Basic Principles of Physics Applied to Earth Sciences

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By

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To Matteo & Giulietta

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Preface

This is an introductory textbook on Physics for first-year undergraduate students in Geological Sciences. The book covers all the topics that are traditionally given in a Basic Physics Course from major international programs. It emphasizes the fundamental concepts of Physics to explain a variety of geological aspects. While keeping arguments mathematically rigorous, the textbook aims to explain Physics concepts by using basic-intermediate mathematical skills to facilitate effective reading comprehension. Today's science requires the broadest possible understanding of scientific disciplines if we are to solve problems that cannot be confined to one field. The Earth's system is not an exception to this, which is why the scientific background of a graduate in Geological Sciences and related disciplines should be shaped by fundamental knowledge in Physics, as well as Mathematics, Chemistry, and Biology.

One of the notable features of this edition is the methodology proposed to consolidate the comprehension of Physics in students from Geological Sciences. Unlike more traditional textbooks that start from a specific Geological problem and try to explain it by introducing the needed Physics concepts, the principal novelty of this textbook is that it first introduces and accurately explains the basic Physics theories. These notions are then applied to solve specific Earth Science problems. The textbook begins with an introductory chapter (Chapter 1) on physical magnitudes, units, dimensional analysis, vector properties, trigonometric functions, and a short summary of basic mathematical techniques for derivation and integration. The book covers most of the fundamental topics in Physics, including mechanics (Chapter 2), gravity (Chapter 3), waves (Chapter 4), stress and strain (Chapter 5), thermodynamics (Chapter 6), electricity (Chapter 7), magnetism (Chapter 8), and light (Chapter 9). Each chapter is accompanied by a wealth of figures and a number of solved numerical problems. Furthermore, every chapter starts with an *Abstract*, highlighting the major topics covered, and concludes with a *Key Point Summary* that lists the significant elements, as well as major mathematical formulas discussed in the chapter, for quick review. Additionally, each theme concludes with a series of end-of-chapter solved numerical problems, covering key concepts discussed throughout the chapter.

The introduced Physics notions are used to explain a variety of techniques and concepts of interest for the Earth scientist community. These include the Earth’s angular rotation and its moment of inertia, pyroclastic bomb ejection from a volcano, density determination of unknown minerals, the escape velocity of water from a dam, satellite exploration, Earth’s shape (geoid, reference ellipsoid), the deflection of the oceanic lithospheric thin plate, maximum lift load of a mining elevator, deformation of a drilling pipe in an oil well, Earth as a thermodynamic system, enthalpy of serpentinization in the oceanic crust, Gibbs free energy involved in the oxidation of magnetite to hematite, electric resistivity of the subsurface (VES technique), the Earth's magnetic field, seismic waves, and seismic exploration for the oil industry (refraction and wide-angle reflection and vertical reflection techniques).

This textbook is suitable for any first-year Physics course in Geological Sciences such as Geology and Geological Engineering. The book is not written for a specific region and can be used by students and professors from any country. Additional uses of this book can potentially be found in other similar scientific disciplines where Physics is given at an introductory level (*i.e.*, Earth and Marine Sciences, Environmental Science, Geography, and Engineering). The book is designed for a one-semester fundamental course on Physics, although it could be easily adapted for a two-semester course. Depending on the level of mathematics proficiency of the students, certain parts can be shortened or expanded accordingly.

Feedback on ideas, suggestions, corrections, and omissions are welcome both in person and via email.

Maurizio Mattesini
Madrid
July 2023

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CHAPTER I: **BASICS CONCEPTS**

This is the introductory 'physicsmath' chapter where the most basic mathematical tools and physics concepts are presented to the reader. It includes fundamental and derived quantities, dimensions, dimensional analysis, and the International System of Units. An introduction to vectors is provided by first defining their mathematical structure and algebraic properties. Cross and dot products are then explained, along with various physical and geometric applications of vector formalism. The chapter ends with a short summary of basic mathematical techniques for derivation and integration.

