Viscous Flow Environments in Oceans and Inland Waters

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Preface

It was a glimpse into some kind of wonderful world where electricity and mathematics and engineering and nice diagrams all came together
~ Edward M. Purcell

MY PRIMARY AIM in this book is to broaden access to quantitative treatments of biologically, chemically, and geologically relevant flow environments at low Reynolds numbers—regimes in which a fluid's viscosity dominates inertial effects. The most famous, central and seminal occupant of this genre is a stunningly and durably effective paper, originally a talk (Purcell 1977). It launched ever-expanding investigations of the physics of swimming and nutrient acquisition at low Reynolds numbers and entrained both biologists and fluid dynamicists into studies at these scales. Earlier and later contributions to biological fluid dynamics at low Reynolds numbers abound, but most are highly technical and inaccessible to the upper-level undergraduate students, graduate students, researchers and autodidacts who constitute my primary target audience. I anticipate that the information provided here will be particularly useful to developing aquatic (marine and freshwater) scientists without prior exposure to fluid dynamics. The audience explicitly comprises science students who are not fluid mechanicians (biologists, chemists, and geologists).

Fluid mechanics are less accessible than mechanics of rigid, discrete bodies (treated as point masses) in part because continuum mechanics are largely omitted from introductory physics lectures and texts. Continuum mechanics analyze mechanical behaviors of materials modeled as continuous masses rather than discrete particles. Their omission is a bad idea for progress in biology because organisms are bathed internally and externally in moving, fluid media (and in the case of mud-dwelling organisms, deformable solids that can fail abruptly by fracture and deform slowly in creep). Omission is due in part to historical and largely accidental relegation of continuum mechanics and material properties to engineering—rather than physics—curricula (Gollub 2003), and primarily at the graduate level. Hope of broader exposure has arrived, however, as nanosciences focus physicists, engineers, chemists and biologists on ways that bulk continuum properties depend on nanoscale arrangements and properties of constituents.

More succinctly, this book summarizes what I wish I had known earlier about fluid dynamics and mass transport relevant to aquatic organisms < 10 cm long. In the interest of accessibility, it does so informally.

Edward M. Purcell (1912, Taylorville, Indiana, U.S.A. – 1997, Cambridge, Massachusetts, U.S.A.) found easy access to low Reynolds number flows because the electromagnetic fields that he studied are treated with the same mathematical approaches (Schey 2005). A pioneering Nobel laureate

(1952) in electromagnetics, Purcell found many useful analogies in methods between electromagnetics and low Reynolds-number flows. His long-term collaborator, Howard C. Berg, produced an insightful volume (Berg 1983) that expands access to understanding of mass transport at bacterially relevant scales and that touches on low Reynolds-number issues of drag and sedimentation. Although it deservedly remains useful for students of organisms that live by molecular diffusion, the book focuses largely upon that mass transport mechanism and upon bacteria with length scales up to a few micrometers. My intent here is to increase the range of Reynolds numbers covered to include upper fringes of low Reynolds-number behaviors into the realms of large phytoplankton chains, filamentous algae, small planktonic and benthic invertebrates and the flow environments that they inhabit. I gratefully acknowledge invariably lucid, thoughtful and constructive responses of both Drs. Berg and Purcell to my and my research colleagues' occasional inquiries about unpublished details of their pioneering work.

Vogel (1981, 1994), together with Denny (1988, 1993), opened expansive windows on fluid statics, kinematics and dynamics. Their books attest that continuum mechanics can be made accessible (Gollub 2003). They, however, devoted far greater emphasis to examples at high Reynolds numbers than at low. For about 17 yr, I used Vogel (1994) as a text and Denny (1993) as a reference for upper-level undergraduates in a course aimed at understanding how momentum, mass and information transfer literally and figuratively shape organism form and function. My first try followed Vogel (1994) chapter by chapter, but students got confused, especially by d'Alembert's paradox and drag coefficients. I backed up to cover low Reynolds number drag on a sphere (including pressure distribution), and confusion disappeared. Thereafter, instead of relegating low Reynolds numbers to special cases near the end of the course, I began with low Reynolds numbers. This order eased students into fluid dynamics in large measure because the mathematics describing laminar flows are linear and easier to apply.

This book is a drag-coefficient-free zone. Students have much better intuitions for drag forces than they do for drag coefficients, and in my experience become much less confused if they learn about the former first. Drag coefficients find greatest utility over Reynolds number ranges where they remain roughly constant. They are therefore least useful at low Reynolds numbers, where these coefficients vary strongly with Reynolds number and give scant insight. Neither is it clear that introducing d'Alembert's paradox is a good idea for an introductory course whose students are not intending to specialize on fluid dynamics. In environmentally relevant Reynolds number ranges in air and water, fluid pressures on the downstream faces of obstacles in the flow remain below ambient levels; in both water and air, viscosity directs net pressure forces on an object downstream over the full range of natural Reynolds numbers. Moreover, arbitrarily close to any living or nonliving object in a Newtonian fluid, flow is laminar, and I find that student intuition is developed more reliably by dealing with this universal case first, well before treating much more idealized, inviscid, potential flows that can give a great deal of insight for high Reynolds numbers, but can never give an accurate picture of flows very near a body. And we biologists are very concerned with forms and functions of bodies.

I developed this book as a companion to Vogel (1994) in a particular course format that began with much of the current contents of Chapters 1-5 herein. Juniors and seniors spent one day a week with me for an entire 16 wk semester in the University of Maine's Semester-by-the-Sea Program. Afternoons comprised lectures and discussions, whereas mornings were devoted to laboratory explorations (in flow tanks) of the prior week's lecture topic and additional discussion and reading material. In weekly, essay-form analyses of their laboratory results, students combined information from readings, lectures and their own observations. My use of the book material concentrated in the first third of the course and tapered off in the middle third, with Vogel (1994) taking up most of the slack. In the last third, I intertwined concepts from Dusenbery (1992) and subsequent developments in sensory ecology to emphasize both how information transfer may depend heavily on details of mass or momentum transfer and that a very small transfer of mass or momentum may carry all the important information. A great deal of information transfer in water relies on sensors of velocity and chemical cues, anchoring the last third to the rest of the course.

