# Finite Element Analysis of Prestressed Concrete Structures Using Post-Tensioning Steel

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Yu Huang and Thomas Kang

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It is also noted that the unit primarily used in this book is the customary US unit, with conversion to the SI unit available in some of the figures.

# ABSTRACT

This research discusses and investigates possible approaches for modeling post-tensioned (PT) prestressed concrete structures via the finite element method (FEM). The challenge of modeling PT prestressed concrete structures lies in the treatment of the interface between the concrete and prestressing tendons. The generic modeling techniques that are discussed were based on general purpose finite element packages. Two strategies for modeling the interface are presented in detail. For the first method, a series of linear spring elements was introduced to approximate the sliding behavior of prestressing tendons at the interface. For the second method, the interface was modeled directly through contact formulation. Additionally, the corresponding material constitutive relations, element preference and solution algorithm are discussed in depth. The generic modeling schemes were validated against experiments and proved to be robust and reliable for modeling PT structures. However, slightly overestimated tendon stresses were observed at the ultimate state of structures in many numerical simulations. A preliminary scientific analysis yields the conclusion that the phenomenon is likely caused by the frictionless assumption which neglects the frictional-induced prestress loss in the modeling.

Besides the issue of overestimated prestressing tendon stress, it is difficult to apply the generic modeling schemes to simulate large-scale PT structures such as a PT frame system. The involvement of the solid element combined with the explicit dynamic algorithm becomes a large barrier to modeling large-scale structures due to the computational anxiety. The later part of the research zeros in on the development of an innovative nonlinear finite element formulation which incorporates contact techniques and engineering elements to considerably reduce the need of computational power. A nonlinear prototype program was developed to model PT prestressed concrete frames in two-dimensional space accordingly. The stress solution of prestressing tendons was also improved by considering frictional effects in the formulation. The proposed formulation was validated against analytic solutions and experimental data via several numerical studies. The prototype program was also demonstrated to be versatile and robust for analyzing PT prestressed concrete frames. Although the currently implemented material constitutive relations and

beam element in the prototype program limit its applications, advanced material models and beam elements could be implemented into the current formulation with trivial works in the future.

In addition to the study of modeling techniques, three practical engineering problems were investigated through the proposed FEM. The investigated problems include: (1) punching shear failure of two-way PT prestressed concrete slabs; (2) prestress increment in the prestressing tendon at the service stage of structural members; and (3) influence of the PT systems (i.e., bonded vs. unbonded) on structural performance of typical PT members. A series of numerical simulations based on general purpose finite element packages were conducted according to the documented experiments. The extensive analyses of the numerical and experimental data lead to the following conclusions: (1) the eccentric shear stress model proposed in ACI 318 predicts reasonable moment-shear interaction mechanisms for PT interior, edge, and, corner slab-column connections; (2) in the case of PT edge and corner connections, the punching shear provisions in ACI 318 are overly conservative in some cases, more research is suggested to quantify the prestressing effect on punching shear capacity and to relieve some of the provisions; and (3) the bonding condition of prestressing tendons has no effect on flexural strength of PT one-way slabs and beams, or the moment-shear interaction in two-way PT interior slab-column connections.

# CHAPTER ONE

# INTRODUCTION

Prestressed concrete is a structural concept that was first introduced for overcoming concrete's natural weakness in tension in the early 1940s. Prestressed concrete is essential in many applications today in order to fully utilize concrete compressive strength, and through proper design, to control cracking and deflection. Due to those benefits, the prestressed concrete industry has been experiencing breathtaking developments and a construction boom ever since the first prestressed concrete project, Walnut Lane Memorial Bridge, was built in Philadelphia, Pennsylvania in 1951. The efforts toward development made by many engineers over the past sixty years have made prestressed concrete an essential and powerful technique in the modern construction industry. Generally, prestressed concrete can be applied in one of three ways: as pre-tensioned prestressed concrete; as unbonded post-tensioned (PT) prestressed concrete and as bonded PT prestressed concrete. The first two have become the most popular types of prestressed concrete systems in North America.