1.1 Fundamental quantities and units

A **physical quantity** is a value associated with a physical property or a measurable characteristic in a physical system. It denotes a feature that can be quantified or measured using appropriate instruments or methods. Historically, the relationships between different physical quantities have been developed since the early theories of physics. Different schools had different units for each quantity based on comparison with a standard. The measurement of any physical quantity requires comparing it to a certain unit value. For instance, to measure the distance between two points, we compare it to a standard unit of distance, such as the meter. When we say that a certain distance is 10 meters, it means that it is equivalent to 10 times the length of the meter standard.

Many of the quantities that we will study can be expressed in terms of three fundamental units: **length**, **time**, and **mass**. The selection of a standard or reference unit for these fundamental quantities determines the system of units. In the **International System of Units** (**SI**), which is the system universally used in the scientific community, the reference unit for length is the meter, the reference unit for time is the second, and the reference unit for mass is the kilogram. The unit for length, the **meter** (symbol m), is defined as the distance travelled by light in a vacuum during a time of 1/299792458 seconds. The unit of time, the **second** (symbol s), is specified in relation to a characteristic frequency associated with the caesium atom (Cs). The second is defined such that the frequency of the light emitted during a specific Cs transition is 9192631770 cycles per second. The unit of mass, the **kilogram** (symbol kg), is defined in a way that it corresponds to the mass of a specific prototype body preserved at the *International Bureau of Weights and Measures*, based in Saint-Cloud, near Paris, France. When studying thermodynamics (Chapter 6) and electricity (Chapter 7), we will need three additional fundamental physical units: the unit of temperature, the **kelvin** (symbol K), the unit of amount of substance, the **mole** (symbol mol), and the unit of electric current, the **ampere** (symbol A). There is another fundamental unit, the **candela** (symbol cd), which is the unit of luminous intensity, but we do not have the opportunity to use it in this book. These seven fundamental units are shown in Table 1.0 and constitute the SI Units. These fundamental quantities and their units are independent and cannot be defined in terms of other physical quantities.

Table 1.0: Fundamental quantities in Physics along with their SI units.

Quantity	Symbol	Units (Symbol)
Time	t	$Second\ (s)$
Mass	m	$Kilogram\ (kg)$
Length	l	$Meter\ (m)$
Temperature	T	$Kelvin\ (K)$
Electric current	I	$Ampere\ (A)$
Amount of substance	N	$Mole\ (mol)$
Luminous intensity	J	$Candela\ (cd)$

For the sake of completeness, it is important to mention that in 2019, a significant change occurred in the definition of four out of the seven base units outlined in the International System of Units.

Rather than relying on human-made artifacts like the standard kilogram, these units were redefined in terms of natural physical constants. Effective from 20th May 2019, kilogram, ampere, kelvin, and mole are now precisely determined by assigning exact numerical values, when expressed in SI units, to the **Planck constant** ($h = 6.62606876(52) \cdot 10^{-34}\, J \cdot s$), **elementary electric charge** ($e = 1.602176462(63) \cdot 10^{-19}\, C$), **Boltzmann constant** ($k_B = 1.3806503(24) \cdot 10^{23} J/K$), and **Avogadro constant** ($N_A = 6.02214199(47) \cdot 10^{23}\, particles/mol$), respectively. The numbers in parentheses represent the errors in the last two digits. For example, for the Planck constant, $h = 6.62606876(52) \cdot 10^{-34}\, J \cdot s$, it means that h is equal to $(6.62606876 \pm 0.00000052) \cdot 10^{-34}\, J \cdot s$.

1.2 Most common derived quantities in Physics

By referring to the fundamental quantities and units outlined in Table 1.0, we can establish the definitions of all other quantities and units found within the field of physics. Some of them are widely known, such as volume units given in m^3 or area units in m^2. In many cases, **multiples** and **submultiples** are used when the values of the quantities are extremely large or small, making it impractical to work with the standard unit.