As book projects are wont to do, however, this one took on additional roles. It addresses student blind spots in rudimentary fluid dynamics that I discovered in 40 yr of teaching of biological oceanography to beginning graduate students and extends advection-diffusion coverage in nutrient uptake to a level slightly past where I took my undergraduates. My secondary target audience thus also includes instructors who use Vogel's (1994) text. In order to make the text useful as a reference my revisions also focused on giving autodidacts the ability to perform calculations relevant to common problems in mass and momentum transfer.

Reasons for attempting to improve access to fluid dynamics at low Reynolds numbers go far beyond pedagogy. Organisms freely living in fluids span roughly eight orders of magnitude in length, from about 0.5 μm to about 30 (whales and giant kelps) or 100 m (redwoods). In aquatic realms and on a logarithmic scale, fully one-half of that range is dominated by effects of viscosity. If we include parasitic marine viruses, the size range gets even wider, well into the colloidal range, down to radii of order 10 nm. Here, water defies human experience. Humans live in a literally turbulent, (gaseous) fluid world, and swim in water by creating inertial, high Reynolds-number flows. Forward progress results from throwing toroidal vortices backward. Well above the scale of diffusive molecular motions, however, flows of water are orderly and, for the most part, laminar. The majority of aquatic organisms, even some of the largest fishes such as tunas and gadids—and the largest seaweeds—spend all or parts of their life cycles in a world where viscosity dominates inertia. Hence I focus on describing their viscous environments.

My concentration on low Reynolds numbers tilts this book decidedly toward water rather than air. Although laminar flows certainly do occur in air and are relevant to processes such as gas exchange across leaf surfaces, wind pollination within flower structures and dry deposition of dust, the lower kinematic viscosity of air limits the range of length scales over which laminar flows occur. Moreover, risk of desiccation, paucity of dissolved or suspended foods, and limited buoyancy apparently have barred paths in gaseous media to evolution of active propulsion mechanisms that operate at truly low Reynolds numbers. Passive seed, pollen and some animal dispersal does occur, however, at low Reynolds numbers (*e.g.*, small caterpillars and spiders "ballooning" or "hang gliding" with the aid of spun silk), and some orb-weaving spiders do feed on pollen depositing from suspension in air. Hopefully my bias toward marine rather than freshwater examples will be as obvious as it is unjustified; although steep mountain streams can be even more turbulent than oceans, lakes, ponds and puddles are generally less so. My sole reason for choosing primarily marine examples is my greater ignorance of inland waters.

Most flow-organism interactions that I treat involve external surfaces of organisms. I do introduce flow regimes (*e.g.,* flow through a cylindrical tube) applicable to both internal and external settings. I do not develop them as much as a physiologist might like, however, and for the most part I avoid discussing physiological processes wherein, from digestion to respiration, interplay between internal chemical kinetics and physical transport becomes key.

Although viscous worlds fall outside everyday human experience, they are well within reach of everyday mathematics and physics. Because the math is linear and the physics, simple (albeit alien), low Reynolds-number flows provide ideal arenas for biologists, chemists and geologists to exercise skills in mathematics and physics in order to develop physical intuition. I need assume only a rudimentary introduction to solid mechanics and to the concepts of differential and integral calculus. Viscous flows provide a compelling suite of joint applications of physical and mathematical skills to important scientific questions in multiple disciplines. When I was an undergraduate I was told—repeatedly, and with grave emphasis—to learn math and physics because they would be useful to me someday. Although that advice was sound, students should not have to wait as long as I did to profit. If this book reduces the wait for a few, it will have been worth the effort. To paraphrase the preface-leading quotation from Purcell, my goal is to provide a glimpse into some kind of wonderful world where biology, chemistry, geology, mathematics and physics and nice diagrams all come together.

Many excellent texts treat fluid dynamics, and a substantial number of specialized ones analyze low Reynolds-number flows. A typical route to low Reynolds numbers begins with an introduction to continuum mechanics, reviews the vector and tensor calculus to be used, goes through basic physical laws of flow, introduces the Navier-Stokes equations, then simplifies them in sequence for inviscid flows (ones without viscosity) and only then for low Reynolds-number applications (no effective inertia), before treating more complex flows at intermediate Reynolds numbers where both inertia and viscosity play important roles. Neither space nor target audience commends this approach here. My introduction to continuum mechanics is brief and informal toward a goal of accessibility. Many interesting phenomena are omitted. For the sake of simplicity, all calculations assume incompressibility along with uniform viscosity and density of the fluid. Also for the sake of simplicity and consistency, all unidirectional flows in all the diagrams proceed from left to right except where vertical motion is treated.

Mathematical concepts are introduced where needed, with greater reliance on geometrical visualization (*cf*. Weinreich 1998) than on mathematical formalism. My most radical departure from a typical fluids text is to begin with the simpler equations of viscous flow and stop well short of the full Navier-Stokes equations, giving only qualitative ideas of what happens "next." The flow equations in this short monograph treat only simple, laminar flows and slightly more complex vortical flows accessible through linear mathematics. The focus is on physics relevant to understanding environments of small organisms, with mathematics in a supporting role.

I emphasize simple equations in the contexts of these flows that provide easily calculable solutions to common problems posed by oceanographers and limnologists. My embrace of simplicity diversifies the mathematics that the reader will encounter. To help keep the mathematics from intimidating, most equations travel alongside written, equivalent, physical interpretations. Readers facile with mathematics will therefore experience some redundancy.

This book aims to whet, not slake, curiosity about fluid dynamics. Fluid dynamicists will be frustrated by my "too little, too late" introduction and application of vorticity. Although I appreciate the centrality of the concept and its applications in fluid dynamics, all my many attempts to introduce it to undergraduates early in an introduction to fluid dynamics failed—left too many behind. This book is for a preparatory course in fluid dynamics, not a course. It is intended to stimulate curiosity in a majority of undergraduates in a hard-science curriculum (including calculus, physics, chemistry, biology and geology) with regard to fluid behaviors and their consequences in natural environments. They should finish the book with a desire to learn more about vorticity.

Because it is so difficult to pull up short, it is best done deliberately, systematically, and with explicit consumer warnings. I was tempted to include some simple equations for inviscid fluids, but inclusion of a few only multiplied temptations. Equations for flows at intermediate Reynolds numbers were easier to avoid for they are in general more complicated than equations that throw out either inertia or viscosity. A few empirical fits to equations for intermediate Reynolds numbers are given, primarily to make clear where low Reynolds-number behavior breaks down. Perhaps the most rational way to learn fluid dynamics is to start with mathematically linear, low Reynolds-number flows, then look at inviscid flows, and then zero in on a topic of interest where inertia and viscosity are both important. I hope that readers will take comfort with my treating only the first step of these three, knowing that the equations in this book apply to real flows, whereas equations for inviscid flows skirt reality but provide real insight on the path to understanding of common flows that combine inertial and viscous effects.

