Introduction to Particle Physics

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Contents

Foreword

One of the methods of studying Nature is to penetrate deeper and deeper in the structure of matter ever increasing the spatial resolution, i.e., studying smaller and smaller objects. In the history of natural sciences new and new particles appeared which were thought to be *elementary*: the four atoms (a-tom = not divisible) of Anaximenes and Democritus, the elements/atoms of Dalton and Mendeleev, the atomic nucleus of Rutherford and the so-called *elementary* particles of which the proton, the neutron, the electron and the neutrino are the most well-known. Between 1930 and 1960 hundreds of such particles were discovered, thus a new level of elementariness was needed and the quark model appeared. We will see that, in fact, the proton and the neutron are also composite particles although the electron stays elementary. This development was crowned by the standard model (SM) in the late sixties and it is still the uncontested global theory of matter, supported by all available theoretical and experimental evidence.

In this textbook we summarize the present knowledge of particle physics at an introductory level. Particle physics is a very broad subject including many different sub-fields. While we mention many of these, a detailed account on all is impossible. Our clear focus is on high energy collider physics that is among the most widely pursued subfields where the threshold of current research is high. With the Large Hadron Collider in operation new results appear regularly. Our goal here is to keep the level introductory, yet help students reach this high threshold making them acquainted with both the experimental and theoretical minimum needed to comprehend current research at colliders. Our treatise is detailed on established results of collider physics while mostly marginal on current developments with the exception of the discovery of the Higgs particle due to its utmost importance.

The first part (written by D. Horváth) is planned to be accessible for advanced BSc or freshmen MSc students, while the second part (written by Z. Trócsányi) on the theory is intended for advanced MSc or freshmen PhD students in particle physics, with some attempt to go into the rather complex mathematical formalism of the field. Our aim is to provide concise but hopefully comprehensive account on the subject and also try to help students in their decision whether to orient themselves towards experiment or theory. We assume that the book can be covered during a full academic year with about 10 hours of serious effort per week. Although the reader may be confused on several occasions when not all details are given, the theory is very precisely elaborated and its predictions beautifully agree with the experimental observations. All present day experimental evidence is summarized in the biennial *Particle Physics Review* of the Particle Data Group [Beringer et al., 2012]; for a theoretical introduction we recommend the textbooks of Halzen and Martin [Halzen and Martin, 1984], Collins, Martin and Squires [Collins et al., 1989], and Perkins [Perkins, 1982]. The theory provides a nice glance at the key experiments as well.

Experimental particle physics is also called high-energy physics, because of its basic method of study. Energy is measured in units of electroplate, eV, the energy gained by a particle of unit charge (e.g. an electron or proton) when crossing a gap of 1 V voltage. One of the earliest means of structural studies was the optical microscope. Its resolution is limited by the wavelength of visible light (corresponding to an energy of $\sim$ 1 eV) to the size of bacteria, 10^{-5} m. The smaller a detail, the shorter wavelength is needed to see it. For the atoms (10^{-10} m) we need X-rays or electron beams of keV (1 keV = 1000 eV) energies, for the atomic nucleus of 10^{-14} m electrons or protons between MeV (10^6 eV) and GeV (10^9 eV) energies, and for the quarks (below 10^{-18} m) up in the TeV (10^{12} eV) region. Higher energy means smaller distances and so studying finer details of the structure of matter. As far as we know at present, the basic particles of the standard model are really elementary: point-like and structureless.

Throughout this book we use the *natural* units of particle physics where the speed of light c and Planck's reduced constant $\hbar = h/(2\pi)$ are both equal to unity and thus both distance and time can be expressed in inverse energy units, i.e. in GeV^{-1} (see Table 1).

We shall quote only selected, but not all original publications as this is intended to be an introductory textbook, not a monograph and we do not want to overwhelm students. Yet we encourage the reader to consult some trustworthy sites on the Web (like Wikipedia *http://en.wikipedia.org/*, Hyperphysics *http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html* and Google Scholar *http://scholar.google.com/*) for simple explanations and the inSpire (*http://inspirehep.net/*) publication data base for reviews and original publications. Typing in the author's name and the date will produce all kind of information at any depth for the reader. We consider especially useful the web version of Particle Physics Review by the Particle Data Group (*http://pdg.web.cern.ch/pdg/*) which provides reliable and comprehensive reviews. Reliability is important in high energy physics as it is in the very front of knowledge and as such it is full of speculations, untested ideas and unconfirmed experimental findings. At some points we shall only mention some examples of those: Earth-absorbing black holes produced in high-energy particle collisions, faster-than-light neutrinos and pentaquarks.

