Critique of Constructal Theory

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CONTENTS

ACKNOWLEDGMENTS

This book is dedicated to my whole family, which gives me a wonderful and beautiful life.

My respected parents brought me up, and support me all the time. My respected parents in law always help me selflessly and warmly.

My dear wife, YunXiao Liu, left our hometown and came to me from the southwest to the east of China and from the upstream to the downstream of the Yangtze River, giving me deep and lasting love and support. I cannot imagine a world without her.

My lovely children, NeYan Cheng and NaYan Liu, always bring me endless happiness and power. I also would like to tell them with the present book: In the world of science, there is no authority, but truth, flaws or fallacies.

Thank them all.

PREFACE

Why do the roots of plants have so many branches? Why are snails small and soft, but elephants big and strong? Why do the crisscrossing streets in a city look like a net? How do rivers evolve into the present structures? How are the cracks in the soil formed during drought? How is the fluttering snow formed in the sky? Why are the structures in many engineering applications very similar to those in nature? These questions seem as simple and naive as those a little child may ask. However, they are never easy to answer. As is known to all, the physical mechanisms for the formations of these shapes and structures may really be very complex. For different environments or different shapes or structures, there may be different physical mechanisms. Therefore, if we are told that one universally applicable theory could govern the evolutions of all kinds of shapes and structures in living and nonliving systems, it must be great news that can attract the attention of the whole world, and the theory may even deserve a Nobel Prize.

About twenty years ago, it was deemed that the constructal theory could do the work. In some applications, the constructal structures could be deduced theoretically and deterministically from small to large, so that it was claimed that the evolutions of natural structures could be governed by the constructal method and the constructal results could give the optimal system performances. With this consideration, the constructal method was regarded as a universally applicable law, and has already been used to explain or optimize many shapes and structures in biology, physics, technology and social science. The constructal tree-shaped structures were also defaulted to be optimal, and have been used in many engineering cases. Obviously, the constructal theory seems to be the unique approach that we want. If it were true, that would be really wonderful and exciting.

However, we still have some questions here: Is the constructal theory always applicable for analyzing or optimizing all kinds of shapes and structures? If the physical mechanisms of the evolutions of different structures are totally different, how could the constructal theory govern the evolutions? Can the constructal shapes or structures always give the best system performances? Can the constructal theory always provide satisfactory explanations for natural shapes or structures? It is known that there are definite application scopes for almost all scientific theories. This is the reason why we ask these questions to challenge the applicability and generality of the constructal theory. Accordingly, we prefer analyzing every detail of the constructal theory before accepting that it is universally applicable.

In this book, we focus on the questions above, and try to check whether the constructal theory is really omnipotent for analyzing or designing all kinds of shapes and structures in nature and engineering or not. The models, methods, applications and results of the theory to the explanations or optimizations of the shapes and structures are reviewed and discussed in detail with theoretical analyses and some typical application cases. The relationships and differences between the constructal theory and some other theories, including the fractal theory, the entropy generation minimization and the equipartition principles, are also reviewed and analyzed. Meanwhile, the abuses of the theory are directly pointed out and discussed. With our analyses and discussions, one can understand the constructal theory more deeply, and easily evaluate the scientificalness, correctness and generality of the constructal approach.

CHAPTER ONE

INTRODUCTION

There are various shapes and structures in the world. In our daily life, we can see trees with branches, slender and tall bamboos, meandering rivers, crisscrossing streets and different buildings and animals. With an astronomical telescope, we can see the complex structures in the distant galaxies. With an electron microscope, we can further find different shapes of microorganisms. On a smaller size, we can even detect the internal structures of molecules and atoms. In nature and engineering, different functions can be achieved with these shapes and structures. For instance, cars can run with round wheels, fluid can flow though hollow tubes, and turtles can be protected by their thick shells. Obviously, the shapes and structures play important roles in living and nonliving systems, so that we can have a beautiful and wonderful world.

Here, one may ask: What are the physical mechanisms for the evolutions or designs of the shapes and structures? Can we use one theory to explain all these physical mechanisms? To answer these questions, Bejan[1] proposed the constructal theory, which was described as

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[1] a) Bejan A. Street network theory of organization in nature. J Adv Transport, 1996, 30: 85-107.

b) Bejan A. Constructal-theory network of conducting paths for cooling a heat generating volume. Int J Heat Mass Transfer, 1997, 40: 799-816.

c) Bejan A, Lorente S. Thermodynamic optimization of flow geometry in mechanical and civil engineering. J Non-Equilib Thermodyn, 2001, 26: 305-354.

d) Lorente S, Bejan A. Heterogeneous porous media as multiscale structures for

> "For a finite-size system to persist in time (to live), it must evolve in such a way that it provides easier access to the imposed (global) currents that flow through it."

Later, two more descriptions were presented[2],

> "For a system with fixed global size and global performance to persist in time (to live), it must evolve in such a way that its flow structure occupies a smaller fraction of the available space."

> "In order for a flow system with fixed global resistance and internal size to persist in time, the architecture must evolve in such a way that it covers a progressively larger territory."

It can be seen that the constructal structure should give the minimum global resistance for fixed internal and global sizes, the minimum internal size for fixed global resistance and size, and the maximum global size for fixed global resistance and internal size, respectively.

When the constructal theory was used in some application cases, the analysis or optimization began with the smallest element, in which the geometrical parameters were theoretically deduced to obtain the minimum flow resistance. Then, a larger element composed of a number of the smallest elements was considered and optimized. By repeating the optimization step by step, the final constructal flow network could be obtained when the whole volume for flow was reached. From above, it can

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maximum flow access. J Appl Phys, 2006, 100: 114909.

e) Reis A H. Constructal theory: From engineering to physics, and how flow systems develop shape and structure. Appl Mech Rev, 2006, 59: 269-282.

[2] a) Bejan A, Lorente S. The constructal law and the thermodynamics of flow systems with configuration. Int J Heat Mass Transfer, 2004, 47: 3203-3214.

b) Lorente S, Bejan A. Svelteness, freedom to morph, and constructal multi-scale flow structures. Int J Thermal Sci, 2005, 44: 1123-1130.

be seen that the constructal flow networks were deterministic. With this consideration, the constructal theory was even compared with the second law of thermodynamics, and it was claimed that[2]

> "There is a time arrow in how flow configurations evolve, and it points toward better flowing configurations. This time arrow is distinct from the time arrow of the second law (i.e., an isolated system evolves toward equilibrium). The constructal law is the law of geometry generation, while the second law is the law of entropy generation…the constructal law has been invoked to predict the occurrence of a variety of natural flow features."

With the constructal method, many shapes and structures in nature and engineering have been analyzed or optimized[1,2]. At first, the pattern and formation of street networks were explained from the constructal viewpoint[1a]. Later, the constructal theory was applied to the heat transfer optimization[1], the optimization of flow structures[1], the disc-to-point flow problems[1,2], the design of fuel cells[3], the analyses of power systems[4], etc. In these application cases, it was concluded that the constructal theory could give the optimal structures which lead to the best system performances. Furthermore, the constructal method was also used to

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[3] a) Vargas J V C, Ordonez J C, Bejan A. Constructal flow structure for a PEM fuel cell. Int J Heat Mass Transfer, 2004, 47: 4177-4193.
b) Vargas J V C, Bejan A. Thermodynamic optimization of internal structure in a fuel cell. Int J Energy Res, 2004, 28: 319-339.
c) Vargas J V C, Ordonez J C, Bejan A. Constructal PEM fuel cell stack design. Int J Heat Mass Transfer, 2005, 48: 4410-4427.

[4] a) Kim Y S, Lorente S, Bejan A. Distribution of size in steam turbine power plants. Int J Energy Res, 2009, 33: 989-998.
b) Koonsrisuk A, Lorente S, Bejan A. Constructal solar chimney configuration. Int J Heat Mass Transfer, 2010, 53: 327-333.
c) Lorente S, Bejan A, Al-Hinai K, Sahin A Z, Yilbas B S. Constructal design of distributed energy systems: Solar power and water desalination. Int J Heat Mass Transfer, 2012, 55: 2213-2218.

explain the physical mechanisms for the formations of many natural shapes and structures[5]. For instance, the reasons why we have a bronchial tree with twenty three levels of bifurcation in the lungs were successfully explained with the constructal approach[6]. The constructal answer for the question, why swimmers must spread their fingers and toes, was also presented[7]. Moreover, the constructal explanations for the global circulation and climate[8], the structures of river basins[9], the droplet impact geometry[10], the structure of the nervous system[1b], the generation of the patterns of pedestrian crowds and stony corals[11] and the unification of biological and geophysical design[12] were all put forward. The constructal theory was

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[5] a) Bejan A. The constructal-law origin of the wheel, size, and skeleton in animal design. Am J Phys, 2010, 78: 692-699.

b) Charles J D, Bejan A. The evolution of speeds, size and shape in modern athletics. J Exp Biol, 2009, 212: 2419-2425.

c) Bejan A, Marden J H. Unifying constructal theory for scale effects in running, swimming and flying. J Exp Biol, 2006, 209: 238-248.

d) Bejan A, Lorente S. The constructal law and the evolution of design in nature. Phys Life Rev, 2011, 8: 209-240.

e) Bejan A, Errera M R. Complexity, organization, evolution, and constructal law. J Appl Phys, 2016, 119: 074901.

f) Bejan A, Lorente S. The constructal law of design and evolution in nature. Phil Trans R Soc B, 2010, 365: 1335-1347.

g) Bejan A, Lorente S. Constructal theory of generation of configuration in nature and engineering. J Appl Phys, 2006, 100: 041301.

[6] Reis A H, Miguel A F, Aydin M. Constructal theory of flow architecture of the lungs. Med Phys, 2004, 31: 1135-1140.

[7] Lorente S, Cetkin E, Bello-Ochende T, Meyer J P, Bejan A. The constructal-law physics of why swimmers must spread their fingers and toes. J Theor Biol, 2012, 308: 141-146.

[8] Reis A H, Bejan A. Constructal theory of global circulation and climate. Int J Heat Mass Transfer, 2006, 49: 1857-1875.

[9] Reis A H. Constructal view of scaling laws of river basins. Geomorphology, 2006, 78: 201-206.

[10] Bejan A, Gobin D. Constructal theory of droplet impact geometry. Int J Heat Mass Transfer, 2006, 49: 2412-2419.

[11] Miguel A F, Bejan A. The principle that generates dissimilar patterns inside aggregates of organisms. Physica A, 2009, 388: 727-731.

[12] Bejan A, Marden J H. The constructal unification of biological and geophysical design. Phys Life Rev, 2009, 6: 85-102.

even extended to analyze many social phenomena, including different S curve phenomena for spreading and collecting[13]. It was deemed that these natural shapes and structures and social phenomena could all be predicted by the constructal theory, and the constructal law could govern the evolutions in physics, biology, technology and society.

The relationships and differences between the constructal theory and some other theories have also been analyzed and discussed. For instance, when the relationship between the constructal theory and the entropy generation minimization was discussed, it was claimed that the constructal results were in accordance with those of the entropy generation minimization[1b]. Meanwhile, the difference between them is also very clear because the entropy generation minimization cannot directly give a structure, while the constructal theory can. Moreover, in the constructal applications, different equipartition principles have been obtained, including the equipartition of time principle in the constructal design of street networks[1a], the equipartition principle of temperature drop in the volume-to-point heat conduction optimization[1b], etc. Obviously, the constructal theory became more amazing with these beautiful equipartition principles. Furthermore, the constructal theory has also been compared with the fractal theory. The comparison showed that the generation direction of the constructal structures was opposite to that of the fractal theory, and it was written that

> "What is wrong with the fractal description-and by wrong I mean totally upside down-is the time arrow of the description…The oldest geometric feature is the smallest, and the youngest (most recent) feature is the largest-after all, this is the meaning of growth." [1a]

[13] Bejan A, Lorente S. Constructal law of design and evolution: Physics, biology, technology, and society. J Appl Phys, 2013, 113: 151301.

> "In fractal geometry, any tree network can be generated by repeating a suitably designed algorithm and interrupting it at a small and finite scale. Since the algorithm and the smallest scale (inner cutoff) have to be postulated, fractal geometry is descriptive, not predictive…It was shown that determinism vanishes if the direction is…from large to small. To emphasize the link between determinism and the direction from small to large, and as a reminder that theory (determinism) runs against fractal thinking, the geometric optimization approach…was named constructal theory."[14]

We can see that the fractal structures are not deterministic because they cannot be obtained through pure theoretical deductions. Therefore, it was claimed that the constructal theory was more reasonable for describing the evolutions of natural structures.

As above, it can be found that the constructal theory seems universally applicable for analyzing or designing all kinds of shapes and structures. If this were true, it must be great news for the whole scientific community. However, before accepting an omnipotent theory, we must carefully check every detail of it because everyone knows that there are definite application scopes for almost all scientific theories. For instance, in the analysis of the movements and interactions of some research objects, the particle physics and quantum mechanics may be necessary when the objects are small enough. When the objects reach the size of general molecules, there may be chemical interactions, and chemistry would be necessary. When the size of the objects gets bigger and the velocity of the objects is low, the Newtonian mechanics may be useful. If the velocity of the objects is close to that of light, we need Einstein's theory of relativity. When the size is big enough, astronomy may become the appropriate

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[14] Bejan A. Constructal tree network for fluid flow between a finite-size volume and one source or sink. Rev Gén Therm, 1997, 36: 592-604.

choice. If the objects are organism and the corresponding surroundings, biology and ecology may play important roles in the research. If people are our research objects, we may further need physiology and psychology. In addition, different mathematical theories and tools may be used in the cases above. Obviously, it is quite normal for one scientific theory to have limitations. Furthermore, with the three descriptions of the constructal theory, we can see that both the directions for the evolutions or optimizations of the constructal shapes and structures and the application preconditions of the constructal theory are definite. Accordingly, when the design objectives are inconsistent with constrcutal directions or the application preconditions are not satisfied, the applicability of the constructal theory should be carefully checked because the theory may not be suitable for application. Therefore, we certainly can doubt the constructal theory before making sure that it really works for all cases.

In fact, there are different viewpoints on the constructal theory. For instance, Ghodoossi[15] discussed three constructal applications and found that the flow performance may not be improved if the internal branching of the flow field is increased. Kuddusi and Egrican[16] obtained the same conclusion when they reviewed fourteen different applications involving constructal flow structures. Therefore, the generality of constructal theory is in question. Furthermore, for the evolutions of the shapes and structures in nature, we can accept that

> "…flow systems seek and find configurations that provide progressively greater access to their currents. Existing flow configurations are replaced by better flowing configurations, smoothly or stepwise, in animal design,

[15] Ghodoossi L. Conceptual study on constructal theory. Energy Convers Manage, 2004, 45: 1379-1395.

[16] Kuddusi L, Egrican N. A critical review of constructal theory. Energy Convers Manage, 2008, 49: 1283-1294.

river basin design, automobile design, and geopolitical design."[17]

However, we still have some questions: Is the generation direction of the constructal structures the same as the evolution direction of the natural structures? Is the flow resistance minimization what the nature only concerns in the evolutions of natural shapes and structures? Can the constructal theory really lead to the best configurations with the minimum flow resistances? Is there any other structure that results in better flow performance than that of the constructal structure? Before accepting that the constructal theory is universally applicable, these questions should be answered satisfactorily.

In this book, the questions above are all analyzed in detail. The main application fields of the constructal theory and the corresponding results are reviewed and discussed. With the consideration of the different viewpoints on the constructal theory, the details of the constructal optimization method are carefully checked with theoretical analyses and different application cases, and the corresponding problems are pointed out and discussed. As the constructal theory has already been related to the entropy generation minimization and the equipartition principles, an additional discussion is presented to show the relationships and differences between the theories. In addition, the abuses of the constructal theory are also directly pointed out. With our analyses and discussions, one can easily judge whether the constructal theory is omnipotent for the analysis, design or optimization of all kinds of shapes and structures or not.

[17] Bejan A. Science and technology as evolving flow architectures. Int J Energy Res, 2009, 33:112-125.

CHAPTER TWO

APPLICATIONS TO STREET DESIGN AND ECONOMICS

In this chapter, the application of the constructal theory to the design and optimization of street networks is discussed. Every step in the constructal design is analyzed in detail, and the corresponding problems are directly pointed out. When the constructal theory was extended to economics, it is shown that the main idea, the equations and the conclusions are all similar to those in the application to street networks. Therefore, the application to economics is also discussed in this chapter although the designs of street networks and economics are two different academic fields.

Part I Design of street networks

The constructal theory was proposed in the analyses of street networks, and it was declared that the theory could explain why quasi-similar street patterns exist, how they form, and how they grow in time[1a,18]. In the physical model for analyzing the street patterns, the function of street was considered to be the paths that connect a finite area to a single destination point, and the design or evolution of street patterns was claimed to be the solution of the fundamental access optimization problem, which can be

[18] Bejan A, Ledezma G A. Streets tree networks and urban growth: Optimal geometry for quickest access between a finite-size volume and one point. Physica A, 1998, 255: 211-217.

expressed as[1a]

> "Consider a finite-size geographical area A, and a point M situated inside A or on its boundary…Each member of the population living on A must travel between his point of residence P(x, y) and a point M. The latter serves as common destination for all the individuals who live on A…Determine the optimal 'bouquet' of paths that link the points P of area A with the common destination M, such that the time of travel required by the entire population is the shortest."

When solving the problem, Bejan[1a] introduced a definite direction: From the smaller subsystem of area A, to the larger subsystem, and ultimately to area A itself. In the smallest (innermost) area element, the first level street was obtained with the optimization objective of the minimum travel time for fixed traveling population and population density. As shown in Fig. 2-1, it was assumed that the area of the element, S_1, was fixed, the shapes of the element and the first level street were both rectangles, and the point O_1 was the only outlet of the population. With the consideration of the symmetry, it can be seen that the street should be located on the axis of symmetry of the area element. In the area element, the travel velocity of the population in the element, V_0, is low, while that in the street, V_1, is high. For the individual who must travel from P(x, y) to the point O_1, the fastest way was assumed to be: First, move vertically to the street, and then walk along the street after arriving at the street. With the assumptions above, the problem was highly simplified, and the average travel time of the population can be calculated by[1a]

$$t = \frac{1}{H_1 L_1}\int_{-\frac{H_1}{2}}^{\frac{H_1}{2}}\int_0^{L_1}\left(\frac{x}{V_1}+\frac{y}{V_0}\right)\mathrm{d}x\mathrm{d}y = \frac{L_1}{2V_1}+\frac{H_1}{4V_0}, \qquad (2\text{-}1)$$

where H_1 and L_1 are the width and length of the area element, respectively, and there should be $H_1 < L_1$. As the area of the element is fixed, Eq. (2-1) can be minimized and the optimal geometry of the area element can be obtained[1a],

$$H_{1\text{-opt}} = \left(\frac{2V_0}{V_1} S_1 \right)^{\frac{1}{2}}, \tag{2-2}$$

$$L_{1\text{-opt}} = \left(\frac{V_1}{2V_0} S_1 \right)^{\frac{1}{2}}, \tag{2-3}$$

where $H_{1\text{-opt}}$ and $L_{1\text{-opt}}$ are the optimal width and length of the element, respectively. Correspondingly, the first level street can also be obtained. Then, it was considered that some first area elements could make up a larger rectangular area, which was called the second area element. With similar optimization steps, the optimal geometry of the second area element and the corresponding second level street were also derived theoretically. In this way, the optimal street network could be obtained by repeating the former steps several times, each time for a larger area element, until the whole area was reached[1a].

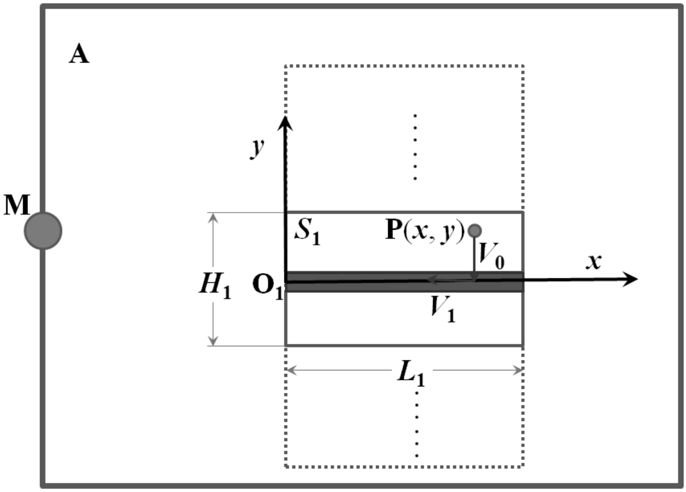

Fig. 2-1 Sketch of the optimization problem of street networks

In Ref. 18, V_0 was not assumed to be perpendicular to V_1, and one more degree of freedom, the angle between V_0 and V_1, was taken into consideration in the optimization design of the areas. In this case, it is very clear that the area elements may not fill the whole area perfectly. However, this point has been ignored, so that the accuracy and correctness of the minimum travel time resulting from the corresponding structures of the areas and streets should be carefully checked. In other words, the corresponding structures obtained with the steps above may not give the real minimum travel time.

With the method above, the structure of the street network can be deduced theoretically from the smallest element to the largest one. Hence, the method was named the constructal theory and thought to be[1a]

> "…a completely deterministic theory of optimized & organized systems that evolve in time...the purely deterministic step that had been ruled impossible by contemporary physics and mathematics…a purely theoretical and analytical alternative worthy of attention."