An artifice that I use, because it simplifies the mathematics and makes flows easier to visualize and illustrate, is to stay almost exclusively with two-dimensional and axisymmetric flows. In particular, this deliberate choice makes the relation between velocity vectors and stream functions easy to define. For similar reasons, I stay primarily with simple geometries in terms of objects in the flow, *i.e.,* spheres, spheroids and cylinders. Despite the obvious oversimplifications, flow around a cylinder is a good approximation to flow around a worm tube, diatom chain or suspension-feeding appendage, and flow around a sphere or spheroid is a first approximation to flow around a fully three-dimensional body of more arbitrary shape. Indeed the chapters cumulatively provide striking contrasts between two- and three-dimensional flows.

I follow traditional book format, a coarsely woven compromise among diverse threads of disparate learning styles. To entrain the visual learner, illustrations play a large role. Equations engage some readers, even while they generate anxiety in others. To inspire homocentric readers, I interject a little context about significant contributors. My compromise caters to no particular style of learner very well, and for this reason needs substantial supplementation when teaching.

In keeping with the spirit of an introductory text, I limit citations and rely heavily on textbooks and reviews that provide easy access without misleading oversimplification, but I also add a few citations that demonstrate applications of the concepts developed in each chapter. Some are cited in the text, whereas others are included in short lists of advanced topics, with brief notation about their contents.

Two books appeared at about the same time to fill portions of the prior void in accessible introductions to biologically relevant, low Reynolds-number flows. I was lucky to have been a formal reviewer of both books, but unwise to have dawdled on mine. Thus the following content has been revised to cover most thoroughly material that is treated lightly or omitted in these two books. Kiørboe (2008) in a succinct volume focused on mechanisms and consequences of organism-organism encounter, primarily from bacterial to macroplanktonic (copepod) scales. At these scales encounter rate is fundamentally a function of abundance, detection distance and relative velocity, all intimately woven into the milieu of mostly laminar flows. Dusenbery (2009), in turn, gave book-length treatment of the phenomena covered by Purcell (1977)—and more—in a strikingly unhurried assessment of physical processes affecting bacteria, *e.g.,* the interaction of Brownian rotation with bacterial motility. The book includes flashbacks to fundamental physical discoveries underlying each process. Both of these books contain enough introductory fluid dynamics to give readers access. Dusenbery (2009) does range beyond bacterial scales to treat fertilization and in particular the fitness advantages of sex differences in gamete size. I omit detailed treatment of motility at low Reynolds numbers in part because it is well covered by Dusenbery (2009), and for brevity. My focus is on environments as context for organisms, solute and particle chemistry, and sediment dynamics.

Both Kiørboe (2008) and Dusenbery (2009) are organism centric. Neither covers a large range of environments where low Reynolds numbers dominate, and neither works through simpler one- and two-dimensional flows before moving to three. Despite the dominance of vortical flows in upper mixed and bottom boundary layers, neither text treats vortical flows. For that reason, I have refocused on those marine environments where low Reynolds number flows dominate and on flow interactions with structures of common geometries (*i.e.,* spheres, spheroids and cylinders), including simple analytic formulations to describe those flows.

A few readers will be interested in technical details. Software versions listed in this paragraph are the most recent ones used to generate material for this book. The book was camera readied in Adobe *InDesign CS6* (San José, CA) on a succession of Macintosh computers running *Mac OSX* 10.13.6. Mathematical solutions and statistical fits were developed primarily in Wolfram's *Mathematica* 11.110 (Somerville, MA). A few more complicated geometries and processes were visualized in *COMSOL* 5.3 (Burlington, MA), a finite-element modeling package that implements fundamental equations of fluid dynamics and molecular diffusion in realistic geometries. Like the book, my *COMSOL* simulations stay out of fully turbulent flow regimes; they assume incompressibility but otherwise use the full Navier-Stokes equations. Simulations using the laminar flow option extend somewhat beyond the creeping-flow regime (where Reynolds numbers are << 1) to Reynolds numbers of about 5 or 10—and a little higher where stated. To maintain stylistic standards, all figures were post-processed within Adobe *Illustrator CS6*. Despite aesthetic appeal of one font for figures and their captions and another for text, to reduce ambiguity in the meanings of symbols and italic letters, I used the same Times font throughout except for Greek letters, which are produced in the Symbol font. The exceptions are my use of Helvetica for annotation of citations and Monotype Corsiva for chapter-opening quotations.

I am grateful to the National Science Foundation and the Office of Naval Research for four decades of generous support that allowed me with colleagues to explore the flow environments covered herein. I also thank the cohorts of student guinea pigs who let me know what did not work in introducing low Reynolds numbers. Several generous colleagues provided informal reviews: Mark Denny (Ch. 4), Kevin Du Clos (Ch. 1-8), George Jackson (Ch. 8), and Don Webster (Ch. 7). I gratefully acknowledge my wife's, Mary Jane Perry's, patience with my work on "the book."

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Chapter 1

A fluid mechanics without beginning or end
Introduction to continuum concepts and table of symbols

One never steps twice into the same river [unless it flows at low Reynolds number].
~Heraclitus

MY COMPUTER PRESENTATIONS do not fire bullets one at a time. No secrets lurk at the middle or back of this book. Life's real problems hold interest enough, absent artifice of suspense. Right here—up front—I can suggest what you might learn if you proceed. As the title of this book implies and your intuition based on high Reynolds numbers no doubt will dispute, flows at very low Reynolds numbers are very orderly: They are mostly laminar, *i.e.*, they follow the contours of surfaces that bound them like the type of this justified text follows the margins or layers of onion follow contours of adjacent layers within and without. The proximate driver of many laminar flows is pressure, but instead of leaving them free to flow precisely down the pressure gradient, viscosity ("stickiness") constrains flows to parallel (stick to) the system's solid boundaries. Fluids are defined as substances that lack fixed shape and respond to pressure, including both gases and liquids. It is premature here to define a Reynolds number, but if the fluid parcel observed is small enough, if the fluid is highly viscous such as honey, if it moves slowly enough, or if fluid density is low enough, the Reynolds number will be small.

