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Cambridge
Scholars
Publishing

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This book first published 2019

Cambridge Scholars Publishing

Lady Stephenson Library, Newcastle upon Tyne, NE6 2PA, UK

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

ISBN (10): 1-5275-4469-9
ISBN (13): 978-1-5275-4469-7

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FOREWORD

Analysis of worldwide aeronautical equipment enhancements convinces us of the necessity to design and implement new types of aircraft crew support systems. Such systems should provide new functions for crew monitoring and intellectual support, based on the automatic analysis of the information, obtained from the onboard equipment, yet maintaining the crew's role as the main aircraft control element.

The issue of flight safety remains a priority all over the world, including the EU countries. Nevertheless, consideration is given mainly to the systems designed to avoid air collisions, and on the ground and at ground proximity. Less work has been dedicated to aircraft actual status monitoring systems and the intellectual support of the crew in abnormal situations [1-15].

In our opinion, none of the Russian or foreign analogs of such systems fully implement the crew support functions by detecting and forecasting abnormal situations, incurred by the following:

- the non-normal functioning of the aircraft systems: despite the equipment developers' significant efforts to increase the equipment elements' reliability, their malfunctioning and failures remain the most frequently encountered reasons for abnormal flight situations, amounting to about 20% of flight accidents and catastrophes;
- the insufficient attentiveness of the pilot, which means that he/she does not pay attention to or even ignores certain signals while handling the aircraft: the instrumental pattern of flight, which the pilot forms, does not correspond to the current flight situation, which incurs irrelevant control actions; insufficient attentiveness is also one of the most frequently encountered reasons for flight accidents, amounting to 22% of accidents due to human factors;
- the insufficient coherence of the pilot's actions, which is evidenced by the incoherent displacement of control devices (the throttle lever, the control stick, the pedals); according to the statistics, the insufficient coherence of control device displacements was the cause of flight accidents and catastrophes in 18% of cases.

In this study we develop a concept, built upon applying a so-called ellipsoidal model of the aircraft-pilot closed ergatic system (APCES), as the intellectual basis for the crew support. This ellipsoidal model is developed and then refined, using the flight data accumulated during simulated or real-life flight modes, executed normally. In other words, such a model defines, for any instant of a typical flight mode, the statistical connection between the aircraft status parameters and the control devices' displacements, which guarantees the normal execution of a typical flight mode. It turns out that applying the APCES ellipsoidal model allows us to realize the following monitoring and crew support functions in order to decrease the human factor impact in flight safety:

1) identification of the moment when an abnormal situation, due to the above-listed reasons, initiates;

2) identification of the exact source of its initiation (non-normal functioning of the onboard systems, insufficient pilot attentiveness, insufficient coherence of control actions) and forecasting of the flight safety hazard generated by it;

3) implementation of the relevant crew support measures.

Let us note that the problem of building up such an ellipsoidal model is not trivial and may be interpreted as a fundamental problem associated with the building up of dynamic systems' attainability sets. Its rigorous solution in the case of non-linear dynamic systems does not exist, and we can only speak of using this or that attainability set approximation method. One of the most attractive methods is ellipsoidal approximation, allowing us to take account of the statistical relations between the APCES parameters most fully. The situation is intensified by the absence of relevant mathematical models, allowing us to account for and describe the pilots' control actions by mathematical means. This fact makes it practically impossible to use the existing theoretical and algorithmic groundwork, based on mathematical models, in order to build up the APCES confidence model.

In these conditions, the data obtained from simulated or real flights, allowing us to take into account all specifics of the pilot's control actions during a typical flight mode execution, might be the only reliable base to form the APCES ellipsoidal model.

These data, when duly processed, allow us to set up the statistical characteristics of the non-controllable factors, influencing the system. In practice, the volume of this information, growing during the aircraft operation, is limited. As a result, the building up of a completely relevant statistical model turns out to be impossible.

The solution in this situation is to use the so-called probability-guaranteeing (confidence) approach, the theoretical basis of which is described briefly in Chapter 1. This approach makes it possible to take into account the objective physical nature of the factors influencing the APCES status in operation. It is important that, in this case, it is not necessary to introduce a fully relevant statistical model, describing the characteristics of the non-controllable factors. Formulation of the problem of the APCES status evaluation in the process of operation is presented in the same chapter.

The models and algorithms, providing the foundations for the ellipsoidal model in question, using the flight data collected by the flight data registration system (FDR), are considered in the next chapter. The authors suggest a method for the compact parametric representation of the ellipsoidal model, as well as the algorithms, providing an evaluation of its parameters. Using the real flight data to evaluate the ellipsoidal model parameters allows us to refine it continuously, taking into account the growth of the statistics accumulated during the aircraft operation. Moreover, these data may be accumulated during many repeats of the same typical flight mode on an aircraft of a certain type by different pilots. In this case, the APCES ellipsoidal model will focus on the support offered an "average" pilot executing this typical flight mode on a certain type of aircraft. However, if the data are accumulated as a result of multiple repeats of the same typical flight mode by one and the same pilot, the ellipsoidal model becomes individually adapted and reflects the specifics of this pilot's control actions during the typical flight mode.

