Methods and Tools for Simulation and Quality Control of Design and Production of Microwave Devices

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Edited by

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This book is based on the results of research papers, doctoral theses, preprints, and books by the authors Sergey Viktorovich Savel'kaev and Sergey Borisovich Danilevich.

The first part of the book, written by S. V. Savel'kaev and consisting of five chapters, describes principles for designing a simulator/analyzer. This measuring system provides simulation modeling of microwave amplifiers and oscillators and subsequent accurate and adequate measurements of the *S*-parameters of their active components, including the complex load reflection coefficient of these devices under actual operating conditions, to check that the simulated microwave device meets the specifications for its production.

Accurate and adequate measurements of the S-parameters of the active component and the complex reflection coefficients of its loads provide a significant increase in the cost efficiency of the computer-aided design and, hence, production of amplifiers and oscillators. This is achieved by eliminating the need for multiple technological tests of prototype microwave devices, resulting in a reduction in the amount of development work needed.

The proposed design solutions and calibration methods of the simulator/analyzer provide transmission of measurement results from its coaxial measuring line to a microstrip line (normalization with respect to the microstrip line).

The importance of the first part of this book lies in the fact that simulators/analyzers are widely used in the computer-aided design of microwave amplifiers and oscillators with controlled quality of design.

The second part of the book, from the sixth to the ninth chapters, is written by S. B. Danilevich and deals with an important component of quality management: the planning of engineering measurements and measuring tests (both continuous and sampling) of product quality. It addresses the development of effective methods for controlling the quality of complex technical products (including microwave amplifiers and oscillators). The use of effective control methods provides results with specified confidence levels (probabilities of errors of the 1st and 2nd kind) at the lowest cost. This is consistent with the concept of the risk-based ISO 9001:2015 international standard.

The simulation method and control simulation algorithms proposed in the second part of the book can be used to analyze existing and develop new effective measuring test procedures that minimize the costs associated with product inspection.

Generally, the book may be useful for developers of measuring devices and systems and quality experts engaged in the design and production of technical products. In addition, it may be useful for students and postgraduates.

CONTENTS

Part II: Effective methods of quality control of technical products

List of Abbreviations

AC – active component;

AD – adapter;

BPF – bandpass filter;

BSF – bandstop filter;

CA – coaxial adapter;

CAD – computer-aided design;

CMT – measuremrnt techniques certified;

CP – control procedures;

CRC – complex reflection coefficient;

CTC – complex transmission coefficient;

CTF – coaxial test fixture;

CVC – current-voltage characteristic;

DB – directional bridge;

DC – direct current;

DR – dielectric resonator;

EC – equivalent circuit;

FB – feedback;

FRF – frequency response functions;

GSI – State System for Ensuring Uniform Measurements;

LPF – low-pass filter;

MCP – multivariate control procedures;

MDS – multilayer dielectric structure;

MN – matching network;

MSL – microstrip line;

MT – measurement technique;

MWO – Microwave Office software;

NA – network analyzer;

PC – personal computer;

PSA – reflective phase shifter-attenuator;

PSCU – power supply and control unit;

RC – detector response curve;

RS – requirement specification;

S – switch;

SA – simulator/analyzer;

SBFET – Schottky-barrier field-effect transistor;

SD – standard deviation;

SE – systematic error;

SM – simulation modeling;

SS – sounding and reference signal synthesizer;

SWR – standing-wave ratio;

TBNS –DR material;

TE – test equipment;

TMT – tunable matching transformer;

ИМ – имитационное моделирование;

КИП – контрольно-измерительные приборы;

МК – методика контроля;

ММК – методика многопараметрового контроля;

МО – математическое ожидание;

РФ – Российская Федерация;

СИ – средство измерения;

СКО – среднее квадратичное отклонение;

ССФ – стационарная случайная функция.

INTRODUCTION

At present, the calculation and design of microwave microstrip amplifiers and oscillators [1–3] widely used in terrestrial and satellite broadcasting and communication, radar, and radio navigation systems are carried out in the space of calculated [1] or measured [4–13] *S*-parameters of the active components (ACs) of these devices, e.g., a strip transistor. The high accuracy and adequacy of measurement of the *S*-parameters of such devices, compared to their approximate calculation, provide a factor of two to three reduction in the period of development of these devices by eliminating the need for multiple technological tests of their prototypes and, thus, provide an increase in the cost efficiency of the computer-aided design (CAD) and production of amplifiers and oscillators.

Accurate and adequate measurement of the *S*-parameters of ACs designed for operation in microstrip lines requires methods and precision instruments that extend the regulations of the State System for Ensuring Uniform Measurements (GSI) to microstrip lines.

The problem of adequately measuring the *S*-parameters of an AC (which is generally an active nonlinear device) is due to the fact that the measured *S*-parameters depend on the operating characteristics of the AC, such as:

- the complex reflection coefficients (CRCs) $\Gamma_{\mathrm{ld}j}$ of the loads (load CRCs $\Gamma_{\mathrm{ld}j}$) and supply voltage $U_{\mathrm{s}i}$ of the AC, which determine its amplification or oscillation modes;
- the input power P_{in} of the AC in the amplification mode;

- the discrete frequencies f within the bandwidth Δf in the amplification mode and the oscillation frequency f in the oscillation mode.

The set of possible values of the above operating characteristics of an AC corresponds to the set of values of its measured *S*-parameters in the amplification mode (for amplifiers) or oscillation (for oscillators). As a result, the *S*-parameters of the AC measured, e.g., in the matched ($\Gamma_{\mathrm{ld}j} = 0$) coaxial measuring line of a microwave network analyzer (NA) [4, 5], in which the AC operation mode is different from that in a real microstrip amplifier or oscillator, are different from the *S*-parameters of this AC connected to the mismatched microstrip line of such devices [1–3].

To solve the problem of adequately measuring the *S*-parameters of an AC, it should be connected to the mismatched $\Gamma_{\mathrm{ld}j} \neq 0$ (coaxial) measuring line of a simulator/analyzer (SA) [8–15] in which this AC is characterized by the CRCs Γ_i at its input ($i = 1$) and output ($i = 2$), its complex transmission coefficients (CTCs) T_{ij}; $i, j = 1, 2 \quad i \neq j$ for the same inputs, and the load CRCs $\Gamma_{\mathrm{ld}j}$ set by means of tunable matching transformers (TMTs) of the SA [16].

The CRCs Γ_i and the CTCs T_{ij} should be measured for specified operating characteristics of this AC (including its measured load CRCs $\Gamma_{\mathrm{ld}j}$). The operating characteristics of the AC should be selected by simulation modeling of an amplifier or oscillator using a SA. The purpose of the simulation modeling is to determine and optimize the AC operating characteristics in such a manner that the performance characteristics of the amplifier or oscillator simulated by the SA, such as

- the band width Δf for the amplifier,
- the oscillation frequency f for the oscillator,
- the power gain K_{pg} and the noise figure K_{n} for the amplifier,
- the phase noise for the oscillator,
- and the required output power P_{out} for the amplifier or oscillator

meet the requirement specification (RS) for the design of these devices, whose AC (in contrast to that of frequency multipliers) operates in the linear region of its current-voltage characteristics (CVC), which excludes the possibility of its multimode operation.

The CRCs Γ_i and $\Gamma_{\mathrm{ld}j}$ and the CTCs T_{ij} of an AC measured in the mismatched $\Gamma_{\mathrm{ld}j} \neq 0$ coaxial measuring line of a SA [10–15] can be used to determine the measured $S = f(\Gamma_i, \Gamma_{\mathrm{ld}j}, \mathrm{T}_{ij})$-parameters of this AC, which it would have in a matched $\Gamma_{\mathrm{ld}j} = 0$ coaxial line.

To transfer the measured CRCs Γ_i and $\Gamma_{\mathrm{ld}j}$ and CTCs T_{ij} of the AC and the $S = f(\Gamma_i, \Gamma_{\mathrm{ld}j}, \mathrm{T}_{ij})$-parameters of the AC from the coaxial measuring line of the SA to the microstrip line to which this AC is to be connected for operation, it is necessary to calibrate the SA using microstrip calibration standards [6, 7, 10–12, 14, 15] (normalization of the measured AC parameters with respect to the microstrip line). In addition, a coaxial test fixture (CTF) [17] is needed to provide connection between strip ACs and coaxial standards, including strip and microstrip calibration standards, to the coaxial measuring line of the SA.

The problem of accurate measurement of the *S*-parameters of an AC is related to the need for amplitude and phase adjustment of the SA [10–12, 14, 15] to the measured CRCs Γ_i and $\Gamma_{\mathrm{ld}j}$ and CTCs T_{ij} of the AC in wide dynamic and frequency ranges using the methods proposed in [18].