More generally, fluids display three flow regimes. At very low Reynolds numbers, such as those I target in this book, flows are simple, predictable, and orderly because viscosity, the cohesion between water molecules, reins in inertia, dissipating it to heat through fluid friction. At intermediate Reynolds numbers, the reins are loosened but still held, and flows are characterized by periodic features indicating spatially or temporally alternating dominance by the fluid's cohesive versus inertial forces. The inspiring sight of a flag waving or fluttering in the wind is emblematic of this unwon fight between stabilizing and destabilizing influences. It is a product of a continuing series of eddies or vortices shed from the flagpole and passing alternately on either side of the flag. Examples of intermediate Reynolds numbers are ubiquitous in the waving of branches and fluttering of leaves. Flows at high Reynolds numbers are much more chaotic; the reins have been dropped. Flow velocity at a point varies widely in both magnitude and direction. The fluid is a motley collection of vortices of diverse sizes rotating around axes in random orientations and continually modifying each other's shapes and dynamics by tangling with one another.

Life at very low Reynolds numbers labors under very different rules than you can observe, day to day, in your entangling, high Reynolds-number world. When a particle sinks in low Reynolds number interaction with the fluid, flow velocity is affected significantly out to of order 100 particle diameters. As a consequence, most particles under it will get pushed by fluid motion out of the way, and the particle itself is exceedingly difficult to catch by any physical means. Although horribly inefficient in encounter, laminar flow-produced collisions of prey and predators are nevertheless efficient enough for predator survival, and some protists feed by adjusting their buoyancies and rising or falling into collisions with food particles. A very practical consequence of these long-range flow effects is the impossibility of obtaining an accurate measure of particle settling velocity in a container fewer than 100 particle diameters wide: The flow field dragged along by the particle itself experiences friction with the walls and in return slows the particle. Another consequence is that bacteria in open waters are very abundant, at 10^5 to 10^6 individuals per cubic centimeter, simply because there is no efficient means to catch them. Fundamental stability of planktonic microbial ecosystems of oceans and inland waters results. Evolution cannot improve efficiency of bacterivores much further because no physical mechanism of encounter at low Reynolds is efficient. Physics constrains life.

And now, pretty much up front, you know my dirty little secret; I think in cgs (centimeters, grams and seconds) units rather than SI (from the French Système international d'unités) mks (meters, kilograms and seconds). To help prevent similar contamination of the next generation, however, in this book I give SI units first and do selective conversion to cgs where the latter may provide more intuition or more compact expression.

My conventions are hybrid among physics, biology, oceanography, engineering and the biological fluid dynamics popularizations that I mention in the preface, and so bear some explanation. My primary criterion for choosing notation is to avoid confusing those without prior exposure to fluid dynamics. Square brackets $[\mathrm{M}^b\ \mathrm{L}^c\ \mathrm{T}^e\ \mathrm{N}^f\ \underline{\mathrm{H}}^g]$ indicate primary dimensions independent of the units in which the quantities are measured. M is mass, L is length, T is time, N is number, and H is thermodynamic temperature, where the lower-case exponents are in general small integers or rational numbers comprising small integers. Some purists might object that N is inherently dimensionless, but I carry it along because of the diversity in kinds of objects that interest biologists, from molecules measured in moles to plankton quantified as numbers per unit of volume.

Although an overbar (*e.g.*, $\overline{x}$) is more frequent in biology, I use angle brackets, $\langle x \rangle$, to indicate statistical means because the intended extent of an overbar can be ambiguous, particularly in the neighborhoods of exponents and subscripts. I use very similar but more acutely angled symbols (<your.url.here>) to give explicit web addresses. Curly brackets {type me into your search engine} contain useful web search terms. Although web addresses

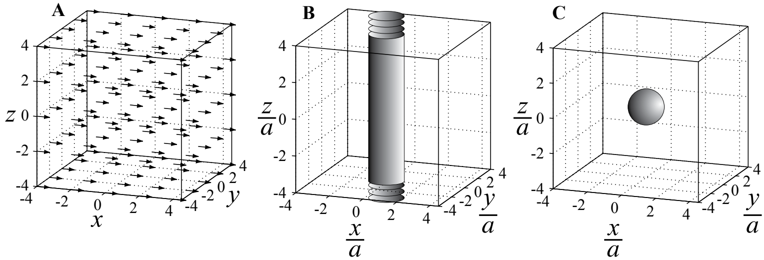

Fig. 1.1. Flow and object geometries often used in the text, all set in Cartesian coordinates with origins at (0,0,0). **A.** Vector plot showing the usual rotation of the coordinate system to yield maximal flow speed in the positive *x* direction. The uniform length and orientation of all the vectors corresponds with a (spatially) uniform flow field. If it also is invariant over time, it is called a steady, uniform flow, an ideal toward which many laboratory flow tanks strive. **B.** An infinite (in length) cylinder, as suggested by the elliptical ellipsis; if exposed to the flow field in **A,** it would be called a cylinder in "cross flow." Note that flow would be diverted only in the *x-y* plane; there would be no *z* components of velocity. **C.** A sphere. Note that for a sphere no location or orientation needs to be specified when the flow field is uniform, but flow will be diverted in any plane that cuts through the sphere's center and is oriented parallel to the upstream flow direction, *i.e.,* in all three dimensions. Assessments of flow fields around the cylinder and the sphere occupy substantial portions of this book. Flow through a tube (hollow cylinder) will also be introduced. Cartesian coordinates are not the natural ones for cylinders (**B**) or spheres (**C**) but are more familiar to most people than the cylindrical and spherical alternatives. Length dimensions in **A** are arbitrary, but those in **B** and **C** are nondimensionalized in dividing by the radius, *a,* of the cylinder and sphere, respectively.

will become obsolete even before this book is printed, useful search terms change much more slowly yet can be difficult to guess because they need to be just general enough to capture the topic of interest. Italic letters from the Times font in which the text is set and (non-italic) Greek letters from the Symbol font indicate variables or constants. Variables are defined locally in the text, with a few exceptions that are introduced in this chapter and included in the master table (Table 1.1) of variable definitions, dimensions, and SI units. With these tabled exceptions, the same lower-case italic letter may be used for different variables in different parts of the book and then will be defined locally. Vector notation is used sparingly and indicated with an arrow (*e.g.,* $\vec{x}$): both magnitude and direction are given or implied.