In Chapter 3 we describe the methods and algorithms, allowing us to identify the exact sources of flight safety hazards, based on the APCES ellipsoidal model. Let us underline that using the ellipsoidal model as the intellectual base for the crew support system allows us to unify the procedures to detect the moments of abnormal situations' initiation and the identification their reasons. This unification is achieved by an analysis of the ellipsoidal model flat sections and positions of the points in the phase space, the coordinates of which are defined by the current aircraft status parameters and control devices displacements.

As a result of such analysis, it becomes possible for us to:

- evaluate the hazard of abnormal situations due to the non-normal functioning of aircraft equipment;
- evaluate the hazard of abnormal situations caused by the insufficient attentiveness of the pilot;
- identify the signals which the pilot ignores in the process of operation;
- evaluate the hazard of abnormal cases caused by the insufficient coordination of the pilot's controlling actions;
- identify the control devices that are not duly coordinated by the pilot.

In order to support the crew in the abnormal situations incurred by the equipment elements' non-normal functioning, we suggest an expert system with a knowledge base, formed with reference to the aircraft Flight Operation Manual (FOM). The novelty of this expert system is that it uses a combined hierarchical model of knowledge representation which applies a formalized criteria, based on the ellipsoidal model, in order to identify the moment of an abnormal case initiation and its source, and the production rules, based on the FOM, in order to elaborate the crew support measures.

In Chapter 4 we describe the architecture of the functional-program prototype (FPP) of the APCES status monitoring and crew support system, based on the APCES ellipsoidal model.

The FPP intellectual core is the so-called electronic passport of the APCES, which includes two main elements:

1) the flight database, growing during aircraft operation, containing the implementations of the APCES status vector, accumulated up to the current moment of operation;

2) the parameters of the APCES ellipsoidal model, formed under the conditions of the normal execution of the typical flight mode in question.

Interaction between the FPP and the FDR is carried out in two modes: post-flight and online.

In the post-flight processing mode the data, related to the APCES status, are downloaded from the flight data files contained in the blackbox, and are recorded in the database. Then the information, accumulated in the database, is used to refine the parameters of the APCES ellipsoidal model.

In the online mode, i.e. only during the flight, the abnormal situation hazards are evaluated, their sources are identified and the crew support measures are elaborated continuously based on the current APCES status parameters with the assistance of the ellipsoidal model.

Let us point out the special features of the suggested system, which introduce some new functions in respect of crew support:

1) the possibility of evaluating the flight safety hazards, incurred by the equipment's non-normal functioning, insufficient pilot attentiveness and the insufficient coherence of the pilot's actions, by applying the unified formal base;

2) expansion of the potentials of the "simplified" and "minimal" back-up control modes, implemented in the fly-by-wire system of the civil aircraft, with the assistance of the following:

   - the indication, attracting the pilot's attention to the information signal, which he/she ignores in the process of control;

   - adaptive limitations of the control device displacements, based on the APCES ellipsoidal model, not only preventing it from entering critical flight modes, but also providing execution of the typical flight mode, meeting the existing accuracy requirements.

3) continuous refinement of the APCES ellipsoidal model, based on the data, accumulated in the process of a certain type of aircraft operation.

We provide the preliminary evaluation of the consistency of the suggested concept and the workability of the algorithms implementing it using a hardware-software flight simulator.

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CHAPTER ONE

PROBABILITY-GUARANTEEING APPROACH TO DYNAMIC SYSTEMS' ANALYSIS: EVALUATION OF THE APCES CURRENT STATE

As we know, both experimental practice and the further normal operation of the aircraft require continuous monitoring of the aircraft-pilot closed ergatic system (APCES) status based on the information supplied by the FDR.

When duly processed, these data allow us to reveal the statistical characteristics of the uncontrolled factors which influence the system. In practice, the flight data volume, gathered during the aircraft operation, is limited. As a result, it becomes impossible to build a fully relevant statistical model for the uncontrolled factors.

In this situation, the solution is to use the so-called probability-guaranteeing (confidence) approach [1.1]. This approach allows us to take into account the objective physical nature of the factors which influence the APCES status in operation. It is noticeable that a fully relevant statistical model, describing the uncontrolled factors properties, is not required in this case [1.2-1.6]. The problem of the operating APCES status evaluation, based on the probability-guaranteeing approach, is set up mathematically in Section 1.1.

1.1 Formalization of the problem of the APCES status probability-guaranteeing evaluation

The final purpose of the APCES status evaluation is to analyze the possibility of the system to fulfill its mission successfully in accordance with the flight plan. Further on we suppose that the entire aircraft trajectory consists of separate stages (typical flight modes) $j = 1,...,m$, and their consecutive and successful execution allows for the fulfilment of the flight mission, i.e. lead the aircraft to the specified destination point with the required quality (precision) (Fig. 1.1).

Let us introduce a block vector $Z_j(t)$ which describes the APCES status during *m* typical flight modes, $j = 1,..,m$:

$$Z_j(t) = \begin{pmatrix} X_j(t) \\ \dots\dots\dots \\ U_j(t) \end{pmatrix} \qquad (1.1),$$

where block $X_j(t)$ is $n \times 1$ vector of the aircraft motion and the onboard systems' state parameters at an arbitrary instant t of a typical flight mode;

block $U_j(t)$ is 5×1 vector of the control devices (the roll, the pitch, the throttle lever, two pedals) positions. The FDR is the source of the current $Z_j(t)$ vector component values.

It is obvious that the APCES status vector $Z_j(t)$ is subject to random variations, emerging because of the environmental influence and changes in the aircraft systems' status, as well as the pilot's actions. Let us emphasize that the pilot's actions are also subject to random variations depending on his/her psychophysiological status, the level of informational load and the flight conditions.

Let $E_j(t)$ be an area in the space of the APCES state parameters at any instant t of the typical flight mode, moving out of which the flight mode is executed successfully at a confidence probability β , close to unity.

Such areas are called the attainability sets. The precise definition of these sets, for large-scale non-linear systems in particular, such as the APCES, is a complex problem. Theoretical and computational difficulties arise while seeking the solutions to such problems.

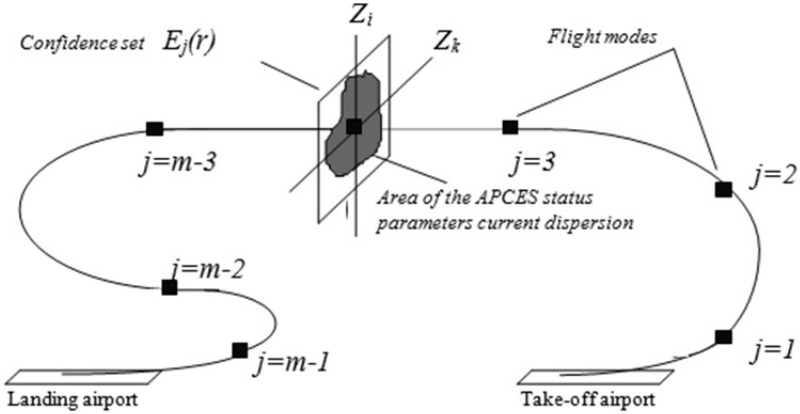

Figure 1.1. The geometry of the APCES status probability-guaranteeing evaluation.

Multiple studies are dedicated to the research of the attainability sets' properties, detailed analysis of which is made, for example, in [1.11]. The methods of the attainability sets' definition for linear dynamic systems are most fully elaborated [1.7 – 1.10].

The problems, related to the attainability sets' specification in the presence of random and random-indeterminate factors, called the confidence sets in this case, are considered in books [1.11, 1.12]. The guaranteeing (minimax) approach to the confidence sets' specification is developed in these studies.

All the above methods of the confidence sets' specification are united by the fact that they are based on the existence of a relevant mathematical model of the dynamic system in question. This fact limits substantially the potential of using the existing apparatus to specify the APCES attainability sets. The point is that the building-up of the relevant mathematical models, allowing us to account for and describe the pilots' actions, still remains an unresolved problem. This fact makes it practically impossible to use the existing theoretical and algorithmic groundwork, based on mathematical models, to specify the confidence sets for the APCES.

Thus several ways are possible for the confidence set approximation, acceptable for experimental purposes [1.11]. It is feasible to consider a parallelepiped in the space of controlled parameters with a specified probability measure, a sphere or a correlation ellipsoid.

For the purposes of the parametric representation, the confidence set $E_j(t)$ at any instant t of a typical flight mode depends on parameter r_{max} and is designated analytically as a description of its boundary:

$$\Phi(Z_j(t)) = r_{max} \tag{1.2}$$

Furthermore,

$$E_j(t,r_{max}) = \{Z_j(t) : \Phi(Z_j(t)) \le r_{max}\} \tag{1.3}$$

The examples of this parametrical representation could be the following:

- a cube in the space of controlled parameters, r_{max} on edge;
- a sphere with radius r_{max} (in this case $\Phi(Z_j(t)) = Z_j^T(t)Z_j(t)$ and $E_j(t,r_{max}) = \{Z_j(t) : Z_j^T(t)Z_j(t) \le r_{max}^2\}$);
- a correlation ellipsoid (in this case $\Phi(Z_j(t)) = Z_j^T(t)K_Z^{-1}(t)Z_j(t)$ and $E_j(t,r_{max}) = \{Z_j(t) : Z_j^T(t)K_Z^{-1}(t)Z_j(t) \le r_{max}^2\}$).

Further on, in Chapters 2 and 3, we shall consider the particular methods of specifying the confidence sets (1.3), reflecting the features of the APCES control application problem.

The characteristic dimension r_{max} of the confidence set is defined so that $\Phi(Z_j(t)) \le r_{max}$ with a specified probability β, defined by the aircraft safe operation requirements:

$$P\{\Phi(Z_j(t)) \leq r_{max}\} \geq \beta \tag{1.4}$$

As a result, we can assess the APCES state at an instant t by vector $Z_j(t)$ belonging to set $E_j(t, r_{max})$:

$$Z_j(t) \in E_j(t, r_{max}) \tag{1.5}$$

$Z_j(t)$ components being outside the $E_j(t, r_{max})$ area may be considered as an integrated factor of the APCES non-normal functioning. The pilot's incorrect actions, jeopardizing flight safety, might be one of the reasons for such a situation.

Taking into account that $Z_j(t)$ vector is formed under the influence of many random factors, the most relevant base for testing condition (1.5) are the probabilistic criteria, which have the following sense in the problem under consideration [1.2]:

1) Direct (probability) criterion $P_j(t)$ is the probability that $Z_j(t)$ vector's components at an instant t of the j-th typical flight mode are within the attainability set:

$$P_j(t) = P\{\Phi(Z_j(t)) \leq r_{max}\} \tag{1.6}$$

In this case, the current APCES status may be evaluated based on testing the condition:

$$P_j(t) \geq \beta \tag{1.7}$$

Condition (1.7) not holding means that the functioning of the APCES elements during the j-th typical flight mode does not meet the specified safety requirements.

2) Indirect (quantile) criterion $\varphi_j(t)$ defines the minimum dimension of the APCES parameters' dispersion area at an instant t of the j-th typical

flight mode, which ensures the specified probability condition of the aircraft safe operation:

$$\varphi_j(t) = \min_r \left(P\left\{ \Phi(Z_j(t)) \leq r \right\} \geq \beta \right) \tag{1.8}$$

In other words, condition (1.8) defines the probability-guaranteeing dispersion of the APCES parameters, corresponding to its status. As a result, the condition, under which the APCES status at an instant t of the j-th typical flight mode meets the specified safety requirements, is defined by the following inequality:

$$\varphi_j(t) \leq r_{max} \tag{1.9}$$

The APCES status integral estimation by testing conditions (1.7) or (1.9) refers to applying certain calculation methods in respect of the probability criteria (1.6), (1.8).

At this point, we have to emphasize that the computation of direct and indirect criteria comes into sharp focus from the point of view of the flight safety improvement issue for large-scale problems with hundreds or thousands of parameters under control.

This key Chapter of this book presents a detailed description of the approaches, methods and algorithms of the probability criteria evaluation.

The next part describes the related methods of the probability criteria evaluation with respect to APCES status.

1.2. Probability criteria evaluation algorithms

The known computation algorithms for criteria (1.6) and (1.8) are quite fully described in the modern bibliography [1.1, 1.12, 1.13]. In this book, we narrow it down in order to analyze the potential of their implementation in the APCES status evaluation problem.

In order to unify further formal records, we assume that the APCES status vector $Z_j(t)$ at an instant t of a typical flight mode is represented as a random vector $\omega \in R^{n+m}$.

In this case, condition of the APCES normal operation is:

$$\Phi(\omega) \le r_{max} \qquad (1.10).$$

The direct probability criterion (1.6) for the conditional test (1.10), taking into account the above designation, is written as

$$P_{\varphi} = P\{\Phi(\omega) \le \varphi\} \qquad (1.11)$$

with $\varphi = r_{max}$.

The indirect criterion (1.8) is written as

$$\varphi_{\beta} = \min_{r}\left(P\{\Phi(\omega) \le r\} \ge \beta\right) \qquad (1.12).$$

We shall describe the particular numerical algorithms, providing criteria (1.11) and (1.12) evaluation, below.

1.2.1. Probability criteria evaluation algorithms based on probability density integration

This group of algorithms is based on resolving the problem (1.11) through computation of the multidimensional integral of the density function $p(\omega)$ which is written as follows:

$$P_{\phi} = \int_{\omega \in E} p(\omega) d\omega \qquad (1.13),$$

where $E = \{\omega : \Phi(\omega) \le \varphi\}$ is a set in the space of random factors ω, limited by the contour line $\Phi(\omega) = \varphi$.

This approach may be of unconditional interest in cases when we succeed in obtaining an exact resolution of the above integral equation. In particular, this is possible when ω is a scalar random value with distribution falling under one of the available statistical laws. The class of distributions, the properties of which were studied fully enough and their distribution function values were tabulated, is rather wide. First of all, it's the normal distribution, the *t*-distribution (or Student distribution), the χ^2-

distribution, the F-distribution and others. Moreover, analytical dependencies, describing the probability density function, are available, which significantly facilitates further integral computation. Up-to-date mathematical statistics apparatuses possess a number of goodness measures (KS-test, χ^2, ω^2), allowing us to confirm the fact of the experimental distribution correspondence to a certain theoretical statistical law. If such a confirmation is obtained we can use the direct approach to obtain rather precise evaluations of the probability criteria in question.

For example, if ω is a scalar random value with distribution suiting the normal statistical law with mathematical expectation m_ω and variance σ_ω^2: $\omega \in N(m_\omega, \sigma_\omega^2)$, then the problem of the probability functional P_φ (1.11) computation resolves itself to the evaluation of the probability that ω belongs to interval $|\omega| \leq \varphi$:

$$P_\varphi = P\{|\omega| \leq \varphi\} = F\left[\frac{\varphi - m_\omega}{\sigma_\omega}\right] + F\left[\frac{\varphi + m_\omega}{\sigma_\omega}\right],$$

where $F[\bullet]$ is the standard normal distribution function (the Laplace function), with tabulated values.

Let us note that finding the quantile functional (1.12) in this case supposes resolving a transcendental equation written as follows: $P_\phi = \beta$.

In situations when ω is a vector random value, we can also distinguish a limited number of cases for which implementing the direct approach in question is possible based on the acquainted analytical solutions. Nevertheless, all these solutions may be applied only when ω is a normally distributed random vector with mathematical expectation m_ω and covariance matrix K_ω, i.e. $\omega \in N(m_\omega, K_\omega)$. In particular, an analytical solution may be obtained if the range of integration E is a sphere in R^{n+m}:

$$E = \{\omega : (\omega - m_\omega)^T (\omega - m_\omega)\}.$$

We can calculate the probability and the quantile, based on the analytical relation between sphere radius φ and probability P_φ that the normal Gaussian variable belongs to it, by transition from the initial random vector $\omega \in N(m_\omega, K_\omega)$ to its normally distributed analog $\omega^* \in N(0, E_n)$, where E_n is a unity matrix; $\omega^* = K_\omega^{-1/2} (\omega - m_\omega)$; $K_\omega^{-1/2} K_\omega^{-1/2} = K_\omega$.

This relation is defined by the following equations, depending on the dimension of the space of random factors n:

1) odd n, n=2k, k=1,2,….

$$\frac{1}{2^{\frac{n-2}{2}}(k-1)!}\int_0^{\varphi} t^{2k-1}\exp\left(-\frac{t^2}{2}\right)dt = P_\varphi \ ;$$

2) even n, n=2k+1, k=0,1,2,

$$\sqrt{\frac{2}{\pi}}\,\frac{1}{(2k-1)!}\int_0^{\varphi} t^{2k}\exp\left(-\frac{t^2}{2}\right)dt = P_\varphi \ .$$

Another analytical solution is related to the case when the integration range E is represented as a correlation ellipsoid in R^n:

$$E = \left\{\omega : (\omega - m_\omega)^T K_\omega^{-1} (\omega - m_\omega)\right\}.$$

In the majority of cases, however, calculation of the integral over probability-density function $p(\omega)$ is possible only by the numerical approach. As it is noted in [1.14], the scheme with the integration range E_ω subdivision into a class of non-intersecting elementary ranges turns out to be the most effective. This scheme allows us to obtain numerical solutions for both direct (1.11) and indirect (1.12) problems.

At the same time, it is noted that the volume of computational load increases with the growth of the random factors' space dimensionality $n+m$. Ref. [1.14] shows that this functional connection is exponential: V^{n+m}, where V is the amount of computational work in a one-dimensional case. This feature of the algorithms, based on the direct approach, restricts their usability for the APCES status evaluation problems significantly.

1.2.2 Probability criteria computation algorithms based on Monte Carlo sampling

This group of algorithms is based on the statistical test method which is focused on obtaining the random values realizations $\mu^j = \Phi(\omega^j), j=1,\dots,N$ according to the probability density function $p(\omega)$. The estimate of probability P_φ is calculated using the obtained sample:

$$P_\phi^* = \frac{1}{N}\sum_{j=1}^{N}\chi_j \qquad (1.14),$$

where χ_j is the indicator function: $\chi_j = 1$, if $\mu^j \le \phi$, otherwise $\chi_j = 0$. Thus, the estimate P_ϕ^* at $N\to\infty$ converges with the true probability P_φ.

The advantage of this approach is that there is no obvious dependency between the amount of computational work and the random factors' vector dimension. However, in this case, we encounter other difficulties. First of all, it is the number of statistical tests required to obtain the estimate of probability close to unity. Such values are of specific practical interest for the purposes of the technical system status evaluation.

Let us consider one of the ways to decrease the volume of sample N to obtain the estimates of probability P_φ, using the Monte Carlo method described in [1.15]. It is based on cluster sampling and essential sampling.

The first method (cluster sampling) suggests that $(n+m)$ random factors' space ω is divided into a system of mutually disjoint sets $\Omega_1, \Omega_2, \dots, \Omega_l$. In this case, the event $A(\varphi)=\{\omega: \Phi(\omega)\le\varphi\}$ can be represented as an intersection and union of the events $\{A(\varphi)\},\{G_l\}, l=1,\dots,L,$ where $G_l=\{\omega: \omega \in \Omega_l\}$,

$$A(\varphi) = \bigcup_{l=1}^{L}[G_l \bigcap A(\varphi)] \qquad (1.15).$$

According to the formula of total probability,

$$P_\varphi = P(A(\varphi)) = \sum_{l=1}^{L} P(G_l)P(A(\varphi)/G_l) \qquad (1.16).$$

Thus to create the P_φ estimates computation, it is necessary to define the probabilities, included in (1.16). Let us note that the probabilities $P(G_l),\ l=1,\dots,L$ can be calculated analytically if the sets $\Omega_l,\ l=1,\dots,L$ are selected in a certain way. For example, it becomes possible if the random factors' space is divided by ring-shaped spheres put one into another, and the obtained ring-shaped sets are divided into symmetrical fractions (Fig. 1.2).

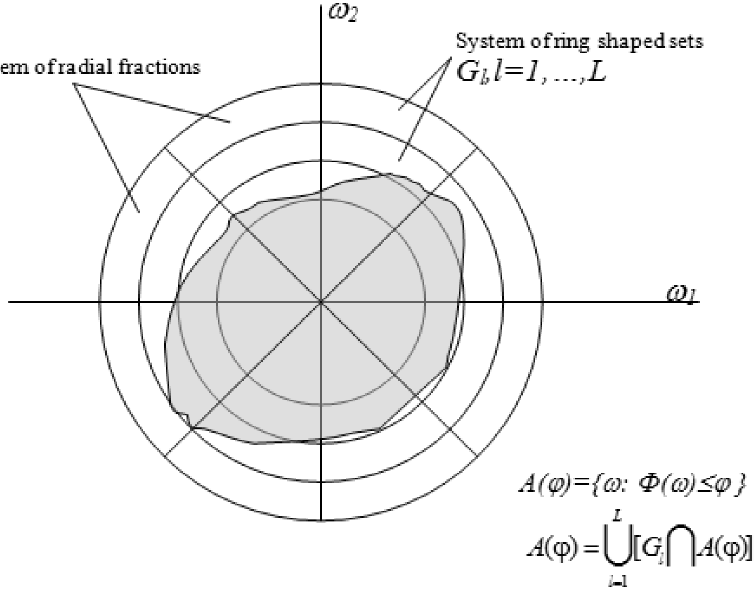

Figure 1.2. Geometry of the random factors' space division

Another way to define the sets $\Omega_l,\ l=1,\dots,L$ is to divide the space R^{n+m} into rectangles, oriented in either direction, not necessarily coinciding with that of the axes of coordinates.

We can obtain the estimates of probabilities $P\{A(\varphi)/,G_l\},l=1,\dots L$ by specifically elaborated statistical modeling. These estimates differ from unity even at rather large values of φ, which is attained by modeling random points $\omega^{il},\ l=1,\dots,L$ in each of the sets $\Omega_l,\ l=1,\dots,L$. The corresponding probability density $p_l(\omega)$ is defined as:

$$p_l(\omega) = \begin{cases} \dfrac{p(\omega)}{p(G_l)}, \omega \in \Omega_l \\ \hline 0, \omega \notin \Omega_l \end{cases}.$$

Let us note that, unlike the traditional Monte Carlo scheme, statistical modeling by this method doesn't require the computing of a large number of $\mu^j = \Phi(\omega^j)$ to obtain the value of P_φ, close to unity.

Another method (the so-called essential sampling method) is based on the following representation:

$$P_\varphi = \int\limits_{\omega\in E_\omega} p(\omega)d\omega = \int\limits_{\omega\in E_\omega} p(\omega)[\frac{p_M(\omega)}{p_M(\omega)}]d\omega = $$
$$= \int\limits_{\omega\in E_\omega} p_M(\omega)[\frac{p(\omega)}{p_M(\omega)}]d\omega \qquad (1.17),$$

where $p_M(\omega)$ is the probability-density function determining the "mechanism" of ω^j realizations' occurrence.

Taking into account (1.17), the formula for calculation of probability P_φ can be written down as follows:

$$P_\varphi = \frac{1}{N}\sum_{j=1}^{N}[\frac{p(\omega^j)}{p_M(\omega^j)}]\chi_j \qquad (1.\ 18),$$

where χ_j is the indicator function, $\chi_j = 1$, if $\mu^j \le \varphi$, 0 otherwise.

Within the frames of the cluster and essential sampling methods in question, we can realize the idea of the consequent (adaptive) statistical modeling scheme planning. In the cluster sampling method it can be achieved by adjustment of the class of sets Ω_l, $l=1,\dots,L$. This will allow the selection of the most effective modeling ranges in the space of random factors from the point of view of probability estimate P_φ. In the essential sampling method this can be achieved through the adjustment of the probability-density function $p_M(\omega)$.

Based on the results obtained in [1.11], we can affirm that the optimum solution (division Ω_l, $l=1,\dots,L$, function $p_M(\omega)$) is defined by the $\Phi(\omega)$ functional contour lines. It becomes impossible to implement it precisely, since the difficulties of computation in this case are comparable to those appearing when using the traditional Monte Carlo scheme.