In this book, we consider methods and precision instruments that provide an increase in the accuracy and adequacy of measuring the CRCs Γ_i and $\Gamma_{\mathrm{ld}j}$, CTCs T_{ij}, and $S = f(\Gamma_i, \Gamma_{\mathrm{ld}j}, \mathrm{T}_{ij})$-parameters of ACs.

1. A principle for designing a SA [10–12, 14, 15] for simulation modeling of amplifiers and oscillators according to the RS for their design is considered, as well as design solutions for its main functional units, e.g., a CTF [17] which provides connection between strip ACs and coaxial standards, including strip and microstrip calibration standards, to the coaxial measuring line of the SA.

2. A mathematical model of the SA that provides an adequate measurement of the CRCs Γ_i and $\Gamma_{\mathrm{ld}j}$ and CTCs T_{ij} of ACs in the mismatched $\Gamma_{\mathrm{ld}j} \neq 0$ (coaxial) measuring line of the SA for simulation of amplifiers and oscillators is developed, and a method for determining the measured $S = f(\Gamma_i, \Gamma_{\mathrm{ld}j}, \mathrm{T}_{ij})$-parameters of ACs [10–15] is considered.

3. A mathematical model for calibrating the SA is developed which provides normalization of the measured CRCs Γ_i and $\Gamma_{\mathrm{ld}j}$, CTCs T_{ij}, and $S = f(\Gamma_i, \Gamma_{\mathrm{ld}j}, \mathrm{T}_{ij})$-parameters of the As with respect to the microstrip line to which this device is to be connected for its operation.

4. A mathematical model of the TMTs of the SA and a mathematical model of their calibration [16] are developed. The mathematical model of the TMTs relates CAD initial approximations of the load CRCs $\Gamma^{**}_{\mathrm{ld}j}$ at discrete frequencies f (for simulation of amplifiers) or at the oscillation frequency (for simulation of oscillators) in narrowband matching and

CAD initial approximations of the inductance L_{kj}^{**} and capacitance C_{kj}^{**} of the $L_{kj}^{**} C_{kj}^{**}$-sections of the TMT represented by a low-pass filter (LPF) [1] in broadband matching to the required positions l_{kj} and insertion depth h_{kj} of the k-th stub of the TMT for determination of the load CRCs $\Gamma_{\mathrm{ld}j}$ of the AC required for simulation modeling of amplifiers and oscillators. This makes it possible to automate the TMT operation in simulations of amplifiers and oscillators using the SA.

5. Methods of phase and amplitude adjustment of the SA to the measured CRCs Γ_i and $\Gamma_{\mathrm{ld}j}$ and CTCs T_{ij} of the AC that provide an increase in the accuracy of measuring these parameters in wide dynamic and frequency ranges based on the results of [18] are considered.

6. The variational method developed in [19] for estimating the limiting total error in measuring the CRCs Γ_i and $\Gamma_{\mathrm{ld}j}$ and CTCs T_{ij} was used to determine the metrological characteristics of the SA based on the minimum error in measuring these parameters.

7. In addition, a method for stability analysis of an AC in the space of load CRCs $\Gamma_{\mathrm{ld}j}$ [9] which facilitates the selection of this AC in the simulation of amplifiers and oscillators is considered, and a library of mathematical models is presented for an amplifier [1], an unstabilized oscillator [2, 3] and an oscillator stabilized by a multi-layer dielectric structure (MDS) based on a dielectric resonator (DR), including a procedure for calculating the resonant frequency f_0 of the MDS [20]. The library provides automation of the simulation of amplifiers and oscillators using the SA in CAD design.

An important element of product quality management is testing and control (quality control of raw materials and components, process control, output quality control of manufactured products [48-51]).

In accordance with the legislation of the Russian Federation (RF) on technical regulation, technical regulations should contain “rules and forms of conformity assessment..., determined taking into account the degree of risk of harm.” The technical regulations should specify the requirements for the accuracy and reliability of metrological procedures such as measurement, inspection, and testing. The regulations should also define the requirements for the metrological support of production, since only in this case are effective measurements, inspection, and testing during conformity assessment work possible.

The Russian legislation obliges users to apply only certified measurement techniques (CMTs). This provides measurement results with known error (uncertainty) and obliges developers to certify the developed CMTs. However, the standards regulate only the general requirements for the development and certification of CMTs, but do not specify methods for analyzing and developing product quality control procedures (CPs). By CPs is further meant a set of control operations and rules that provide control results with specified reliability indicators. Such indicators are the probabilities of control errors of the 1st and 2nd kind, as well as the risks of the manufacturer and the customer.

An important problem in the metrological support of production is the development of methods for analyzing and certifying existing CPs and planning new effective CPs. Effective CPs should be used in both continuous control (inspection of each manufactured product) and sampling control in mass and large-scale production.

Of particular significance is the analysis and development of multivariate control procedures (MCPs) for complex technical products.

Analytical methods for planning effective MCPs for continuous or sampling control are complex and can be used to obtain specific numerical results only with significant limitations [51]. For example, the reliability indicators for tolerance control recommended in [52, 53] can be easily calculated only for one-parameter control. For multivariate control, typically used for complex products, these indicators are difficult to calculate [51].

It is therefore important to develop a new method for solving the following problems:

- assessing the reliability of control when using existing MCPs (assessing the validity and effectiveness of these procedures);
- developing new effective MCPs providing the required reliability of control at the lowest cost.

The method of analysis and synthesis of MCPs should implement a systematic approach, i.e., take into account virtually all factors affecting the quality of control. The method should be universal and allow the development of effective procedures for both continuous and sampling control.

The Monte Carlo simulation modeling (SM) method has been proposed as such a method. Usually, computer simulation of the control procedure (taking into account accepted models of measurement error) and subsequent processing of the results using a specified control algorithm [54–58] involve no difficulties.

An advantage of the SM method is that it allows one to simulate control procedures on a computer and evaluate the results without performing physical experiments (in the absence or in the presence of a small amount of test objects). This provides considerable opportunities for developers to analyze and synthesize MCPs.

Usually, available a priori data and the capabilities of the SM method make it possible to develop adequate mathematical models of test objects and measurement errors. The quality control procedure model proposed in the book allows the development of MCPs without large-scale experimental studies [55–58].

It is expedient to solve the problems of analysis and subsequent synthesis of MCPs in two steps.

In the first step, using some basic statistic model of the tested parameters of products, the quality of several variants of the MCP is evaluated and the most suitable variant (according to the criterion used) is selected. The basic model of parameters should allow analyzing the effectiveness of the MCP. Such a model can be developed on the basis of available a priori information on the possible values of tested parameters of products and the experience of developers.

In the second step (after the manufacture of a batch of products), it is advisable to update the basic statistical model of the tested parameters by taking into account new (experimental) information about the products. Then, the updated model of product parameters is used to assess the reliability of the quality control results obtained using the basic quality control procedure.

If the reliability of the quality control results is significantly higher than that required by the customer or regulatory documents, the quality control procedure can be adjusted to account for the actual quality of the products to be inspected. If the products have stable quality and statistical controllability, the MCP procedure can be simplified and the cost of control can be reduced [58–60].

The objective of the study was to develop a method for analyzing existing and designing new effective MCPs for both continuous and sampling quality control. To achieve this objective, the following tasks were performed:

- an algorithm for modeling continuous and sampling procedures and approaches to the development of stochastic models of measured parameters and measurement errors was developed;
- a SM method for the metrological analysis of existing and designing of new effective MCP was developed;
- an algorithm for studying the effect of more stringent (control) tolerances for tested parameters on control reliability indicators was proposed;
- the dependence of the control reliability indicators on the quality of the inspected products was investigated, showing that improving the quality of products leads to a significant reduction in the risk of the customer, but, at the same time, to an increase in the probability of control errors of the 2nd kind;
- a method for estimating the costs and losses associated with the organization and implementation of control was proposed, which allows optimizing the control procedure.

The developed method and algorithms for simulating quality control can be used to analyze existing and design new effective MCPs.

PART I

THEORETICAL BASIS FOR DESIGNING ANALYZERS/SIMULATORS FOR MICROWAVE DEVICES

SERGEY V. SAVEL'KAEV

1

MICROWAVE NETWORK ANALYZERS

1.1 Principle of operation and advantages

Significant advances in the automated high-precision measurement of the *S*-parameters of microwave networks, compared to analog or vector prototypes [21], have been achieved by measuring the powers P_k ; $k = \overline{1,3}$ at the information ports of the measuring $2n$-ports of NAs [22] with subsequent determination of the CRCs Γ, as is illustrated in Fig.1-1, *a* and *b*.

Subsequently, such NAs served as prototypes for more advanced NAs with tunable $2n$-ports [23], whose operating principle is based on measuring the powers P_k ; $k = \overline{1,3}$ at the output ports of their $2n$-port in its k states, and for two-signal NAs [18, 24], whose operating principle is based on measuring these powers P_k at the k-th discrete phase shifts between the probing a and reference a^0 signals.

According to Fig. 1-1, *a* and *b*, the limiting error Δ_Γ^1 in measuring the CRC Γ depends both on the dynamic range of the standing wave $\Delta = 10\lg(P_{\max} / P_{\min})$ with minimum and maximum values $\Delta_{\text{min, max}}$ and on the discrete increment $\theta_{2,3}$ of the phase shift φ_k ; $k = \overline{1,\ 3}$ of its measured powers P_k [18]. If the bilateral amplitude limitation $\Delta_{\min} \le \Delta \le \Delta_{\max}$ and the phase condition $\theta_{2,3} \cong \theta_0$ are satisfied, the limiting measurement error Δ_Γ^1 does not exceed its tolerance limit $\Delta_\Gamma^1 < [\Delta_\Gamma]$, as shown in Fig. 1-1, *b*, where $\Delta_{\text{min, max}}$ and θ_0 are the minimum and maximum allowable values of the dynamic range Δ and the optimal phase shift of the standing wave for which the minimum limiting error Δ^1_Γ of the CRC Γ has a minimum $\Delta^1_\Gamma = \min\min \Delta^1_\Gamma$.

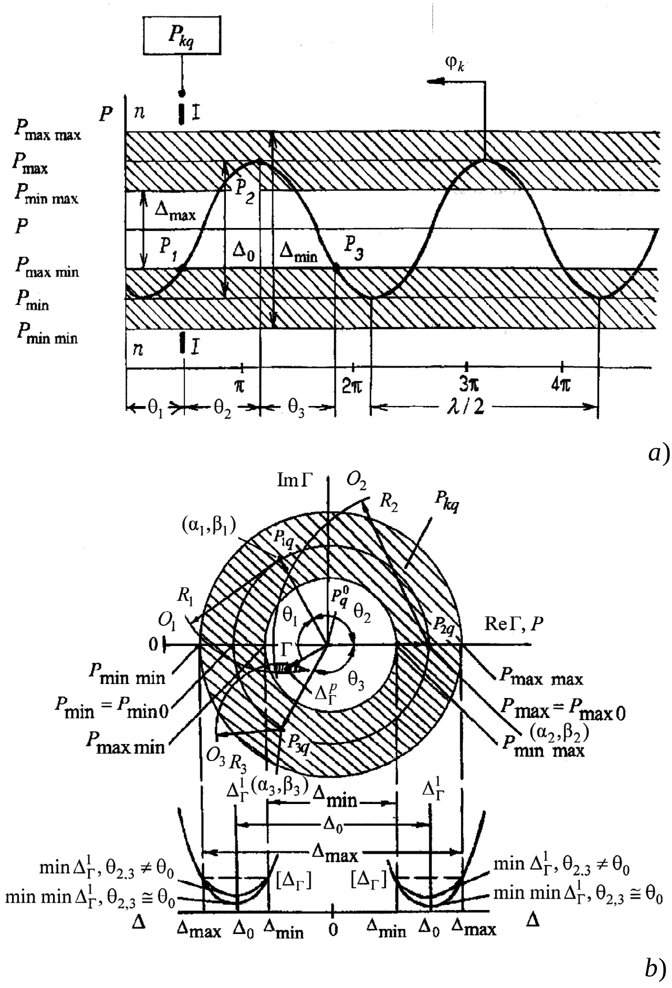

Fig. 1-1. Standing wave–*a*) and a graphical determination of the measured CRC Γ –*b*)

Thus, we can introduce the concept of amplitude adjustment for a NA as its property to maintain the bilateral amplitude limitation $\Delta_{\min} \le \Delta \le \Delta_{\max}$ in a wide dynamic range, i.e., for any measured CRC Γ with a magnitude $0 < |\Gamma| \le 1$ and the concept of phase adjustment for a NA as its property to maintain the phase condition $\theta_{2,3} \cong \theta_0$ in a wide frequency range.

A disadvantage of classical one-signal NAs [22] which have a source of one probing signal a is that they can maintain only their amplitude adjustment in the form of the unilateral amplitude limitation $0 \le \Delta \le \Delta_{\max}$, under which the limiting measurement error Δ_Γ^1 of a CRC Γ with a magnitude $|\Gamma| < 0.3$, after reaching its tolerance limit $[\Delta_\Gamma]$, increases without bound $\Delta_\Gamma^1 >> [\Delta_\Gamma]$. In addition, the ability of such NAs to maintain the phase condition $\theta_{2,3} \cong \theta_0$ in a wide frequency range is determined by the frequency properties of their $2n$-port, necessitating its replacement in operation in a wide frequency range.

To improve the adjustability properties of one-signal NAs, an attempt has been made to replace their $2n$-port by a tunable one [23]. However, the further development of such NAs has been hindered by difficulties in the technical implementation of their tunable $2n$-port.

In this book, as the base NA we consider a two-signal NA [18, 24] which has sources of two signals: a probing signal a and a reference signal a_{kq}^0 shifted in phase by φ_k^0. The design of its $2n$-port is much simpler than that of the well-known one-signal NAs with $n \ge 4$ information terminals [22] and NAs with a tunable $2n$-port [23].

Using a mathematical model of a two-signal NA [18, 24], we will discuss methods for its amplitude and phase adjustment in the form of the bilateral amplitude limitation $\Delta_{\min} \le \Delta \le \Delta_{\max}$ and the phase condition $\theta_{2,3} \cong \theta_0$ and a method for its calibration that provides an improvement in the measurement accuracy of reflection parameters in wide dynamic and frequency ranges with a possibility of correcting measurement errors in real time based on the results of multiple measurements.

The high accuracy provided by two-signal NAs in measuring the CRCs Γ, combined with their ease of calibration, was the main reason for choosing such NAs as a prototype for designing SA with description in the space of *D*-matrices [25].

1.2 Generalized equation of physical conversion in the space of *D*-matrices

Currently, there is a great variety of mathematical models of $2n$-port NAs in the space of *S*-parameters [26]. A disadvantage of such mathematical models is that in the space of *S*-parameters, the number of independent complex parameters of these models is redundant and equal to n^2+n-1, where n^2 is the size of the matrix of *S*-parameters of a $2n$-port network with n ports; $n-1$ is the total number of CRCs $\Gamma_i;\, i=\overline{1,\ n-1}$ of the loads at its l input ports and $p=n-l-1$ information ports, excluding the measured CRC Γ_n at its n-th measuring port. This redundancy of mathematical models largely complicates their analytical description and the calibration of NAs.

One of the most promising approaches to the reduction in the redundancy of the mathematical model of a $2n$-port NA is a functional description of its $2n$-port in the space of D-matrices [25], in which it is considered loaded.

To derive a generalized equation for the physical conversion of a $2n$-port NA, we represent it with respect to the $n-n$ plane of its measuring port in the form of a tunable loaded $2n$-port as shown in Fig. 1-2, where, as before, $l, p = n-l-1$, and n are the numbers of the input and information ports of the $2n$-port and their total number, including its n-th measuring port which is connected to the tested load with the CRC Γ.

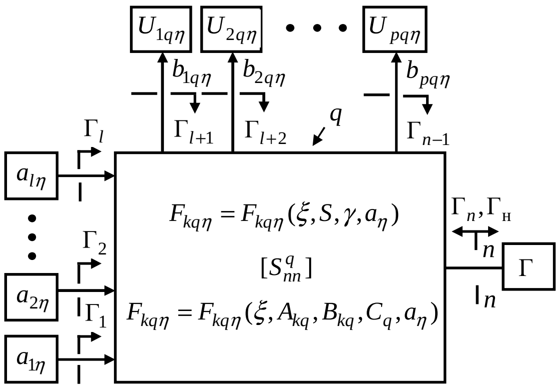

Fig. 1-2. Measuring channel of the NA represented in the form of a tunable loaded $2n$-port network