The authors acknowledge the support of the Hungarian Scientific Re-

Quantity	MKS	particle physics	natural unit	$\hbar = c = 1$
Energy	1 J	$6.24 \cdot 10^9$ GeV	GeV	GeV
Momentum	1 kg m/s	$5.61 \cdot 10^{26}$ GeV/c	GeV$/c$	GeV
Mass	1 kg	$5.61 \cdot 10^{26}$ GeV	GeV$/c^2$	GeV
Distance	1 m	$5.07 \cdot 10^{15}$ GeV^{-1}	$\hbar c/$GeV	1/GeV
Time	1 s	$1.52 \cdot 10^{24}$ GeV^{-1}	$\hbar/$GeV	1/GeV
Electric charge (e)	0.16 aC	$\sqrt{4\pi\epsilon_0\alpha\hbar c}$	$\sqrt{4\pi\epsilon_0\hbar c\alpha}$	$\sqrt{4\pi\epsilon_0\alpha}$

Table 1: Natural units of particle physics. $\alpha = 1/137$ is the fine structure constant, $\epsilon_0 = 8.8 \cdot 10^{-12}$ F/m is the electric permittivity of vacuum. In the last column only energy units appear, which permits the use of the power of GeV to characterize the unit as "mass dimension", e.g., the unit of length, GeV^{-1} is referred to as mass dimension minus one. *In this book we use the units of the last column, except for two cases.* One is momentum, which has natural units GeV/c, to become GeV if $c = 1$. However, in order not to confuse it with energy, we keep the natural unit for momentum. The other exception is the electric charge, for which we use the conventional notation $\sqrt{4\pi\alpha}$, i.e., we use $\epsilon_0 = 1$, which should not cause any confusion.

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Part I

Particle phenomenology

Chapter 1

Particles and symmetries

MOTTO:

Central to that theory is the concept of spontaneously broken gauge symmetry. According to this concept, the fundamental equations of physics have more symmetry than the actual physical world does.

(Frank Wilczek)

1.1 Symmetries in particle physics

Symmetries in particle physics are even more important than in chemistry or solid state physics. Just like in any theory of matter, the inner structures of the composite particles are described by symmetries, but in particle physics everything is deduced from the symmetries (or invariance properties) of the physical phenomena, or from their violation: the conservation laws, the interactions and even the masses of the particles. Symmetries that are not connected to space and time are called *internal* symmetries. Continuous symmetries can be *global*, i.e. independent of the space-time coordinates, or *local*. The latter means that we can choose the orientation of the (external or internal) coordinate axes freely at any space-time point. These symmetries are also called *gauge symmetries* or *gauge invariance* laws.

In field theory, according to the theorem of *Emmy Noether* any continuous global symmetry leads to a conservation law. Thus the freedom to choose the origin and orientation of our coordinate system leads to the conservation of momentum and angular momentum, that of the origin of time measurement to the conservation of energy. The conserved quantities due to continuous global internal symmetries are called conserved charges. An example is the free choice of the phase of the electron wave function as it is not measurable experimentally. This global phase invariance is also a continuous symmetry and the emerging conservation law is the conservation of the fermion charge in general. The global gauge invariance of electrodynamics leads to the conservation of the electric charge.

Gauge symmetries have far reaching consequences as they lead to interactions between particles. In the theory part (Part IV) of this book we formulate gauge symmetries in a precise mathematical sense to *discover* the three fundamental interactions of particles (the strong, weak and electromagnetic forces). Local symmetry always implies the existence of a related global symmetry as well, but a global symmetry may not necessarily imply a local symmetry, hence an interaction.

1.2 Symmetry groups and spin

The characteristic features of particles are described in terms of *symmetry groups*. The language of physics is mathematics: the mathematical formulation makes the difference between theory and speculation in physics as that allows for making quantitative predictions, which then can be checked experimentally. A new physical theory is accepted if the predictions agree with *all* available experimental information.

