Sociothermodynamics

Sociothermodynamics:

Segregation and Evolution in a Mixed Population

By

Ingo Müller

Cambridge
Scholars
Publishing

Sociothermodynamics: Segregation and Evolution in a Mixed Population

By Ingo Müller

This book first published 2019

Cambridge Scholars Publishing

Lady Stephenson Library, Newcastle upon Tyne, NE6 2PA, UK

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

ISBN (10): 1-5275-3766-8
ISBN (13): 978-1-5275-3766-8

Prologue in Heaven

(A.E. and I.N. and C.D. on a cloud. A symposion on *Deduction by Analogy*. Occasionally an apple passes through the scene, -- flying upwards!)

A.E.	Listen Isaac and Charles, I have unified your theories.
I.N. and C.D.	What? Gravitation and evolution? Marvellous Albert. Explain!
A.E.	Yeah. It was easy: At the beginning apples fell from the trees isotropically, in all directions. But only those hitting the ground made new apple trees.
C.D.	Ha! Splendid! Survival of the fittest.
I.N.	Ouch!

TABLE OF CONTENTS

Synopsis of Chapters

Intended for readers who are afflicted by *horror mathematicae* in order to tempt them to cure that affliction

Chapter I. *Games which birds, people, and molecules play*

The game of hawks and doves is popular in biology as a strategy of a mixed population of birds which can feed itself and reproduce successfully while engaged in a contest for a resource needed by both species. The rules of the game guarantee a certain gain for a bird in an encounter with another bird. It is tempting to apply those rules to the conduct of a mixed human society of allegorical hawks and doves, and to invent minor modifications by which the society can react to varying economic conditions by adjusting the contest strategies. For that purpose the resource is given a value, represented by its price. The price is high in difficult times when resources are scarce and it is low in good times when resources are abundant. Two phenomena can thus be described and analyzed for their efficacy. (i) Segregation: For a fixed overall composition of hawks and doves the society may react to shortages by segregating into colonies of different compositions and with different strategies. (ii) Evolution: For a variable composition the society may undergo a process of social evolution leading to an optimal hawk fraction. Those phenomena -- segregation and evolution -- occur on different time scales: Segregation takes a small part of the lifetime of the birds and social evolution takes a few generations. Both phenomena, however, are driven by the quest for maximal gain, so that segregation and social evolution are both seen as beneficial for the population, because they increase the gain.

The pursuit of gain by segregation of the hawk/dove population bears a strong resemblance to thermodynamics of a binary solution, where the molecules of the constituents segregate in their quest for minimal energy. A well-known example from the kitchen is a nourishing broth in which oil droplets segregate from the watery solution and float on its surface.

Chapter II. *First and second laws of thermodynamics and sociothermodynamics*

The consilience of thermodynamics and the hawk/dove game is not complete, however, because the stochastic aspect of thermodynamics is missing in the rules of the game. That stochastic aspect finds its expression in thermodynamics in the concept of entropy and its growth, and it is ignored -- so far -- in game theory. We find it represented by the random motion which the birds perform in their habitat in search of the resource on which they live. That motion is more vigorous in times of scarcity than in times of affluence, so that its intensity is proportional to the price of the resource. In order to formulate a veritable sociothermodynamics, the first and second laws of thermodynamics must be extrapolated to the population of the birds. The new first law relates the gain rate of the birds to the trading of resources across the boundary of the habitat and to the working which the birds exert on the boundary; it is analogous to the first law of thermodynamics which relates the rate of change of internal energy of a fluid to heating and working. And for the second law of sociothermodynamics I propose an easy paraphrase of the second law of thermodynamics. The latter was discovered Clausius who said: *Heat cannot pass by itself from cold to warm*. My paraphrase for the population of birds reads: *Value cannot pass by itself from cheap to dear*. I attempt to make this new axiom plausible by referring to simple self-evident rules of trading in an open market. The exploitation of the new laws leads to the concept of an entropy for a population of birds and to the realization that the entropy tends to grow in sociothermodynamics, -- just as it does in thermodynamics.

In thermodynamics a process in approach to equilibrium can be seen as a competition of internal energy U, which tends to a minimum and entropy S which tends to a maximum. And it is the temperature T which determines the relative significance of those trends. The three quantities form the Gibbs free energy U-TS, which reaches a minimum in equilibrium. Sociothermodynamics is just like that: The gain and the entropy strive toward maxima, and in this case it is the price of a resource, which is the arbiter in that strife. In that way the price of sociothermodynamics is equivalent to the temperature of thermodynamics. The analogy between the two fields is made more striking, if we replace the gain by the shortfall of gain from its maximal possible value: Thus a growth of gain is equivalent to a decrease of shortfall. And the Gibbs free shortfall becomes minimal in equilibrium.

Chapter III. *Thermodynamics of mixtures of fluids*

In order to formulate and appreciate sociothermodynamics of a mixed population of hawks and doves we need to know thermodynamics of binary mixtures, i.e. mixtures of two constituents capable of a decomposition into different phases: solid, liquid and vapour. This means to be familiar with the free energies of Helmholtz or Gibbs type, with energies and entropies of mixing and -- above all -- with chemical potentials, the paradigmatic quantities characterizing the presence of the constituents in the mixture; the chemical potentials become equal in all phases when equilibrium is approached. Such familiarity was acquired over many generations of alchemists and chemists who first dabbled at, and then succeeded at distilling wine or other fermented fruit and vegetable juices into hard liquor for human consumption. Nowadays the task has largely shifted to the conversion of coal or mineral oil into efficient fuels for industrial applications, and it is done by chemical engineers. Important tools of their trade are phase diagrams into which the acquired knowledge about transitions between solids and liquids and between liquids and vapours is encoded.

Even if there are no phase transitions, thermodynamics is needed for understanding mixtures. Thus thermodynamics describes and explains the phenomenon of osmosis at semipermeable membranes in plant and animal physiology -- and in technology.

This chapter provides the reader with a modicum of thermodynamics of mixtures, often called physical chemistry. It is here that the *horror mathematicae* may turn out to be most virulent, although I have attempted to replace formal mathematical arguments by plausible graphical constructions invented by chemists. Actually chemists themselves are not known for a high tolerance of mathematics.

Chapter IV. *Entropy, a most plausible concept*

It was Clausius who discovered the entropy and some of its properties, in particular the growth property. But Clausius did not recognize the stochastic nature of entropy growth. That recognition came later and Boltzmann was its pioneer. The formula reads $S \propto \ln W$, where S stands for the entropy of a body or a population, and W is the number of possibilities in which the state of a body can be realized by rearrangements of the constituent molecules or birds. That formula is easily the second most important formula of physics, or maybe I mean *the* most important formula of all. Since W is continually changing randomly in the course of the thermal molecular motion, it is the state with most realizations which a body will approach in time when it tends toward equilibrium. And since the motion is random, that approach is probabilistic, or stochastic. Thus the trend of entropy toward a maximum is also only probabilistic, not strict. Once this is recognized, entropy can be calculated for any system composed of many elements. And if the system´s elements are in random motion, the growth of its entropy is guaranteed by the rules of probability calculus. Here I apply that notion to a monatomic ideal gas in thermal motion and to a population of birds, -- either hawks or doves --, milling around in their habitat in search of the resources that feed them.

Even in equilibrium, where entropy has reached a maximum, the random motion of the elements does not stop. Thus random fluctuations are continually testing the stability of the equilibrium state. It is therefore possible to identify the states where a homogeneous mixture loses its stability and becomes segregated into phases or colonies. That happens when unequal next neighbours among the elements -- belonging to different constituents or species -- cannot be accommodated in large numbers, because they feel too uncomfortable with such neighbours.

The growth of entropy is one of the most efficient driving forces of nature. And yet it is ignored, -- ignored in the double sense of either completely unknown or conciously neglected --, in all fields of human knowledge except physics, chemistry and engineering. And even engineers .... Well, let us not go into this! The neglect must be due to the stochastic character of entropy which makes it difficult to understand for people who prefer a strict relation between cause and effect. Anyway, I have taken this situation as an incentive to write a little more about entropy than is strictly neccessary for my consideration of hawks and doves. In particular, I make the point that the concept of entropy -- as a universal property of a system composed of large numbers of elements -- is much easier to grasp than the concept of energy whose basic nature, if the truth were known, is completely in the dark.