Coordinate systems used here are usually Eulerian (fixed in space) and Cartesian (orthogonal $\vec{x}$, $\vec{y}$ and $\vec{z}$), with the conventions that $\vec{z}$ is distance along the vertical axis (Fig. 1.1) and that $\vec{u}$, $\vec{v}$ and $\vec{w}$ are components of velocity in the *x, y* and *z* directions, respectively. When only the magnitude of the velocity (*i.e.,* speed) is important, I will use *u, v* and *w* without arrows for simplicity, even if in some cases the statement or equation may also hold for the corresponding vector. Although positions in Cartesian, cylindrical and spherical coordinates without doubt are vectors, I will also drop their arrows unless doing so might cause confusion; retention usually provides no more clarity and causes more anxiety for those uncomfortable with vectors. Engineers, oceanographers and limnologists often rotate the Eulerian coordinate system so that positive values of x and the greatest magnitudes of $\vec{u}$ correspond with the dominant flow direction (positive downstream), y and $\vec{v}$ are cross stream, and z and $\vec{w}$ are positive upward. Such is my practice here. Beware that oceanographers and limnologists concerned with the upper mixed layer, however, more often choose z to be positive downward distance from the air-water interface. All the bounded flows in this text are unidirectional along the x axis from left to right—in the positive x direction. For calculation of flow fields and forces, I also rotate the coordinate system for flow around objects to have a uniform flow approach the object from the negative x direction ($\vec{u}$ positive downstream). Illustrations of flows produced by settling objects, however, will be illustrated with the object traveling in the $-z$ direction.

To avoid immediate confusion, I should point out that Vogel (1994) used U for both u and $\vec{u}$, making distinction between speed and velocity apparent from context. U is often used in engineering as the magnitude of the velocity vector when direction is of no concern. I use lower case, however, because it is more conventional in both oceanography and engineering and because upper case for y and z components of velocity is so rare that it would certainly confuse. For brevity, I use indicial notation, $\vec{u}_i$, to include all three flow components, $\vec{u}$, $\vec{v}$ and $\vec{w}$. Some generalizations can be made, however, for any continuous velocity field without these directional conventions. Equations that hold for such cases will use $\vec{u}_{\mathrm{L}}$ to designate local velocity, *i.e.,* velocity at a point.

Far from any boundary, fastidious attention to vertical versus horizontal components of flow in the absence of density stratification can be an unnecessary complication. In many cases, only one velocity scale will be needed. I then use $\vec{u}$ for velocity and u for speed. Where only this single component is given, coordinates implicitly have been rotated to maximize the positive magnitude of $\vec{u}$, implying $v = w = 0$.

Leonhard Euler (1707, Basel, Switzerland – 1783, St. Petersburg, Russia) first formalized continuum mechanics, initially in analyzing material properties of solids, such as elasticity, and later in analyzing interactions of fluids with bodies such as ship hulls. Continuum mechanics is a branch of mechanics that analyzes kinematics and the mechanical behavior of materials modeled as continuous media rather than as discrete, separate masses. A key concept and methodology in continuum mechanics is an accounting of the forces acting on the matter within a bounded volume together with the forces acting on the surfaces bounding that specified volume. That focal volume is called a control volume, and its surfaces are called control surfaces. It is a more difficult accounting perspective than is usually presented in introductory physics courses, which rarely go beyond the mechanics of "point masses," *i.e.,* bodies in space clearly delineated from other such bodies and subject to analysis by treating them as having masses at their (point) centers of mass. Sir Isaac Newton (1643, Woolsthorpe, England – 1727, London, England) relied primarily on point masses in his mechanical derivations of the laws of motion but did touch on a few continuum concepts.

I have barely been able to put off this long naming the process whereby fluids move in organized, macroscopic flows and in which the idea of a directed velocity makes sense. In oceanography, advection is the generic term and convection is a special case in which motion is due to density differences in the fluid that cause it to sink in some locations, rise at others and move horizontally to keep the seas from parting (maintaining a continuum). There is some potential for confusion, however, because in engineering usage convection is the generic term for organized fluid motion, with a distinction sometimes made between natural (density- or wind-driven) and forced (engineered) convection. Internal to this book there should be no confusion because I treat neither salinity nor temperature gradients and so consider no density gradients. I use "advection" throughout.

Fig. 1.1**A** gives the first opportunity to consider the important idea of an advective flux attributable to the velocity. More generally fluxes are usually quantified per unit of area, and then are called flux densities, or are integrated over the entire area of an object or section of interest, in which case they are called total fluxes. Fluxes in both cases are perpendicular to the surfaces of interest. Because the only flow in Fig. 1.1**A** is in the *x* direction, it produces fluxes through *y-z* planes. Advective fluxes per unit of area of *y-z* plane are uniform because velocities are uniform, and four resultant kinds of fluxes will be of interest. Total volumetric flux through an area, *S,* of the plane, with the subscript V for volume, is

$$\vec{Q}_{\mathrm{V}} = \vec{u}S. \tag{1.1}$$

Total volumetric flow rate or flux is the product of velocity times the area through which it flows. Check the dimensions: $[\mathrm{L\,T^{-1}}] \times [\mathrm{L^2}] = [\mathrm{L^3\,T^{-1}}]$. If a given volumetric flow rate, $\vec{Q}_{\mathrm{V}}$, at average velocity $\langle\vec{u}\rangle$ through a pipe of cross-sectional area S into a pipe of cross-sectional area $S/2$, its average velocity in the narrower pipe must be $2\langle\vec{u}\rangle$. Multiplying by the density of the fluid, ρ $[\mathrm{M\,L^{-3}}]$, gives the total mass flux, $\vec{Q}_{\mathrm{M}}$ $[\mathrm{M\,T^{-1}}]$,

$$\vec{Q}_{\mathrm{M}} = \rho\vec{Q}_{\mathrm{V}} = \rho\vec{u}S. \tag{1.2}$$

Total advective mass flux is the product of fluid density and total volumetric flow rate. Often one is interested not in mass flux of liquid but in molar or numerical flux of some dissolved or suspended chemical or biological constituent (*e.g.*, phytoplankton or bacteria) at concentration, C, [N L^{-3}]. This total molar or numerical flux, $\vec{Q}_C$ [N T^{-1}], is

$$\vec{Q}_C = C\vec{Q}_V = C\vec{u}S. \qquad (1.3)$$

Total molar flux is molar concentration times volumetric flow rate. Finally, because momentum is mass times velocity, total momentum flux, $\vec{Q}_E$ [M L T^{-2}] (momentum traveling through the cross section of interest per unit of time), is

$$\vec{Q}_E = \vec{Q}_M\vec{u} = \rho\vec{u}^2 S. \qquad (1.4)$$

Total momentum flux is total mass flux times velocity, giving it a dependence on the square of velocity. Total momentum flux thus has dimensions of a force.

