Optical Metrology with Interferometry

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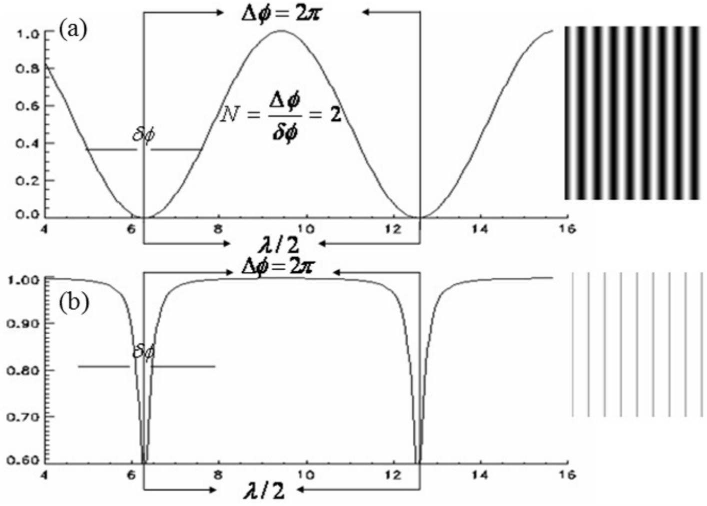
1.1 Introduction

