3.1. Rigorous stigmatism (perfect)

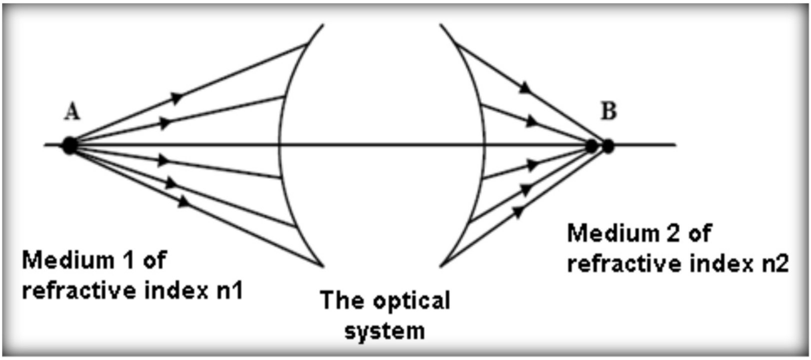

Figure No. 9: Image of a point by an optical system

We consider the following notations **S**: An optical system of some kind, **A**: The object and **B**: The image of **A** Through **S**.

The object **A** and its image **B** are said to be conjugate concerning system **S**.

- ✓ If the emerging rays are going by **B**, then the image is real.
- ✓ If it is the extensions of the emerging beams that cross B, then the image is virtual.

An optical system **S** stigmatic is if any object **A** admits a single image **B** across the optical system **S**. That is to say; an optical system is said rigorously stigmatic if any ray passing by **A** goes, after having crossed **S,** by **B**.

In the example in the figure above, this stigma condition is not verified.

Example of rigorous stigmatism:

- The parabolic elliptical mirror is strictly stigmatic for the focal points of the ellipse (**Figure No. 10**).

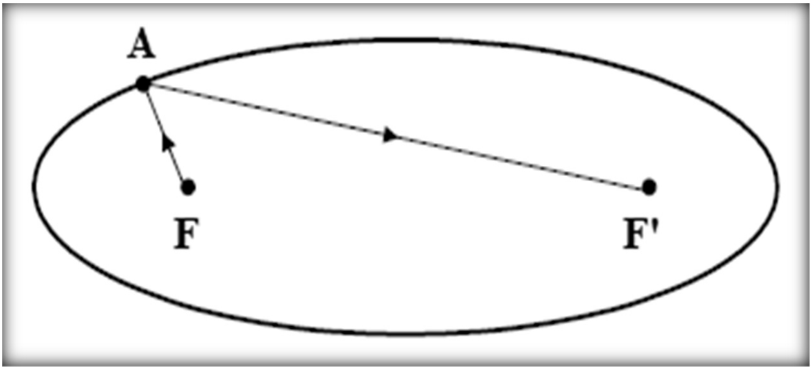

Figure No. 10: Rigorous stigmatism of the elliptic mirror for the focal points of the ellipse

- The parabolic mirror is rigorously stigmatic for any point lying at infinity.
- The plane mirror is rigorously stigmatic at every point.

III.3.2. Approximate stigmatism and aplanetism

The systems that are rigorously stigmatism are rare, which is why systems with so-called approached stigmatism are widely used and highly valued.

An optical system is said to have an approximate stigmatism if any light ray passing through a point A, after having crossed the optical system s, passes in the vicinity of another same point B.

Centred optical system: This is an optical system that is rotationally symmetrical about an axis called the optical axis.

Aplanetism: This is a property of optical systems that are capable of retaining a stigmatic image of any object point adjacent to a stigmatic point, in a plane perpendicular to the optical axis. Therefore, the object and image planes are parallel and they are said to be conjugate.

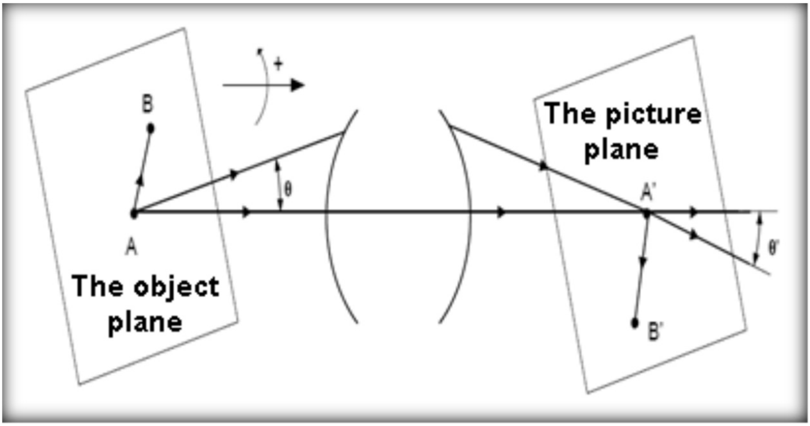

Figure No. 11: Object and image plans

III.4. Approximation and conditions of Gauss

(**Johann Carl Friedrich Gauss** (**1777 - 1855**) is a German mathematician, astronomer and physicist).

The conditions of Gauss: Let be a centred optical system, it is said used in the conditions of the Gauss approximation, when the rays which cross it form a weak angle with the optical axis of the system (low compared to the angle under which the opening of the optical system is observed) and when they are not distant from the optical axis (short distance in front of the characteristic scale of the optical system). Under these conditions (**Figure No. 11**) the following properties are verified:

- ✓ **Relation of conjugation**: The system verifying the conditions of Gauss ensures an approached stigmatism and an approximate aplanetism.
- ✓ **Transverse magnification**: Under Gauss conditions, the transverse magnification Γ is constant. That is, any pair of conjugate points (A, A') and (B, B') satisfies the relation:

$$\frac{\overline{\mathbf{A'}B'}}{\overline{\mathbf{AB}}} = \mathbf{\Gamma} = \mathbf{cte}$$

Angular magnification: In the conditions of Gauss, the angular magnification λ is constant. That is to say, the incident angle and emerging angle verify the following relation: