Reliability of Stochastic Stress-Strength Models

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Dedicated
to
My Beloved
Parents

Contents

NOTATIONS

R_n: Reliability after n cycles of loading

$R(t)$: Reliability at time t

R: Reliability, independent of the cycle number (fixed)

X_i: Stress random variable for thei^{th} cycle

Y_i: Strength random variable on the i^{th} cycle

E_i: Event that no failure occurs on the i^{th} cycle

$f_i(x_i)$: Probability density function for stress random variable x_i, $i = 0,1,2,...$

$g_i(y_i)$: Probability density function for strength random variable y_i, $i = 0,1,2,...$

$\pi_i(t)$: Probability of i cycles occurring in the time interval [0, t]

p: Probability of success in any trial

$q = 1 - p$: Probability of failure

α: mean number of occurrences per unit time

INTRODUCTION

In the present scenario of global competition and liberalisation, it is imperative that Indian industries become fully conscious of the need to produce reliable products meeting international standards. Reliability is one of the performance measures of a physical item used in industry. The concept of reliability is as old as humankind itself. Humans have long been concerned with the problem of the unreliability of the products they use. Reliability means the probability that a failure may not occur in a given time interval. The government and industrial agencies recognise the need and importance of reliability studies as unreliable systems would result in schedule delays, inconvenience, customer dissatisfaction and perhaps even the loss of national security. Hence reliability should provide suitable techniques, methods, information, and procedures to ensure the most effective balance between the cost and benefits accomplished in the design, development, and production of a system.

Reliability theory is a well-established scientific discipline with its own principles and methods for solving its problems. Probability theory and mathematical statistics play a major role in most of the problems in reliability theory. In fact, reliability is often defined in terms of probability. It deals with the inter-disciplinary use of probability, statistics, and stochastic modelling, combined with engineering insights into the design and scientific understanding of failure mechanisms, to study the various aspects of reliability. It also encompasses issues such as (i) reliability modelling, (ii) reliability analysis and optimization, (iii) reliability engineering, (iv) reliability science, (v) reliability technology, and (vi) reliability management.

The notion of a quantitative analysis of reliability dates back to about the 1940s, when mathematical techniques, some of which were quite new, were applied to many operational and strategic problems in World War II. Before this period, the concept of reliability was primarily qualitative and subjective, based on intuitive notions. Actuarial methods had been used to estimate survivorship of railroad equipment and in other applications early in the twentieth century (Nelson 1982). These were the forerunners of the statistical and probabilistic models and techniques that form the basis of modern reliability theory.

The needs of modern technology, especially the complex systems used in military and space programmes, led to the quantitative approach, based on mathematical modelling and analysis. In space applications, high reliability is especially essential because of the high level of complexity of the systems and the inability to repair or change most systems once they have been deployed in an outer space mission. This gave impetus to the rapid development of reliability theory and methodology beginning in the 1950s. As the space programme evolved and the success of the quantitative approach became apparent, the analysis was applied in many non-defence/space applications as well. Important newer areas of application are biomedical devices and equipment, aviation, consumer electronics, communications, and transportation.

A more quantitative (or mathematical) and formal approach to reliability grew out of the demands of modern technology, particularly out of experiences in the World War II with complex military systems (Barlow and Proschan1965, 1). Barlow (1984) provides a historical perspective of mathematical reliability theory up to that time. Similar perspectives on reliability engineering in electronic equipment, space reliability technology, nuclear power system reliability, and software reliability can be found in Coppola (1984), Cohen (1984), Fussel (1984), and Shooman (1984), respectively.

The 50th Anniversary special publication of the IEEE Transactions on Reliability (vol. 47, no. 3-SP, September 1998) contains several historical papers dealing with various aspects of reliability (e.g., reliability prediction, component qualification) applications, including space systems, physics, electronics, and communications.

As the study of reliability is applicable not only to products, including hardware and software, but also to services, reliability should provide suitable techniques, methods, information, and procedures to ensure the most effective balance between the cost and benefits accomplished in the design, development, and production of a system.

CHAPTER 1

RELIABILITY MODELS

Definition and Concept of reliability

A large number of definitions of reliability are found in the literature. The variety of such definitions is because the interpretation of the word *reliability* depends very much on the context in which it is used. One such definition is: "Reliability is the probability of a device performing its purpose adequately for the period intended under the given operating conditions" (see Bazovsky [1961] 2004, 11). This definition brings into focus four important factors:

(i) The reliability of a device is expressed as a probability.
(ii) The device is required to give an adequate performance.
(iii) An adequate performance duration is specified.
(iv) The importance of environmental conditions or operating conditions, such as temperature, humidity, stress, shock, vibrations, etc., is considered.

Reliability and hazard models

The prediction of system reliability is based on a number of factors, such as life characteristics, operating conditions, and failure distribution. Thus, the initial step in reliability prediction is the determination of life characteristics.