Chapter V. *Sociothermodynamics. Segregation and infiltration*

The strategies of game theory of hawks and doves provide the quest for high gain, and the stochasticity of the birds´ random motion guarantees entropic growth of the mixed population whose species compete for the same resource. The value of the resource determines the relative intensity of the two trends. They co-operate with each other or oppose each other depending on the contest strategies which the birds adopt, which in turn are dictated by the price of the resource. If they co-operate, which happens in good times when resources are cheap, the population will form a homogeneous mixture and, if they oppose each other, which happens in bad times when prices are high, the population will segregate into colonies of different hawk/dove ratios employing different contest strategies. In that latter situation the birds encounter primarily other birds of their own kind. The results are most conveniently summarized in strategy diagrams which bear a strong resemblance to the phase diagrams of a binary mixture of fluids. It is true that the entropy effect is a relatively minor one, -- at least for the parameter values chosen here --, but it is there. And it confirms the ubiquitous observation that small minorities can always be integrated by a population while large minorities segregate, especially in economically difficult times when resources are dear. The striking analogies between phase diagrams of mixtures of fluids and strategy diagrams of mixed populations supports the main motif of this work. Namely: To show that the simple laws of thermodynamics are applicable to any system which is composed of many elements in random motion; and, in particular, to a population of many birds. This conclusion can be further extrapolated to describe the process of infiltration of one species into the domain of the other one, -- by osmosis.

Chapter VI. *Social evolution*

On the time scale of a few generations the composition of a hawk/dove population can change by social evolution, if that change leads to a growth in gain -- decrease of shortfall -- and a growth of entropy. The final destination of the evolution is the state of minimal Gibbs free shortfall, which forms a triad of shortfall, entropy and price. It is true that during the process of evolution the population may pass through a period of segregation, where hawks and doves were divided into distinct colonies with different compositions employing different contest strategies. But sociothermodynamics implies that in the final optimal stable state the population is homogeneous, exhibiting the same composition and employing the same strategy throughout the habitat. Also it turns out that a mixed population cannot turn into a pure population of only one species by social evolution. It is true that the fraction of hawks or doves can become small, but it can never vanish;

the entropy forbids that, because the entropy growth becomes infinite in the neighbourhood of a pure species.

Some wit with an original turn of mind has expressed this last observation by drawing an analogy to book printing, where a text may be seen as a mixture of letters in their regular positions and misprints: Entropy forbids a large book with many letters and without misprints. That is one reason why this book contains only 173538 letters in 35215 words. The other reason is that sociothermodynamics is not longer, -- not at the present time.

Chapter VII *Human society, game of hawks and doves and thermodynamics*

Erwin Schrödinger, the illustrious physicist and founder of quantum mechanics, gives apt advice to scientiest when he counsels them not to touch a subject in which they are not experts. I disregard that advice when I argue that the human society is akin to a thermodynamic system of two constituents. My defence is threefold: (i) the temptation is great, (ii) Schrödinger himself did not heed his advice when he wrote about biology, and (iii) I find that professional sociologists and historians do not see the wood for trees.

Of course there are problems: Sociothermodynamics and the hawk/dove game cannot be more than models for society, and the question is whether the models are good ones. For instance, what about the implied assumption that a mixture of only two constituents, -- the metaphorical hawks and doves --, is good enough to represent a human society?

That question is easy to answer seeing that all men struggle for gain and status: Do we not easily position ourselves among our neighbours, who are pursuing their quest in a more aggressive -- hawkish -- manner than ourselves, or in a less aggressive -- dovish -- one? Thus we have no difficulty to recognize the hawks and doves among the neighbours.

What then about the quest for gain and status as such? Is that the only game for humans? Obviously not; unlike our birds men can play many other games: go on a pilgrimage, separate glass and plastics in the garbage, build sand castles, or play ping-pong. The point is, however, that the quest for gain and status is the only *universal* game, essential for survival and successful procreation of all persons. All the other games are pastime activities.

The more humanity grows in numbers, and the more the liberal doctrine spreads across the globe, -- making man less of a slave to authority, less gullible, and more self-reliant --, the more obvious it becomes that playing ping-pong, or attending church is not an essential activity, whereas the pursuit of gain and status *is* essential. It is the one and only game that *is* essential. All others are ephemeral. At least that is my thesis, and by that thesis the hawk/dove model is a good one for representing the quest of a human society for a decrease of the free Gibbs free shortfall.

Is that a bleak picture? Maybe it is. But then, there is always ping-pong to detract us from realizing the bleakness, -- for a while.

I. GAMES BIRDS PLAY

Topics: Segregation and integration. Biological and social evolution

When resources become scarce in a population, dearth and starvation occur. Crises of that sort are accompanied by social changes and the often deplored phenomenon of segregation of social groups is one of them. However, that phenomenon may not be merely a fortuitious concomitance of an economic crisis: Like a fever in an infected body,—when a sick body runs a temperature in order to fight the infection—segregation may be a symptom which shows that a population attempts to improve an adverse situation or to make the best out of it.

The behaviour of individuals in a population is largely dictated by the competition of constituent groups for a limited amount of resources, essentially and ultimately food. Such groups may represent social classes, or ethnic and racial groups, or religious sects, etc.

If resources are abundant and consequently prices are low, the competition is more or less friendly and relaxed, and there is room for social niceties and tolerant intercourse between the groups. Granted that there is always competition, yet in times of abundance the contest strategy is dictated by minimal envy and some good will, and the population finds it easy to integrate members of a foreign group.

On the other hand, when resources are scarce and therefore expensive, the competition becomes more serious, or even fierce. A new strategy—a more competitive one—may be employed by all social groups and the mutual tolerance between groups is strained, or altogether abandoned. Those are the conditions under which segregation occurs in a population. The population falls apart into different colonies so that members of a group have social contact primarily among themselves.

In some intuitive way we all know this, although, following the lead of politicians, we usually deplore segregation. It is true that segregation complicates the life of politicians, because it obliges them to make different promises to different colonies belonging to their electorate. Therefore they try to discourage segregation, advise against it, lament over it, and lambaste the population for it. However, usually their disapproval achieves nothing, and in this book I attempt to show why this is so. It is *not* due to obstinacy on the part of the population.

Rather the reason is that in economically difficult times, when resources are scarce and expensive, segregation into colonies with different strategies and different compositions is more conducive to individual gain—for members of all groups—than an integrated population with either strategy, the relaxed one or the fierce one. I shall describe a sociological model to substantiate that claim.

In order to avoid the minefield of a discussion of the complex human relations, I take recourse to a behavioural model from game theory. The model is one of two species, hawks and doves, both competing for the same resource, food (say). That model is quite popular in *biology*; in fact, it was invented by the biologists Maynard-Smith and Price [1]. In that field the model is used to prove that in a mixed population of two species both may coexist and be stable in the evolutionary process. Indeed, stable fractions of hawks and doves will be approached in the course of biological evolution, because the species with the higher gain expectation—in the contest for the resource—will have a more numerous progeny. Thus the fractions will shift by natural selection of the fitter species, as it were. In this way the fractions will approach an evolutionarily stable state which the biologists call an ESS.[1]

The hawks and doves are always metaphorical birds, of course, even when the model is employed in biology. Thus the hawks of game theory never eat the doves. Dawkins [2] uses the model as a convenient tool in order to support his subtle doctrine of the selfish gene, which is the prime mover of biological evolution in his view.

Applied to *society* the hawk/dove model is even more metaphorical, if one may say that. To be sure, the hawks still do not eat the doves, and I shall assume that the individuals still employ the contest strategies of hawks and doves that proved useful for biology. However, natural selection is largely invalidated in a modern society, whose members are somewhat intelligent creatures. I stipulate that their collective intelligence is strong enough to curb selfishness of the birds to the extent that they strive for optimal gain of the population as a whole, rather than optimal gain individually. In this manner the population will in time tend—by social evolution—toward an optimal stable state rather than an ESS; I shall call that state an OSS.