By employing multiples, which are larger units, or submultiples, which are smaller units, we can conveniently express these values in a more manageable and understandable form. This approach simplifies calculations and facilitates communication in various scientific and engineering disciplines. For instance, micrometres ($1\, \mu m = 10^{-6}\, m$) are employed to measure the dimensions of objects observed through a microscope, while kilometres ($1\, km = 10^3\, m$) are used to denote larger distances. The most common **prefixes** for multiples and submultiples are summarized in the following Table 1.1.

Table 1.1: Most common prefixes for multiples and submultiples in the SI system.

Prefix	Symbol	Multiplication factor
Yotta	Y	10^{24}
Zetta	Z	10^{21}
Exa	E	10^{18}
Peta	P	10^{15}
Tera	T	10^{12}
Giga	G	10^{9}
Mega	M	10^{6}
Kilo	k	10^{3}
Hecto	h	10^{2}
Deca	*da*	10^{1}
Deci	d	10^{-1}
Centi	c	10^{-2}
Milli	m	10^{-3}
Micro	μ	10^{-6}
Nano	n	10^{-9}
Pico	p	10^{-12}
Femto	f	10^{-15}
Atto	a	10^{-18}
Zepto	z	10^{-21}
Yocto	y	10^{-24}

There are also derived quantities from well-known physics equations in everyday life, such as velocity, given in m/s or in units of different systems like km/h. Others, like electric potential given in volts (V), are commonly known, but their relationship with fundamental units requires knowledge of the formula from which the definition of voltage is derived.

Finally, there are quantities whose units do not have their own name and are expressed in terms of other derived and fundamental quantities. In any case, understanding derived quantities requires knowledge of the physical quantity's definition, which will be the objective of this book. Table 1.2 provides a summary of some of the most common physical quantities that will be presented in this book and have their own name in the SI.

Table 1.2: The most common physical quantities that have their own unit names (in the SI system) and their relationship with other derived units or fundamental units.

Magnitude	Symbol	Units (Symbol)	Derived units
Frequency	f	Hertz (Hz)	$cycles/s$
Force	F	Newton (N)	$kg\ m\ s^{-2}$
Pressure	P	Pascal (Pa)	$N/m^2 = kg\ s^{-2}\ m^{-1}$
Energy	E	Joule (J)	$N\ m = kg\ m^2\ s^{-2}$
Work	W	Joule (J)	$N\ m$
Heat	Q	Joule (J)	$N\ m$
Power	P	Watt (W)	$J\ /s = kg\ \ m^2/s^3$
Electric charge	Q	Coulomb (C)	$A\ s$
Voltage	V	Volt (C)	J/C
Electric resistance	R	Ohm (Ω)	V/A
Electrical capacitance	C	Farad (F)	C/V
Magnetic induction	B	Tesla (T)	$V\ s\ m^{-2}$
Magnetic flux	Φ	Weber (Wb)	$V\ s$

1.3 Dimensions and dimensional analysis

Dimensional analysis is a mathematical technique used to analyse physical quantities and their relationships by examining their **dimensions.** Most physical quantities can be expressed in terms of combinations of five fundamental dimensions. These are **mass** (M), **length** (L), **time** (T), **electric current** (I), and **temperature** (θ). It is extremely useful for understanding the consistency of physical relationships between quantities. Certain algebraic operations, such as addition and subtraction, can only be performed between quantities with the same dimensions. Additionally, there must be dimensional consistency between a quantity and its uncertainty.