However, it seems that the scientists, researchers and engineers in the related fields have not realized the usefulness and importance of the constructal theory in the past two decades, so that there are still not many literatures on the application of the theory to the transportation planning and construction of roads in urban communities. Why? This question will be discussed below.

In the optimization above, it was assumed that there is only one outlet for every area element of every size level. Therefore, the travel routes of the individuals have to start from their points of residence to the first level street, then to higher level streets level by level, and finally reach the destination of the whole area, the point M. Furthermore, the movement of the population in the area, which is a two-dimensional problem, was simplified to be one-dimensional with the assumption of the travel route.

Obviously, the assumptions are unreasonable and inconsistent with the actual situations. In practical applications, it is very common that the communities in cities have more than one outlet, and the individuals at different locations may have different choices for leaving the area, so that the optimization problem should be two-dimensional. Furthermore, it is not necessary for the individuals to get to the streets level by level. If possible, they can move directly to high level streets, so that much time can be saved.

As shown in Fig. 2-2, let us analyze the travel routes of the individuals in the first area element, in which the unreasonable assumptions are dropped, the population can move directly to the second level street and there is more than one outlet for the individuals. First, let us assume that V_0 should be perpendicular to the streets. In this case, it can be seen in Fig. 2-2 that there are two travel routes for the individuals at the point P to leave the area. If the original travel route is chosen and the outlet of the second area element is in the $+y$ direction, the travel time for the individuals at the point P to arrive at the intersection point of two routes, the point Y, can be calculated by

$$t_{\mathrm{O}} = \frac{x}{V_1} + \frac{y}{V_0} + \frac{y}{V_2}, \tag{2-4}$$

where V_2 is the travel velocity of the individuals in the second level street. If the individuals move directly and vertically to the second level street, the travel time of the new route is

$$t_{\mathrm{N}} = \frac{x}{V_0}. \tag{2-5}$$

When $t_{\mathrm{O}} > t_{\mathrm{N}}$, there is

$$\frac{y}{x} > \frac{1 - V_0/V_1}{1 + V_0/V_2} \quad (2\text{-}6)$$

Obviously, when Eq. (2-6) is satisfied, the second travel route is a better choice. On the other hand, if the outlet of the second area element is in the –y direction, the times for the individuals to arrive at the intersection point of two routes, the point O_1, can be calculated by

$$t_{\mathrm{O}} = \frac{x}{V_1} + \frac{y}{V_0}, \quad (2\text{-}7)$$

$$t_{\mathrm{N}} = \frac{x}{V_0} + \frac{y}{V_2}. \quad (2\text{-}8)$$

When $t_{\mathrm{O}} > t_{\mathrm{N}}$, there is

$$\frac{y}{x} > \frac{1 - V_0/V_1}{1 - V_0/V_2}. \quad (2\text{-}9)$$

In this case, when Eq. (2-9) is satisfied, it can also be seen that the original travel route cannot give the minimum travel time, either, so that the individuals would not choose this route. Therefore, when Eq. (2-6) or Eq. (2-9) holds, it is very obvious that the calculation result should be larger than the actual travel time if the travel time is still calculated with the original travel route.

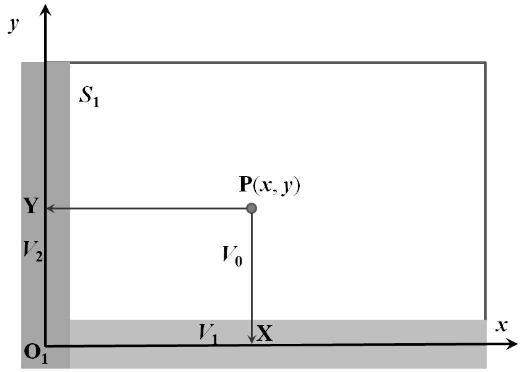

Fig. 2-2 Sketch of the first area element

Second, we further assume that V_0 can be not perpendicular to the streets. In this case, if the outlet of the second area element is in the +y direction and the individuals can move directly to the second level street, it is clear that the shortest time can also be expressed by Eq. (2-5). On the other hand, if the individuals have to arrive at the first level street first, the travel time for arriving at the point Y can be calculated by

$$t_O = \frac{y}{V_2} + \frac{x_1}{V_1} + \frac{\sqrt{y^2 + (x - x_1)^2}}{V_0}, x_1 \in [0, x) \quad (2\text{-}10)$$,

where x_1 is the abscissa value of the intersection point between the travel route and the x axis.