As symmetries usually appear at transformations of our coordinate systems the mathematical apparatus is chosen accordingly. A trivial example is the rotation of a 2-dimensional coordinate system around its origin by an angle Θ. As shown by Fig. 1.1 the new (x', y') coordinates of point P are obtained in the rotated system from the old (x, y) coordinates by the transformation

$$\begin{aligned} x' &= a + b = x\cos\Theta + y\sin\Theta\,, \\ y' &= y'' - c = y\cos\Theta - x\sin\Theta\,. \end{aligned}$$

The point P, just as any two-dimensional vector, undergoes the following

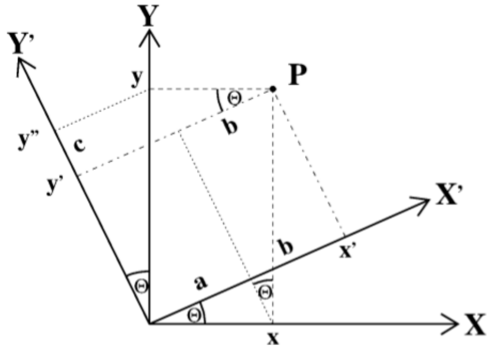


Figure 1.1: Rotation of a coordinate system in two dimensions: coordinate system $[X', Y']$ is obtained by rotating system $[X, Y]$ by an angle Θ.

coordinate transformation:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\Theta \cdot x + \sin\Theta \cdot y \\ -\sin\Theta \cdot x + \cos\Theta \cdot y \end{pmatrix} = \begin{pmatrix} \cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} .$$

This means that the vector $\begin{pmatrix} x' \\ y' \end{pmatrix}$ is obtained by multiplying the vector $\begin{pmatrix} x \\ y \end{pmatrix}$ with the matrix in front of it. An important property of these rotation transformations is that they do not change the length of the vector pointing to P (its absolute value) as

$$x'^2 + y'^2 = (x^2 + y^2) \cdot \left(\cos^2\Theta + \sin^2\Theta\right) = x^2 + y^2 .$$

The condition that the length of the vector remains unchanged demands that the complex transformation matrix be unitary:

$$U^\dagger U = \begin{pmatrix} U_{11}^* & U_{21}^* \\ U_{12}^* & U_{22}^* \end{pmatrix} \cdot \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix} =$$

$$\begin{pmatrix} U_{11}^2 + U_{21}^2 & U_{11}^* U_{12} + U_{21}^* U_{22} \\ U_{12}^* U_{11} + U_{22}^* U_{21} & U_{12}^2 + U_{22}^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Rotations of this type have the following mathematical properties:

- they are additive: rotation by Θ_1 and then by Θ_2 is equivalent to a rotation by $\Theta = \Theta_1 + \Theta_2$;
- their addition is associative: $(\Theta_1 + \Theta_2) + \Theta_3 = \Theta_1 + (\Theta_2 + \Theta_3)$;

- they have a unit element: rotation by $\Theta = 0$ which does not do anything;
- the rotations can be inverted ($\Theta - \Theta = 0$ and the inverse elements are also members of the set.

Sets with operation among its elements obeying these properties are called *groups*. Spin is a three-dimensional quantity with the properties of the rotation group and its mathematical description (*representation*) is called the $SU(2)$ group of special (determinant = 1) unitary complex 2×2 matrices. $SU(2)$ can be applied not only for spin, but for any physical quantity with similar symmetry properties, like for example the *isospin* to be introduced later.

When we increase the degrees of freedom we get higher symmetry groups of similar properties. The next step, $SU(3)$, which is also used in particle physics, is the symmetry group of special unitary complex 3×3 matrices. It has three possible *eigenstates* which can be interpreted as three corners of a triangle with an $SU(2)$ symmetry between any two of its corners (see Chapter 3).

In case of complex quantities we can also decrease the degrees of freedom of rotations, then we get the $U(1)$ group of 1×1 unitary matrices, i.e. $e^{i\phi}$ complex phases. That is the symmetry group of the *gauge transformations* of the electromagnetic interaction. The simplest manifestation of the gauge symmetry of electromagnetism is that we can freely choose the zero point of the electrostatic potential as demonstrated by the birds sitting on high-voltage wires. The global $U(1)$ symmetry of Maxwell's equations leads to the conservation of the electric charge. In the more general case, the $U(1)$ symmetry of the Dirac equation [Dirac, 1931], the general equation describing the motion of a fermion, causes the conservation of the number of fermions or *fermion charge* [Halzen and Martin, 1984].