To represent the dimensions of a quantity, we often use square brackets. For example, the dimension of length is $[l] = L$, the dimension of mass $[m] = M$, and dimension of time is $[t] = T$. Similarly, the dimensions of area can be written as $[area] = L^2$, and those of volume are $[volume] = L^3$. Table 1.3 summarizes the dimensions of the most common physical quantities. In a simple case, such as the dimensional analysis of velocity, which is the ratio of length to time, it can be represented as:

$$[v] = \frac{L}{T}$$

where $[v]$ indicates the dimensions of velocity, and L and T are the basic dimensions of length and time, respectively. Thus, when using SI units, velocity is measured in meters per second (m/s).

Table 1.3: Dimensions of common physical quantities.

Quantity	Symbol	Dimensions
Area	A	L^2
Volume	V	L^3
Velocity	v	L/T
Acceleration	a	L/T^2
Force	F	ML/T^2
Pressure (F/A)	p	M/LT^2
Density (M/V)	ρ	M/L^3
Energy	E	ML^2/T^2
Power (E/T)	P	ML^2/T^3

A more complex exercise would involve performing dimensional analysis for force, which, as we will see from Newton's second law in Chapter 2, is the product of mass and acceleration:

$$[F] = [M][a]$$

The dimension of mass $[M]$ is already a fundamental dimension associated with the quantity of mass (m), but the dimensions of acceleration correspond to its units (m/s^2). The dimensions associated with the unit meters is length (L), and the unit associated with seconds is time (T). Upon substitution, we obtain:

$$[F] = MLT^{-2}$$

By considering the dimensions of various quantities involved in a more general problem, dimensional analysis can help verify equations, identify missing terms, check the correctness of derived formulas, and simplify complex calculations. It is particularly useful in physics, and other scientific disciplines for performing unit conversions, validating equations, and solving problems.

1.4 Basic trigonometric functions

In the triangle shown in Figure 1.1, the basic trigonometric functions are defined as the following ratios or proportions between its sides:

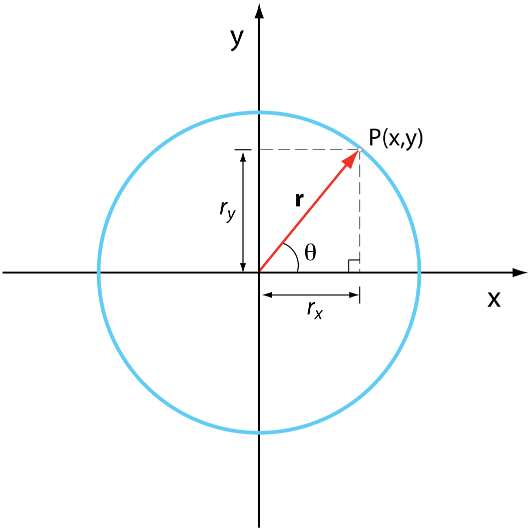


Figure 1.1: In trigonometry, a unit circle is commonly used to define the trigonometric functions. The unit circle is a circle with a radius of one unit, cantered at the origin of a coordinate system.

The **sine** function (sin) relates an angle to the length of the opposite side divided by the hypotenuse in a right triangle. For an angle θ, it is defined as:

$$sin\,\theta = \frac{r_y}{r} \tag{1.1}$$

The **cosine** function (cos) links an angle to the length of the adjacent side divided by the hypotenuse in a right triangle:

$$cos\,\theta = \frac{r_x}{r} \tag{1.2}$$

The **tangent** function (tan) correlates an angle to the length of the opposite side divided by the adjacent side in a right triangle. When considering an angle θ, it is defined as:

$$tan\,\theta = \frac{r_y}{r_x} = \frac{sin\,\theta}{cos\,\theta} \tag{1.3}$$

These three functions are the most fundamental and common in trigonometry. Additionally, there are other trigonometric functions derived from them, such as **cosecant** (csc), **secant** (sec), and **cotangent** (cot), which are the inverses of sine, cosine, and tangent, respectively.