However, the main issue in this work is not evolution; rather it is segregation and integration in a mixed population, because those phenomena exhibit a striking analogue to thermodynamics of mixtures, solutions and alloys. Evolution, the original main subject of the hawk/dove game, plays a secondary role here.We shall take it up in Chap. VI.

In this book I do not always draw a strict line between the behaviour of a population of hawks and doves and the conduct of human society. This is intentional, because I believe that the two have a lot in common. Or rather: If common features were not always apparent throughout past history, human society is becoming more and more like a population of birds in our times—as the "end of history" draws near, so to speak. Later on in the book, in Chap. VII, I shall attempt to bolster up this view, and I come up with the conclusion that *the human society is akin to a thermodynamic system*.

Game theory

Two strategies

In order to describe two possible contest strategies—called A and B—of a population of hawks and doves I have adapted a model from game theory due to Maynard-Smith and Price [1]. It describes the behaviour of birds who compete for the same resource. Let us consider strategy A first.

Hawk-dove policies in strategy A.

> If two hawks meet over a resource, they fight until one is injured. The winner gains the value τ, while the loser, being injured, needs time for healing his wounds. Let that time be such that the losing hawk must buy two resources, worth 2τ, to feed himself during convalescence. Two doves do not fight; they merely engage in a symbolic conflict over the resource, posturing and threatening but not actually fighting. One of them will eventually win the resource—always with the value τ—but on average both lose time such that, after every dove-dove encounter, they need to catch up by buying part of a resource, worth 0.2τ. When a hawk meets a dove over a resource, the dove walks away, while the hawk wins the resource; there is no injury, nor is any time lost.

[1] In the work of Maynard-Smith and Price ESS stands for evolutionarily stable *strategy.* I have changed the meaning of that acronym but not significantly.

Assuming that winning and losing the fights or the posturing games is equally probable, we thus conclude that the elementary expectation values for the gain per encounter are given by the arithmetic mean values of the gains in winning and losing, i.e.

$$\begin{aligned} e_A^{HH} &= 0.5(\tau - 2\tau) = -0.5\tau \\ e_A^{HD} &= \tau \\ e_A^{DH} &= 0 \\ e_A^{DD} &= 0.5\tau - 0.2\tau = 0.3\tau \end{aligned} \qquad \text{(I.1)}$$

for hawks meeting a hawk or a dove and for doves meeting a hawk or a dove respectively.

My adaptation consists in giving the resource an adjustable value, expressed by its price τ. For Maynard-Smith and Price τ was equal to one. In *my* model τ influences the behaviour of the birds as I shall explain. The price is out of control of the birds, much like the weather or the stock market is for humans, and we may think of τ as a measure for the economic climate: In good times—times of abundance—τ is small, while in hard times—times of scarcity—τ is large.

From the elementary expectation values (I.1) we may derive the expected gains e_A^H and e_A^D for a hawk or a dove in an encounter with any other bird. They are given by weighed mean values with the hawk- and dove-fractions as weighing factors

$$e_A^H = Xe_A^{HH} + (1-X)e_A^{HD} \quad \text{and} \quad e_A^D = Xe_A^{DH} + (1-X)e_A^{DD} \quad \text{and by (I.1)}$$

$$e_A^H = \tau - 1.5\tau X \quad \text{and} \quad e_A^D = 0.3\tau - 0.3\tau X \,. \qquad \text{(I.2)}$$

Here X is the fraction of hawks and 1-X is the fraction of doves..

Finally, the gain expectations e_A per bird—hawk or dove—and per encounter reads

$$e_A = Xe_A^H + (1-X)e_A^D \qquad \text{or, by (I.2)}_1 \qquad \text{(I.3)}$$

$$e_A = \underbrace{e_A^{HH} X}_{\text{gain of hawks}} + \underbrace{e_A^{DD}(1-X)}_{\text{gain of doves}} + \underbrace{(e_A^{HD} + e_A^{DH} - e_A^{HH} - e_A^{DD})X(1-X)}_{\text{gain of mixing}}. \qquad \text{(I.4)}$$

In this latter form the first term is proportional to the hawk-fraction X and the second term is proportional to the dove-fraction (1-X). Therefore those terms may be called the gains of hawks and doves, respectively, while the third term may be called the gain of mixing, since it is proportional to X(1-X),—an expression proportional to the probability of unequal pairs of neighbours in the mixture.[2] This interpretation is indicated in equation (I.4) below the braces.

By use of (I.1) the equation (I.4) may be written explicitly as

$$e_A = \underbrace{-0.5\tau X}_{\text{gain of hawks}} + \underbrace{0.3\tau(1-X)}_{\text{gain of doves}} + \underbrace{1.2\tau X(1-X)}_{\text{gain of mixing}} \qquad \text{or} \qquad \text{(I.5)}$$

$$e_A = -1.2\tau X^2 + 0.4\tau X + 0.3\tau \quad . \qquad \text{(I.6)}$$

The gain e_A may be graphically represented by concave parabolas as a function of X with the parameter τ. Fig. I.1 and Figs. I.2a through I.2f exhibit such graphs as dashed lines.

Note that both the fighting of the hawks and the posturing of the doves detract from the gains. Thus both species would do better without these activities. From the point of view of gain that aspect of the birds´ behaviour must be considered a luxury. Its rationale is based on ulterior motifs

[2] Here and in the sequel, for simplicity, I speak of "gains" rather than of "expected gains" or of "gain expectations".

like, perhaps, to gain status by impressing a possible sexual partner. Therefore the game of hawks and doves may be seen as a quest for gain and status. We have to keep that interpretation in mind at all times.

However, when times are bad and food is scarce and expensive, the birds may decide that it is more important to eat than to seem sexually attractive. So they may choose to cut down on fighting and posturing, or abandon that behaviour altogether. And what is more: The meekness of a dove confronted with a hawk may be considered overcautious in bad times. Such considerations have led to the formulation of strategy B.

Hawk and dove policies in strategy B:

The hawks adjust the severity of the fighting—and thus the gravity of an injury—to the prevailing value τ of the resource. If τ is higher than 1 (say), they fight less, so that the mean time of convalescence in case of a defeat is shorter and the value to be bought during convalescence is reduced from 2τ to $2\tau(1\text{-}0.3(\tau\text{-}1))$. Likewise the doves adjust the duration of their posturing game so that the penalty for lost time is reduced from 0.2τ to $0.2\tau\ (1\text{-}0.3(\tau\text{-}1))$. But that is not all: To be sure, in strategy B the doves will still not fight when they find themselves competing with a hawk, but they will try to grab the resource and run. Let them be successful 4 out of 10 times. However, if unsuccessful, they risk injury from the enraged hawk and may need a period of recovery at the cost of $2\tau\ (1+0.5(\tau\text{-}1))$.

The elementary expectation values for gain under strategy B thus read

$$\begin{aligned}
e_B^{HH} &= 0.5(\tau - 2\tau(1-0.3(\tau-1))) = (0.3\tau - 0.8)\tau \\
e_B^{HD} &= 0.6\tau \\
e_B^{DH} &= 0.4\tau - 0.6\cdot 2\tau(1+0.5(\tau-1)) = -(0.2+0.6\tau)\tau \\
e_B^{DD} &= 0.5\tau - 0.2\tau(1-0.3(\tau-1)) = (0.06\tau+0.24)\tau\,. \qquad \text{(I.7)}
\end{aligned}$$

As before we derive the gains e_B^H and e_B^D of hawks and doves, respectively with any other bird

$$e_B^H = 0.6\tau + (-1.4\tau + 0.3\tau^2)z \quad \text{and} \quad e_B^D = (0.24\tau + 0.06\tau^2) - (0.44\tau + 0.66\tau^2)z\,. \text{ (I.8)}$$

Therefore, in analogy to (I.5), the gains e_B per bird and per encounter under strategy B is given by

$$e_B = \underbrace{(0.3\tau - 0.8)\tau X}_{\text{gain of hawks}} + \underbrace{(0.06\tau+0.24)\tau(1-X)}_{\text{gain of doves}} \underbrace{-\,0.96\tau(\tau-1)X(1-X)}_{\text{gain of mixing}}\,, \qquad \text{(I.9)}$$

$$e_B = 0.96\tau(\tau-1)X^2 - (0.72\tau + 0.08)\,\tau X + (0.06\tau + 0.24)\tau\,. \qquad \text{(I.10)}$$

We conclude that the gains e_B for $\tau > 1$ are representred by convex parabolas. Graphical representations are given in Fig. I.1 and Figs. I.2a through I.2f by the solid graphs.

Obviously the gain e—that is to say e_A or e_B—is the average gain that a bird can acquire from an encounter. It may be considered as the gain which the population as a whole can expect from its birds in an encounter. If e is maximal, the common good is best served.

τ=1 is a reference value for which both strategies coincide except for the grab-and-run feature of strategy B. Penalties for either fighting or posturing should never turn into rewards for whatever permissible value of τ. This condition imposes an upper bound on τ: τ<4.33. That constraint could be avoided, if I permitted non-linear penalties which, however, I avoid for the sake of simplicity.

It is obvious from (I.7)$_3$ that the grab-and-run policy is not a wise one for the doves, because, on average, they get punished for it. So why should they adopt that policy? I explain that to myself by assuming that doves are no wiser than people, who have often in history started a conflict with the expectation of a quick gain and then met disaster.

Long-term effects of strategies. Biological and social evolution

The strategies as such and their gains do not by themselves lead to predictions of the development of the populations which employ them. An additional stipulation is required to predict the future. And that stipulation is different in biology and sociology.

In *biology* it is easy,—nearly automatic—to assume that HH-descendants will grow in number, or decrease depending on whether $e^H \substack{>\\<} e^D$ holds; natural selection favours hawks or hurts them as dictated by the relative size of e^H and e^D. Mutatis mutandis the same is true for DD-descendants, and there are no others. Therefore both groups tend toward the fraction X^{ESS} for which $e^H(X) = e^D(X)$ holds, the evolutionarily stable state called ESS. Fig. I.1a indicates these trends by the arrows on the straight lines representing $e^H(X)$ and $e^D(X)$.

In Fig. I.1a, which represents the case of strategy A for τ=1, we have $X^{\text{ESS}} = 7/12$. It is this result that explains the general interest in the hawk/dove strategy of Maynard-Smith and Price: Both species may coexist. This situation was not always accepted in evolutionary theories; indeed for some time and in some circles a primitive slogan prevailed: "the fitter species takes it all." Evidently the advocates of that slogan did not realize that being fitter than your competitors may depend on the sizes of the competing fractions.

In *sociology* it is not quite so easy to predict the long-term evolution of the population. Sociology deals with people and hawk/dove pairs may even have common descendants. What is *their* character? Also HH-descendants and DD-descendants are not necessarily hawks and doves themselves. I stipulate that the new descendants of any pair—be it HH, or HD, or DD—may *choose* at an early stage what they want to be for the rest of their lives, hawkish or dovish. Also, people possess some intelligence, and maybe foresight, but certainly hindsight. And that influences their choice. Suppose that during some period in the past they all chose "hawk". According to Fig. I.1a that choice eventually landed them in a pure hawk environment with the lowest possible gain: $e_A^{HH} = -0.5$. That was bad and they have never forgotten the hardships of that state; so they will not make the same choice again. Or, let us suppose that they all chose "dove". That was better, because, again according to Fig. I.1a, they ended up in a pure-dove environment with an acceptable gain of $e_A^{DD} = 0.3$. That was not bad at all, but it was not optimal. On many different occasions with less uniform choices they may even occasionally have hit the ratio $X = \frac{1}{6}$ with the optimal gain of $e = \frac{1}{3}$. Those were their best times and they are stored in the collective memory of the population. Therefore they will henceforth adjust their choice so as to approach that optimal gain.

Surely that is plausible. Anyway it is plausible for me, and therefore that is what I stipulate as part of the game strategy applied to sociology: The population will arrange its choices so as attain a higher gain *e*. Thus in the long run it will reach an optimal stable state, which I call an OSS. What I have described here as the stipulated behaviour of societies is not natural selection, of course. Since we shall need to refer to it, I call it *collective selection*, because the process makes use of the collective memory of the population over all choices in the past and their effects.

In summary: Biological evolution proceeds by natural selection toward an ESS, and social evolution proceeds by collective selection toward an OSS.

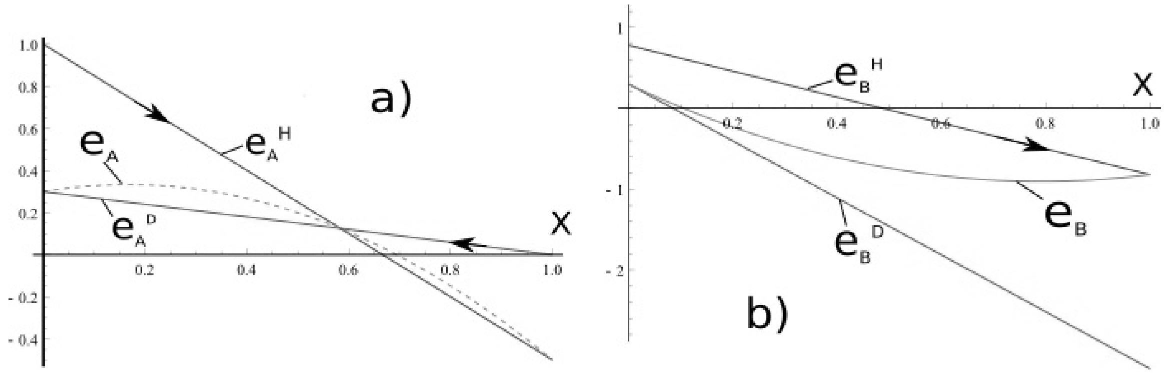

Fig. I.1. Gains e^H and e^D for hawks and doves per encounter, and gain e per bird and encounter; all as functions of X. a) strategy A, τ=1. b) strategy B, τ=2

Evidently my stipulation leaves some questions open. Because it implies—in the situation of Fig. I.1a—that, while all birds are better off in the OSS than in the ESS, few hawks are much better off than the majority of doves. Who will be allowed to be the lucky ones? And how do they cope with the ensuing inequality? Obviously feudal societies have no problem with this. And even egalitarian societies tolerate the privileges of soccer players, pop stars, and top models with no more than minimal resentment; even their scandals are accepted with a good deal of friendly understanding. So, maybe some inequality in the population is not a problem.

Fig. I.1a shows graphs and values relevant to this discussion, at least for strategy A and for the exemplary case $\tau = 1$. Let us also discuss strategy B, the case for $\tau = 2$. I have plotted $e_B^H(X), e_B^D(X)$, and $e_B(X)$ in Fig. I.1b and we see that the graphs are rather different from those of Fig. I.1a. First of all, in this case there is no ESS with 0<X<1, because the two graphs for $e_B^H(X)$ and $e_B^D(X)$ do not intersect within that interval. And since the former is higher, biologists will conclude that hawks have the upper hand, irrespective of the extant value of X, until they end up—by *natural selection*—in an all-hawk population, see the arrow on $e_B^H(X)$. Their unrestrained selfishness has landed the population in a state of minimal gain.

What about the significance of this plot for sociology? There are *two* maxima of $e_B(X)$ in this case—actually they are end-point maxima, but maxima all the same. And I have argued that the population strives toward such maxima by means of what I have called *collective selection*. So we conclude that the gain of the population,—if it insists on using strategy B—will follow the $e_B(X)$-curve in Fig. I.1b, always upwards, but toward the left or right maximum depending on where it started. If it started left of the minimum of the $e_B(X)$-curve, it will end up as a pure-dove population. Otherwise, if it started right of the minimum, it will become pure-hawk.

Time scales

For *natural* selection to have a sizable effect it takes a long time. Indeed, if hawks have the evolutionary advantage at some time, the next generation may have increased its hawk fraction by a few percent at best and it takes several, or many generations to make a significant approach toward X^{ESS}.

The process which I have called *collective* selection is faster than that. In fact in an extreme case, when all newly born birds decide to be hawks, the whole population will consist of hawks after only one generation. That tallies well with our intuitive knowledge that social evolution,—which employs collective selection—is quicker than biological evolution which employs natural selection. We shall have to keep that in mind.

What concerns us most, however, is that even social changes are rare compared to the frequency of the encounters between the birds seeking the resources. Those encounters may occur daily, or many times daily, while a significant shift in hawk fractions by collective selection typically requires a sizable part of one generation.

Strategy diagram

Preferred strategies

The Figs. I.2a through I.2f show plots of $e_A(X,\tau)$ as dashed graphs in (*e,X*)-diagrams and for increasing values of τ. All graphs exhibit maxima in the range 0<*X*<1. Those maxima are the optimal stable states,—OSS—if the population employs strategy A. Given time for what I have called social evolution, these maxima will eventually be reached by the population, if it continually employs strategy A.

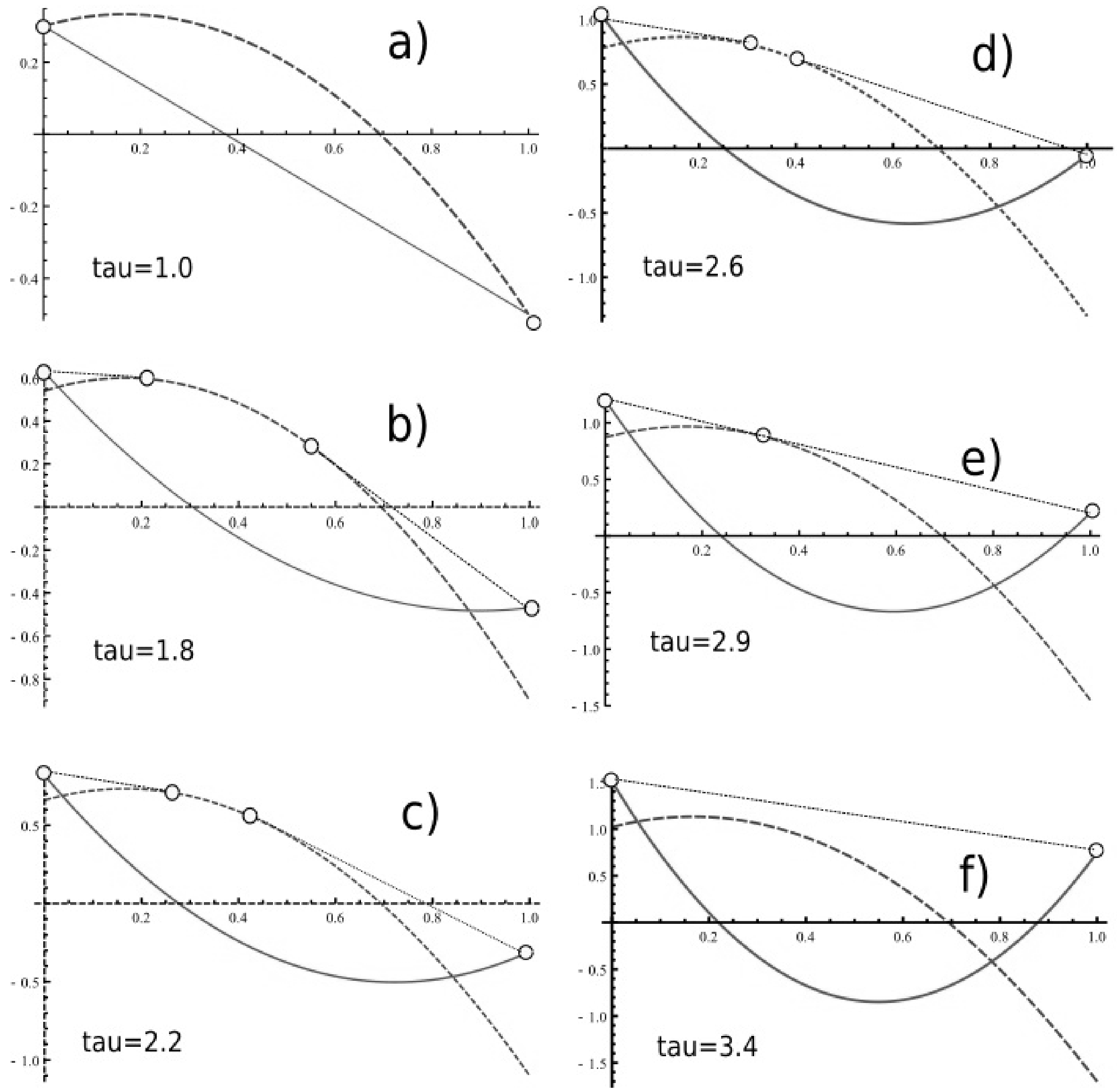

Fig. I.2a through I.2f. Gains e_A (dashed) and e_B (solid) as functions of hawk fraction X and for different values of the price τ. Concavifying tangents (dotted)

The figures also show plots of $e_B(X,\tau)$ as solid graphs. All graphs have end-point maxima and, if the population employs strategy B, it will evolve toward an all-dove population when $\tau = 1$ holds. For $\tau > 1$ there are *two* end-point maxima, one each at X=0 and at X=1. Therefore, given time, the population evolves toward an all-dove population, if its initial state lies left of the intermediate mimimum of the $e_B(X)$ -curve. Otherwise it will evolve toward an all-hawk population.

So much for the cases when it is either strategy A or strategy B which the birds can adopt. But then, I have argued earlier that the birds may switch strategies in order to adopt *the* strategy which provides the higher gain. This means that strategy A will occur for $\tau = 1$ irrespective of the hawk fraction, since $e_A > e_B$ holds in the whole range 0<X<1 for this value of τ. But for values $\tau > 1$ the situation is different: For growing values of τ the gain e_B is bigger than e_A for increasingly wide intervals near X=0 and X=1. Therefore populations with a hawk fraction in these intervals should employ strategy B whereas in the broad domains between those intervals strategy A should prevail.

Once all of this is understood, and once it has been verified by inspection of Figs. I.2a through I.2f, we are ready for the next—and decisive—step in my argument and that concerns segregation.

Segregation by concavification

I refer to Fig. I.3 which is an enlarged version of Fig. I.2c, the one for $\tau = 2.2$. Let us suppose that the population initially has the hawk fraction X=0.82. From what has been said before we should then expect that it adopts strategy A, since $e_A(0.82) > e_B(0.82)$. And in the long run it could then evolve—by collective selection—along the graph $e_A(X)$ en route to the maximum of that curve. But that takes time, at least part of the duration of one generation as I have argued. In the short term the population would seem to be stuck at X=0.82 and $e_A = -0.39$.

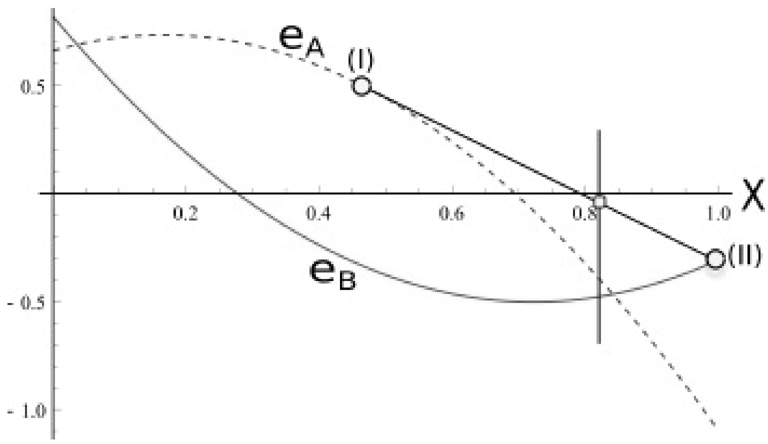

Fig. I.3 Concavification. Graphs appropriate for τ=2.2

However, the population of hawks and doves may be cleverer than that. Indeed, the population can increase its gain by giving up homogeneity, i.e. by segregation into colonies. To wit: By having its state lie on the common tangent of $e_A(X)$ and $e_B(X)$, see Fig. I.3.[3] The population achieves that state by splitting into the colonies (I) and (II) whose states are given by the points

[3] A subtle point: The end-points of e_B at X=0 and X=1 have infinitely many tangents and therefore a person unfamiliar with convex analysis may tend to say that those are not "real" tangents. However that may be, for the present purpose it suffices to say that only one of those infinitely many tangents is also tangent to e_A or -- in the case of Fig. I.2f -- to the opposite point of e_B.

where the common tangent touches the curves. Within the colonies the hawk fractions have values X^I and X^{II} and we have

$$X = 0.82 = z^I X^I + (1 - z^I) X^{II} \quad \text{and} \quad e = z^I e_A(X^I) + (1 - z^I) e_B(X^{II}) \quad , \qquad \text{(I.11)}$$

where z^I and $z^{II} = 1 - z^I$ are the fractions of birds—hawks *and* doves—in the two colonies. From Fig. I.3 we either read off the pertinent values, or we calculate them as properties of the common tangent

$$X^I = 0.46\,, \quad e_A(X^I) = 0.51\,, \qquad X^{II} = 1\,, \quad e_B(X^{II}) = -0.31 \quad , \qquad z^I = 0.34\,.$$

Thus both gains $e_A(X^I)$ and $e_B(X^{II})$ are bigger than before segregation, when we had $e_A = -0.39$. And, of course, the average gain $e = -0.03$ is bigger than both $e_A(X)$ and $e_B(X)$ for X=0.82. Therefore in this case the segregation is good all the way round.

Given time for collective selection the state of the segregated population will slowly move along the common tangent keeping the hawk-fractions X^I and X^{II} of the colonies fixed, merely changing the fractions z^I and z^{II} of the birds in the colonies. Obviously all along the tangent the gain of the segregated population is bigger than the gains of the homogeneous populations employing either strategy A or B. And that is why segregation occurs. It provides bigger gains.

The construction of the common tangent is called concavification of the graph max[e_A, e_B], because it creates a concave envelope of that graph. Concavification can be used whereever the two curves e_A and e_B intersect each other as indicated in Figs. I.2.b through I.2d. The state of the colonies is indicated in the figures by little circles. Thus for τ=2.2 it is only in a rather small interval with intermediate values of X where a homogeneously mixed population can exist. That interval of X shrinks further for growing τ until it vanishes altogether for τ=2.9, where the two lateral tangents come together to form a straight line.

For still larger τ's the concavifying tangent of the graph max[e_A, e_B] ignores $e_A(X)$ altogether: It is then represented by the common tangent of the end points of the graph $e_B(X)$ alone as shown in Fig. I.2f. Thus for large values τ the birds in the colonies employ strategy B for all X and the colonies contain pure-dove and pure-hawk populations; a homogeneously mixed population cannot exist in these cases.

Fig. I.3 demonstrates a particularly favourable case of segregation, because $e_A(X^I)$ and $e_B(X^{II})$ are both bigger than the value $e_A(X)$ before segregation. This is not always the case, not in all cases of concavification shown in Fig. I.2. However, the average value $e = z^I e_A(X^I) + (1 - z^I) e_B(X^{II})$ *is* always bigger than the original one, and so is either $e_A(X^I)$ or $e_B(X^{II})$. Therefore segregation is benificial for the population as a whole, even though it may not be so for the birds in one of the two colonies. Some spirit of solidarity is demanded from the birds—or maybe the authorities enforce that spirit for the sake of the common good.

Strategy diagram

A convenient way to represent the states (τ,X) where segregation occurs is by constructing a strategy diagram as shown in Fig. I.4: The concavifying tangents for 1<τ<4 of Figs. I.2b through I.2f are projected onto the appropriate horizontal lines τ=const in a (τ,X)-diagram and the end points of the projections are connected. In this way the strategy diagram comes to exhibit four regions denoted by (I) through (IV) in Fig. I.4. They contain

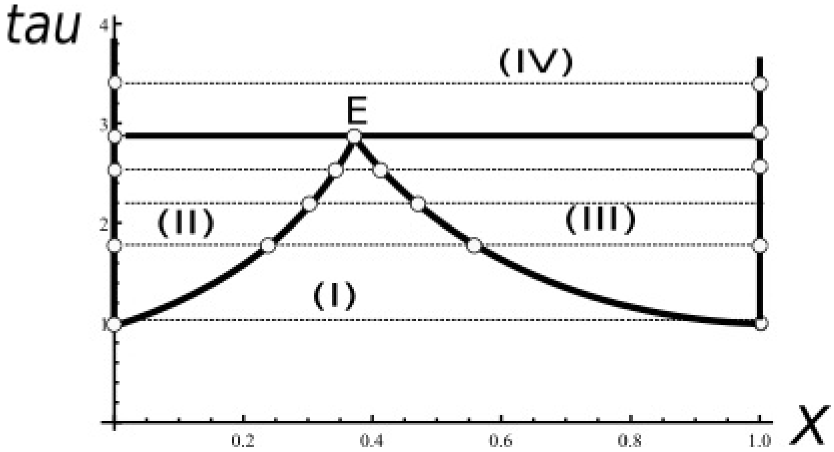

Fig. I.4 (τ, X) - strategy diagram

(I) Homogeneously mixed—"integrated"—populations employing strategy A for all hawk fractions *X*.
(II) Segregated populations with two types of colonies: Pure-dove colonies employing strategy B and colonies with moderate hawk-fractions employing strategy A.
(III) Segregated populations, again with two types of colonies: Pure-hawk colonies employing strategy B and colonies with appropriate hawk-fractions employing strategy A.
(IV) Segregated populations with pure-hawk and pure-dove colonies all employing strategy B.

Thus integration prevails when resources have a low price τ, while segregation is unavoidable for large prices. This confirms the intuitive expectation—expressed in the introduction of this chapter—that in hard times with high prices the more competitive strategy prevails and segregation occurs, while in good times with low prices the competition is less acute and integration prevails. The information encoded in the strategy diagram, however, goes beyond this intuitive expectation; it describes what happens in-between those two extreme ranges: The transition is not abrupt, except when the hawk fraction is equal to X_E. If it is, the change from strategy B to strategy A is easiest, i.e. it occurs at the highest price, namely τ_E. Therefore I call the point E the *eutropic* point.[4]

For $X \neq X_E$ the change from integration to segregation is more complex. Let us consider this by inspecting Fig. I.5. We start at the bullet point and proceed by increasing τ. Nothing much happens at first. The population is mixed with hawks and doves in the proportion $\frac{X_\bullet}{1-X_\bullet}$ and employs strategy A. This remains so until the border of region II is reached, whereupon a colony,—or colonies—of pure doves with strategy B form inside the mixed population of hawks and doves with strategy A. Upon further increase of τ the state of the latter population moves upwards along the curved border line of region II, becoming more hawk-rich in the process. The proportion of birds with strategies A and B in this range is given by the ratio of the lengths $\frac{a}{b}$, see Fig. I.5.

When eventually τ_E is reached, pure-hawk colonies begin to form—employing strategy B, just like the already existing pure-dove colonies. The two types of colonies coexist, and they coexist

[4] In Greek: eutropic -- easy to change.

with the remaining mixed hawk-dove population at (τ_E, X_E) of strategy A. At a still higher price the mixed hawk-dove population vanishes and gives rise to pure-hawk colonies and pure-dove colonies in the proportion $\frac{X_\bullet}{1-X_\bullet}$. The arrows in Fig. I.5 indicate the changes of state in the process.

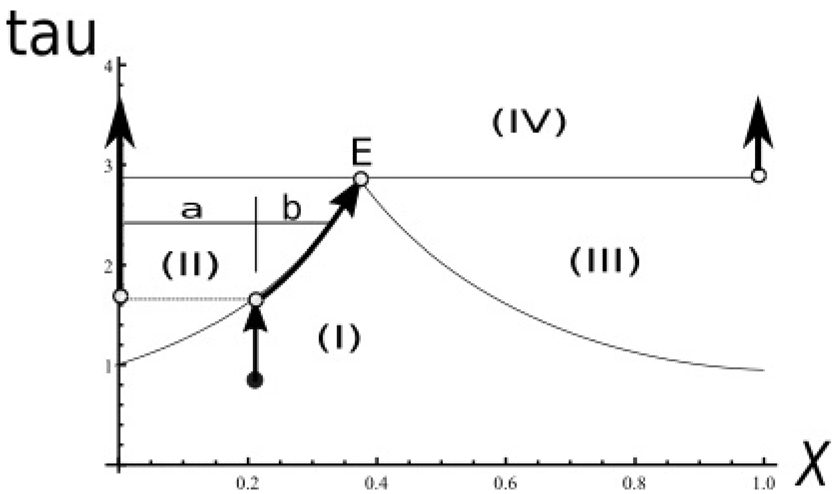

Fig. I.5 Change of strategy upon a rise of price

All of this complex behaviour is dictated, of course, by the trend of the population with a given overall $X_\bullet$ toward the largest possible gain. It is noteworthy, perhaps, that integration persists in part of the population—with an increasing hawk fraction—even after the first all-dove colonies appear; it ceases, however, when τ_E is surpassed.