To find the minimum value of Eq. (2-10), we have

$$\frac{dt_O}{dx_1} = \frac{1}{V_1} + \frac{x_1 - x}{V_0\sqrt{y^2 + (x - x_1)^2}} = 0 \quad (2\text{-}11)$$,

and

$$x_1 = x - \frac{V_0/V_1}{\sqrt{1-(V_0/V_1)^2}}\,y \quad (2\text{-}12)$$

.

Therefore, we can obtain that

$$t_{\text{O-min}} = \frac{y}{V_2} + \frac{x}{V_1} + \frac{\sqrt{1-(V_0/V_1)^2}}{V_0}\,y \quad (2\text{-}13)$$

,

where $t_{\text{O-min}}$ is the corresponding minimum time. When $t_{\text{O-min}}$ is larger than the time expressed by Eq. (2-5), there should be

$$\frac{y}{x} > \frac{1/V_0 - 1/V_1}{1/V_2 + \sqrt{1-(V_0/V_1)^2}\,/V_0} \quad (2\text{-}14)$$

.

Obviously, when Eq. (2-14) is satisfied, moving directly to the second level street is a better choice for the population to leave the area, and some travel time can be saved. On the other hand, if the outlet of the second area element is in the $-y$ direction, the travel times for arriving at the point O_1 through different routes can be calculated by

$$t_{\text{N}} = \frac{y_1}{V_2} + \frac{\sqrt{x^2+(y-y_1)^2}}{V_0}, y_1 \in [0, y) \quad (2\text{-}15)$$

,

$$t_{\text{O}} = \frac{x_1}{V_1} + \frac{\sqrt{y^2+(x-x_1)^2}}{V_0}, x_1 \in [0, x) \quad (2\text{-}16)$$

,

where y_1 is the ordinate value of the intersection point between the travel route and the y axis. To find the minimum value of Eq. (2-15), we have

$$\frac{\mathrm{d}t_{\text{N}}}{\mathrm{d}y_1} = \frac{1}{V_2} + \frac{y_1 - y}{V_0\sqrt{x^2+(y-y_1)^2}} = 0 \quad (2\text{-}17)$$

,

and

$$y_1 = y - \frac{V_0/V_2}{\sqrt{1-\left(V_0/V_2\right)^2}}x \quad (2\text{-}18)$$

Therefore, we can obtain that

$$t_{\text{N-min}} = \frac{y}{V_2} + \frac{\sqrt{1-\left(V_0/V_2\right)^2}}{V_0}x \quad (2\text{-}19)$$

where $t_{\text{N-min}}$ is the corresponding minimum travel time. For Eq. (2-16), it can be seen that Eq. (2-12) can also give the minimum value and

$$t_{\text{O-min}} = \frac{x}{V_1} + \frac{\sqrt{1-\left(V_0/V_1\right)^2}}{V_0}y \quad (2\text{-}20)$$

When $t_{\text{O-min}} > t_{\text{N-min}}$, there should be

$$x\left(\sqrt{\frac{1}{V_0^2}-\frac{1}{V_2^2}}-\frac{1}{V_1}\right) < y\left(\sqrt{\frac{1}{V_0^2}-\frac{1}{V_1^2}}-\frac{1}{V_2}\right) \quad (2\text{-}21)$$

When Eq. (2-21) is satisfied, it is obvious that moving to the first level street first is never a good choice because more travel time will be taken when this route is chosen.

As above, it can be seen that the travel time calculated by the original model in the constructal design of street network is not the minimum when we drop the unreasonable assumptions in the first element area. Correspondingly, the constructal optimization result may not be really optimal. In our analyses, the optimization problem is treated as a real two-dimensional problem because not all the individuals have to move to the first level street and they may have different choices for leaving the area. In the area element, there is more than one outlet, which is also reasonable because this kind of communities is very easy to find in cities. If we consider larger area elements and assume that the individuals can move directly to higher level streets, it is obvious that more travel time can be saved. Especially, it is never necessary for the population that is close to the point M to move to the streets level by level. In fact, if possible, they may move directly to the point M. With the discussion above, we can see that the constructal theory cannot give the minimum travel time. When the constructal approach is used, it is very sure that some travel time of the population will be wasted for leaving the area.