1.3 Fermions and bosons

Particles are categorized according to various properties. The most important one is spin, the intrinsic angular momentum. Spin cannot be interpreted as related to actual rotation, but it is added to the orbital momentum. Its natural unit is the reduced Planck constant $\hbar = h/(2\pi)$. Spin is strange: the spin of the electron is added to its orbital momentum, but it has two eigenstates only: it is either left or right polarized as compared to its momentum (or points either up or down in a vertical magnetic field) in any coordinate system. Thus spin is characterized by two independent quantities in three dimensions: by its length and one of its vector components.

The particles with half-integer spin ($S = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}...$) follow the Fermi–Dirac statistics and they are called *fermions*, whereas those with integer spin ($S = 0, 1, 2$ ...) follow the Bose–Einstein statistics and called *bosons*. They have very different symmetry and other properties. The wave function describing a system of fermions changes sign when two fermions switch quantum states whereas in the case of bosons there is no change; all other differences can be deduced from this property. Fermion number is conserved whereas one can create and absorb bosons: a lamp can irradiate any number of photons ($S = 1$) and an antenna can absorb them, assuming that the energy and momentum are conserved. At the same time the electrons ($S = \frac{1}{2}$) in order to illuminate a lamp or a cathode tube of an old-fashioned TV set have to be brought there and then conducted away. Another very important difference is that any number of bosons can be put in any particular quantum state, but only one fermion: this is *Pauli's exclusion principle*. That is why the electrons of the atom fill discrete energy levels and this is the force which prevents the atoms of matter and the nucleons in the nucleus from penetrating each other; it provides macroscopic forms for our material objects.

Actually, the mathematics behind Pauli's exclusion principle is extremely simple. The state function for a two-fermion system changes sign when you exchange its particle states, $\psi_F(1,2) = -\psi_F(2,1)$, while that of a bosonic system does not, $\psi_B(1,2) = +\psi_B(2,1)$. Thus if the two fermions would have exactly the same quantum numbers, their state function should be zero as that is the only function which does not change at changing its sign.

Some basic properties of fermions and bosons are compared in Table 1.1. The elementary fermions of the standard model are the six leptons and six quarks, the elementary bosons are the mediators of the three particle interactions and the Higgs boson.

The standard model assumes that our world is constructed of three fermion families, each consisting of a pair of quarks and a pair of leptons, and their interactions, deduced on the basis of symmetry principles, are mediated by vector bosons of spin 1. All fermions have *antiparticles* with opposite charges and otherwise the same properties. The interaction of a particle with its antiparticle leads to their annihilation. When an electron and its antiparticle, the *positron* annihilate they produce two or three photons, whereas the annihilation of a proton with an antiproton releases so much energy, almost 2 GeV, that half a dozen particles (mostly pions, the lightest mesons) could be emitted.

The lack of antimatter in the Universe [Cohen et al., 1998] implies a

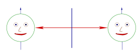
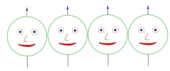
Property	fermion	boson
Spin	half-integer ($\frac{1}{2}$, $\frac{3}{2}$ $\ldots$)	integer (0, 1, 2, $\ldots$)
$\psi(1,2) = \pm\psi(2,1)$	$-$	$+$
Pauli exclusion	yes	no
Particle number conserved	yes	no
Condensation	 no	 yes
Statistics	Fermi-Dirac	Bose-Einstein

Table 1.1: Comparison of the properties of fermions and bosons

possible asymmetry between particle and its antiparticle, and that is one of the great mysteries of physics. Were there antimatter galaxies, they would emit antiparticles and they would be encircled by a halo of annihilation when meeting the particles emitted by neighbouring galaxies of ordinary matter, but the astronomers do not see such phenomena anywhere.