1.5 Vectors

A vector is a mathematical object that it is commonly used in physics, engineering, and mathematics to describe quantities such as displacement, velocity, force, magnetic field, and more. Vectors are represented by **arrows** and are defined by their **magnitude**, which is also known as the **modulus**, and their **direction**. The magnitude of a vector represents the numerical value or size of the quantity it represents. On the other hand, the direction of a vector is determined by its orientation and sense. Consequently, the direction of a vector indicates the orientation or where the quantity is pointing or acting, while sense refers to the distinction between two possible directions along the same line. In this book, a generic vector 'a' is indicated by using bold letters ($\boldsymbol{a}$) or, in some cases, by placing an arrow above the letter ($\vec{\boldsymbol{a}}$). Figure 1.2 shows a two-dimensional vector $\boldsymbol{a}$ with its two rectangular components, a_x and

a_y. The magnitude of vector **a** represents the size (length) of the arrow, its direction is determined by the straight line that forms an angle θ with the x-axis, and its sense is indicated by the direction in which the arrow points.

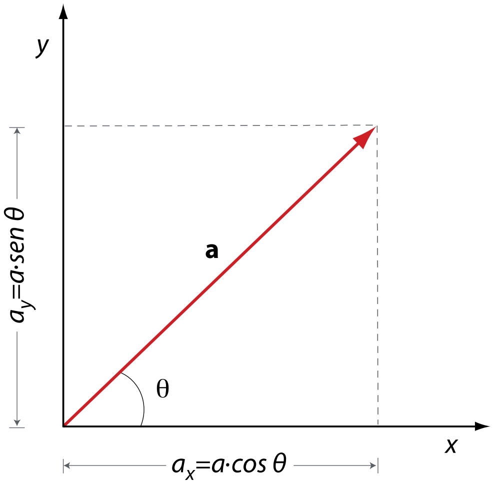


Figure 1.2: Geometrical representation of a 2D vector $\boldsymbol{a}$ with its rectangular components a_x and a_y.

If we know a_x and a_y, the angle θ is obtained through this trigonometric relationship:

$$\theta = arctan\ \frac{a_x}{a_y} \tag{1.4}$$

and the **magnitude** (or **modulus**), $|\boldsymbol{a}|$, is determined using the **Pythagorean theorem**:

$$|\boldsymbol{a}| = \sqrt{a_x^2 + a_y^2} \tag{1.5}$$

Generalizing the above for an n-dimensional vector, the magnitude is given by:

$$|\boldsymbol{a}| = \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2} \tag{1.6}$$

The **unit vectors** $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$, and $\widehat{\boldsymbol{k}}$ are dimensionless vectors with a magnitude of one that point in the x, y, and z directions of a rectangular coordinate system, as shown in Figure 1.3a.

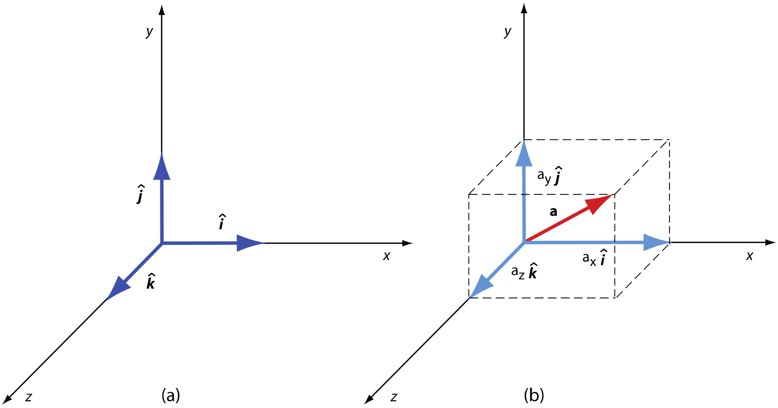


Figure 1.3: (a) In a Cartesian coordinate system, three-unit vectors are commonly used: $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$, and $\widehat{\boldsymbol{k}}$. The unit vector $\hat{\boldsymbol{\imath}}$ points in the positive x-direction, $\hat{\boldsymbol{\jmath}}$ points in the positive y-direction, and $\widehat{\boldsymbol{k}}$ points in the positive z-direction. (b) A 3D general vector $\boldsymbol{a}$ as a function of the unit vectors.