Although design methods have been developed over the decades, an understanding of the ultimate mechanism in the prestressed concrete system is still greatly needed in many aspects. Such aspects include the intricate problems of punching shear failure of a prestressed two-way slab system, stress increase in the prestressing strand under service loads, and discrepancies of structural behaviors between bonded and unbonded post-tensioned systems. However, to perform extensive experimental tests on each subject is extremely expensive and time-consuming. The finite element method, on the other hand, was introduced into structural analysis in the late 1960s. The efforts and developments made by many pioneering researchers over the past five decades have enabled the finite element method to become a versatile and powerful approach in structural analysis. The principal goals of this study are establishing finite element models of prestressed concrete systems subject to particular problems and investigating those problems in accordance with current building codes.

Chapter 2 summarizes previous experimental tests regarding particular engineering problems. The problems considered include punching shear failure of two-way post-tensioned slabs, stress increase in prestressing tendons at the failure of the structural members and an investigation about the influence on the behaviors of prestressed concrete systems by different bonding (i.e., bonded PT vs. unbonded PT). Related literatures of finite element modeling with respect to prestressed concrete systems are provided.

Chapter 3 briefly presents the hypothesis and objectives of the book.

Chapter 4 provides descriptions of modeling prestressed concrete systems via existing general-purpose finite element packages. The modeling of PT tendon systems presents a great challenge in terms of finite element discretization due to the boundary nonlinearity. Two possible approaches to modeling PT tendon systems are discussed. The first and traditional approach is a so called ‘spring system method’ which literally utilizes a large number of rigid springs to approximate the mechanical behavior of an unbonded PT system. The second approach formulates the PT tendon systems as contact problems. The nonlinear boundaries are directly handled by the second approach which yields a better flexibility and accuracy of modeling a variety of PT structures. The spring method is found in many literatures whereas the contact formulation is rarely applied to finite element analysis in the context of PT structures. The reason behind this is that the generic contact problems are difficult to model and still remains a very active research area today. In addition, directly formulating nonlinear boundary problems via contact approach usually requires much more computational power than the traditional spring method.

Followed by the general modeling approaches discussed in Chapter 4, a large number of documented tests are numerically simulated to validate the proposed modeling approaches as well as to conduct further researches in Chapter 5. The numerically modeled specimens (19 slabs and 3 beams) are grouped to two parts where the first part of the simulations is used to analyze the punching shear failure of a prestressed two-way slab system. The second part is used to study discrepancies of structural behaviors between bonded and unbonded PT systems. The numerical results are well agreed with experimental data and are used to investigate the aforementioned practical engineering problems.

The drawback of using a general-purpose finite element package is that one has less modeling flexibility. For example, prestress loss is hard to model at several prestressing stages. Therefore, a two-dimensional nonlinear finite element formulation incorporating contact techniques is proposed in Chapter 6. The formulation can analyze inelastic behavior of PT beams. Several elements are developed in order to assemble the complete PT beam system. The proposed contact element deals with the interaction between concrete and prestressing tendons, the nonlinear beam element simulates conventional reinforced concrete (RC) beams; the nonlinear truss element assembles the prestressing tendon; the embedding element embeds prestressing tendons into concrete beams, and a special anchorage element is also proposed to accommodate the ability of simulating post-tensioning procedures. With all these elements assembled together, a complete PT beam system can be successfully modeled along with appropriate material constitutive models. The solution strategy employs a modified Newton-Raphson approach with a line search technique that improves the robustness of the method. Accordingly, all tangent stiffness matrices of aforementioned elements are analytically derived in accordance with the proposed nonlinear solver. Both frictionless and frictional contacts are formulated in simulating unbonded and bonded PT beams. Furthermore, in the proposed finite element framework it is easy to implement other types of elements such as elements with higher order beam theory (e.g., a Timoshenko beam element) to address different kinds of practical engineering problems. Enough accuracy can be obtained with a relatively coarse mesh under the current formulation to compare to the modeling approach with solid elements. This leads to a very cost-efficient solution to studying engineering problems and aid practical designs. All the proposed formulations are programmed and implemented by MATLAB language (MATLAB, 2010b). Several numerical studies are carried out to inspect the performance of the proposed model.

