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Cambridge
Scholars
Publishing

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This book first published 2020

Cambridge Scholars Publishing

Lady Stephenson Library, Newcastle upon Tyne, NE6 2PA, UK

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

ISBN (10): 1-5275-4677-2
ISBN (13): 978-1-5275-4677-6

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PREFACE

This monograph sets out the theory of the propagation of infrasound waves (frequencies below 20 Hz) in a real atmosphere with its inherent mesoscale wind velocity and temperature fluctuations with periods from 1 min to several hours. The theory explains the effect of mesoscale fluctuations on the parameters of infrasound waves propagating in the atmosphere, including the atmospheric boundary layer, the stratosphere, the mesosphere, and the lower thermosphere.

At present, there is an extensive literature on theoretical and experimental studies of sound propagation in the atmosphere with turbulent fluctuations in the inertial scale range, which are described by the model of locally homogeneous and isotropic turbulence. As for the statistical properties of mesoscale wind velocity and temperature fluctuations caused by internal gravity waves and vortex structures in the atmosphere, and the effects that these fluctuations have on the propagation of sound, they have only begun to be intensively studied in the last three decades. We tried to generalize for the first time the theoretical and experimental results of these studies and to present them consistently in this monograph.

The current practical problems of atmospheric acoustics that have arisen, together with the development of the methods for infrasound monitoring of tests of nuclear explosions and life-threatening natural phenomena (volcanic eruptions, tsunamis, hurricanes, earthquakes, tornadoes, meteorites, etc.), have also given rise to theoretical problems. We encounter these problems every time when it becomes necessary to take into account the influence of the structure and dynamics of the real atmosphere on the processes of sound propagation. It became clear that modern problems of acoustics and atmospheric dynamics are interconnected and require a joint solution.

The experimental data of observations of infrasound signals from ground-based explosions and volcanic eruptions accumulated in recent decades indicate a significant influence on these signals of the so-called fine-scale layered structure of the atmosphere. This structure consists of anisotropic inhomogeneities of temperature and wind velocity with horizontal scales significantly exceeding their vertical scales.

In the infrasound frequency range, the wavelengths are comparable with the characteristic vertical scales of anisotropic inhomogeneities. Such inhomogeneities, therefore, significantly scatter infrasound. In addition, the random fluctuations of the phases and amplitudes of the infrasound waves caused by inhomogeneities lead to errors in determining the location and power of the sources, which must be taken into account when monitoring explosions and other infrasound sources.

Obviously, to determine the statistical characteristics of fluctuations in the parameters of infrasound signals (azimuth and propagation time, amplitude and duration), it is necessary to know the statistical characteristics of the anisotropic fluctuations themselves: their spatio-temporal spectra, correlation and structure functions. However, the very nature of the occurrence and the statistical properties of anisotropic fluctuations have been very poorly studied to date compared with the statistical characteristics of locally homogeneous and isotropic turbulence. The need to fill the gap in this area led the authors of this book to turn first to theoretical and experimental studies of statistical characteristics of the anisotropic turbulence in a stably stratified atmosphere, and using a model of such turbulence take into account its influence on the infrasound propagation in the atmosphere. Some new results of these studies have been obtained recently. The authors decided to present them in this monograph from a single point of view, which, in our opinion, could help to significantly advance progress toward solving the problem of the propagation of infrasound waves in a real fluctuating atmosphere.

Chapters 1 and 2 of this monograph are devoted to the description of the wave theory of the propagation of low-frequency acoustic waves in the atmosphere, as in a moving plane-layered medium without anisotropic fluctuations. In Chapter 5, this approximate model of the atmosphere is compared with its more realistic model, which takes into account the permanent presence of anisotropic fluctuations in the entire atmosphere, from the atmospheric boundary layer (ABL) to the heights of the lower thermosphere (100–140 km).

Chapter 3 is devoted to the description of the first experiments in which the effect of the scattering of infrasound signals on a fine-scale layered structure of wind velocity and temperature in the stratosphere, mesosphere and lower thermosphere was discovered. This effect has changed our ideas about the distribution of the zones of acoustic shadow and audibility along the surface of the earth, which existed from the beginning of the last century when the structure of the atmosphere was intensively studied using infrasound pulses generated by explosions. Based on the scattering effect of infrasound waves in the acoustic shadow region, the authors developed a new method of remote sensing of the atmosphere (described in Chapter 6) using powerful pulsed sound sources (ground explosions, volcanoes and detonation generators). With this method, the instantaneous vertical profiles of wind velocity were retrieved in the stratosphere and lower thermosphere (up to an altitude of 140 km). This method made it possible to obtain new data on temporal variability, vertical wave number spectra, and coherence of anisotropic wind velocity fluctuations at altitudes of the upper stratosphere (30–50 km) and the lower thermosphere (90–140 km) (Chapter 6). Such data are necessary for refining modern models of general atmospheric circulation and impurity transport.

A theoretical model for the formation of a fine-scale layered structure in a stably stratified atmosphere, taking into account non-resonant interactions between internal waves and between waves and horizontal vortex motions, is discussed in Chapter 4. For the first time, this theory has made it possible to obtain, based directly on the equations of hydrodynamics, a three-dimensional (3-D) spatial spectrum of anisotropic wind velocity and temperature fluctuations caused by the ensemble of internal waves and horizontal vortex motions, and with its help parameterize the statistical characteristics of the fluctuations of infrasound signals (frequency spectrum of the scattered signal field, its coherence, the travel time and angle of arrival).

The adequacy of the theory of the formation of anisotropic fluctuations in the atmosphere presented in this book is confirmed by the agreement of the model vertical and horizontal wave number spectra of fluctuations with the observed spectra in the upper troposphere and stratosphere obtained from radar and aircraft measurements. It should be noted that the forms of 3-D spatial spectra of mesoscale pulsations of temperature and wind velocity in stably stratified atmospheric layers have already been used both in problems of forecasting the statistical characteristics of fluctuations in the parameters of infrasound waves and in fluctuations of light intensity in the problems of optical sounding of the atmosphere.