Furthermore, it can be seen that the street networks obtained by the constructal theory are tree-like structures[1a,18], which are loopless patterns, and the links do not cross[18]. Unfortunately, the tree-like structures are not often used in real street design, so that such street structures are very difficult to find in any city in the world. In fact, the practical structures of streets in cities are mainly reticulate, and loop patterns are very common in the design of streets in almost all cities. Accordingly, we can conclude that the constructal street networks are not what we really need although the results are deterministic and the geometrical features of the streets can be deduced theoretically. Why? This question can be simply explained below.

From above, it can be seen that the constructal street design only considers one function of streets, which is the network that connects a finite area to a single destination point. Obviously, this is not appropriate because the streets have more functions in actual life. For instance, in commercial areas, the streets themselves can be the destinations because there may be shops on both sides of the streets. Therefore, the streets may satisfy the shopping needs of the citizens. In a city, there are many destination points or areas, and the streets can also be a connection between them. For instance, the schools, hospitals, factories and other buildings can also be the destinations, which should be all connected by streets. Hence, even if we only consider the function of transportation, it is still not suitable to consider only one function in the constructal design. In a city, as the individuals at any location need to reach any other location through the streets efficiently, the design objectives should include the efficient transportation among major living areas, business areas, work areas, etc. When the ground traffic cannot meet the need of transportation, we even have to consider the spatial transportation and underground traffic. Obviously, these design problems cannot all be attributed to the optimization of the connection between an area and one point. Therefore, the constructal explanation for the evolution of street network is never satisfactory. If we consider more functions of streets and the corresponding conditions and constraints, the street design can be more complex, so that the constructal theory may be even more unsuitable for the design. This is the reason why the constructal theory has rarely been applied to the real street design.

Part II Constructal economics

The constructal theory was also applied to economics[19,20]. In the application, the optimization problem was described as[19,20]

> "Consider a stream of goods that proceeds from one point (the producer or distributor) to every point of a finite-size territory (containing the consumers). The objective is to minimize the total cost associated with the given point-to-area or area-to-point stream…considering the area A in which goods must arrive at (or depart from) every point at the uniform rate...The total stream of goods flows between A and a single point M (producer, or collection point)…Several means of transport are available, and they are represented by the sequence of costs K_i…such that $K_0 > K_1 > K_2$…Each K_i represents the cost per unit of goods transported and per unit of distance travelled. The problem consists of covering A with trajectories of various unit costs such that the total cost…is minimum. One way to approach this problem is by allocating an area element to each K_i link, and

[19] Bejan A, Badescu V, de Vos A. Constructal theory of economics. Appl Energy, 2000, 67: 37-60.

[20] Bejan A, Badescu V, de Vos A. Constructal theory of economics structure generation in space and time. Energy Convers Manage, 2000, 41: 1429-1451.

connecting the links such that the entire area A is connected to M."

It can be seen that this optimization problem is very similar to the constructal optimization of street networks. In this problem, K_i plays the same role as the inverse of the travel speed V_i in the street design, and the cost is also proportional to the distance traveled[19,20]. It was assumed that the stream of goods flows with the highest cost K_0 first, and then with lower costs level by level. Moreover, in the optimization, it was assumed that the smaller areas assembled into a larger area communicate with each other only through the root points, while the rest of their rectangular perimeters are impermeable[19,20]. In other words, there was also only one outlet of goods for each area element of every size level. Therefore, the optimization steps of the present problem and the corresponding mathematical expressions of the optimal results are also very similar to those of the constructal optimization of street networks. For instance, when the first area element shown in Fig. 2-1 was optimized, there is[19,20]

$$\begin{aligned} C_1 &= \gamma \int_{-\frac{H_1}{2}}^{\frac{H_1}{2}} \int_0^{L_1} \left(K_1 x + K_0 y \right) \mathrm{d}x\mathrm{d}y \\ &= 2\gamma \int_0^{\frac{H_1}{2}} \int_0^{L_1} \left(K_1 x + K_0 y \right) \mathrm{d}x\mathrm{d}y \\ &= \gamma \left(\frac{1}{4} K_0 H_1^2 L_1 + \frac{1}{2} K_1 H_1 L_1^2 \right) \end{aligned} \tag{2-22}$$,

where C_1 is the total cost of the area, and γ is the uniform rate of the goods per unit area. As the elemental area and its total stream, γS_1, are fixed, there is

$$\frac{C_1}{\gamma S_1} = \frac{1}{4} K_0 H_1 + \frac{1}{2} K_1 L_1 \tag{2-23}$$.

Therefore, the optimal geometry of the area element can also be obtained[19,20],