In Chapter 7, all materials are summarized together. Conclusions are given regarding finite element modeling techniques of PT structures as well as the practical engineering problems analyzed and evaluated by the proposed modeling scheme.

# CHAPTER TWO

# INVESTIGATION OF EXISTING STUDIES

## 2.1 Punching Shear Failure of PT Slab-Column Connections

The following review covers previous experiments conducted regarding PT slab-column connections or PT flat plates under quasi-static gravity and/or lateral loadings. Some of them were selected as the experimental basis of the following numerical analyses in Chapter 5.

Scordelis et al. (1959) tested a two-way unbonded PT flat plate system with four panels supported at nine points (eight points along edges and one point at the center). The slab was post-tensioned with twelve cables in each direction, uniformly distributed with a draped parabolic profile to balance desired moments. Each cable consisted of a single 1/4 in. high strength steel wire greased and placed in a plastic sheathing. Four tests were performed, one each under uniform prestress; under unequal prestress (1.8:1 for column to middle strip), skip loading (live load was only presented on one panel) and uniform loading to failure (live load was presented on four panels). A punching shear failure occurred at the center support after extensive flexural cracking. Theoretical calculations were also performed to predict moment and deflection within the elastic range by the beam theory and elastic plate method. The design method was evaluated by the experiment.

Brotchie and Beresford (1967) conducted an experimental test of an unbonded PT flat plate system. The overall slab dimensions were 26 ft by 42 ft in footprint and 3 in. in thickness, with supporting columns spaced on a 12 ft by 9 ft grid. Prestressing tendons were single 0.276 in. wires unbonded and draped to follow overall panel moment profiles and tensioned to balance these moments at the sustained loading. A 30-month long test was performed under sustained loading followed by a short term uniform loading test until flexural failure occurred. The slab system failed by a process of folding and yield rotation initialization where actual shear capacity was unknown.

An experimental study followed Scordelis' study (1959) and was performed by Odello and Mehta (1967). The test specimen was identical to the specimen in Scordelis' study except that five drop panels were added at the edge and center supports, respectively. The slab was loaded and unloaded at several levels and monotonically loaded to failure. Flexural failure was observed first followed by an ultimate failure of combined flexure and shear failure with a slightly higher load. The beam theory was investigated with this experiment and yielded a satisfactory elastic and ultimate analysis. Cracking and ultimate load carrying capacities of the slab were increased significantly with the presence of drop panels. These three experiments were related to large scale unbonded PT flat plate systems. However, conclusions on the punching shear capacity of slab-column connections under such prestressed concrete systems have not been reached.

Other researchers (Gamble, 1964; Burns and Hemakom, 1977; Burns and Hemakom, 1985; Kosut et al., 1985) conducted experiments on unbonded PT flat plate systems with attempts to investigate the punching shear strength. A two bay by three bay continuous unbonded PT flat plate was tested by Gamble (1964). The plate system consisted of two spans of 12 ft in the transverse direction and three 9 ft spans in the longitudinal direction along with twelve square column supports underneath the slab. The lightweight concrete slab was post-tensioned in both directions using 0.276 in. diameter high strength wire with a straight profile. The wires were spaced at 4 in. in the direction of the 12 ft span and 6 in. in the direction of the 9 ft span. Additional non-prestressed mild steel was also provided. Long term and short term tests were performed. The specimen was loaded uniformly until all panels failed. A very brittle shear failure occurred at one of the interior slab-column connections before flexural failure. The moment distribution was investigated and was in a reasonable agreement with the elastic moment before cracking. The flexural strength was predicted by using the yield line theory but was not able to be verified by the experiment. Punching shear strength was evaluated based on several available equations for the interior slab-column connection, whereas no investigation was made to the corner connections.

Burns and Hemakom (1977) tested a one-third scale unbonded PT flat plate with nine panels. The slab was three bay by three bay with 10 ft spans in each direction and was post-tensioned with 68 1/4 in. diameter seven-wire strands in each direction with a draped profile. The tendons were distributed 70% in the column strip and 30% in the middle strip. In addition, non-prestressed mild steel was provided at the column regions. The slab was loaded by a whiffletree system producing sixteen load points on each main panel and four load points on each span of overhang. A total of 15 tests were performed. The first test was about the instrument check. Nine of them were related to flexural tests and five of them were subjected to punching shear failure tests. The experimental observations indicated that the shear capacity was relatively consistent from column to column. The moment distribution observed from the experiments was compared with the elastic plate theory and equivalent frame method, indicating the design method performed well. In addition, the flexural capacity was accurately predicted by the yield line theory. Another half scale unbonded PT flat plate with nine panels was tested by Burns and Hemakom (1985). The slab had three spans of 10 ft in each direction and was post-tensioned with 23 1/4 in. diameter tendons in the north-south direction and 24 1/4 in. diameter tendons in the west-east direction. The tendons were uniformly distributed in the north-south direction and banded in the column strip in the west-east direction. All tendons had a draped profile. Additional non-prestressed mild steel was provided at the column regions. A similar loading procedure was employed in a total of 12 tests. The first test was performed to check instruments. Eight of the tests were flexural tests and three of the tests were punching shear tests. In all failure load tests, the slab failed by flexure followed by a punching shear failure with the same load. Burns and Hemakom (1985) concluded a large deflection and curvature at the negative moment yield line might have triggered the shear failure after the flexural failure.

Kosut et al. (1985) conducted a test of half scale unbonded PT flat plate with four panels. The test slab was 20 ft square with two spans of 10 ft in each direction and nominally 2.75 in. in thickness. The 0.25 in. diameter, 244 ksi strength wires were uniformly distributed in one direction and banded in the column strip in the other direction. All tendons had a draped profile. Auxiliary non-prestressed bonded reinforcement was provided at the connection. Several connections were further reinforced by shear stirrups. A similar loading system used by Burns and Hemakom (1977; 1985) was employed in this test. A total of 13 tests were performed of which eight were related to flexural tests and four were tested to assess the shear strength of individual slab-column connections. Test results showed use of vertical reinforcement at the exterior slab-column connection did not increase the shear strength.

Martinez-Cruzado (1993) investigated two 3/7 scale isolated unbonded PT edge slab-column connections and two corner connections. The slabs with the corner connection had an overall length of 7 ft 1-1/2 in. in each direction and 3-5/8 in. in thickness. The corner connection was in the south-west corner, whereas other corners of the slab were supported by pin connections simulating an inflection boundary in the prototype structure. The slab with the edge connection had an overall length of 13 ft 3 in. in the lateral loading direction and 6 ft 11-11/32 in. in the transverse direction, and a slab thickness of 3-5/8 in. Similar boundary conditions were used for the edge connection specimens. Five 3/8 in. diameter prestressing strands were banded in the north-south direction concentrating in the column strip, while another two and three were banded and uniformly distributed inside and outside the column strip along the west-east direction, respectively, in the slabs. A similar arrangement of tendon layouts was used for the slab with the edge connection except the banded tendons were grouped in the west-east direction. All prestressing strands were drape shaped. Supplemental bonded reinforcing bars were provided at the negative moment region around the column. Additional dead loads were applied to the slab before testing to achieve the desired gravity load in the column at the initialization of the test. A cloverleaf displacement loading pattern was applied to the column top with several cycles of different drift ratios for the purpose of simulating multi-directional seismic loading. One of the conclusions reached in the study was that the presence of high compressive stress in the slab-column connection region increases the shear strength of the connection. The increase of tensile stress was very small in the strand under constant gravity and increased lateral load.

Gardner and Kallage (1998) tested a two bay by two bay unbonded PT flat plate. The slab had two spans of 8 ft 11-7/8 in. in each direction and 3.54 in. in thickness. Three of the edge columns were circular and the others were square shaped. The 0.51 in. diameter, greased, seven-wire strands were uniformly distributed in one direction and banded in the column strip in the other direction. All tendons had a draped profile. No other supplementary bonded reinforcement was provided. The slab was uniformly loaded to failure in increments with forty-point loads. The punching shear failure occurred in a circular edge slab-column connection through a uniformly distributed load increase. After the initial failure, additional supports were placed at the failed edge connection to further load until another punching shear failure occurred at a rectangular interior slab-column connection. By shoring the interior connection, load was applied again until a rectangular corner slab-column connection failed by punching shear. Gardner and Kallage (1998) proposed a method to predict the punching shear strength and concluded that the presence of precompression in a slab might affect the shear strength of edge and corner slab-column connections.