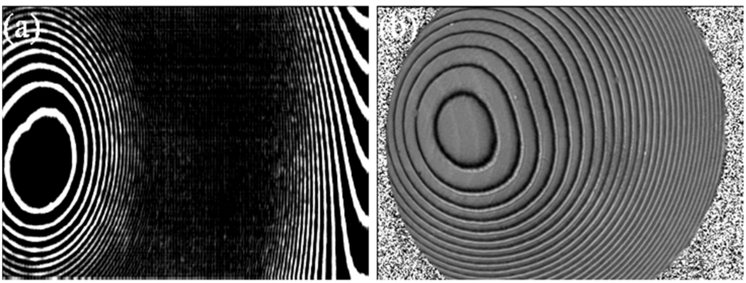
λ

N

λ

λ





$n\lambda$ μt θ λ μ

θ n

θ $\mu =$ $n\lambda$ t

$n\lambda$ μt θ

	t		
μ	θ		
	θ	$t\ \lambda$	
λ	t	t θ λ	

t dt

dn

dn

$\lambda/$ λ

$n\lambda$ μt θ

t

1.2 Multiple-beam interferometry at transmission

λ

1.3 Intensity distribution in the parallel plate case

$$I_{\infty} = \frac{T}{-R}\frac{I}{+\frac{R}{-R} \qquad \delta}$$

I

δ

δ λ μt θ β μ

λ

t β

β

β $n\,k$ n n k , n n k

λ β β

β

π

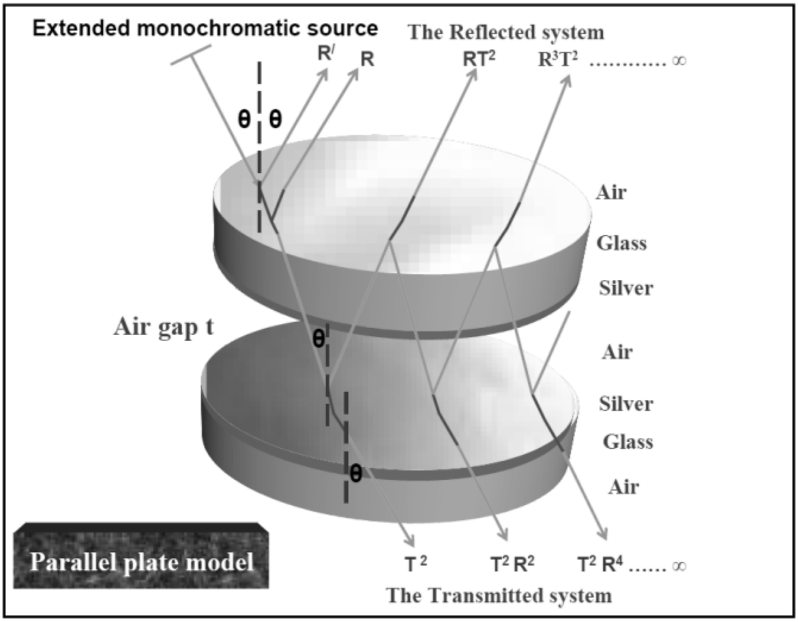
π

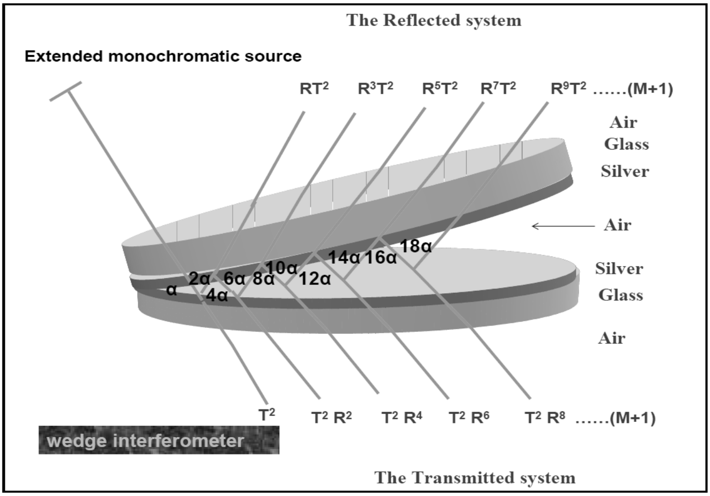
β β π

λ β β

T R

1.4 Intensity distribution in the wedge case


Extended monochromatic source
The Reflected system
R/
R
RT2
R3T2 .... ∞
θ θ
Air
Glass
Silver
Air gap t
θ
Air
Silver
Glass
θ
Air
Parallel plate model
T 2
T2 R2
T2 R4 ...... ∞
The Transmitted system


The Reflected system
Extended monochromatic source
RT2
R3T2
R5T2
R7T2
R9T2 ......(M+1)
Air
Glass
Silver
Air
α 2α 4α 6α 8α 10α 12α 14α 16α 18α
Silver
Glass
Air
T2
T2 R2
T2 R4
T2 R6
T2 R8 ......(M+1)
wedge interferometer
The Transmitted system

$$I_{M+} = T_g T \frac{-R^{M+} \quad + \quad R^{M+} \qquad M+ \quad \delta}{-R \quad + \quad R \qquad \delta}$$

T

M

M

W W R R R

R R

R

R R W

R

λ

n

nt $n\ \varepsilon\ t$ t

ε

R

$n\ \varepsilon\ t$ λ t

t

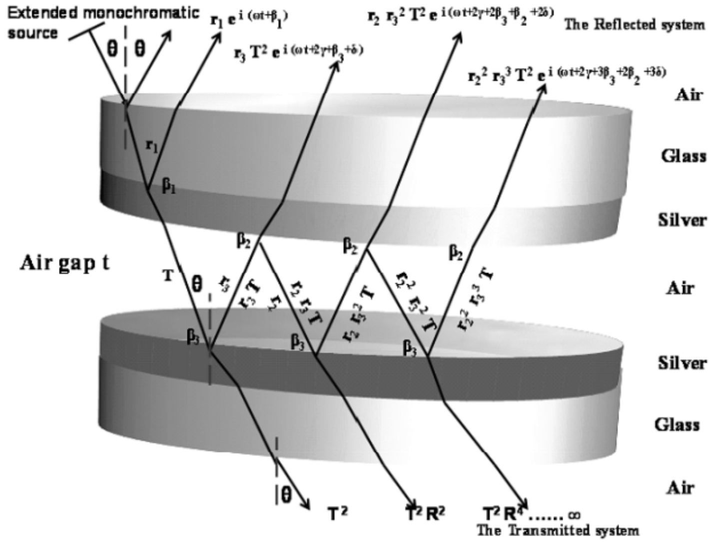
1.5 Multiple-beam interferometry at reflection