In three dimensions, a generic vector $\boldsymbol{a}$ can be written as the sum of three vectors, each of them parallel to one axis of the reference system, as shown in Figure 1.3b:

$$\boldsymbol{a} = a_x\hat{\boldsymbol{\imath}} + a_y\hat{\boldsymbol{\jmath}} + a_z\widehat{\boldsymbol{k}} \tag{1.7}$$

Therefore, the sum of two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ can be written in terms of unit vectors as follows:

$$\boldsymbol{a} + \boldsymbol{b} = \left(a_x\hat{\boldsymbol{\imath}} + a_y\hat{\boldsymbol{\jmath}} + a_z\widehat{\boldsymbol{k}}\right) + \left(b_x\hat{\boldsymbol{\imath}} + b_y\hat{\boldsymbol{\jmath}} + b_z\widehat{\boldsymbol{k}}\right) \tag{1.8}$$

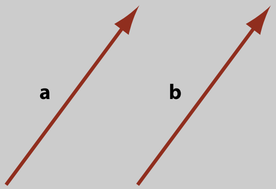
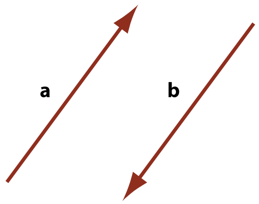
to result in,

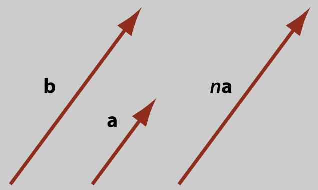
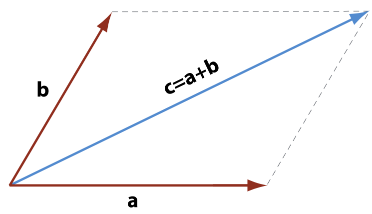
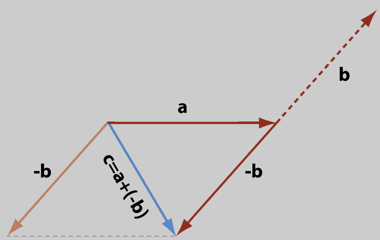
$$\boldsymbol{a} + \boldsymbol{b} = (a_x + b_x)\hat{\boldsymbol{\imath}} + (a_y + b_y)\hat{\boldsymbol{\jmath}} + (a_z + b_z)\hat{\boldsymbol{k}} \tag{1.9}$$

1.5.1 The properties of vectors

Vectors satisfy a series of simple algebraic properties that are summarized in Table 1.3. These properties form the foundation of vector algebra and allow for various operations and manipulations involving vector quantities.

Table 1.3: Summary of the general properties of vectors.

Property	Characteristics	Graphical representation
Equality	$\boldsymbol{a} = \boldsymbol{b}$ The two vectors are equal in magnitude, and direction. $a_x = b_x$ $a_y = b_y$ $a_z = b_z$	
Negative of a vector	$\boldsymbol{a} = -\boldsymbol{b}$ If $\lvert\boldsymbol{a}\rvert = \lvert\boldsymbol{b}\rvert$ and have opposite directions. $a_x = -b_x$ $a_y = -b_y$ $a_z = -b_z$	