Failures are a way of life in the modern technological world; the penalties paid by people in terms of money, time, and security are becoming more and more severe, due to the increasing application of complexities and automation. This does not mean that systems cannot be made more reliable. Better understanding of failures, improved manufacturing techniques, careful planning and designing of new systems, and the proper selection of components are some of the approaches that can be tried to reduce the unreliability level of systems.

If a random sample of items is taken from a population and is put to the test under a set of fixed or given environmental or operating conditions, and the number of samples that fail successively each time is counted, the data so obtained will represent the life length of each item. The life length can be measured depending on whether the item is repairable (e.g., computers, aeroplane, television), or non-repairable (e.g., fuses). For repairable items, life can be measured by failure rate or meantime between failures, whereas for non-repairable items, life can be measured by the length between meantime and failure.

Many factors can lead to the failure of a product or can contribute to the likelihood of the occurrence of a failure. Such factors include design, materials, manufacture, quality control, shipping and handling, storage use, environment, age, the occurrence of related previous failures, the failure of an interconnected component, part, or system, quality of repair after a previous failure, and so forth. Furthermore, because of the nature of the many factors that may be involved, the time of occurrence of a failure (i.e., the lifetime of the item) is unpredictable. Reliability and maintenance deal with the estimation and prediction of the probability of failure (i.e., the randomness inherent in this event), as well as many related issues such as prevention of failure, cost of prevention, optimisation of policies for dealing with failure, and the effect of environment on failure rates.

Since failure cannot be prevented entirely, it is important to minimise both its probability of occurrence and the impact of failures when they do occur. This is one of the principal roles of reliability analysis. Increasing reliability entails costs to the manufacturer, the buyer, or both. There is often a trade-off between these costs. Increasing reliability by improving design will lead to fewer failures and may decrease maintenance costs later on.

Failure rate

The failure rate λ is expressed in terms of failures per unit time. It is computed as a simple ratio of the number of failures f during a specified test interval to the total test time T of the items undergoing test. Thus

$$\lambda = \frac{f}{T} \tag{1}$$

where

$$\lambda = failure\ rate$$
$$f = number\ of\ failures\ during\ the\ test\ interval$$
$$T = total\ test\ time$$

When the design is mature, the failure rate is fairly constant during the operating or useful life of the system. The small failure rate system gets a higher level of reliability.

Meantime to failure (MTTF)

It is often more interesting to know the mean time to failure of a component than the complete failure details. This parameter is assumed to be the same for all the components that are identical in the design and that operate under identical conditions. The meantime to failure is used where a system is replaced after a failure. If we have life-test information on a population of N items with failure times $t_1, t_2, \dots, t_n$ then the MTTF is defined as

$$MTTF = \frac{1}{N}\sum_{i=1}^{n} t_i \quad (2)$$

The MTTF measures average times to failure with the modelling assumption that the failed system is not repaired.

Meantime between failures (MTBF)

The meantime between failures is the predicted elapsed time between inherent failures of a system during operation. The MTBF can be calculated as the arithmetic mean (average) time between the failures of a system. The MTBF is typically part of a model that assumes the failed system is immediately repaired, as a part of a renewal process. The failure rate is simply the multiplicative inverse of the MTBF$\left(\frac{1}{\lambda}\right)$.

For each observation, the *down time* is the instantaneous time it went down, which is after (i.e., greater than) the moment it went up (the *up time*). The difference is the amount of time it was operating between these two events.

The MTBF is the sum of the operational periods divided by the number of observed failures. If the down time (with space) refers to the start of the down time (without space) and the up time (with space) refers to the start of the up time then:

$$MTBF(\theta) = \frac{\sum(start\ of\ down\ time - start\ of\ up\ time)}{number\ of\ failures}$$

Reliability function

The reliability function is the complement of the cumulative distribution function. If modelling the time to failure, the cumulative distribution function represents the probability of failure and the reliability function represents the probability of survival.

The probability of failure as a function of time can be defined by

$$P(T \leq t) = F(t), \quad t \geq 0$$

where T is a random variable denoting the failure time. If we define reliability as the probability of success or the probability that the system will perform its intended function at a certain time t, then the reliability function is expressed as

$$R(t) = 1 - F(t) = P(T > t)$$

If the time to failure random variable T has a density function $f(t)$ then

$$R(t) = 1 - F(t) = 1 - \int_0^t f(\tau)d\tau = \int_t^\infty f(\tau)d\tau \qquad (3)$$

Hazard function

The hazard function is defined as the limit of the failure rate as the interval approaches zero. Thus the hazard function is the instantaneous failure rate.