1.6 Reflected system at infinite number of beams collected

$e^{i\omega t}$ β

β β

γ



r r

r

T

R

$$R_r = r^{i\ \omega t+\beta} + T\ r e^{i\ \omega t+\ \gamma+\beta\ +\delta} +$$

$$T\ r\ r\ e^{i\ \omega t+\ \gamma+\beta\ +\ \beta\ +\ \delta} +$$

$$R_r = e^{i\ \omega t+\beta} \left\{ r + T\ r e^{i\ F+\delta} \begin{bmatrix} +\ r\ r\ e^{i\delta} & + \\ r\ r\ e^{i\delta} & + \end{bmatrix} \right\}$$

$$F = \ \gamma - \beta\ - \beta$$

$$\delta = \frac{\pi}{\lambda}\ \ \mu t Cos\ \ \theta\ \ + \beta\ + \beta\ = \Delta + \beta\ + \beta$$

$$\left[\ + r\,r\,e^{i\delta}\ + r\,r\,e^{i\delta}\ \ + r\,r\,e^{i\delta}\ \ + r\,r\,e^{i\delta}\ \ +\ \ \right]$$

$$\frac{}{-r\,r\,e^{i\delta}}$$

$$R_r = e^{i\ \omega t+\beta}\ \left\{ r + T\ \ r\,e^{i\ \ F+\delta}\ \ \frac{}{-r\,r\,e^{i\delta}} \right\}$$

$$R_r = e^{i\ \omega t+\beta}\ \left\{ r + T\ \ r\,e^{i\ \ F+\delta}\ \ \frac{-r\,r\,e^{-i\delta}}{-r\,r\,e^{i\delta}\ \ \ -r\,r\,e^{-i\delta}} \right\}$$

$$e^{i\delta} = Cos\ \delta\ + iSin\ \delta \qquad e^{-i\delta} = Cos\ \delta\ - iSin\ \delta$$

$$R_r = e^{i\ \omega t+\beta}\ \left\{ r + T\ r\,e^{i\ F+\delta}\ \frac{-r\,r\ \ \ \ \delta - i\ \ \ \delta}{-r\,r\ \ \ \delta - i\ \ \ \delta\ \ \ -r\,r\ \ \ \ \delta - i\ \ \ \delta} \right\}$$

$$R_r = e^{i\ \omega t+\beta}\ \left\{ r + T\ \ r\ \frac{F+\delta\ + i\ \ \ \ F+\delta\ \ \ -r\,r\ \ \ \ \delta - i\ \ \ \delta}{-r\,r\ \ \ \ \delta - i\ \ \ \delta\ \ \ -r\,r\ \ \ \ \delta - i\ \ \ \delta} \right\}$$

$$R_r = e^{i\ \omega t+\beta}\ \left\{ r + T\ \ r\ \frac{\begin{matrix} F+\delta\ -r\,r\ \ \ \ \delta\ \ \ \ F+\delta\ -r\,r\ \ \ \ \delta\ \ \ \ F+\delta \\ +i\ \ r\,r\ \ \ \ \delta\ \ \ \ F+\delta\ -r\,r\ \ \ \ \delta\ \ \ \ F+\delta\ +\ \ \ \ F+\delta \end{matrix}}{-r\,r\ \ \ \ \delta\ + r\ \ r} \right\}$$

$$Cos\ \delta\ =\ \ -\ \ Sin\ \ \delta$$

$$Sin\ \ \alpha \pm \beta\ \ = Sin\ \ \alpha\ \ Cos\ \ \beta\ \ \pm Cos\ \ \alpha\ \ Sin\ \ \beta$$

$$Cos\ \ \alpha \pm \beta\ \ = Cos\ \ \alpha\ \ Cos\ \ \beta\ \ \mp Sin\ \ \alpha\ \ Sin\ \ \beta$$

$$R_r = e^{i\ \omega t+\beta}\ \left\{ \frac{r + T\ \ r\ \ \ \ \ F+\delta\ -r\,r\ \ \ \ F\ \ +i\ -r\,r\ \ \ \ F\ +\ \ \ \ F+\delta}{-r\,r\ \ \ \ \delta\ + r\ \ r} \right\}$$

$$re^{i\theta} = a + ib \qquad r = \sqrt{a\ \ +b} \qquad I_R = A_R = a\ \ \ +b$$

$$I$$

$$I_R = \left\{ \frac{r + T\ r \quad F+\delta - r\,r \quad F}{-r\,r \quad \delta + r\ r} \right\} + T\ r\ \left\{ \frac{-r\,r \quad F + \quad F+\delta}{-\ r\,r \quad \delta + r\ r} \right\}$$