1.4 Mirror reflection: parity

Changing signs of all three coordinates, i.e. changing the directions of all three axes of a rectangular (Cartesian) coordinate system is equivalent to changing just one sign: we call it going from the usual right-handed coordinates to left-handed ones as in the usual coordinate system rotating the x-axis to y defines the direction of z according to a right-handed screw. This is explained in Fig. 1.2.

Parity is a general property of the mathematical functions of physics. Any function $f(x)$ can be written as a sum of two functions of odd and even parities, e.g. by separating even and odd terms in its Taylor or trigonometric expansion or simply by writing $f(x) = \frac{1}{2}[f(x) + f(-x)] + \frac{1}{2}[f(x) - f(-x)]$. The parity operator is of course unitary,

$$P\psi(r,t) = \psi(-r,t); \quad P^2 = 1 .$$

Any interaction with a spherically symmetric potential, e.g. the Coulomb

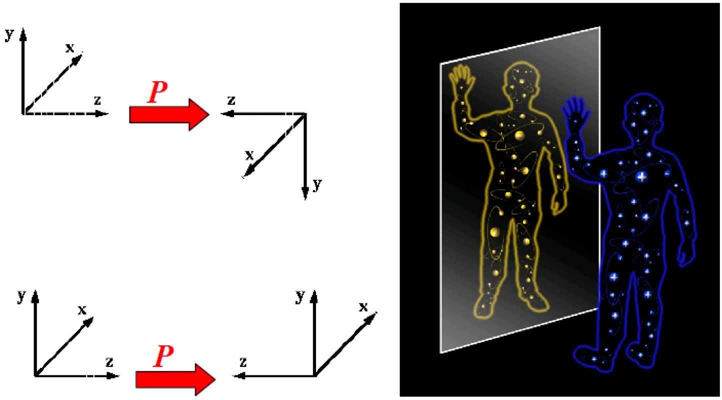


Figure 1.2: Mirror reflection and parity change (after D. Kirkby, APS, 2003) from right-handed to left-handed coordinates.

potential of a point-like electric charge, conserves parity:

$$V(r) = V(-r) \Rightarrow H(r) = H(-r) \Rightarrow [P, H] \equiv PH - HP = 0\,.$$

As a result, parity is a good quantum number[1] of particles. For instance, the wave function of the hydrogen atom is a parity eigenstate:

$$PY_{\ell m}(\Theta, \phi) = Y_{lm}(\pi - \Theta, \pi + \phi) = (-1)^{\ell} Y_{\ell m}(\Theta, \phi)$$

with an eigenvalue of $P_{\mathrm{em}} = (-1)^{\ell}$ where ℓ is the orbital momentum. As the minimal electromagnetic transition, $E1$ means a change of orbital momentum $\Delta\ell = \pm 1$, the photon emitted in electromagnetic reactions should have at least $P_{\gamma} = -1$, the photon has a negative intrinsic parity (it can have an orbital momentum as well, of course). Anti-fermions have the opposite parities as fermions, whereas particles and antiparticles of composite bosons[2] have the same parity determined by the parity and angular momentum of its constituents.

Parity quantum numbers are multiplicative, e.g. for a system of three particles: $P(123) = P_1 \cdot P_2 \cdot P_3$. As mesons are bound states of quarks and

[1]Quantum number: such a physical quantity which can change in definite quanta only, like charge or angular momentum, and a set of which can uniquely characterize a physical state.

[2]Fundamental bosons do not have antiparticles as the existence of antiparticles is related to the Dirac equation of the fermions.

anti quarks, their ground state ($\ell = 0$) parity is $P(q\overline{q}) = P(q) \cdot P(\overline{q}) = -1$. By definition for the nucleons $P_p = P_n = +1$, thus the parities of quarks are $+1$, and of anti-quarks -1. For the particles spin J and parity P are denoted as J^P, e.g. for charged pions $\pi^\pm : 0^-$.

We shall see later that parity is not conserved by the weak interaction, it is a broken symmetry. Stephen Weinberg calls such symmetries *accidental symmetries*.