We could confine ourselves to a simpler solution if we tried to approximate the objective function $\Phi(\omega)$, for example, with a quadratic form $\Phi^*(\omega)$ using the current information $\Phi(\omega^j)$, $j=1,\dots,N$, gathered as a result of N statistical tests:

$$\Phi^*(\omega)=\omega^T A\omega+B^T\omega+C, \qquad (1.19),$$

where A is an $(n+m)\times(n+m)$ matrix; B is a $(n+m)\times 1$ vector; C is a scalar, all parameters are refined in the process of the statistical tests while information about $\Phi(\omega)$ is accumulated.

Approximation $\Phi^*(\omega)$ may become more detailed if we carry out several quadratic approximations in single ranges of the space R^{n+m}, instead of the gross approximation (1.19). But we have to bear in mind that any complication of the approximation method entails growth of the statistical modeling computational costs.

We can calculate parameters A, B and C in (1.19) using the least square method (LS method):

$$J=\sum_{j=1}^{N}[\Phi(\omega^j)-(\omega^{jT}A\omega^j+B^T\omega^j+C)]^2\to\min_{A,B,C}\dots \qquad (1.20).$$

We can obtain the solution of the optimization problems (1.20) analytically if we introduce a vector of parameters with dimension $n^*=((n+m)^2+(n+m)+1)$:

$$\theta=(A^{sq}(s=1,\dots,n+m;q=1,\dots,n+m),\ B^s(s=1,\dots,n+m),C).$$

Then

$$J=(\Phi^N(\omega)-\Lambda^N(\omega)\theta)^T(\Phi^N(\omega)-\Lambda^N(\omega)\theta)\to\min_{\theta} \qquad (1.21),$$

where $\Phi^N(\omega)$ is the N-dimensional observation vector $\Phi^*(\omega)$:

$$\Phi^N(\omega) = \begin{bmatrix} \Phi(\omega^1) \\ \dots\dots\dots \\ \Phi(\omega^N) \end{bmatrix} (N \times 1);\ \Lambda^N(\omega) \text{ is the matrix of the observations}$$

linear model:

$$\Lambda^N(\omega) = \begin{bmatrix} \varphi^T(\omega^1) \\ \dots\dots\dots\dots \\ \varphi^T(\omega^N) \end{bmatrix} N \times \left((n+m)^2 + (n+m) + 1 \right) .$$

$$\varphi^T(\omega^j) = (\omega^{jS}\omega^{jq}\left(s = 1,..,n+m; q = 1,\dots,n+m\right), \omega^{jS}\left(s = 1,\dots,n+m\right), 1)$$

The estimate of the optimum parameters vector θ is written as

$$\theta* \ = \ [(\Lambda^N(\varphi))^T \Lambda^N(\varphi)]^{-1} (\Lambda^N(\varphi))^T \Phi^N(\varphi). \qquad (1.\ 22).$$

Let us note that, in practice, only the diagonal elements of matrix A can be defined, in this case the vector dimensionality n^* decreases significantly (to $(2(n+m)+1)$).

It is feasible to refine the estimates θ^* periodically while the volume of the $\Phi(\omega)$ realizations grows. Based on the information about the objective function $\Phi(\omega)$ in the form of $\Phi^*(\omega)$, we can build a system of ring-shaped sets Ω_l, $l=1,\dots,L$:

$$\Omega_l, = \{\omega :\ \Phi^*(\omega) \le \varphi_l\},$$

where levels φ_l are selected so that the set Ω_l probability measure is sufficiently close to unity.

In the essential sampling method, the modeling density $p_M(\omega)$, refined after the $\Phi(\omega)$ function realization is obtained, is written as:

$$p_M(\omega) = k_1 exp\{\ -k_2\Phi^*(\omega)\},$$

where coefficient k_1 is defined following the normalization requirement, coefficient k_2 is refined empirically and may be used as an additional optimization parameter.

The suggested scheme allows us to outline the process of the mathematical modeling systematically, with the purpose of increasing the accuracy of the probability estimates P_φ, as new information becomes available at an acceptable computation cost.

1.2.3. The algorithms, based on the confidential approach

The confidential approach algorithms are based on transferring from the initial problem of the probability criterion's (1.12) estimation to an equivalent minimax problem:

$$\phi_\beta = \min_{E \in E_\beta} \max_{\omega \in E} \Phi(\omega) ,$$

where E_β is the class of sets with probability measure β.

While seeking the solution to this problem we define not only the value of quantile ϕ_β in question, but also the so-called "optimal" confidence set E_β in the space of random factors ω.

The comprehensive description of the algorithms, realizing the general minimax approach, is provided in [1.2]. Let us consider one of these algorithms, illustrating the procedure of the initial confidence set parametric optimization.

The guaranteed quantile computation algorithm.

To obtain the two-sided estimates for the quantile $\Phi(\omega)$, convex with respect to at least one of the disturbance vector elements, we use the algorithm of the minimum and the maximum search at the boundary of a sphere-shaped confidence set in R^{n+m} space. The algorithms of directional and non-directional search can be used here. The pattern of non-directional search is as follows.

Step 1. The required probability value β is specified.

Step 2. A sphere S with probability measure $P(S) = \beta$ is used as the confidence set.

Step 3. A regular grid of M points is built on the surface ∂S of the sphere S. One of the ways to form it is described in [1.12].