On the one hand the theoretical and experimental results presented in the book are important for the development of the theory of mesoscale atmospheric turbulence, which has so far turned out to be very poorly developed in comparison with the theory of locally homogeneous and isotropic turbulence. On the other hand, we hope that these results will be of practical use in solving the problem of infrasound monitoring of explosions and natural hazards, to improve models of the long-range propagation of sound and light in the atmosphere and parameterize the effect of internal waves on the general circulation of the atmosphere.

This book may be useful for specialists in the fields of acoustics and optics of the atmosphere, remote sensing of the atmosphere, the dynamics of internal waves, nonlinear acoustics, infrasound monitoring of explosions and atmospheric storms.

Igor Chunchuzov and Sergey Kulichkov

CHAPTER ONE

THE PROPAGATION OF LOW-FREQUENCY SOUND WAVES IN A STRATIFIED MOVING ATMOSPHERE: THEORY AND OBSERVATIONS

In this Chapter we present a number of theoretical and experimental results obtained in the field of sound propagation in a stratified moving atmosphere. The atmosphere will be considered as a layered medium unperturbed by anisotropic wind velocity and temperature fluctuations. Later (in Chapter 5) such an approximate model of the atmosphere will be compared to its more realistic model taking into account the permanent presence of anisotropic fluctuations in the entire atmosphere: from the stably stratified atmospheric boundary layer (ABL) to the heights of the lower thermosphere (100–140 km).

The fundamentals of the theory of sound propagation in a moving inhomogeneous medium, that was developed significantly during the Second World War, are presented consistently in Blokhintsev's famous monograph (1956). Later, especially in the 1980s and 1990s, the wave theory of sound propagation in moving media started to develop intensively using well-developed wave methods of acoustics of layered media at rest and generalizing them to the case of a moving medium with its inherent azimuthal anisotropy. See for example, the books by Pierce (1981), Brekhovskikh and Godin (1989), Ostashev (1997), Brekhovskikh and Godin (2007), Ostashev and Wilson (2015) and the works by Pridmore-Brown (1962), Chunchuzov (1983, 1984, 1985), Goncharov (1984), Ostashev (1984, 1986, 1987), Razin (1985, 1995), Grigoriev and Yavor (1986), Godin (1987a,b, 1990), Pierce (1965, 1967, 1990) and Vdovicheva et al. (1990).

In this Chapter we will use wave theory of the acoustics of moving layered media to explain the wave effects observed in experiments on the propagation of low-frequency sound waves in the real boundary layer of the atmosphere (Bush et al. 1985; Otrezov and Chunchuzov 1987; Nesterova et al. 1987).

In the ABL the wave effects are significant for infrasound waves with frequencies below 20 Hz. This is due to the fact that the typical vertical scales of changes in wind velocity and temperature in the surface layer of the atmosphere, which are usually tens of meters, are comparable to the wavelengths of the acoustic waves. In this case, the methods of geometric acoustics are not applicable for the calculation of the acoustic field. Based on the exact analytical solutions of the Helmholtz equation for a number of vertical wind velocity and temperature profiles, the method of normal modes applied to a moving stratified medium proved to be very efficient for such calculations (Chunchuzov 1983, 1984, 1985, 1992; Brekhovskikh and Godin 1989, Chap. 3.2). This allows us to determine the main parameters of the sound field in a wide range of frequencies. This is especially important when analyzing the propagation in the atmosphere of pulsed acoustic signals with a broadband frequency spectrum.

Naturally, the set of model profiles of wind velocity and temperature that allow exact analytical solutions of the Helmholtz equation does not cover all the diversity of stratification conditions in the real atmosphere with its inherent spatial and temporal variability of wind and temperature fields. Various numerical methods of finding solutions to wave problems of the acoustics of a moving stratified atmosphere (Pierce 1965, 1967, 1981), which have recently been developed intensively (Nijs and Wapenaar 1990; Salomons 2001; Waxler 2002; Waxler et al. 2006, 2008; Wexler and Assink 2019) help to cover such diversity in the modeling of long-range sound propagation in the atmosphere.

Below we obtain a wave solution in general form for a harmonic point source of sound in a stratified moving atmosphere with arbitrary profiles of wind velocity and temperature. The exact solution of the Helmholtz equation will be obtained for exponential profiles. Its importance lies in that it allows us to analyze the continuous evolution of the acoustic field when transiting from short wavelengths relative to the thickness of the atmospheric inhomogeneous layer to very long wavelengths (Chunchuzov 1992). Despite the particular nature of this solution it does not depend at low frequencies on the detailed shape of the chosen profile and depends mainly on the vertical jump in the acoustic refractive index in the moving atmospheric layer multiplied by the ratio of the layer thickness to the wavelength.

There are many factors that have a significant impact on the propagation of high-frequency sound waves in the ABL (frequencies above 100 Hz), such as classical molecular and relaxation absorption (Kallistratova 1994), scattering by turbulent wind velocity and temperature fluctuations (Wilson et al. 2015; Ostashev and Wilson, 2015), scattering on trees in the forest (Muhlestein et al. 2018), the effect of ground surface impedance (Piercy et al. 1977; Wilson 1997; Attenborough et al. 2011). However, these factors have a very weak effect on the propagation of infrasound waves. At the same time there are various anisotropic vortex and wave structures in the atmosphere, the scales of which are comparable with the wavelengths of infrasound waves (Gossard and Hooke 1975; Chimonas 1999; Danilov and Chunchuzov 1992; Anderson 2003; Chunchuzov 2004; Lyulyukin et al. 2015; Chunchuzov et al. 2017). Such structures have a significant impact on the infrasound wave propagation.