The complex amplitude of the signals $b_{kq\eta}$ at any of the $k=\overline{1,\ p}$ information ports of the $2n$-port network in any of its $q=\overline{1,\ r}$ state is defined as

$$b_{kq\eta} = F_{kq\eta}(\Gamma_n)\,, \tag{1.1}$$

where $\Gamma_n = \Gamma$ is the CRC of the microwave network measured in the $n-n$ plane of its n-th measuring port; $F_{kq\eta}$ is the scale of physical conversion of the $2n$-port network (physical conversion of the CRC Γ_n into signals $b_{kq\eta}$)

$$F_{kq\eta} = F_{kq\eta}(\xi, S, \gamma, a_\eta)\,; \tag{1.2}$$

$\xi = \{\xi_1, \xi_2, ..., \xi_v\}$ is the v-dimensional random factor which generally includes random factors of measurement and calculation tools; $S = \{[S^1_{nn}], [S^2_{nn}], .., [S^r_{nn}]\}$ and $\gamma = \{\Gamma_1, \Gamma_2, .., \Gamma_{n-1}\}$ are the vector of the matrices of the *S*-parameters of the $2n$-port network and the vector of its load CRCs $\Gamma_i; i = \overline{1,\ n-1}$ at its l input ports and $p = n - l - 1$ information ports; $a_\eta = \{a_{1\eta}, a_{2\eta}, .., a_{i\eta}\}$ and $\eta = \overline{1,\ r-l}$ are the η-dimensional vector of the complex amplitudes of signals at $m = \overline{1,\ l}$ input ports of the $2n$-port and the serial number of its state; r is the limiting number of states q and η.

The physical conversion scale $F_{kq\eta}$ (1.2) is written in the *S*-space and has

$$G_S = rn^2 + n - 1 \tag{1.3}$$

degrees of freedom for the complex parameters *S* and γ, which depend on the design of the $2n$-port network, its states *q*, and the design properties of its loads.

The functional description is chosen so as to satisfy the condition

$$G\big|_{x=S,\ D,..} = \min\,, \tag{1.4}$$

where $G = \{G_S, G_D, ..\}$ is the vector of the degrees of freedom $G_S, G_D, ..$ of the functional descriptions in the $S, D, ..$ spaces, of which we choose the functional description that has the minimum number of degrees of freedom, e.g., G_D.

In order for condition (1.4) to be better satisfied, we define the equation of physical conversion (1.1) in the space of D -matrices [25]

$$b_{kq\eta} = \sum_{m=1}^{i} \frac{D_{ksm}^{q}}{D^{q}} a_{m\eta} \ , \tag{1.5}$$

where D_{ksm}^{q} is the determinant of the matrix $[D_{nn}^{sq}]$ obtained by replacing the k -th indicator column of the matrix $[D_{nn}^{q}]$ by the m -th generator column of the matrix $[S_{nn}^{q}]$; D^{q} is the determinant of the matrix $[D_{nn}^{q}]$, whose elements are given by

$$D_{ii}^{q} = 1 - S_{ii}^{q}\Gamma_{i}, D_{ij}^{q} = -S_{ij}^{q}\Gamma_{j}, i \neq j; \tag{1.6}$$

Γ_i and Γ_j are the CRCs at the $i,\ j = \overline{1,\ n}$ ports.

The matrix $[D_{nn}^{q}]$ has two important properties:

1. decomposition

$$D^{q} = D_{ii}^{q} - \Gamma_{i} D_{isi}^{q}, D_{ii}^{q} = D_{ii,\,jj}^{q} - \Gamma_{j} D_{ii,\,jSj}^{q}, i \neq j; \tag{1.7}$$

2. reduction

$$D_{isj}^{q} = D_{isi,\,jj}^{q} - \Gamma_{j} D_{isj,\,jSj}^{q} = D_{isi,\,jj}^{q}, i \neq j, \tag{1.8}$$

where D_{ii}^{q} and D_{jj}^{q} are the determinants of the matrix $[D_{nn}^{q}]$ with the eliminated i -th or j -th row and i -th or j -th column, respectively; $D_{isi,\,jj}^{q}$ is the determinant of the matrix $[D_{nn}^{sq}]$ with the eliminated i -th row and j -th column.

Using the properties (1.7) and (1.8), we transform (1.5) to

$$b_{kq\eta} = \sum_{m=1}^{I} \frac{D_{ksm,nn,mm}^{q} - \Gamma_{n} D_{ksm,nsn,mm}^{q}}{D_{nn}^{q} - \Gamma_{n} D_{nsn}^{q}} a_{m\eta} \ , \tag{1.9}$$

Dividing the numerator and denominator (1.9) by D_{nn}^{q} leads to