Foutch et al. (1990) tested four isolated PT slabs with edge connections. Four slabs had the same dimensions of 80 in. in one direction, 60 in. in the other direction and 4 in. in thickness. The slabs were only supported by the edge connections located at the center of the exterior edge. Slab1 and Slab2 had banded tendons perpendicular to the exterior edge while tendons were uniformly distributed in the other direction. Tendons were banded parallel to the exterior edge in Specimens Slab3 and Slab4. All tendons were Grade 270, 3/8 in. diameter, seven wire strands. In addition, non-prestressed mild steel was used as the crack control reinforcement in the vicinity of the column and was also placed as the top and bottom edge reinforcement around the perimeter of the slab to prevent the splitting crack. The loading pattern of Slab1 and Slab2 were different from Slab3 and Slab4. The loading positions varied by the moment-to-shear ratio. All slabs were monotonically loaded to a failure. It was supported by the test results that the shear strength of a slab-column edge connection benefited from the precompression in the concrete.

Warnitchai et al. (2004) tested a 3/5 scale bonded PT slab with an interior slab-column connection. The square slab had a length of 18 ft 8-2/5 in. along each direction and 4.72 in. in thickness. The column was right located at the center of the slab with a section dimension of 9.84 in. x 19.69 in. and extended 35.43 in. above and below the slab mid-plane. The slab was supported by pin connections at two sides in the direction of lateral loading. The clear span between the pin connections of two sides was 16 ft 4-4/5 in. The slab was post-tensioned with eight bonded straight, Grade 270, 1/2 in. diameter, seven-wire strands. The tendons were banded in the lateral loading direction and uniformly distributed in the other direction. Additional non-prestressed reinforcement was provided at the slab top and bottom. The top slab bars were concentrated only at the connection region. The bottom steel mat was provided throughout the whole slab. Additional gravity loads were applied to the slab by sand bags in order to obtain the desired gravity-to-shear ratio. The lateral load was applied on the top of the column with several cycles of different drift ratios until the punching shear failure occurred. The brittle failure occurred at the drift ratio of 2%.

Most of the experimental programs reviewed above, however, provided insufficient data to assess the punching shear strength in the PT slab-column connections. The experiments carried out by Martinez-Cruzado (1993), Foutch et al. (1990), and Warnitchai et al. (2004) were isolated systems which had enough data from tests. Therefore, the FEM models were constructed for those specimens. The descriptions of the modeling are addressed in Chapter 4.

## 2.2 Comparative Study of Prestressing Tendon Bonding

The following reviews previous investigation of bonding influence of prestressing tendons. They were selected as the experimental basis of the following numerical analyses in Chapter 5.

Comparative studies regarding the prestressing tendon bonding influence on the structural behavior have been rarely reported in the past. Mattock et al. (1971) conducted an experimental study of seven simple supported single-span beams and three continuous two-span beams. All beams were categorized into three groups for the test as three T-beams (CB1, CU1 and CU2), continuous over two spans of 28 ft each; three simply supported T-beams (TB1, TU1 and TU2) of 28 ft each; and three simply supported rectangular beams (RB1, RU1 and RU2) of 28 ft span each. An additional unbonded PT T-beam was tested which was identical to the TU1 and TU2, except that a single 3/8 in. diameter non-prestressed seven wire strand was provided as additional bonded reinforcement. Each of the beams was post-tensioned by two, Grade 270, 1/2 in. diameter seven wire strands. In the first beam of each group, the tendons were bonded by grouting after the post-tensioning. The tendons were left unbonded in the other two beams of each group. The tendons were draped parabolically in all simply supported beams with an effective depth of 10 in. at the mid-span and zero eccentricity at the ends of the beams. In all continuous beams, the tendons were draped with an effective depth of 10 in. at both the mid-spans and zero eccentricity at the ends of the beams. In addition, different non-prestressed bonded mild steel was provided for each beam in accordance with ACI 318-63 (1963). Dead weight of all test specimens was increased by 100 percent since all beams were half-scaled. Each span was subjected to four equal point loads which were applied at points 1.5 ft and 5.5 ft away from the mid-span on each side. The point loads were increased monotonically until the failure of test beams occurred. For the simply supported beams, it has been realized that the ultimate strength of the unbonded beams might have been up to 30% greater than that of the bonded beams even though they were with the same design strength according to ACI 318-63. The test results from both simply supported and continuous beams also revealed that the crack spacings of the unbonded beams were equivalent to or larger better than those of the bonded ones which implied good serviceability characteristics. Mattock et al. (1971) also concluded that the unbonded beams with non-prestressed bonded reinforcement behaved more like a flexural member than a hinged tied arch.