$$I_R = r \quad + \frac{T\ \ r\,r \qquad F+\delta\ \ -r\,r \qquad F}{+r\ \ r\ -\ \ r\,r \qquad \delta} +$$

$$\frac{T\ \ r \qquad F+\delta\ \ +r\ \ r \qquad F\ -\ \ r\,r \qquad F+\delta \qquad F}{+r\ \ r\ -\ \ r\,r \qquad \delta} +$$

$$\frac{T\ \ r \qquad F+\delta\ \ +r\ \ r \qquad F\ -\ \ r\,r \qquad F+\delta \qquad F}{+r\ \ r\ -\ \ r\,r \qquad \delta}$$

$$I_R = r\ \ + \frac{T\ r \quad +r\ r\ -\ r\,r \qquad \delta}{+r\ r\ -\ r\,r \qquad \delta} + \frac{T\ r\,r \qquad F+\delta\ -\ T\ r\,r\,r \qquad F}{+r\ r\ -\ r\,r \qquad \delta}$$

$$I_R = r\ \ + \frac{T\ \ r\ +\ T\ \ r\,r \qquad F+\delta\ -\ T\ \ r\,r\,r \qquad F}{+r\ \ r\ -\ \ r\,r \qquad \delta}$$

I

$dI\ d$

$$\frac{dI}{d\delta} = \quad r\,r\,T \qquad F + \quad r\,r\ T\ \ -\ \ r\,r\ r\ T \qquad F \qquad \delta$$

$$r\,r\ \ r \qquad F + \quad r\,r\,T \qquad F \qquad \delta$$

$$-\ \ r\,r\,r\ T \qquad F =$$

$$M \quad \delta + N \quad \delta \cong P$$

$$\phi\ = M\ \sqrt{M\ \ + N} \qquad \phi\ = N\ \sqrt{M\ \ + N} \qquad \psi\ = P\ \sqrt{M\ \ + N}$$

$$\phi + \delta\ = \quad \phi \qquad \delta\ + \quad \phi \qquad \delta$$

$$\frac{N}{\sqrt{M\ \ + N}} \qquad \delta\ + \frac{M}{\sqrt{M\ \ + N}} \qquad \delta\ = \frac{P}{\sqrt{M\ \ + N}} = \qquad \psi$$

$$\phi + \delta\ = \qquad \psi$$

$$\delta = \quad n\pi + \psi - \phi \qquad \delta = \quad n + \quad \pi - \psi - \phi$$

δ $\qquad$ I_R

$$\psi = \quad i\ e \qquad \psi = \qquad P = \quad r\,r\,r\ T \qquad F = \qquad\qquad F =$$

F π

F $n\pi$

$$I_R = r + \frac{T\ r + T\ rr\quad \delta - T\ rrr}{+r\ r - rr\quad \delta}$$

δ $n\pi$

$$I_R = r + \frac{T\ r + T\ rr - T\ rrr}{-r_r} = r_+ \frac{T\ r}{-r\ r} = I_R$$

δ n π,

$$I_R = r + \frac{T\ r + T\ rr - T\ rrr}{+r\ r} = r_- \frac{T\ r}{+r\ r} = I_R$$

F n π

$$I_R = r + \frac{T\ r - T\ rr\quad \delta + T\ rrr}{+r\ r - rr\quad \delta}$$

δ $n\pi$

$$I_R = r + \frac{T\ r - T\ rr + T\ rrr}{-r\ r} = r_- \frac{T\ r}{-r\ r} = I_R$$

δ n π

$$I_R = r + \frac{T\ r + T\ rr + T\ rrr}{+r\ r} = r_+ \frac{T\ r}{+r\ r} = I_R$$

$< R <$

$F = \quad n + \quad \pi$

$F = \pi\ r\ = r\ = R\ t\ = t\ = T$ $\pi + \delta = - \quad \delta$

$$I_R = R + \left\{ \frac{T \; R + \; TR \; - \; TR \qquad \delta}{+R \; - \; R \qquad \delta} \right\}$$