Under stable stratification of the ABL the anisotropic structures create at some heights above ground significant vertical gradients of wind velocity and temperature reaching values of the order of 0.1 s^{-1} and 0.1 deg/m, respectively. Recently, the partial (Fresnel) reflections of acoustic signals from thin layers in a stably stratified ABL with extremely large wind speed gradients were detected at horizonal distances of more than 2 km from artificial pulse sound sources (Perepelkin 2006; Perepelkin et al. 2013; Chunchuzov et al. 2017).

In this Chapter we will limit ourselves to considering the effect of the mean wind and temperature stratification on the field of a harmonic source of low-frequency sound. In the next Chapter the field of a pulsed sound source will be analyzed. The effects of sound scattering on anisotropic inhomogeneities of wind velocity and temperature in a stably stratified atmosphere will be considered later, in Chapter 4. Particular attention in this Chapter will be paid to the comparison of theoretical solutions with experimental data obtained by continuous monitoring of stratification of wind speed and temperature in the ABL. Such control for the first time made it possible to check the adequacy of the analytical solutions.

1.1. Equations of the acoustics of an inhomogeneous moving medium

Equations for acoustic-gravity waves in a stratified moving atmosphere

To derive the basic equations of the acoustics of a stratified moving atmosphere we will start from the general system of equations of fluid motion, the state of which is characterized by density ρ , pressure p , entropy S and velocity $\vec{v}$, which are functions of the coordinate $\vec{x} = (x, y, z)$ of a point of space and time t. This system includes the equation of fluid continuity in the presence of sources of volume velocity q ,

$$\frac{\partial \rho}{\partial t} + div(\rho \vec{v}) = \rho q, \tag{1.1}$$

the equation of motion of a unit mass of a viscous and heat-conducting fluid in a field of gravity $\vec{g}$ and other arbitrary external forces $\vec{f}$ per unit mass,

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\nabla)\vec{v} = -\frac{\nabla p}{\rho} + \vec{g} + \vec{f} + \nu_k \Delta \vec{v} + \frac{\nu_k}{3}\nabla(\nabla \vec{v}), \tag{1.2}$$

where $\nu_k = \frac{\mu}{\rho}$ is kinematic viscosity of the fluid, μ is its dynamic viscosity, $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is Laplasian, and the equation of heat flow

$$\frac{\partial S}{\partial t} + (\vec{v}\nabla)S = c_v \chi \frac{\Delta T}{T} + \frac{Q}{\rho T}, \tag{1.3}$$

where Q is the dissipative function, T is the temperature connected with ρ and S by the equation of state $T = T(\rho, S)$, c_v is the heat capacity of the medium at a constant volume, and χ is its thermal conductivity coefficient.

Suppose that in the atmosphere, the unperturbed state of which is described by the fields of wind velocity $\vec{V}$, density $\overline{\rho}$, pressure $\overline{p}$ and entropy $\overline{S}$ satisfying the system of equations (1.1)–(1.3), the small disturbances $\vec{v}', \rho'$, p' and S' arise relative to their unperturbed values. Below we focus on the important case, when the unperturbed atmosphere is considered as a layered medium in which all fields are considered only as functions of z, and the wind velocity vector $\vec{V}(z) = (V_x, V_y, 0)$ has only horizontal components. In this case, pressure $\overline{p}(z)$ and density $\overline{\rho}(z)$ are related by the static equation: $\overline{\rho}(z) g = -\frac{d\overline{p}(z)}{dz}$.

Substitute the values of $\vec{v} = \vec{V} + \vec{v}\,'$, $\rho = \overline{\rho} + \rho'$, $p = \overline{p} + p'$ and $S = \overline{S} + S'$ into the system of equations (1.1)–(1.3), and linearize it with respect to small perturbations, neglecting in the equations the influence of irreversible processes on these perturbations caused by viscosity and thermal conductivity of the medium. In this approximation, the propagation of disturbances in the medium is an adiabatic process, since any particle of gas does not exchange heat with the environment and retains entropy: $\frac{dS}{dt} = 0$. Taking into account the equation of the state $p = \rho^{\gamma} \frac{\overline{p}}{\overline{\rho}^{\gamma}} \exp(\frac{S - \overline{S}}{c_v})$ relating pressure with density and entropy, the equation of adiabaticity of motion takes the form $\frac{d(p/\rho^{\gamma})}{dt} = 0$. Then, in the linear approximation we obtain the following system of equations for perturbations of velocity $\vec{v}\,' = (v_x', v_y', v_z')$, pressure p' and density ρ':

$$\frac{D\vec{v}_{\perp}'}{Dt} + v_z' \frac{d\vec{V}}{dz} = -\frac{\nabla_{\perp} p'}{\overline{\rho}} + \vec{f}_{\perp} \tag{1.4}$$

$$\frac{Dv_z'}{Dt} = -\frac{1}{\overline{\rho}} \frac{\partial p'}{\partial z} - g \frac{\rho'}{\overline{\rho}} + f_z, \tag{1.5}$$

$$\frac{D\rho'}{Dt} + v_z' \cdot \frac{d\overline{\rho}}{dz} + \overline{\rho} \cdot \nabla_{\perp} \vec{v}_{\perp}' + \overline{\rho} \cdot \frac{\partial v_z'}{\partial z} = \overline{\rho} \cdot q, \tag{1.6}$$

$$c^{-2} (\frac{Dp'}{Dt} + v_z' \frac{d\overline{p}}{dz}) = \frac{D\rho'}{Dt} + v_z' \frac{d\overline{\rho}}{dz}, \tag{1.7}$$

where $\vec{v}_{\perp}' = (v_x', v_y')$ is the perturbation of the horizontal velocity, $c^2 = (\frac{\partial p}{\partial \rho})_S = \gamma \frac{\overline{p}}{\overline{\rho}}$ is the square of the adiabatic velocity of sound in the unperturbed atmosphere, and $\nabla_{\perp} = \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y}$ and

$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + (V \cdot \nabla)$ are differential operators. In case of horizontal wind, they act only on the horizontal coordinates of the functions included in system (1.4)–(1.7).