Corresponding advective flux densities have dimensions that differ by a factor of L^{-2}, removing the S from each equation above. Dimensions of corresponding equations for flux densities, $\vec{J}$, to Eq. 1.1 - 1.4 would be [L T^{-1}], [M L^{-2} T^{-1}], [N L^{-2}, T^{-1}], and [M L^{-1} T], respectively. Beware that symbols for flux densities versus total fluxes are not standardized. Note also that all fluxes are rates, *i.e.*, include dimensions of inverse time [T^{-1}], so that the term "flux rate" is both misleading (implying dimensions of [T^{-2}]) and redundant, and therefore to be avoided.

Most commonly, vectors are drawn to represent velocities at points where their tails originate. For clarity, only a subset of vectors is drawn. A vector is intended to represent the instantaneous velocity of an infinitessimal volume of water located at its tail. Fluid dynamicists sometimes call these infinitessimal volumes of water "point particles" (with the same implication as the infinitessimal size of a point in space) or just "particles." The latter shorthand can be confusing if one fails to realize that no solid particles are implied or involved. Adding to the confusion, the same term is sometimes applied to approximations for the motion of real, finite-sized, solid particles such as sediment grains, but some of these approximations assume that a particle of finite size would follow the same path as an infinitessimal parcel of fluid located at the particle's center of mass. Flows can interact, however, with real, solid particles in complex ways that are not captured by "point-particle" analyses in either of these contexts.

Two cases of objects in flows dominate examples in this book (Fig. 1.1**B**, **C**). Which is simpler depends on the problem at hand. They are what engineers call an infinite cylinder in uniform, steady cross flow and a sphere in uniform, steady flow. Because no simpler cases of objects in flow are possible, I rely extensively on these two examples to develop familiarity and intuition. The cylindrical coordinate system for cross flow is even simpler than the Cartesian because it is two dimensional (position along the length of the cylinder not affecting the flow pattern observed and so omitted from the explicit coordinate system). Coordinates are r for distance from the cen-

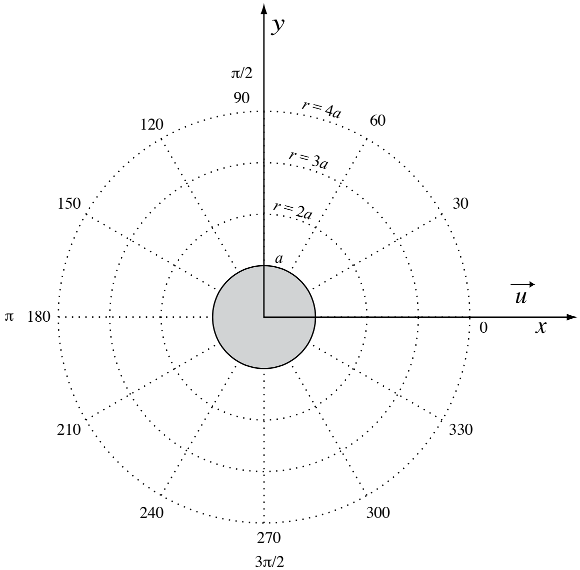

Fig. 1.2. Coordinate system used in this text for a cylinder in uniform cross flow and a sphere in uniform flow. Orientation in the flow field of Fig. 1.1**A** is indicated by the x and y Cartesian axes added to the figure, although coordinates for the cylinder and sphere will be distance from the center, r, and angle from the downstream direction, θ, (shown in both degrees and radians, measured counterclockwise). As in most drawings herein, r is nondimensionalized as the number of object radii (a) from the origin (small integers to the right of and oblique to the y axis). Two-dimensional coordinate systems suffice for infinite cylinders in cross flow and spheres because of their symmetries. All planes parallel to the x-y axis for the cylinder will show the same flow pattern (Fig. 1.1**B**), whereas all planes through the center of the sphere and parallel to the x axis (flow direction) will show the same flow pattern for the sphere (Fig. 1.1**C**). Therefore the z position along the cylinder and the angular position around the sphere in the plane perpendicular to the flow direction are neither needed nor given.

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ter of the cylinder, positive in the outward radial direction, and θ, varying from 0 in the downstream direction to π radians (180°) in upstream angle. Due to symmetry, what happens between π radians and 2π radians is a mirror image, reflected about the x axis of Fig. 1.2, of what happens between zero and π radians. Outward radial and tangential (to local r) velocities are $\vec{u}_r$ and $\vec{u}_\theta$, respectively. For a sphere in uniform flow again in principle there are three spherical dimensions, one radial and two angular (just as one can measure three-dimensional position by distance from the center of the earth, coupled with longitude and latitude). Because flow should be symmetrical around the upstream-downstream axis of the sphere, however, the coordinate system used again simplifies to two dimensions, with the same convention and notation as for the cylinder. These symmetries also explain why many of the flow diagrams in the text can be two dimensional (Fig. 1.2). For the infinite cylinder, they really are two dimensional (no flow components in the direction parallel to the cylinder axis). In the case of the sphere, there are flow components in all three dimensions, but the symmetry keeps us from having to track more than two at a time.

When a problem has a single length scale, I will nondimensionalize by it. Thus the coordinate systems of Fig. 1.1 and 1.2 are both in (nondimensional) multiples of the cylinder or sphere radius. I follow engineering convention by using a for an object's dimensional radius (m), and give nondimensional radial distances from the object's geometric center in terms of r/a. Some other treatments use r_0 in place of a.

It is worth a separate paragraph to emphasize the profound importance of and difference between these two examples. The cylinder in cross flow is the simplest example of flow deflection in two dimensions. The sphere in cross flow, in turn, is the simplest case of flow deflection in three dimensions. Flow can go around the cylinder in only two directions, whereas flow can diverge in any direction around the sphere. Hence there is fundamentally less resistance to flow around a three-dimensional object than to flow around a two-dimensional object. Flow around a cylinder is a crude, first-order approximation to flow interaction with an organism's appendage (especially crude because flow effects of the body supporting the appendage are not included), whereas flow around a sphere is a first-order approximation to flow interaction with the whole organism.