1.5 Charge conjugation

Charge conjugation converts a particle into its antiparticle, $C|p\rangle = \pm|\overline{p}\rangle$. It is also a unitary operator, $C^2 = 1$. It changes the signs of all kinds of charges: electric, baryon, lepton charges, but not the spin. Only neutral particles could be C-eigenstates, the eigenvalue is the C-parity. Strong and electromagnetic interactions conserve it. For instance, in the electromagnetic decay of the neutral pion 2 photons are emitted, $\pi^0 \to \gamma\gamma$ and $C|\pi^0\rangle = C|\gamma\gamma\rangle = |\gamma\gamma\rangle = |\pi^0\rangle$, thus $C_{\pi^0} = +1$. As $C|\gamma\rangle = -1$, the π^0 cannot decay to three photons, $\pi^0 \nrightarrow \gamma\gamma\gamma$.

1.6 *CPT* symmetry

Antiparticles can mathematically be treated as particles of the same properties *going backward in space and time*. This is a very important symmetry of Nature: the physical laws do not change when charge (C), space (P) and time (T) are simultaneously inverted:

- charge conjugation (i.e. changing particles into antiparticles), $C\psi(r,t) = \overline{\psi}(r,t)$;
- parity change (i.e. mirror reflection), $P\psi(r,t) = \psi(-r,t)$, and
- time reversal, $T\psi(r,t) = \psi(r,-t)K$ where K denotes complex conjugation.

This is called *CPT* invariance. As time reversal is an *anti-unitary* operation, *CPT* is also anti-unitary, it *conjugates the phase of the system*, but does not change any measurable properties. The electron-positron annihilation can be mathematically described as if an electron arrived, irradiated two or three photons and left backward in space and time. Using an analogy with the electric current we call this *particle current*; in the above example the incoming electron and positron constitute a lepton current.

In the simplest case of particle collision two such particle currents exchange a boson. This is made possible by the *uncertainty principle* of Heisenberg, as it allows a violation of energy and momentum conservation for very small time and space intervals: $\Delta E \cdot \Delta t \geq \hbar/2$ and $\Delta p \Delta x \geq \hbar/2$, where Δ indicates a very small change in the quantity behind it and $E,\ p,\ t,\ x$ the energy, momentum, time and space position. The very small value of the reduced Planck constant ($\hbar \simeq 1.055 \cdot 10^{-34}$ J·s) ensures that the conservation laws are fulfilled in the macro-world. The boson mediating the interaction can be *real* or *virtual* depending on whether or not it satisfies the on-shell condition $E^2 = m^2c^4 + \vec{p}^2c^2$. Effects of virtual particles can be detected experimentally: in the inelastic scattering of high energy electrons on each other quark pairs could be produced when a virtual photon emitted by one of the electrons is absorbed by a quark of a virtual quark-antiquark pair produced momentarily by another photon emitted by the other electron.

CPT invariance is supported by ample experimental evidence. Its role is so important in quantum field theory that according to some theorists it is impossible to test experimentally: in the case of observing a small deviation one should rather suspect the violation of a conservation law than CPT violation. In spite of this, there are considerable efforts to test it experimentally. The most precise of those tests is the very small possible relative mass difference between neutral kaon and anti-kaon which is less than 10^{-18}. The European Particle Physics Laboratory, CERN has built the *Antiproton Decelerator* facility in 1999 with the aim to test CPT invariance using precision spectroscopy of antihydrogen, the bound state of an antiproton and a positron and also that of anti-protonic atoms where an electron is replaced by an antiproton. The latter measures the mass and charge of the antiproton (*antimatter physics*).

If CPT invariance is indeed a fundamental symmetry of nature, then the violation of time reversal is equivalent to the violation of the combined CP symmetry. We shall see later the weak interaction breaks not only P (maximally), but also CP (a little). As a result, time invariance is also violated by the weak interaction, in contrast to classical mechanics.

1.7 Isospin and strangeness

One of the earliest observations implying an inner structure of particles thought to be elementary is the similarity of the proton and the neutron:

these have almost the same mass and apart from a charge effect the strong interaction within the atomic nucleus affects them identically. *Heisenberg* introduced the concept of the *nucleon* which has two eigenstates, the proton and the neutron. This needed a new quantum number characterizing it; as its symmetry properties are identical to those of the spin he called it the *isospin* I from isotopic spin (isobaric would be more precise as isotopes have different numbers of nucleons whereas isobaric nuclei have the same numbers of nucleons). The nucleon has an isospin $I = \frac{1}{2}$, the proton is the nucleon state with $I_3 = +\frac{1}{2}$ and the neutron[3] is that of $I_3 = -\frac{1}{2}$.