Let us reduce (1.4)–(1.7) to the system of only two equations with respect to perturbations of the vertical velocity $v_z{}'$ and pressure p'. To do this, we act by the operator D/Dt on both sides of equation (1.5) and eliminate $\frac{D\rho'}{Dt}$ from it using (1.7). Given the static equation $\frac{d\overline{p}}{dz} = -g\overline{\rho}$, we get

$$\frac{D^2 v_z{}'}{Dt^2} + N^2 v_z{}' + \overline{\rho}^{-1} \frac{D}{Dt}\left(\frac{\partial p'}{\partial z} + \frac{gp'}{c^2}\right) - \frac{Df_z}{Dt} = 0, \tag{1.8}$$

where

$$N^2 = -g\left(\frac{1}{\overline{\rho}}\frac{d\overline{\rho}}{dz} + \frac{g}{c^2}\right) \tag{1.9}$$

is the square of the Brent-Väisälä frequency that characterizes the static stability of the stratified medium.

In the linear approximation under consideration, when the relative perturbations of the medium are small, we will consider the medium to be statically stable, $N^2 > 0$, which implies a gradient of unperturbed density satisfying the condition $\overline{\rho}^{-1}\frac{d\overline{\rho}}{dz} < -\frac{g}{c^2}$.

After introducing new variables for the velocity and pressure components

$$\vec{u} = \left(\frac{\overline{\rho}}{\overline{\rho}_0}\right)^{1/2} \vec{v}_\perp{}', \; w = \left(\frac{\overline{\rho}}{\overline{\rho}_0}\right)^{1/2} v_z{}', P = \left(\frac{\overline{\rho}_0}{\overline{\rho}}\right)^{1/2} p', \tag{1.10}$$

where $\overline{\rho}_0 \equiv \overline{\rho}(z=0)$ is the unperturbed density at $z = 0$, the Eq. (1.8) takes the form,

$$\frac{D^2 w}{Dt^2} + N^2 w + \bar{\rho}_0^{-1} \frac{D}{Dt}\left(\frac{\partial P}{\partial z} + \Gamma P\right) - (\bar{\rho}/\bar{\rho}_0)^{1/2} \frac{Df_z}{Dt} = 0, \qquad (1.11)$$

where $\Gamma = (2\bar{\rho})^{-1} \frac{d\bar{\rho}}{dz} + \frac{g}{c^2}$ is the so-called Eckart parameter characterizing the effect of compressibility and density gradient on the inertial terms in the equations of motion (Gossard and Hook 1975).

In order to obtain the second equation for w and P we use equations (1.4) and (1.6), eliminating in the latter $\frac{D\rho'}{Dt}$ with (1.7) and making in them the change of variables (1.10). We then get

$$\frac{D\vec{u}}{Dt} + w\frac{d\vec{V}}{dz} = \bar{\rho}_0^{-1}\nabla_\perp P + (\bar{\rho}/\bar{\rho}_0)^{1/2} \vec{f}_\perp, \qquad (1.12)$$

$$\bar{\rho}_0^{-1} c^{-2} \frac{DP}{Dt} + \nabla_\perp \vec{u} + \left(\frac{\partial}{\partial z} - \Gamma\right) w - \left(\frac{\bar{\rho}}{\bar{\rho}_0}\right)^{1/2} q = 0. \qquad (1.13)$$

Acting by an operator $\nabla_\perp$ on (1.12), and $\frac{D}{Dt}$ as an operator on (1.13), subtracting one equation from another, we obtain the second equation connecting P and w:

$$\begin{aligned} &\frac{D^2 P}{Dt^2} - c^2 \Delta_\perp P + \bar{\rho}_0 c^2 \left[\left(\frac{\partial}{\partial z} - \Gamma\right)\frac{Dw}{Dt} - 2\left(\frac{d\vec{V}}{dz}\nabla_\perp\right) w\right] - \\ &-(\bar{\rho}\bar{\rho}_0)^{1/2} c^2 \frac{Dq}{Dt} + (\bar{\rho}\bar{\rho}_0)^{1/2} c^2 \nabla_\perp \vec{f}_\perp = 0 \end{aligned}, \qquad (1.14)$$

which, along with (1.11), constitutes the system of equations with respect to w and P.

In the particular case of constant wind velocity ($\vec{V}(z)$ = const), sound speed ($c(z) = \text{const}$) and Brent-Väisälä frequency ($N(z)$ = const), and in the absence of sources of force ($\vec{f} = 0$) and mass (q =0), the solution of the system of equations (1.11) and (1.14) describes linear acoustic-gravity waves and Lamb wave, which are analyzed in detail, for example, in monographs (Gossard and Hook 1975; Lighthill 1977; Gill 1982). In the case of the atmosphere with wind velocity depending on altitude $(\frac{d\vec{V}}{dz} \neq 0)$, this system, after excluding the vertical velocity w , was reduced by Ostashev (1987) to a single equation of the sixth order in time with respect to pressure perturbations p'.

For a 3D inhomogeneous moving atmosphere, when the wind velocity and temperature in the unperturbed atmosphere depend on all spatial coordinates *x, y* and *z*, it is not possible to obtain a closed equation for p' without using any additional assumptions. Below we consider one of the approximate acoustic equations for a 3D inhomogeneous moving medium obtained by Obukhov (1943). In the particular case of a moving layered medium Obukhov's equation will be compared with the equation obtained from the system of equations (1.11)–(1.14).