Cooke et al. (1981) investigated twelve simply supported PT one-way slabs. Nine of them were prestressed with the unbonded tendons and the rest were prestressed with the bonded tendons. Of the nine unbonded slabs, three (Slab1, Slab2 and Slab3) had a length of 15 ft 9 in. with spans of 15 ft 1-1/8 in.; three (Slab4, Slab5 and Slab6) had a length of 11 ft 9-3/4 in. with spans of 11 ft 1-3/4 in.; and three (Slab7, Slab8 and Slab9) had a length of 7 ft 10-1/2 in. with spans of 7 ft 2-5/8 in. The three bonded slabs (Slab B4, Slab B5 and Slab B6) were identical to Slab4, Slab5 and Slab6, respectively, except the tendons were bonded to the concrete. The slabs in each unbonded group had a width of 1 ft 1-7/8 in., 2 ft 3-3/4 in. and 3 ft 10-1/2 in. All slabs were 7-1/16 in. in the thickness. The first two slabs in each group were prestressed with three straight tendons of 1/2 in. diameter. The last slab in each group was prestressed with three straight tendons of 5/16 in. diameter. All tendons were placed at an effective depth of 4-3/4 in. No additional bonded reinforcement was provided to any of the slabs. All slabs were subjected to a constant bending moment by two line loads which were applied at 4 ft 11-1/16 in., 3 ft 7-5/16 in. and 2 ft 3-9/16 in., respectively, from the supports in each group. All slabs were statically loaded in increments until the failures occurred.

All equations predicting tendon ultimate stress were evaluated to have under-predicted the ultimate tendon stress except the equation from ACI 318-77 (1977). This equation yielded a slightly higher value compared to measured results only for a low prestressing steel index, $q_e$ ( $q_e = \rho_p f_{pe}/f'_c$). All the equations examined for flexural strength produced conservative values except those equations from ACI 318-77 and CP110 which yielded slightly higher values for a low $q_e$. It was recommended that non-prestressed bonded reinforcement should have been provided when $q_e$ is less than about 0.11, otherwise the flexural instability can result in smaller ductility at failure. The difference of behaviors between the bonded and unbonded slabs with similar reinforcement, and material and loading configurations were found to be very small except the slabs with a very low $q_e$. Cooke et al. (1981) also observed that the crack spacings of Slab4 were larger than those of its bonded counterpart, SlabB4. A conclusion was drawn that Slab6 and SlabB6 with a very low $q_e$ behaved more like a hinged tied arch than a flexural member. Some of the specimens in these two studies were modeled by the finite element method (FEM) for further study. The descriptions of modeling are presented in Chapter 4.