With the development of experimental methods many strongly interacting particles, *hadrons* were observed and all had characteristic isospins, i.e. all could be arranged in groups of particles of similar properties but different charges according to their isospins. The nucleon has an isospin $I = \frac{1}{2}$ and two similar states, with $I_3 = \pm\frac{1}{2}$. The lightest hadron, the π*-meson* or *pion* has $I = 1$ with three eigenstates (I_3 = -1, 0 and +1) and three charge states π^+, π^0 and π^-. The Δ hyperon has $I = \frac{3}{2}$:

$$\Delta^-(I_3 = -\frac{3}{2}),\ \Delta^0(I_3 = -\frac{1}{2}),\ \Delta^+(I_3 = +\frac{1}{2}),\ \Delta^{++}(I_3 = +\frac{3}{2})\,.$$

A unit change of I_3 involves a corresponding unit change in charge.

Then a third quantum number, *strangeness* S was discovered. Pairs of particles were produced in collisions of energetic protons with probabilities characteristic of the strong interaction and lived long enough that they must have decayed via weak interactions. They were called V-particles as their tracks curved in the magnetic field of the detectors in opposite directions. To explain this *Murray Gell-Mann*, *Abraham Pais* and *Kazuhiko Nishijima* introduced *strangeness* S as a new additive quantum number which is conserved in strong interactions but not in weak reactions. For instance, the Σ^- hyperon ($S = -1,\ I = 1$) created in π^-p$\rightarrow$ K$^+\Sigma^-$ decays via $\Sigma^- \rightarrow$ nπ^- with a lifetime of $\tau \sim 10^{-10}$ s whereas the Δ^+ hyperon ($S = 0$) decays in $\Delta^+ \rightarrow$ nπ^+ with a lifetime of $\tau \sim 10^{-23}$ s. It was postulated that only the weak interaction can change the new quantum number.

Strangeness and isospin made together an $SU(3)$ group which made it possible to construct a unique frame for all known particles. In order to explain this, *Murray Gell-Mann* and *George Zweig* suggested the *quark* model of hadrons. Using three new elementary fermions, three quarks (Table 1.2), all observed hadrons could be described. Isospin became the

[3]In nuclear physics often a converse convention is used and $I_3 = +\frac{1}{2}$ is associated with the neutron.

quantum number of the two lightest quarks and because of the analogy to the spin the $I_3 = +\frac{1}{2}$ state was named *up quark* with the sign u and the $I_3 = -\frac{1}{2}$ state *down quark*, d. The third quark's quantum number is the strangeness, so that is the *strange* (s) quark. The isospin and strangeness, characterizing the various kinds of quarks are called *flavour* quantum numbers.

The quark model postulates that quarks can bind together only in two ways: in quark-antiquark pairs, those are called *mesons*, and three-quark states, those are the *baryons*. As the quarks have spin $\frac{1}{2}$, naively we expect that mesons are bosons and baryons are fermions. Quarks have baryon number $\frac{1}{3}$ and fractional electric charges: in units of elementary charge e the u quark has charge $+\frac{2}{3}$ while the d and s quarks $-\frac{1}{3}$. This of course gives the proper charges to the proton p = [uud] and the neutron n = [udd] or the pions: $\pi^+ = [\mathrm{u\bar{d}}]$, $\pi^0 = \frac{1}{\sqrt{2}}[\mathrm{u\bar{u}} + \mathrm{d\bar{d}}]$, $\pi^- = [\mathrm{\bar{u}d}]$. Thus the third component of the isospin is directly connected to the charge, as its unit increase means replacing a d by a u quark, i.e. increasing the total charge by $+\frac{2}{3} - (-\frac{1}{3}) = 1$.

quark	J	e_q	B	I_3	S	$Y = B + S$
u	$\frac{1}{2}$	$+\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	0	$+\frac{1}{3}$
d	$\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{2}$	0	$+\frac{1}{3}$
s	$\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$	0	-1	$-\frac{2}{3}$

Table 1.2: The first three quarks (up, down and strange): their spin J, electric charge e_q, baryon number B, isospin third component I_3, strangeness S and hypercharge $Y = B + S$.