In general case, when a 3D inhomogeneous moving medium is non-stationary, but slowly varies in time over scales of the order of wave periods, the wave equations were obtained in different approximations in the works of Abdullayev and Ostashev (1988), Godin (1989), Pierce (1990) and Brekhovskikh and Godin (2007, p.1.2–1.3). Here we will continue the consideration of a layered moving medium, since in this case one can find the exact solutions of the Helmholtz equation, not limited to the high-frequency approximation. These solutions describe the sound field in a wide range of frequencies, including infrasonic frequencies below 20 Hz. Because of this, they are very important for understanding the process of distorting the form of broadband pulsed signals propagating from acoustic sources of an explosive nature (explosions, volcanic eruptions, earthquakes, shock waves from supersonic airplanes, rockets, etc.) in a real stratified moving atmosphere

The relationship between obtained equations and the equations of aeroacoustics

In aeroacoustics, there is a widely known equation that describes sound propagation in moving randomly inhomogeneous media, in particular, in a turbulent jet (Goldstein 1981). It is obtained from the system of acoustic equations of a non-uniform moving medium under the assumption that the average pressure in the medium is constant ($\overline{p}$ = const) and when there are external sources of force per unit volume $\vec{F} = \bar{\rho} \cdot \vec{f}$ and sources of volume velocity q . In the particular case of a layered medium with a horizontal flow $\vec{V}(z) = (V_x, V_y, 0)$ this equation takes the form

$$\frac{D}{Dt}[\frac{c^{-2}D^2 p'}{Dt^2} - \bar{\rho}\cdot\nabla(\frac{1}{\bar{\rho}}\nabla p')] + 2\frac{d\vec{V}}{dz}\nabla_\perp \frac{\partial p'}{\partial z} + \frac{D}{Dt}(\bar{\rho}\cdot\nabla\frac{\vec{F}}{\bar{\rho}}) - \\ -2\frac{d\vec{V}}{dz}\nabla_\perp F_z - \bar{\rho}\frac{D^2 q}{Dt^2} = 0 \qquad (1.15)$$

Eq. (1.15) can also be obtained from the system of equations (1.11) and (1.14) if we neglect the influence of gravity forces putting formally g = 0 and $\frac{d\bar{p}}{dz} = 0$. In this case $N = 0$ which means that the medium has a neutral stratification and $\Gamma = \frac{1}{2\bar{\rho}}\frac{d\bar{\rho}}{dz}$.

Let us take the derivative D/Dt from Eq. (1.14) and eliminate the vertical velocity w from it using the equation resulting from (1.5) and (1.10):

$$\frac{Dw}{Dt} = -\frac{1}{\bar{\rho}_0}[\Gamma P + \frac{\partial P}{\partial z}] + \frac{\bar{\rho}^{1/2} f_z}{\bar{\rho}_0^{\,1/2}}. \qquad (1.16)$$

Expressing P in (1.16) through pressure perturbation p' using (1.10), and taking into account that

$$(\frac{\partial}{\partial z} - \Gamma)\cdot(\frac{\bar{\rho}_0^{\,1/2}}{\bar{\rho}^{1/2}} \cdot\frac{\partial p'}{\partial z}) = (\frac{\bar{\rho}_0}{\bar{\rho}})^{1/2}(-\bar{\rho}^{-1}\frac{d\bar{\rho}}{dz}\cdot\frac{\partial p'}{\partial z} + \frac{\partial^2 p'}{\partial z^2}),\ \vec{F} = \bar{\rho}\cdot\vec{f}, \qquad (1.17)$$

we finally come to the Eq. (1.15)

The equation for a layered moving medium obtained by Brekhovskikh and Godin (1989, Eq. 15.5) can be reduced to the Eq. (1.15) for the case of stationary horizontal flow, density and sound speed.

Obukhov equation for sound field quasipotential

An approximate acoustic equation obtained by Obukhov (1943) for a 3D inhomogeneous moving medium with a vortical flow was used to calculate the acoustic field scattered by turbulent inhomogeneities of wind velocity and temperature in the atmosphere (Blokhintsev 1981).

Chunchuzov (1983, 1984) first applied this equation to calculate the field of a point sound source in the surface layer of the atmosphere with a non-uniform over height wind. Considering that the Obukhov equation played an important role in the development of wave theory in the acoustics of inhomogeneous moving media (Ostashev, 1985, 1997; Godin and Brekhovskikh 2007) we will focus on it in more detail. In deriving this equation, the following assumptions were made:

1. The velocity field of the main flow is incompressible, i.e. $div\vec{V} = 0$;
2. The values of the velocity in the main flow are small compared with the speed of sound in the medium, i.e. the Mach number for the flow $\beta_0 \equiv V_0 / c$ << 1, where V_0 is the maximum velocity value in the flow, and c is the sound speed;
3. The main flow is assumed to be quasi-stationary. The changes in the elements of the main flow over a period of time on the order of the oscillation period of the sound field are negligible;
4. The flow is assumed to have a nonzero vortex, and the intensity of the vortices $\vec{\Omega} = rot\vec{V}$ is a small first-order value compared with the angular frequency ω_0 of the sound wave.
5. The changes in the sound speed c due to temperature inhomogeneity of the medium are small $\left|grad(\lg c^2)\right| \lambda << 1$, where λ is the wavelength of the sound wave.

After the introduction of pressure potential $\Pi = \int dp / \rho$, which implies neglecting the effect of entropy changes ∇S in the flow on pressure changes ∇p, the hydrodynamic equations of an ideal compressible fluid were derived in the following form (Obukhov 1943):

$$\frac{\partial \vec{v}}{\partial t} + [rot\vec{v}, \vec{v}] + \text{grad}(\Pi + v^2 / 2) = 0, \tag{1.18}$$

$$\frac{\partial \Pi}{\partial t} + (\vec{v}, \nabla \Pi) + c^2 \nabla \vec{v} = 0, \tag{1.19}$$

where $c^2 = (\partial p / \partial \rho)_S$ is the square of the adiabatic sound speed.