Multiplication by a scalar n	$\boldsymbol{b} = n\boldsymbol{a}$ The vector $\boldsymbol{b}$ has a magnitude $\lvert\boldsymbol{b}\rvert = n\lvert\boldsymbol{a}\rvert$ and the same direction as $\boldsymbol{a}$ if $n > 0$. If $n < 0$, vector $\boldsymbol{b}$ assumes the direction of $-\boldsymbol{a}$. $b_x = na_x$ $b_y = na_y$ $b_z = na_z$	
Sum	$\boldsymbol{c} = \boldsymbol{a} + \boldsymbol{b}$ Rule of parallelogram[1] $c_x = a_x + b_x$ $c_y = a_y + b_y$ $c_z = a_z + b_z$	
Subtraction	$\boldsymbol{c} = \boldsymbol{a} - \boldsymbol{b}$ Rule of parallelogram $c_x = a_x - b_x$ $c_y = a_y - b_y$ $c_z = a_z - b_z$	

[1] The rule of parallelogram is a geometric method used to add two vectors graphically, also known as the parallelogram law.

1.5.2 The scalar product

The **scalar product**, also known as the **dot product**, is a mathematical operation that combines two vectors to produce a scalar quantity. It is denoted by the dot symbol (·), for example, $\boldsymbol{a} \cdot \boldsymbol{b}$. The scalar product of two non-zero vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is defined as the product of their magnitudes multiplied by the cosine of the angle between them:

$$\boldsymbol{a} \cdot \boldsymbol{b} = |a| \cdot |b| \cos\theta \tag{1.10}$$

where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$. If one of the vectors is zero or if the two vectors are perpendicular (**orthogonal vectors**) to each other ($\theta = 90°$ and $cos\,\theta = 0$), the scalar product $\boldsymbol{a} \cdot \boldsymbol{b}$ will be equal to 0. If the scalar product of two vectors is equal to the product of their magnitudes ($\boldsymbol{a} \cdot \boldsymbol{b} = |a||b|$), they are said to be **parallel** or colinear. This means that the angle θ between them is $0°$ or $180°$ ($cos\,\theta = 1$).

The properties of a scalar product can be summarized as follows:

$$\boldsymbol{a} \cdot \boldsymbol{a} = |\boldsymbol{a}|^2 \tag{1.11}$$

$$\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{b} \cdot \boldsymbol{a} \tag{1.12}$$

$$\boldsymbol{a} \cdot (\boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{a} \cdot \boldsymbol{b} + \boldsymbol{a} \cdot \boldsymbol{c} \tag{1.13}$$

$$(n\boldsymbol{a}) \cdot \boldsymbol{b} = n(\boldsymbol{a} \cdot \boldsymbol{b}) = \boldsymbol{a} \cdot (n\boldsymbol{b}) \tag{1.14}$$

$$\boldsymbol{a} \cdot 0 = 0 \tag{1.15}$$

The scalar product finds particular utility in physics. It is employed, for instance, to determine the component of a vector along a specific direction, calculate work and energy (Chapter 2), determine the angle between two vectors, and test for orthogonality.

1.5.3 The vector product

The **vector product**, also known as the **cross product**, is a mathematical operation that combines two vectors to produce a new vector. It is used to determine the direction and magnitude of quantities such as torque (Chapter 2), and magnetic field (Chapter 8). In terms of mathematical representation, the vector product of two non-zero three-dimensional vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is denoted as $\boldsymbol{a} \times \boldsymbol{b}$ and is defined as follows:

$$\boldsymbol{a} \times \boldsymbol{b} = |a| \cdot |b| \, sin\,\theta \, \hat{\boldsymbol{n}} \tag{1.16}$$

where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$, and $\hat{\boldsymbol{n}}$ is the unit vector perpendicular to both vectors, whose direction is determined by the **right-hand rule**, as shown in Figure 1.4.

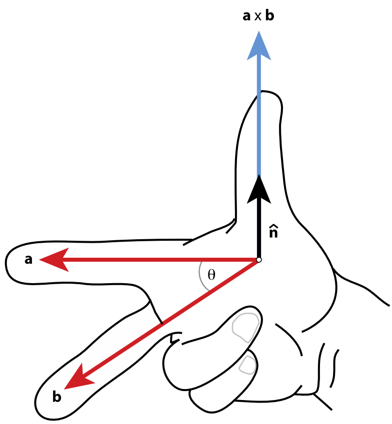


Figure 1.4: Graphical representation of the right-hand rule.