Together with appreciation for the benefits of nondimensionalization, the most important take away for many students studying fluid dynamics for the first time is the importance of attention to dimensions. If you find an inhomogeneous equation in this text, *mea culpa:* Dimensions on each side of and in each term of all the equalities and inequalities should be the same, and this logical convention will be a great aid. Although you might not recall the dimensions of a force, you probably do recall that

$$\vec{F} = m\vec{a} \,. \tag{1.5}$$

Force equals mass times acceleration, where $\vec{F}$ is force, m is mass [M] and $\vec{a}$ is acceleration [L T^{-2}]. Hence dimensions of a force must be [M L T^{-2}]. Similarly, you probably recall that kinetic energy , E_K, can be calculated as

$$E_K = \frac{mu^2}{2} \,, \tag{1.6}$$

so it must have dimensions of [M] × [L T^{-1}]^2 or [M L^2 T^{-2}]. A useful habit is to check the dimensions of any equation before exercising and trusting its quantitative outputs. The text explicitly reinforces this habit.

Welcome (back) also to the delightful world of physics, where pressure, tension, compression, stress and strain all are clearly defined and usually devoid of negative connotation. They all have to do with forces applied to control volumes or surfaces. For simplicity, I start with a solid cube. Body forces are associated with the mass of the object and distributed throughout that whole mass. They include forces of gravity, inertia and magnetism. Because they are distributed throughout the body, these forces often are given per unit of body volume as a "body force intensity" [$M\ L^{-2}\ T^{-2}$]. All other forces act on a body by contact with its surface and so are called surface forces and often are expressed per unit of body surface area [$M\ L^{-1}\ T^{-2}$], in which case they are called stresses. The difference between a force and a stress is made obvious by a pointy object or a blade. For example, stiletto heels have applied enough force per unit of area (stress) to the concrete seaside promenade in Naples to leave heelprints. The point or blade concentrates stress by shrinking the area over which available force acts.

The easiest bookkeeping of surface stresses decomposes them into three, mutually orthogonal components acting on each face, one of them orthogonal to the surface and the other two tangential (Fig. 1.3). A normal stress pushing inward on a body is called compressive, whereas one pulling outward on it is called tensional. Tensional or compressive stresses can be uniaxial (running along only one axis), biaxial or (on a very bad hair day) triaxial. By convention, outward from a body is considered the positive direction, so that tension is positive and compression, negative in sign. Arrows then point in the direction that the force, if sufficient, would move the object. A tangential stress is called a shear stress, and a normal stress (tensional or compressional) is called a pressure. Both shear stresses and pressures are forces per unit of area. A common misconception is that, because the bookkeeping separates tangential from normal components, they must represent different forces or forcing mechanisms. If I press my index finger at 45° into the top of my desk, however, my desk (and my finger) will experience both shear and compressional stresses over the area of contact from my single, futile, action.

If the cube has finite size, a proper accounting of surface forces on it also requires specification of the normal and tangential stresses on the hidden faces of the cube. Although positive remains outward for the normal stresses, convention reverses the positive directions for tangential stresses on those hidden faces, *i.e.,* positive is in the opposite directions to those for the illustrated faces. As we shrink the cube to infinitessimal size, however, the nine illustrated vectors become sufficient for a full description of surface forces or the state of stress at a point.

A square array or matrix of numbers is a compact means to make an accounting of and to summarize this state. This matrix summary is referred to as a second-order stress tensor because each term contains two indices that represent aspects of the stress geometry:

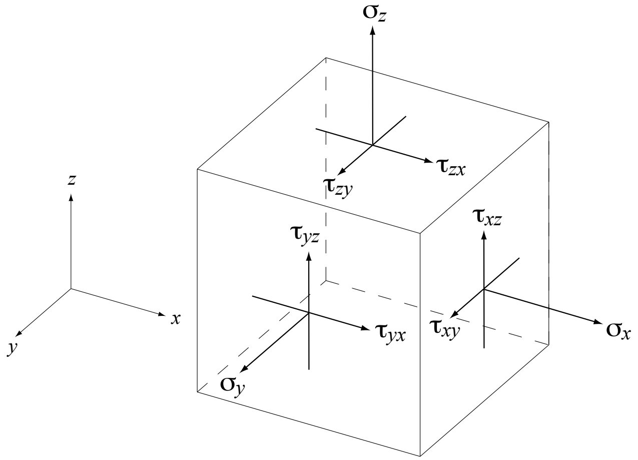

Fig. 1.3. Bookkeeping convention for surface stresses on the faces of a cube, with the Cartesian coordinate system indicated by the inset. For normal stresses, $\vec{\sigma}_i$, a single index (subscript) is sufficient to indicate the stress because convention makes outward forces positive. For shear stresses, $\vec{\tau}_{ij}$, the first index specifies the plane of interest (perpendicular to the i axis), and the second (j) the direction of the shear stress. All arrows above point in directions in which stresses are considered positive.

$$\vec{\sigma}_{ij} \equiv \overrightarrow{\text{stress}}_{ij} \equiv \begin{array}{cl} \begin{bmatrix} \vec{\sigma}_{xx} & \vec{\tau}_{xy} & \vec{\tau}_{xz} \\ \vec{\tau}_{yx} & \vec{\sigma}_{yy} & \vec{\tau}_{yz} \\ \vec{\tau}_{zx} & \vec{\tau}_{zy} & \vec{\sigma}_{zz} \end{bmatrix} & \begin{array}{l} \leftarrow \text{on the } x \text{ plane} \\ \leftarrow \text{on the } y \text{ plane} \\ \leftarrow \text{on the } z \text{ plane} \end{array} \\ \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \text{in the } x \text{ direction} & \text{in the } y \text{ direction} & \text{in the } z \text{ direction} \end{array} & \end{array} \quad . \tag{1.7}$$