The system (1.18) and (1.19) was linearized with respect to small perturbations $\vec{v}\,' = \vec{v} - \vec{V}$ and $\Pi' = \Pi - \Pi_0$ caused by the sound wave in the main undisturbed flow with pressure potential Π_0. In this case, the unperturbed fields in the zero-order approximation are themselves related by equations (1.18) and (1.19). Since the vector $\vec{v}$ is not a potential one it cannot be expressed through the usual potential of the field, as in the case of a stratified medium at rest in the absence of gravity. However, Obukhov introduced the so-called "quasipotential" ψ connected with velocity perturbations $\vec{v}\,'$ as follows

$$\vec{v}\,' = -\nabla \psi + \int [\vec{\Omega}, \nabla \psi] dt, \tag{1.20}$$

which allowed him to reduce the system (1.18) and (1.19) to a single approximate equation with respect to ψ. In the absence of flow vorticity ($\Omega = 0$), the quasipotential ψ becomes a usual potential of the sound field.

When substituting (1.20) into the linearized equation (1.18), we can neglect the terms of the second order of smallness, Ω^2 / ω^2 and $\frac{\Omega}{\omega}\beta_0$, where ω is the angular frequency of the wave, and $k = \omega / c$ is its wave number. This leads Eq. (1.18) to an approximate equation relating the quasipotential to acoustic pressure:

$$\Pi' = \frac{p'}{\bar{\rho}} = \frac{D\psi}{Dt}. \tag{1.21}$$

Linearizing also the second equation (1.19), we obtain

$$\frac{D\Pi'}{Dt} + (\nabla \Pi_0, \vec{v}\,') + c^2 \nabla \vec{v}\,' = 0, \tag{1.22}$$

where the square of the sound speed c^2 is taken at undisturbed density $\rho = \bar{\rho}$.

Let us express $\vec{v}'$ and Π' in Eq. (1.22) with the help of (1.20) and (1.21) taking into account that

$$\nabla\left[rot\vec{V} \times \nabla\psi\right] = (\nabla\psi \cdot rotrot\vec{V}) = -\nabla\psi \cdot \Delta\vec{V}. \tag{1.23}$$

Then, up to the terms of the order of $\frac{\Omega}{\omega}$ and $\left|\frac{\nabla\Omega}{k\Omega}\right| \cdot \frac{\Omega}{\omega}$, we obtain the Obukhov equation

$$\frac{D^2\psi}{Dt^2} - c^2\Delta\psi - (\nabla\Pi_0 \cdot \nabla\psi) - c^2\int dt(\nabla\psi \cdot \Delta\vec{V}) + (\nabla\Pi_0 \cdot \int dt[\vec{\Omega} \times \nabla\psi]) = 0 \tag{1.24}$$

Note that in the resulting equation (1.24) the small terms $\sim \beta_0{}^2$ and $\sim \frac{\left|\nabla c^2\right|}{kc^2}\beta_0$ were neglected taking into account assumptions made during its derivation.

Comparison with the Goldstein equation

If the medium moves with a horizontal velocity $\vec{V}(z) = (V_x, V_y, 0)$ depending only on the *z* coordinate, then in the equation of motion for the unperturbed flow, the advective term $(\vec{V}\nabla)\vec{V}$ vanishes, which in the absence of gravity ($g = 0$) means that the pressure is constant throughout the flow: $\nabla\bar{p} = 0$. In this case, $\nabla\Pi_0 = 0$ in Eq. (1.24), and from the equation of the state of the medium it follows that $\frac{\nabla\bar{\rho}}{\bar{\rho}} = -\frac{\nabla T}{T}$, hence the relative changes in the unperturbed density as well as in temperature are considered small over the wavelength.

To write the resulting equation with respect to the pressure perturbation p' we use its relation (1.21) with the quasipotential ψ. Let us use twice the operator $\frac{D}{Dt}$ on Eq. (1.24) and take into account the relation

$$\frac{D}{Dt}\Delta(\frac{D\psi}{Dt}) =$$
$$= \Delta\frac{D}{Dt}(\frac{D\psi}{Dt}) - 2(\frac{d\vec{V}}{dz}\cdot\nabla)\ \frac{\partial}{\partial z}\frac{D\psi}{Dt} - (\frac{d^2\vec{V}}{dz^2}\cdot\nabla)\frac{D\psi}{Dt}. \qquad (1.25)$$

Then, we can rewrite the resulting equation through acoustic pressure p' using (1.21). Comparing the obtained equation with (1.15) in the absence of sources of mass and forces (q = 0, f = 0), we come to the conclusion that these equations coincide when the small terms of the order $k^{-1}\left|\bar{\rho}^{-1}\frac{d\bar{\rho}}{dz}\right|$ and $(\frac{dV/dz}{\omega})^2$ are neglected. In other words, in the case of a layered medium with a horizontal flow velocity the Obukhov equation (1.24) can be reduced to Eq. (1.15) under the assumption that the temperature (or density) stratification of the medium has a weak effect on the sound field and the flow vorticity is small in magnitude compared to the sound frequency. These conditions, as will be shown below, are well satisfied in the surface atmospheric layer for sound with frequencies above 10 Hz. The dominant influence of the wind stratification on the sound field compared with the influence of temperature stratification is also evidenced by the results of experiments on the propagation of sound in a stably stratified ABL (Otrezov and Chunchuzov 1986, 1987).

For sufficiently low frequencies, for which the change in temperature with height z is no longer small at the wavelength of the sound wave, it is also necessary to take into account the influence of temperature gradients on the sound field. In this case, to calculate the sound field in the ABL, we will use equation (1.15), since the most significant temperature change with height above the ground usually occurs in the near-surface atmospheric layer several tens or hundreds of meters thick, while the average pressure in the atmosphere changes with height much more slowly. Its significant decrease occurs only at the height $\bar{H} = c^2/(\gamma g)$ of a homogeneous atmosphere, which is about 8 km.