$\delta \qquad n \qquad \pi \qquad\qquad\qquad \delta \qquad n\pi$

Minima: $\qquad\qquad\qquad\qquad I_R$

$$I_R \qquad = R + \frac{RT}{-R}\{T + \quad R - \quad\}$$

$R + T =$

I_R

$A \qquad\qquad R+T+A=$

$I_R \;^A \qquad\qquad A \; R \qquad -R$

$$I_R \;^A \quad = \; I_R \qquad + \frac{R}{-R} A$$

$A \; R \qquad -R$

$R + T + A =$

Maxima:

$$I_R \qquad = R + \frac{RT}{+R}\{T + \quad R + \quad\}$$

$R + T =$

$$I_R \qquad = R + \frac{R}{+R}\left\{ \; - \quad R - R \; \right\}$$

$I_R \;^A$

$$I_R \;^A \quad = R + \frac{R}{+R}\left\{ \quad - \quad R - R \quad + \quad A \quad - \quad A \; \right\}$$

A

$A \qquad\qquad A$

$$I_R{}^{A} = R + \frac{R}{+R}\left\{ - R - R - A \right\}$$

$$I_R{}^{A} = I_R - \frac{R}{+R} A$$

A $R+T+A=$

$A\ R\ -R$

1.7 Reflected system at finite number of beams collected

R_r

$$\left[+ r\,r\,e^{i\delta} + r\,r\,e^{i\delta} + r\,r\,e^{i\delta} + r\,r\,e^{i\delta} + \right]$$

$$S_n = \left[\frac{- r\,r\,e^{i\delta}\ {}^{M}}{- r\,r\,e^{i\delta}} \right]$$

$$R_r = e^{i\ \omega t+\beta} \left\{ r + T\ r\,e^{i\ F+\delta} \left[\frac{- r\,r\,e^{i\delta}\ {}^{M}}{- r\,r\,e^{i\delta}} \right] \right\}$$

$$r = r = r \quad r\,r = R$$

$$e^{i\delta} = Cos\ \delta + iSin\ \delta \qquad e^{-i\delta} = Cos\ \delta - iSin\ \delta$$

M

$$\frac{-\quad i\delta \quad M}{-\quad i\delta}\ \frac{-\quad -i\delta \quad M}{-\quad -i\delta} = \frac{-\quad -i\delta - R^{M} e^{-i\delta M} + R^{M+} e^{i\delta\ M-}}{-\ RCos\ \delta + R}$$

$$-\quad -i\delta - R^{M} e^{-i\delta M} + R^{M+} e^{i\delta\ M-}$$

$$= \begin{bmatrix} -R[Cos\ \delta - iSin\ \delta\ ] \\ -R^{M}[Cos\ M\delta + iSin\ M\delta\ ] \\ +R^{M+}[Cos\ M-\ \delta + iSin\ M-\ \delta] \end{bmatrix}$$

$$= \begin{bmatrix} -RCos\ \delta - R^{M} Cos\ M\delta + R^{M+} Cos\ M-\ \delta + \\ i[RSin\ \delta - R^{M} Sin\ M\delta + R^{M+} Sin\ M-\ \delta] \end{bmatrix}$$

$$r + T\ r e^{i\ F+\delta} \left[\frac{-\ r\, r\, e^{i\delta \quad M}}{-\ r\, r\, e^{i\delta}} \right] =$$

$$r + T\ r\ \frac{\begin{bmatrix} Cos\ F+\delta - RCos\ \delta\ Cos\ F+\delta \\ -R^{M} Cos\ M\delta\ Cos\ F+\delta \\ +R^{M+} Cos\ M-\ \delta\ Cos\ F+\delta \\ +iRSin\ \delta\ Cos\ F+\delta - iR^{M} Sin\ M\delta\ Cos\ F+\delta \\ +iR^{M+} Cos\ F+\delta\ Sin\ M-\ \delta \\ +iSin\ F+\delta - iRCos\ \delta\ Sin\ F+\delta \\ -iR^{M} Cos\ M\delta\ Sin\ F+\delta \\ +iR^{M+} Cos\ M-\ \delta\ Sin\ F+\delta \\ -RSin\ \delta\ Sin\ F+\delta + R^{M} Sin\ M\delta\ Sin\ F+\delta \\ -R^{M+} Sin\ M-\ \delta\ Sin\ F+\delta \end{bmatrix}}{-R\ +\ RSin\ \delta}$$