The right-hand rule is applied by pointing the index finger in the direction of vector $\boldsymbol{a}$, the middle finger in the direction of vector $\boldsymbol{b}$, and the thumb represents the direction of the cross product $\boldsymbol{a} \times \boldsymbol{b}$. The magnitude of the vector product is related to the area of the parallelogram formed by vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, and its direction is determined by the right-hand rule. The resulting vector product is a new vector that is perpendicular to both vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. If one of the two vectors is zero or if vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are parallel to each other ($\theta = 0°$ and $sin\,\theta = 0$), the result of the vector product $\boldsymbol{a} \times \boldsymbol{b}$ is zero. On the other hand, if the two vectors are perpendicular to each other ($\theta = 90°$ and $sin\,\theta = 1$), the magnitude of the vector product will be $\mathbf{a} \times \boldsymbol{b} = |a||b|$.

The properties of a vector product are resumed as follows:

$$\boldsymbol{a} \times \boldsymbol{b} = -\boldsymbol{b} \times \boldsymbol{a} \tag{1.17}$$

$$(\boldsymbol{na}) \times \boldsymbol{b} = \boldsymbol{n}(\boldsymbol{a} \times \boldsymbol{b}) = \boldsymbol{a} \times (\boldsymbol{nb}) \tag{1.18}$$

$$\boldsymbol{a} \times (\boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{a} \times \boldsymbol{b} + \boldsymbol{a} \times \boldsymbol{c} \tag{1.19}$$

$$(\boldsymbol{a} + \boldsymbol{b}) \times \boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{c} + \boldsymbol{b} \times \boldsymbol{c} \tag{1.20}$$

1.6 Derivatives and integrals

Derivatives and integrals play significant roles in physics, particularly they provide valuable tools for modelling and understanding the behaviour of physical systems. In this book, readers will encounter a multitude of applications involving derivatives and integrals. In kinematics for example (Chapter 2), the derivative of the displacement function with respect to time gives the velocity of an object, while the derivative of velocity with respect to time gives the acceleration. These derivatives describe the motion of objects and are essential in studying concepts like position, velocity, and acceleration. The derivative of current with respect to time gives the rate of change of electric charge (Chapter 7), while the derivative of temperature with respect to time gives the rate of change of heat energy (Chapter 6). Similarly, the definite integral of force with respect to displacement gives the work done on the object (Chapter 2). These concepts are crucial in understanding the relationship between forces, energy, and motion. In electromagnetism (Chapters 7-8), derivatives and integrals are used to describe electric and magnetic fields. The derivative of the electric field with respect to position gives the electric field strength, while the definite integral of the magnetic field over a closed loop gives the magnetic flux. These concepts are vital for understanding electromagnetic phenomena and Maxwell's equations (Chapter 8). These are just a few examples illustrating how derivatives and integrals are applied in physics to describe and analyse various physical phenomena. The understanding and application of these concepts are crucial in the study of mathematical functions and their utilization in problem-solving across various scientific disciplines such as physics, geology, engineering, and many other scientific disciplines.

1.6.1 Derivatives

Derivatives are generally used to describe the rate of change of physical quantities and provides information about the slope of a function at each point. The derivative of a function $y = f(x)$ is denoted as $f'(x)$ or dy/dx and represents the instantaneous rate of change of the function at a given point. It can be calculated by taking the limit of the incremental ratio between the function and the change in the independent variable:

$$f'(x) = \lim_{h \to 0} \frac{[f(x+h) - f(x)]}{h} \qquad (1.21)$$

Below is a concise overview of the most common derivatives and properties that can assist the readers of this book:

- The derivative of a constant (c) is always zero (**Constant rule**):