1.2. Field of a point harmonic sound source in the atmosphere stratified by temperature and wind velocity

The Helmgholtz equation

For a stationary layered medium, it is convenient to transfer to the spectral Fourier components $\hat{P}(\omega,\vec{\xi},z)$ for pressure perturbations

$$p'(\vec{r},z,t)=\int_{-\infty}^{\infty}d\omega\int_{-\infty}^{\infty}d\xi_x\int_{-\infty}^{\infty}d\xi_y\hat{P}(\omega,\vec{\xi},z)\cdot e^{-i\omega t+i\vec{\xi}\cdot\vec{r}}\ , \qquad (1.26)$$

and for the sources of mass $\bar{\rho}\cdot q(\vec{r},z,t)$ and force $\vec{f}(\vec{r},z,t)$, where $\vec{r}=(x,y)$ is the horizontal radius vector drawn from the origin of coordinates to the observation point, and $\vec{\xi}=(\xi_x,\xi_y)$. Since p' is a real quantity, then $\hat{P}(\omega,\vec{\xi},\mathrm{z})=\hat{P}^*(-\omega,-\vec{\xi},\mathrm{z})$, therefore (1.26) can be replaced by the integral over $\omega\geq 0$ plus its complex-conjugate value. Substituting (1.26) into the wave equation (1.15), we obtain second-order differential equation for the spectral amplitude $\hat{P}(\omega,\vec{\xi},z)$.

$$\begin{aligned}&\frac{d^2\hat{P}}{dz^2}-(\bar{\rho}^{-1}\frac{d\bar{\rho}}{dz}+\Omega^{-2}\frac{d\Omega^2}{dz})\cdot\frac{d\hat{P}}{dz}+(\frac{\Omega^2}{c^2}-\xi^2)\cdot\hat{P}=\\&=i\bar{\rho}\cdot\Omega\cdot\hat{q}+i\cdot\vec{\xi}\cdot\hat{\vec{f}}_{\perp}-(\bar{\rho}^{-1}\frac{d\bar{\rho}}{dz}+\Omega^{-2}\frac{d\Omega^2}{dz})\cdot\hat{f}_z+\frac{d\hat{f}_z}{dz}\end{aligned}\ , \qquad (1.27)$$

where $\Omega=\omega-\vec{\xi}\cdot\vec{V}$, $\hat{\vec{f}}(\omega,\vec{\xi},z)=(\hat{\vec{f}}_{\perp,}\hat{f}_z)$ and $\bar{\rho}\cdot\hat{q}(\omega,\vec{\xi},z)$ are Fourier transforms of the components of external forces and mass sources, respectively.

Using this equation, we will find the field of a point source with a bulk velocity *Q* (*t*) located in a moving stratified layer of the atmosphere at a certain height above the flat ground surface. We assume that the atmospheric layer with arbitrary smooth profiles of wind velocity and temperature is in the upper half-space, and the force sources are absent, i.e. $f=0$ in (1.15) and (1.27).

Using a new function $\tilde{P}(\omega,\vec{\xi},z)$ by replacing

$$\tilde{P}=\alpha^{-1/2}\cdot\hat{P}\,,\;\; \alpha=\bar{\rho}\cdot\Omega^2=\bar{\rho}\cdot(\omega-\vec{\xi}\cdot\vec{V})^2\,, \tag{1.28}$$

we obtain from (1.27) the following inhomogeneous equation,

$$\frac{d^2\tilde{P}}{dz^2}+[k_z{}^2+\frac{1}{2}\cdot\frac{\alpha''}{\alpha}-\frac{3}{4}\cdot(\frac{\alpha'}{\alpha})^2]\tilde{P}=i\bar{\rho}^{1/2}\hat{q}\,, \tag{1.29}$$

where the prime denotes the derivative of $\alpha(z)$ with respect to z, and

$$k_z{}^2=\frac{\Omega^2}{c^2}-\xi^2\,, \qquad \xi^2=\xi_x{}^2+\xi_y{}^2\,. \tag{1.30}$$

Equation (1.27) should be added with a boundary condition on the ground surface and the condition of the absence at infinity of sources of acoustic energy. This implies that the density of acoustic energy tends to zero when $z\to\infty$ and the amplitude of the sound wave is bounded everywhere in the upper half-space

$$\left|\tilde{P}(z)\right|<\infty\;\; . \tag{1.31}$$

For some types of the underlying ground surface, such as grass, for which the sound speed is much lower than in air, the acoustic impedance $Z=-\left[\dfrac{p'}{v_z{}'}\right]_{z=0}$ depends only slightly on the angle of incidence of a plane acoustic wave. Such surfaces are considered locally reactive (Piercy et al. 1977), since the vertical oscillatory velocity at each point of the surface is determined locally by the acoustic pressure at that same point.

Taking into account the relation (1.5) (for the case $f = g = 0$) between the vertical velocity component and the pressure perturbation, the boundary condition at the surface z = 0 takes the form

$$(\frac{\partial}{\partial z}+\gamma)\Big|_{z=0}\hat{P}=0\,, \tag{1.32}$$

and for $\tilde{P}(\omega, \vec{\xi}, z)$ leads to the condition

$$(\frac{\partial}{\partial z} + \gamma + \frac{\alpha'}{2\alpha})\Big|_{z=0} \tilde{P} = 0, \qquad (1.33)$$

where $\gamma = \dfrac{i\omega}{c\tilde{Z}}$, and $\tilde{Z} = \dfrac{Z}{\overline{\rho}(z=0) \cdot c(z=0)}$ is the specific ground surface impedance.