Social evolution

I have argued that social evolution by collective selection is negligible on the time scale of encounters between the birds. But it does happen, albeit slowly. And it leads to a state of maximal gain, the OSS, by varying the hawk fraction *X*. Given the possibility of a decomposition of the population into colonies, we must realize that the OSS lies in the maximum of the convex envelope of the graph max[e_A, e_B]. It is therefore clear from Fig. I.2 that for $\tau > 1$ the OSS occurs in a homogeneous mixture at *X*=1/6 only if $\tau \leq \frac{3}{2}$ holds. In that range the optimal strategy is A. The upper limit $\tau = \frac{3}{2}$ is the price for which the left common tangent is horizontal. Beyond that price the OSS is the pure-dove state at *X*=0 in which the doves employ strategy B.

Analogies with thermodynamics

The above arguments have provided us with a first view of the behaviour of a somewhat intelligent population of hawks and doves according to a model from game theory. Later, in thermodynamics,—in Chap. III—I shall show that the mindless molecules of a binary alloy exhibit a phase diagram which is strongly reminiscent—in most of its qualitative features—of the strategy diagram of Fig. I.4; it describes the phase change between the solid and the liquid phase. To be sure, region (IV) in Fig. I.4—the range of segregation—is called a *miscibility gap* in thermodynamics, and the eutropic point is called the *eutectic point*[5] of an alloy, because it makes melting easy. Also a thermodynamic solution does not strive for maximal gain, rather it tends toward minimal energy. But these may be regarded as minor differences.

[5] In Greek: eutectic -- easy to melt

It is true though that—at this point—the analogy is not complete. Thermodynamics exhibits a somewhat richer behaviour than the game theory which has been considered above. That is due to an element of stochasticity—embodied in the concept of entropy—which is inherent in all thermodynamic systems and which, so far, I have ignored in the exploitation of the games which the birds play. That deficit will presently be corrected.

In the past I have already occasionally presented the above ideas about an emerging sociothermodynamics; usually as comic relief when a serious professional colleague celebrated an important anniversary. But I have also described the theory in academic journals, see [3], [4]. Because I do take sociothermodynamics seriously, and I am not the only one,—not even the first one. Thus Mimkes, a metallurgist, has noticed the analogy between segregation in a society and the miscibility gap in alloys, see [5]. His approach is not based on game theory; it is more phenomenological than mine. Thus Mimkes has studied the integration and segregation of protestants and catholics in Northern Ireland and he came to interesting conclusions about mixed marriages. More recently Mimkes [6] has extrapolated thermodynamic ideas to economics.

It is interesting to note that sociothermodynamics is easily accessible only to chemical engineers and metallurgists. Those are the only people who know phase diagrams and their usefulness. It cannot be expected, in our society,—and in our educational system—that sociologists will appreciate the potential of these ideas. They have never seen a phase diagram in their lives. It is to be hoped that this booklet may serve to change that situation.

II Thermodynamics and Sociothermodynamics— Single Fluids and Pure Populations

Similarities and discrepancies between fluids and populations

There are conspicuous similarities between fluids and populations: Fluids consist of large numbers of molecules and populations consist of many individuals—individual birds in our case. The molecules perform a random thermal motion, while the birds mill around randomly in their habitat in search of resources. Molecules interact with attractive and repulsive forces and birds interact employing their contest strategies over a resource. Both—by virtue of their random motion—exert a pressure on their boundaries and both may carry out exchanges across the boundaries: Fluids exchange heat across the boundary and populations trade for resources with the outside world,—at least that is what I shall assume of "my populations."

What is more however, is that both, fluids and populations,—boundary conditions permitting—relax in time toward a nearly constant and homogeneous equilibrium, even if initially they exhibited rapid changes and strong gradients. If disturbances at the boundary—heating, trading, and working—are slow and benign, the interior of a fluid body or a population settles down to homogeneity in the late stage of the approach to equilibrium. This phenomenon is taken for granted and the laws of near-equilibrium thermodynamics—or *reversible thermodynamics*—are formulated so as to describe it in fluids. Here I advertise the idea that analogous laws govern the comportment of populations near an equilibrium.

The reason behind the intrinsic trend toward equilibrium lies in the large numbers of the constituent molecules or birds, respectively, and in the randomness of their motion. Those features allow a fluid and a population to proceed from initially improbable distributions to successively more probable ones and, eventually, to an equilibrium,—the most probable distribution. Thermodynamics first recognized this element of stochasticity and embodied it in the concept of entropy.

As a historical aside I should mention, however, that the discoverers of entropy did not recognize at first that their new important quantity, the entropy, had anything at all to do with probability or stochasticity. Entropy for them entered thermodynamics in a formal mathematical manner as the generating function of a differential form, and that concept does not lend itself to an intuitively appealing interpretation. Thus entropy, if taught badly, remains a vague concept even today in the minds of many physicists and nearly all students. Actually, if we split the human intellectual activity into "Arts and Sciences", entropy is virtually ignored by the "Arts" despite its role as one of the prime movers of natural phenomena. The concept is largely unknown among "artists" of any extraction.

It is the eye-catching analogy between phase diagrams in thermodynamics of mixtures and strategy diagrams of a population of hawks and doves that has motivated this work. I shall now proceed to draw the analogy closer. First of all, it is irritating that the gains of a population tend to a *maximum,* while the energy of a fluid tends to a *minimum*. And I remedy that discrepancy by introducing a new quantity to replace the gains: The shortfall.

Shortfall as the equivalent of the thermodynamic energy

We imagine a plane on which the resources which birds need are densely distributed. The birds live on an area $\mathcal{V}$ of that plane with the boundary $\partial\mathcal{V}$ and they compete for the resources with different strategies A and B, see Chap. I. The resource has a value, represented by its price τ, which is high when the resource is scarce and low when the resource is abundant. We recall that $e_i(X,\tau)$ (i=A,B) in (I.5) and (I.9) is the gain of a single bird with strategy i in one encounter. If ν_i is the frequency of encounters, the gain of the bird during the time interval Δt is equal to $\nu_i \Delta t\, e_i$ and, if there are $n_i d\mathcal{V}$ birds in the area element $d\mathcal{V}$, we may write the gain rate of the population as $\dot{E}_i = \int_{\mathcal{V}} n_i \nu_i\, e_i\, d\mathcal{V}$.

The maximal gain which a bird can reasonably expect in an encounter is 0.5τ. This value would be realized if the birds abandoned all fighting or posturing. If they do, the gain rate will come out as $\dot{E}_i^{\max} = \int_{\mathcal{V}} n_i \nu_i\, 0.5\ \tau\, d\mathcal{V}$. In reality, since fighting and posturing are instinctive and lie in the nature of the birds, the gain rate will fall short of the maximum by the amount $\dot{E}_i^{\max} - \dot{E}_i$. We call that value the *shortfall* $\mathcal{U}_i$:

$$\mathcal{U}_i = \int_{\mathcal{V}} n_i \nu_i (0.5\tau - e_i) d\mathcal{V} \quad \text{and, under conditions of homogeneity}$$

$$\mathcal{U}_i = \mathcal{N}_i \nu_i (0.5\tau - e_i) \qquad \text{(i=A,B),} \qquad \text{(II.1)}$$

where $\mathcal{N}_i$ is the number of birds with strategy *i.* In sociothermodynamics I choose $\mathcal{U}_i$ as the analogue of the internal energy of thermodynamics. The shortfall is minimal, when the gain rate $\dot{E}_i$ is maximal. For simplicity I take the frequencies ν_i of collisions to be constants, independent of n_i and τ, and equal for hawks and doves, but dependent on the strategies A or B.