$$r + T\ r e^{i\ F+\delta} \left[\frac{-\ r\, r\, e^{i\delta \quad M}}{-\ r\, r\, e^{i\delta}} \right] =$$

$$r + \begin{bmatrix} T\ r[Cos\ F+\delta + iSin\ F+\delta\ ] \\ \dfrac{\begin{bmatrix} -RCos\ \delta - R^{M} Cos\ M\delta + R^{M+} Cos\ M-\ \delta + \\ i[RSin\ \delta - R^{M} Sin\ M\delta + R^{M+} Sin\ M-\ \delta] \end{bmatrix}}{-R\ +\ RSin\ \delta} \\ \\ \end{bmatrix}$$

$$re^{i\theta} = a + ib \qquad r = \sqrt{a\ +b} \qquad I_R = A_R = a\ + b$$

$$I$$

$$a = \frac{\begin{bmatrix} r \quad -R \quad + \quad r\,RSin \quad \delta \qquad + xCos \quad F+\delta \\ - xRCos \quad \delta \; Cos \quad F+\delta \\ - xR^{M} Cos \quad M\delta \; Cos \quad F+\delta \\ + xR^{M+} \; Cos \quad M - \quad \delta \; Cos \quad F+\delta \\ - xRSin \quad \delta \; Sin \quad F+\delta \quad + xR^{M} Sin \quad M\delta \; Sin \quad F+\delta \\ - xR^{M+} \; Sin \quad M - \quad \delta \; Sin \quad F+\delta \end{bmatrix}}{-R \quad + \quad RSin \quad \delta}$$

$$b = \frac{\begin{bmatrix} + xRSin \quad \delta \; Cos \quad F+\delta \quad - xR^{M} Sin \quad M\delta \; Cos \quad F+\delta \\ + xR^{M+} \; Cos \quad F+\delta \; Sin \quad M - \quad \delta \\ + xSin \quad F+\delta \quad - xRCos \quad \delta \; Sin \quad F+\delta \\ - xR^{M} Cos \quad M\delta \; Sin \quad F+\delta \\ + xR^{M+} \; Cos \quad M - \quad \delta \; Sin \quad F+\delta \end{bmatrix}}{-R \quad + \quad RSin \quad \delta}$$

$$Cos \quad \delta \quad = \quad - \quad Sin \quad \delta$$

$$Sin \quad \alpha \pm \beta \quad = Sin \quad \alpha \quad Cos \quad \beta \quad \pm Cos \quad \alpha \quad Sin \quad \beta$$

$$Cos \quad \alpha \pm \beta \quad = Cos \quad \alpha \quad Cos \quad \beta \quad \mp Sin \quad \alpha \quad Sin \quad \beta$$

$$I_R = R \quad = a \quad + b \quad =$$

$$r \quad + \left[\frac{\begin{matrix} r\,r\,T \begin{Bmatrix} F+\delta \; -R \qquad F \; -R^{M+} \qquad M+ \quad \delta+F \\ +R^{M+} \qquad M+ \quad \delta+F \end{Bmatrix} \\ +r \;\; T \left\{ + \;\; R^{M+} \quad - \frac{R^{M+} \begin{bmatrix} M+ \quad \delta+R \qquad M+ \quad \delta \\ -R \qquad M+ \quad \delta-R \qquad M\delta \end{bmatrix}}{-R \quad + \quad R \qquad \delta} \right\} \end{matrix}}{-R \quad + \quad R \qquad \delta} \right]$$

M

$F \quad n\pi \qquad T \qquad r \quad r \qquad r \quad = \quad \% \qquad M$

$F \quad n \quad \pi, n = 1, 2, 3, \ldots. \;\; r \quad r \quad r \qquad T$

$F \quad n \qquad \pi, n = 1, 2, 3, \ldots. \;\; r \quad r \qquad T$

r

$n\pi$ $n = 1, 2, 3, \ldots.$ π
F
F n π

F n π

F n π

$n\pi$ F

β β γ
F γ β β