Strangeness and isospin constitute together an $SU(3)$ group which made it possible to construct a unique frame for all known particles. Gell-Mann and Zweig proposed in 1964 the quark model with the first three quarks (Table 1.2). The quark model postulated that quarks cannot exist free, only in bound states of two combinations: the three-quark bound states make baryons and the quark+antiquark states mesons. As the quarks are fermions ($J = \frac{1}{2}$), the baryons are also fermions with baryon number $B = 1$ while the mesons are bosons with $B = 0$. The proton is a [uud], the neutron a [udd] ground state (i.e. $J = \frac{1}{2}$ with zero orbital momentum) whereas the pions are the lightest $[\mathrm{q\bar{q}'}]$ (q, q' = u, d) combinations.

Exercise 1.1
What invariance principles are violated by the weak, electromagnetic and colour interactions?

Exercise 1.2
What are the analogies and differences between spin and isospin?

Exercise 1.3
How can the three-dimensional spin be characterised by two independent quantities?

Exercise 1.4
What gauge symmetry facilitates the conservation of electric charge and fermion number?

Chapter 2

What is measured in experiment?

MOTTO:

I was brought up to look at the atom as a nice hard fellow, red or grey in colour, according to taste.

(Ernest Rutherford)

Physics is experimental science, in particle physics every single statement has to be based on experimental observations. In high energy physics the two most important experimentally measurable quantities are the cross section and the resonance with its invariant mass and width.

2.1 Cross section

The probability of an interaction in accelerator experiments of nuclear and particle physics is usually characterized by the ratio of the measured transition probability of the reaction in unit time, and the intensity of the bombarding beam, the *flux*:

$$\sigma = W/\Phi,$$

measured in units of *cross section* (see Fig. 2.1).

The flux of a beam, $\Phi = n_b \cdot v_b$ is the number density times velocity of the particles in a beam, number of particles divided by cross section and time. The σ cross section of a reaction is measured in units of *barn* (1 barn = 10^{-28} m^2). That unit comes from atomic physics and it is very large in particle physics (that is how it got its name), so large that in high energy physics we most often express our measured cross sections in pico-barn (1 pb = 10^{-12} barn) or femto-barn (1 fb = 10^{-15} barn).

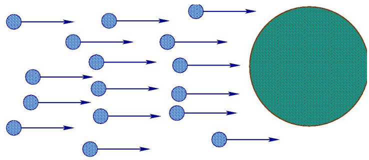


Figure 2.1: Cross section characterizing interaction in particle scattering

When the interaction is perturbative, i.e. the Hamilton operator can be written in the form $H = H_0 + H'$ with eigenvalues $E' \ll E_0$, the transition probability between an initial and a final state can be written as

$$W(i \rightarrow f) = \frac{2\pi}{\hbar} |M_{if}|^2 \rho_f$$

where $\rho_f = \frac{\mathrm{d}n}{\mathrm{d}E_{CM}}$ is the density of final states in unit centre-of-mass energy and $M_{if} \approx \int \psi_f^* H' \psi_i \mathrm{d}\tau$ is the transition matrix element, the overlap between the two approximate wave functions. As the strong and electromagnetic interactions are invariant under CP (charge and parity) reflection, CPT invariance, which is supposed to be valid in field theory, demands that they should also be invariant against time reflection T, and so $|M_{if}|^2 = |M_{fi}|^2$ (the principle of *detailed balance*).

In a centre-of-mass system a non-relativistic $a + b \rightarrow c + d$ particle scattering reaction will have the cross section

$$\sigma = \frac{W}{n_a v_i} = \frac{1}{\pi \hbar^4} |M_{if}|^2 \frac{(2S_c + 1)(2S_d + 1)}{v_i v_f} p_f^2$$

where $p_f = |\vec{p}_c| = |\vec{p}_d|$ is the final state momentum, $n_a = 1$ is the number of scattered particles, $v_i = |\vec{v}_a - \vec{v}_b|$ and $v_f = |\vec{v}_c - \vec{v}_d|$ are the relative velocities. The density of available final states will be

$$\frac{\mathrm{d}p_f}{\mathrm{d}E_0} = \frac{E_c E_d}{p_f E_0} \approx \frac{1}{v_f} .$$