Generrally, the frequency dependencies of the real and imaginary parts of the impedance of different types of the underlying ground surfaces can be obtained only empirically using various methods of its measurement or parameterization (Wenzel 1974; Piercy et al. 1977; Piercy and Embleton 1981; Attenborough et al. 2011). For example, the measurements of the impedance for grass covered surface (Figure 1.1) show that its reactive ($\operatorname{Im}\tilde{Z}$) and active ($\operatorname{Re}\tilde{Z}$) components increase with decreasing frequency (Piercy et al. 1977).

The tendency of impedance growth with decreasing frequency leads to the fact that, at frequencies below 10 Hz, the underlying surface behaves almost like a rigid surface for which $|\tilde{Z}| >> 1$ (Otrezov and Chunchuzov 1987). Therefore, for the low frequencies of interest we first consider the limiting case of an absolutely rigid surface ($|\tilde{Z}| \to \infty$), setting $\gamma = 0$ in the boundary condition (1.32).

Field of a point harmonic sound source above a completely rigid surface

For a point source located at a point $(0, 0, z_0)$ the volume velocity and its Fourier transform $\hat{q}(\omega, \vec{\xi}, z)$ are presented in the form,

$$q(x, y, z, t) = Q(t) \cdot \delta(x) \cdot \delta(y) \cdot \delta(z - z_0), \qquad (1.34)$$

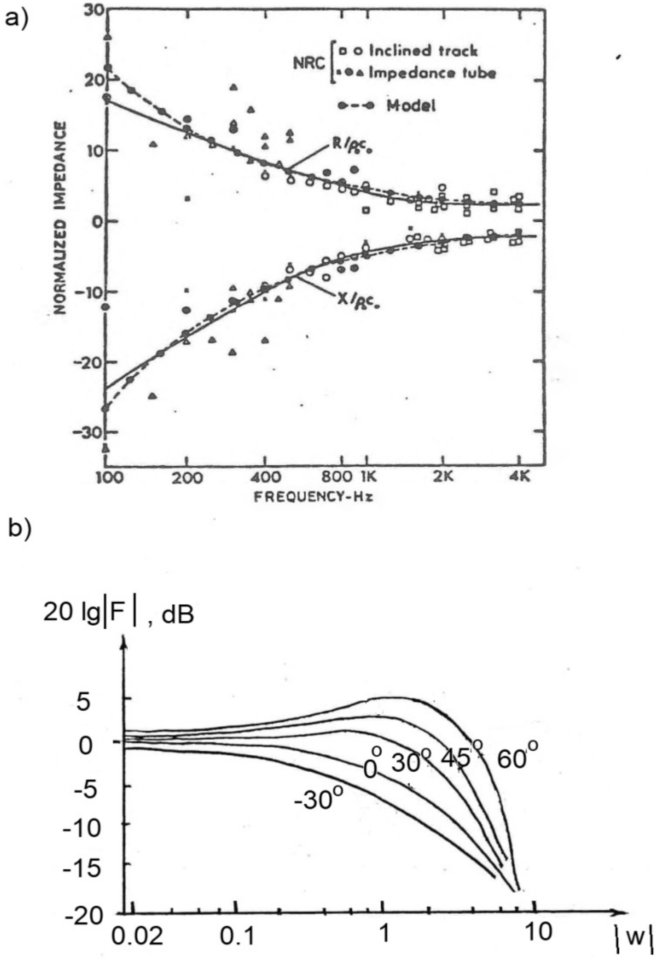

Figure 1.1. The effect of specific ground impedance, $Z = R + iX$, on sound wave attenuation in a homogeneous atmosphere. a) Frequency dependence of the real and imaginary parts of the specific impedance $\tilde{Z}$ for the grass covered surface (Piercy et al. 1977). b) The amplitude of the wave from a point source above the surface with impedance $\tilde{Z}$ normalized to the amplitude of spherical wave depending on the dimensionless distance $w = ikr/(2\tilde{Z}^2)$ and for different values of the phase shift *tan*$^{-1}$(*X/R*) (Wenzel 1974).

$$\hat{q}(\omega,\vec{\xi},z)=(2\pi)^{-2}\cdot\hat{Q}(\omega)\cdot\delta(z-z_0)\,, \tag{1.35}$$

where $\hat{Q}(\omega)$ is the Fourier component of $Q(t)$, and $\delta(x)$ is the delta function.

We will follow the general method of finding the solution of the inhomogeneous Helmholtz equation (1.29) with boundary conditions (Brekhovskikh 1973). Let us denote by $\tilde{P}_1(z)$ and $\tilde{P}_2(z)$ the solutions of the homogeneous equation (1.29) satisfying the condition $\left|\tilde{P}(z)\right|<\infty$ at $z\to\infty$ and the boundary condition at the rigid ground surface, respectively,

$$\frac{d\hat{P}}{dz}\Big|_{z=0}=\alpha^{1/2}\left(\frac{d\tilde{P}}{dz}+\frac{\alpha'}{2\alpha}\tilde{P}\right)\Big|_{z=0}=0\,. \tag{1.36}$$

At a point of the source location $z=z_0$ there is a discontinuity of the derivative $\frac{d\tilde{P}}{dz}$,

$$\frac{d\tilde{P}}{dz}\Big|_{z_0+0}-\frac{d\tilde{P}}{dz}\Big|_{z_0-0}=\frac{i}{(2\pi)^2}\cdot\bar{\rho}^{1/2}(z_0)\cdot\hat{Q}(\omega)\,. \tag{1.37}$$

This can be verified if both sides of Eq. (1.29) are integrated over z from $z_0-\varepsilon$ to $z_0+\varepsilon$ in the small ε neighborhood of the point where the source is found, and then tend ε to zero. Taking into account the boundary conditions and the "matching" condition (1.37) the solution (1.29) can be presented as

$$\tilde{P}(\omega,\vec{\xi},z)=-(2\pi)^{-2}i\bar{\rho}^{1/2}(z_0)\cdot\hat{Q}(\omega)\cdot G(z,z_0,\vec{\xi},\omega)\,, \tag{1.38}$$

where