Step 4. The maximum $\psi(S)$ and the minimum $\chi(S)$ of function $\Phi(\omega)$ are found in the grid points, and they are taken for the quantile upper and lower estimates correspondingly.

We can prove as per [1.2] that the described search mechanism for the two-sided quantile estimates on the surface of a sphere-shaped confidence set appears to be much more effective than the standard Monte Carlo algorithm.

For a further reductionin the computational costs of finding the maximum and the minimum on the sphere surface, we shall use the directional search algorithm, allowing us to find the quantile estimates in the vicinity of the global extremes. The pattern of the maximum directional search with a "directing sphere" [1.12] is written as follows.

Step 1. The required probability β is specified.

Step 2. A sphere S with probability measure $P(S) = \beta$ and radius r is selected as the confidence set E in space R^{n+m} of random disturbances ω.

Step 3. An initial point ϖ^0 in the auxiliary space R_{ϖ}^{n+m} is set, whereby the dimension $(n+m)$ of space R_{ϖ}^{n+m} equals the dimension of space R^{n+m}. The auxiliary space is introduced for solving the problem of the unconditional optimization of function $\Phi(\omega)$ in it. And the problem of the conditional optimization with limitation $\omega \in \partial S$ is solved in the space R^{n+m}.

Step 4. Point ϖ^0 is transferred from space R_{ϖ}^{n+m} to space R^{n+m}, onto the surface of sphere S:

$$\omega_i^0 = \frac{\overline{\omega}_i^0}{\sqrt{\sum_{i=1}^{n} \overline{\omega}_i^{02}}} r, \ i = \overline{1,n} \qquad (1.23),$$

where ω_i^0 is the i-th coordinate of the point in space R^{n+m}. Let us note that we can define the initial point ω^0 in R^{n+m} based on prior suggestions about the point of the extremum or based on the results of the non-directional search. The value of function $\Phi(\omega^0)$ at ω^0 is defined.

Step 5. m sample vectors are formed:

$$\mu^j = \frac{W_0 + k\xi^j}{\left\| W_0 + k\xi^j \right\|}, \ j = \overline{1,m} \qquad (1.24),$$

where W_0 is a unit memory vector (at Step 1 $W_0 = 0$); ξ^j is a unit evenly distributed random vector with dimension *n*, $k > 0$ is some invariable.

Step 6. Coordinates of *m* points in space $R_{\overline{\omega}}^{n+m}$ are defined:

$$\overline{\omega}^j = \omega_0 + h_{np}\mu^j, \ j = \overline{1,m} \qquad (1.25),$$

where $h_{np} \in [0,1]$ is some step.

Step 7. Points $\overline{\omega}^j$, $j = \overline{1,m}$, are transferred from space $R_{\overline{\omega}}^n$ to space R^n: in accordance with (1.23), the coordinates of points ω^j, $j = \overline{1,m}$ are defined on the surface of sphere *S*.

Step 8. The values of function $\Phi(\omega^j)$ at all points ω^j, $j = \overline{1,m}$ are defined.

Step 9. The statistical gradient is defined:

$$d^0 = \sum_{j=1}^{m} \mu^j \left[\Phi(\omega^j) - \Phi(\omega^0) \right] \qquad (1.26).$$

Step 10. The working displacement is defined:

$$\Delta\overline{\omega}^0 = h_p d^0 / \left\|d^0\right\| \qquad (1.27),$$

h_p is the working step.

Step 11. Coordinates of a new point in space $R_{\overline{\omega}}^{n+m}$ are defined:

$$\overline{\omega}^1 = \overline{\omega}^0 + \Delta\overline{\omega}^0 \qquad (1.28)$$

Step 12. The memory vector is updated:

$$W^1 = \frac{W_0 + \gamma\Delta\overline{x}^0}{\left\|W_0 + \gamma\Delta\overline{x}^0\right\|} \qquad (1.29),$$

where $\gamma \geq 0$ is the parameter, describing the method's inertia.

Step 13. Point $\overline{\omega}^1$ is transferred from space $R_{\overline{\omega}}^{n+m}$ to space R^{n+m}: in accordance with (1.23), point ω^1 is defined on the surface of sphere S.

Step 14. Function $\Phi(\omega^1)$ value at ω^1 is defined.

Step 15. The disparity is defined:

$$\varepsilon = \left|\Phi(\omega^1) - \Phi(\omega^0)\right| \qquad (1.30).$$

In case the disparity is small, the maximum $\Phi(\omega)$ value is deemed to have been found on the surface of sphere *S*, and it is set as the quantile ϕ_β upper estimate. Otherwise, go to Step 5.

The degree of convergence of the described algorithm depends on invariables *k* and γ effecting the method's response rate, and also on the value of step h_{np}. Generally, this algorithm doesn't ensure the discovery of the global maximum $\psi(S)$ on surface ∂S. It is necessary to apply this algorithm several times, setting different initial points $\overline{\omega}^0$, and to set the maximum of the attained values as estimate $\psi(S)$. Using the same