The state of stress at a point is equal to the overall vector sum (all nine components) of the single pressure stress and the two orthogonal shear stresses on each of its three orthogonal faces. It is easy to imbue this equation or identity with more meaning than it has. It is simply a compact and exhaustive list of all the stresses of Fig. 1.3 when the cube has been shrunk to an infinitessimal point. Fig. 1.3 and all three identities of Eq. 1.7 are fully substitutable. $\overrightarrow{\text{Stress}}_{ij}$, or more often $\vec{\sigma}_{ij}$ (where it is understood that $\vec{\sigma}$ can stand for either a pressure, $\vec{\sigma}_{ii}$, or a shear stress, $\vec{\tau}_{ij} \equiv \vec{\sigma}_{ij}, i \neq j$), is the shorthand form in index (or indicial) notation. In this text $\vec{\tau}_{ij}$ will be used for shear stresses, but the only normal stress encountered in subsequent chapters will be fluid pressure. Fluid pressure, p, at a point is omnidirectional and re-

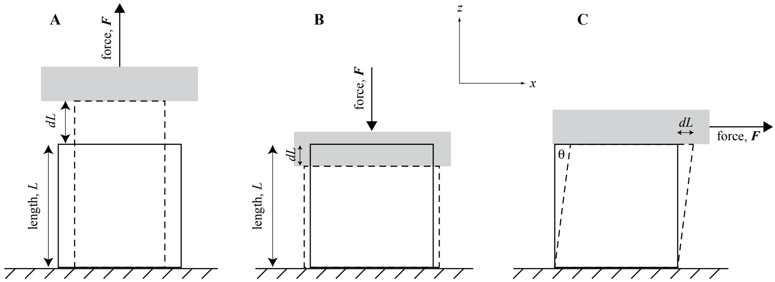

Fig. 1.4. Response of a cube of an ideal solid to uniaxial forces of (**A**) tension, (**B**) compression and (**C**) shear. Hatched regions, following engineering convention, represent immovable boundaries, in this case firmly attached to the cube. Solid lines show initial boundaries. Gray rectangles represent an object or apparatus firmly attached to the block at the opposite end from the immovable boundary for the purpose of transferring the force, $\boldsymbol{F}$, uniformly to the *x-y* plane at the top of the block. This object is drawn centered on and attached to the deformed block (dashed lines). These side views show only the *x-z* plane (inset). In **C**, the deformation is measured as the angle θ, or the maximal linear displacement, dL.

garded as a scalar. It nevertheless results in a vector pressure force on a solid object when pressure stress is integrated over a given area of surface.

What happens to a cube of solid or liquid under a particular combination of body forces and shear stresses? It may translate (move in a particular direction), rotate (about an axis without translating its center of mass), deform (change shape) or undergo any combination of these motions (including none of the above if the material is strong enough and the resisting forces balance those applied). A body is said to deform or be strained when the distance between at least two points within it changes in response to applied stresses.

Because most readers have a more intuitive understanding of solids, strain responses of ideal, or Hookean (for Robert Hooke), elastic solids to stresses are good places to start. These three kinds of stresses are uniaxial tension, uniaxial compression and shear (Fig. 1.4). Under constant stress, ideal solids achieve constant displacement or strain. So long as the force lasts, they remain deformed. When the force is removed, they return to their initial shapes. For small displacements, many solids approximate this ideal behavior, making strain linearly proportional to stress. To exercise the notation of Fig. 1.3, the stresses shown are (**A**) $\vec{\sigma}_z$ (**B**) $-\vec{\sigma}_z$ and (**C**) $\vec{\tau}_{zx}$. Take S as the initial surface area of the cube. Shear stress is force per unit of area, so $\vec{\tau}_{zx} = \vec{F}/S$ [M L^{-1} T^{-2}]. If we measure strain $\vec{\varepsilon}_z$ for Fig. 1.4 **A** or **B** as dL/L, then $E_Y = \sigma_z/\varepsilon_z$ expresses the linear relationship in magnitude between stress and strain, and E_Y [M L^{-1} T^{-2}] is known as the elasticity constant or Young's modulus.

For many solids (again under small deformations), E_Y is similar in magnitude between tension and compression. Under shear stress, (Fig. 1.4**C**), strain magnitude ε_{zx} is more consistently measured as θ (radians), but for small displacements $\theta \cong dl/L$. The shear modulus, G [M L^{-1} T^{-2}], *i.e.,* the linear relationship between stress and strain, then becomes $\tau_{zx}/\varepsilon_{zx}$. If the material is uniform and isotropic over the cube (*e.g.*, a non-crystalline metal or a polymer such as rubber), then E_Y and G do not depend on direction, and vector notation is unnecessary. Shearing the cube in either positive or negative direction along either the *x* or the *y* axis should give the same result for G. For many solids, $E_Y \cong 3G$. It is not hard to anticipate that repeating the measurements of Fig. 1.4 with water instead of a cube of solid in the space between the immovable wall and the moving device would yield very different results; the device (gray block) would continue to move in the direction of the force as long as the force was applied. Developing these results more quantitatively, however, benefits from additional information within the next chapter.

Some fluid behaviors do not depend on geometry, however, and it is useful to introduce them here. I will assume constant fluid density, ρ (M L^{-3}). Although constant density often poorly represents real fluids, I avoid fluid density variations strictly to avoid confusion, leaving them to more advanced treatments. Although water (until the freezing point) becomes denser as it cools, I will in all examples treat it as having uniform density. Water is barely compressible. Perhaps more surprisingly, air over the range of natural flow velocities (as opposed to its state within enclosures such as bicycle tires) is barely compressible, so the assumption I make here of fluid incompressibility is a good approximation for the two most common environmental fluids. A consequence is that all parts of the fluid in all examples in this text contain the same mass per unit of volume and that when water moves from point A to point B, the same volume and mass of water has to come from somewhere to replace the water that left point A. This principle of continuity and mass conservation can be written parsimoniously as

$$\nabla \bullet \vec{u}_i = 0 \,. \tag{1.8}$$

The divergence of velocity at all points is zero. For any control volume, what leaves must be replaced with an equal mass and volume of fluid (cavitation and density changes not allowed), and what enters must displace an equal mass and volume from the region (compression not allowed). The "del" operator or nabla symbol, ∇, in the vector Eq. 1.7 can be written as

$$\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\,, \tag{1.9}$$

where $\vec{i}$, $\vec{j}$, *and* $\vec{k}$ are vectors one unit long in the *x, y,* and *z* directions, respectively. It thus gives the gradient in all three dimensions of the variable of interest. The black dot in Eq. 1.8 denotes scalar multiplication, *i.e.,*