I proceed to develop sociothermodynamics—argument by argument—in juxtaposition to thermodynamics wherever that seems useful for an easy explanation of the basic tenets. And I use similar letters for quantities that play similar roles. Thus U is the internal energy of a fluid and $\mathcal{U}$ is the shortfall of a population; both have a natural tendency to decrease. V is the volume of a fluid and $\mathcal{V}$ is the area of the habitat of a population, p is the pressure of the fluid and p is the pressure of the population, etc.

By (I.4) and the equivalent equation for strategy B we have for the specific shortfalls

$$u_i = \frac{\mathcal{U}_i}{\mathcal{N}_i} = \nu_i (0.5\tau - e_i^{HH}) X + \nu_i (0.5\tau - e_i^{DD})(1-X) - \nu_i (e_i^{HD} + e_i^{DH} - e_i^{HH} - e_i^{DD}) X(1-X) \quad . \quad \text{(i=A,B)} \qquad \text{(II.2)}$$

These shortfalls are plotted in Fig. II.1 as functions of the hawk-fraction and of the price. Recall that the birds will strive for the minima of the shortfalls by collective selection. $u_A(\tau,X)$ has shallow minima for all τ at $X=\frac{1}{3}$, while $u_B(\tau,X)$ has only end-point minima, i.e. minima at X=0 and X=1.

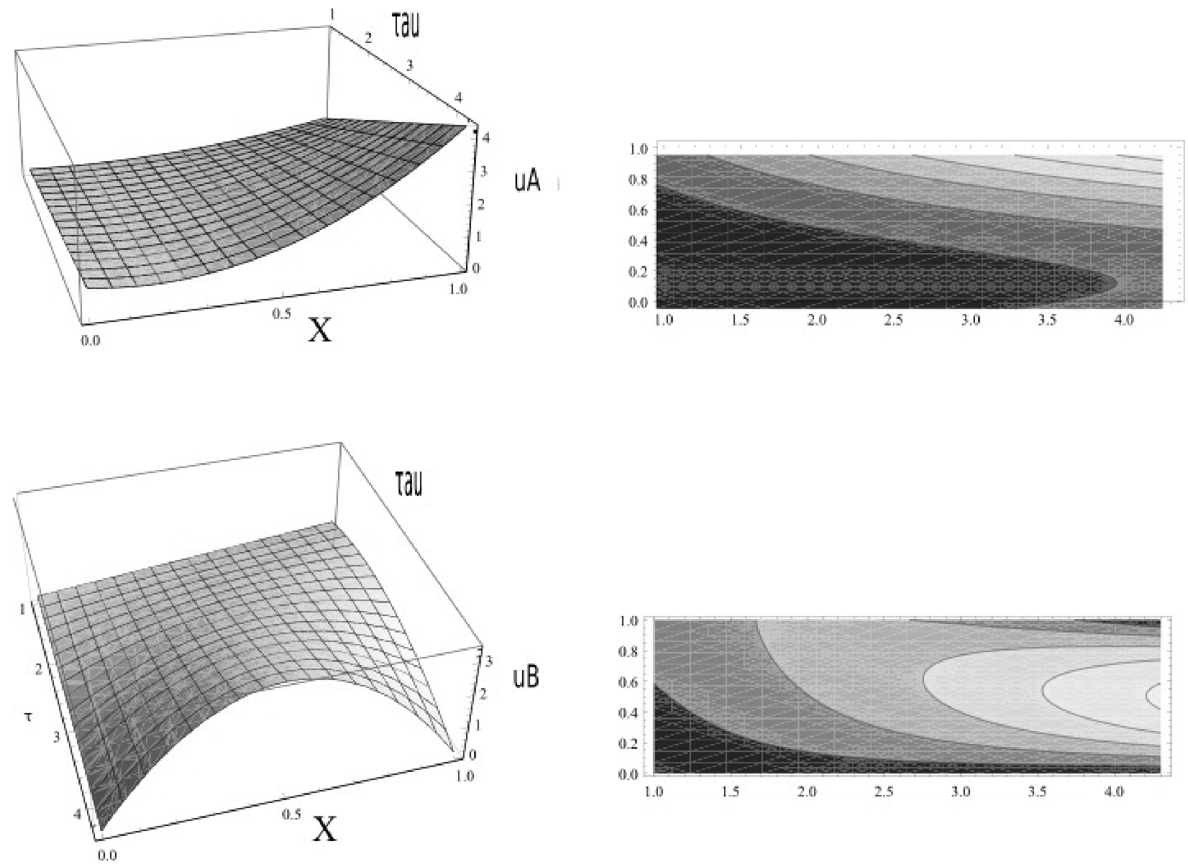

Fig. II.1 Specific shortfalls as functions of hawk-fraction X ($0 \leq X \leq 1$) and price τ ($1 \leq \tau \leq 4.3$). Top: Strategy A. Bottom: Strategy B

To begin with, in the present chapter I restrict the attention to single fluids and pure populations of either hawks or doves. For the birds, by (II.2), we thus have shortfalls of hawks and doves of the forms

$$u_i^H = \frac{\mathcal{U}_i^H}{\mathcal{N}_i^H} = \nu_i(0.5\tau - e_i^{HH}) \quad \text{and} \quad u_i^D = \frac{\mathcal{U}_i^D}{\mathcal{N}_i^D} = \nu_i(0.5\tau - e_i^{DD}). \quad (i\text{=A,B}) \qquad \text{(II.3)}$$

Note that, by (I.1) and (I.7), $\mathcal{U}_A^\alpha$ (α=H,D), the shortfalls of strategy A, are proportional to τ, while $\mathcal{U}_B^\alpha$, the shortfalls of strategy B, have second order terms in τ.

Thermodynamics of single fluids and sociothermodynamics of pure populations

Processes and equations of state

thermodynamics || socio-hermodynamics

We consider a body in a volume V with boundary ∂V . The body consists of N molecules of some fluid, a gas or a liquid, and the internal energy of the body is given by

(II.4) $$U = \int_V nu dV ,$$

where n is the number density of molecules and u is the specific internal energy, the energy per molecule.

The molecules perform a random —*thermal*— motion in V, and the intensity of that motion is measured by the temperature T. The motion of the molecules also determines the fields of pressure p and stress t_{ij} in V .

A process in the fluid is characterized by the fields

(II.5) $n(x,t)$ - density of molecules
$T(x,t)$ - temperature, and
$v_i(x,t)$ - velocity for all $x \in V$.

The first two of these fields determine the fields of pressure $p(x,t)$ and specific internal energy $u(x,t)$ in a manner that depends on the fluid. Thus we have $p = p(n,T)$ and $u = u(n,T)$, and such relations are called the thermal and caloric equations of state respectively of the fluid.

We consider a population in the area $\mathcal{V}$ with boundary $\partial\mathcal{V}$. The population consists of $\mathcal{N}$ birds, either hawks or doves and the shortfall of the population is given by (II.2) such that

$$\mathcal{U} = \int_A nu d\mathcal{V} \quad \text{where} \qquad \text{(II.4)}$$

$$u = \begin{cases} \nu(0.5\tau - e^{HH}) & \text{for hawks} \\ \nu(0.5\tau - e^{DD}) & \text{for doves} \end{cases} .$$

n is the number density of birds and u is the specific shortfall, the shortfall per bird.

The birds are milling around randomly in $\mathcal{V}$ in search of resources with a value determined by the price τ . If the resources are scarce, their value is high and we expect the random motion to be brisk. Thus the intensity of the motion is high, if the price is large and vice versa. The motion of the birds also creates fields of pressure and stress inside $\mathcal{V}$ much as the molecules of a gas do.

A process in the population is characterized by the fields

$n(x,t)$ - density of birds, (II.5)
$\tau(x,t)$ – price of a resource, and
$v_i(x,t)$ - velocity of the birds for all $x \in \mathcal{V}$.

The fields of number density and price determine the pressure field $p(x,t)$ and the field $u(x,t)$ of specific shortfall in a manner that depends on the population, i.e. whether they are hawks or doves and which strategy they employ. Thus we have $p = p(n,\tau)$ and $u = u(n,\tau)$ and such equations are called equations of state of the population.