## 2.3 Finite Element Formulation of Prestressed Concrete Members

Finite element applications of prestressed concrete (PC) have been actively researched for decades since the pioneering work was done by Ngo and Scordelis (Ngo and Scordelis, 1967). The key to simulating the different types of prestressed concrete systems (pre-tension, bonded and unbonded PT) lies in the modeling of the bonding condition between the concrete and tendons. Tendons in a pre-tensioned prestressed concrete member can be idealized as perfect bonding which implies the strain compatibility between the tendons and surrounding concrete. On the other hand, either a bonded or an unbonded PT member requires an unbonded formulation in the jacking stage. Studies regarding the formulations of the bonded, partially and fully unbonded tendons are various and rich in the literature. The following review discusses the previously developed modeling scheme of interactions between prestressing tendons and concrete. The most popular treatment was to utilize link (rigid spring) elements between tendons and correspond sheathing or concrete. The other method was to employ empiric equations for determining the unbonded tendon strain. However, research on contact techniques to directly model the interface between prestressing tendons and corresponding sheathing has not been conducted to the author’s knowledge.

Kang and Scordelis (1980) proposed a nonlinear finite element procedure analyzing prestressed concrete frames with bonded or unbonded tendons. This procedure considers the material and geometric nonlinearities as well as load history, temperature history, creep, shrinkage, and aging of the concrete, and relaxation of the prestress. The complete uniaxial stress-strain relations for the concrete, non-prestressed steel reinforcement and prestressing steel were modeled. The displacement field of the frame element is linear along its local axis and cubic perpendicular to the axis. Reinforcement steel was modeled by a separate layer with the assumption of perfect bonding. The curved prestressing tendons were discretized into several linear segments. The bonded prestress tendons were treated in the same way as non-prestressed bonded reinforcement steel with perfect bonding of the concrete, whereas the strain of the unbonded prestressing tendons was determined by the deformed geometry of the tendons. The prestress loss due to friction and anchorage slip of the PT members at a transfer stage were calculated along with empiric equations and incorporated into the procedure. For the pre-tensioned or PT bonded members, the stiffness of the prestressing tendon was included in the stage of perfect bonding in the analysis, while the stiffness of the prestressing tendon was neglected for the PT unbonded members. The equilibrium equations were formulated as incremental forms of the load. An iterative procedure was applied to solve each load increment. The time domain was divided into a discrete number of time steps to perform a time dependent nonlinear analysis. The aforementioned iterative procedure was applied to each time step to converge the unbalanced load to a prescribed tolerance. This nonlinear finite element procedure was applied to several numerical examples including a bonded PT beam tested by Lin (1955), a bonded and unbonded PT I-beam tested by Breckenridge and Bugg (1964) as well as a pre-tensioned concrete column tested by Aroni (1968). All simulations resulted in good agreement with the experiments.

The finite element procedure proposed by Van Greunen and Scordelis (1983) was able to analyze both the pre-tensioned and PT bonded and unbonded slabs. The procedure was incorporated into a computer program, NOPARC (Van Greunen, 1979) which can analyze the prestressed slabs with the consideration of time dependent effects, material and geometric nonlinearities. A flat triangular shell finite element was used for concrete. The biaxial stress-strain law for concrete was an orthotropic formulation proposed by Darwin and Pecknold (1976; 1977) and adopted in the study. A uniaxial stress-strain relation was used for the non-prestressed reinforcement steel as well as prestressing tendons. The stiffness of the bonded reinforcement steel and tendon was included in the formulation where the deformation field of the steel is the same as that of the concrete. For the unbonded tendons, a modified method utilizing an average extension factor was used to determine the stress-strain state of the unbonded tendons. A step-by-step integration scheme in the time domain was employed to analyze the effects of the time-dependent phenomena such as creep, shrinkage in the concrete, stress relaxation of the prestressing tendons, and temperature changes on the behavior of reinforced and prestressed concrete structures. Two numerical analyses were carried out in the study including a pre-tensioned prestressed concrete column tested by Aroni (1968) and a continuous two-way unbonded PT slab tested by Scordelis et al. (1959). The analysis maintained a good agreement with the experiment for the pre-tensioned column, while the modeling scheme was partially consistent with the slab test data in the inelastic range. Van Greunen and Scordelis (1983) explained that the discrepancies found at the failure stage were caused by the coarse mesh and difficulty of modeling the actual tendon forces at major interior cracks.

El-Mezaini and Citipitioglu (1991) presented a discrete formulation for reinforcement simulating the bonded, fully unbonded and partially unbonded behaviors of the tendons. The key feature of the formulation