The probability of the failure of a system in a given time interval $[t_1, t_2]$ can be expressed in terms of the reliability function as

$$\int_{t_1}^{t_2} f(t)dt = \int_{t_1}^{\infty} f(t)dt - \int_{t_2}^{\infty} f(t)dt$$
$$= R(t_1) - R(t_2)$$

and the failure rate during the interval $[t_1, t_2]$ as

$$= \frac{R(t_1) - R(t_2)}{(t_2 - t_1)R(t_1)} \qquad (4)$$

If the time interval is defined as$[t, t + \Delta t]$, the expression in equation (1) becomes

$$\frac{R(t) - R(t + \Delta t)}{\Delta t \, R(t)} \qquad (5)$$

Therefore the hazard rate is

$$h(t) = \lim_{\Delta t \to 0} \frac{R(t) - R(t + \Delta t)}{\Delta t \, R(t)}$$
$$= \frac{1}{R(t)} \left[\frac{-d}{dt} R(t) \right] \qquad (6)$$

$$= \frac{1}{R(t)}\left[\frac{-d}{dt}(1 - F(t)\right]$$

$$= \frac{1}{R(t)}F'(t)$$

$$= \frac{f(t)}{R(t)} \qquad (7)$$

The importance of the hazard function is that it indicates the change in the failure rate over the life of a population of devices.

Relationship between various functions

From (6)

$$h(t) = \frac{-1}{R(t)}\frac{dR(t)}{dt}$$

By rearranging the above equation and integrating the proper limits, we get

$$h(t)dt = \frac{-dR(t)}{R(t)}$$

$$\int_0^t h(t)dt = -logR(t) \qquad (8)$$

(or)

$$R(t) = exp\left[-\int_0^t h(t)dt\right] \qquad (9)$$

$$F(t) = 1 - R(t)$$

$$= 1 - exp\left[-\int_0^t h(t)dt\right] \qquad (10)$$

$$f(t) = \frac{d}{dt}F(t)$$

$$= \frac{d}{dt}\left[1 - exp\left[-\int_0^t h(t)dt\right]\right]$$

$$= h(t)exp\left[-\int_0^t h(t)dt\right] \qquad (11)$$

$$f(t) = h(t).R(t) \qquad (12)$$

$$h(t) = \frac{f(t)}{R(t)} = \frac{f(t)}{1 - F(t)} \qquad (13)$$

A reliability function and its related hazard function are unique. Thus each reliability function has only one hazard function and vice versa.

Failure models

A failure model (or) a failure distribution represents an attempt to describe mathematically the length of life of a device. The models of possible failures for the items in question will affect the analytical form of the failure distribution. The choice of failure distribution on the basis of physical considerations is difficult and thus the identification of failure distribution with the failure rate is important. Some typical failure models are:

(a) Constant hazard model

This is of the form $h(t) = \lambda$ where $\lambda$is a constant that is independent of time. This characteristic is exhibited by many products, particularly electronic components. The model has been used for many years in reliability studies where the constant rate is observed for failures.

If $h(t) = \lambda$ then

$$f(t) = h(t).exp\left[-\int_0^t h(t)dt\right] \quad (14)$$

$$= \lambda exp(-\lambda t)$$

$$R(t) = 1 - F(t)$$

$$= 1 - \int_0^t f(t)dt$$

$$= 1 - \int_0^t \lambda exp(-\lambda t)dt$$

$$R(t) = exp(-\lambda t) \quad (15)$$

$$F(t) = 1 - R(t) = 1 - exp(-\lambda t) \quad (16)$$

$$MTTF = \int_0^\infty R(t)dt$$

$$= \int_0^\infty exp(-\lambda t)dt$$

$$MTTF = \frac{1}{\lambda} \quad (17)$$

(b) Linear hazard model

Many components that are subject to mechanical stress fail due to wearing out or deteriorating. The hazard rate of such components increases with time. The linear hazard model is the simplest time-dependent model and has the form:

$$h(t) = bt, \qquad t > 0$$

where$b$ is a constant.

Therefore for such a hazard model

$$f(t) = h(t).exp\left[-\int_0^t h(t)dt\right]$$

$$= bt.exp\left[-\int_0^t btdt\right]$$

$$f(t) = bt.exp\left[-\frac{bt^2}{2}\right] \quad (18)$$

This is the density of Rayleigh distribution. Hence, reliability may deal with Rayleigh distribution when the hazard model is a linear one. The reliability function is as follows:

$$R(t) = 1 - F(t)$$

$$= 1 - \int_0^t f(t)dt$$

$$R(t) = exp\left[-\frac{bt^2}{2}\right] \quad (19)$$

$$MTTF = \int_0^\infty R(t)dt$$

$$MTTF = \sqrt{\frac{\pi}{2b}} \quad (20)$$

(c) Nonlinear hazard model

The hazard rate is not always a linearly increasing function of time. It is therefore useful to have a more general form of the hazard model

$$h(t) = at^b$$

where$a$ and $b$are constants. Then

$$f(t) = at^b.exp\left[-\int_0^t at^b dt\right]$$

$$f(t) = at^b.exp\left[-\frac{at^{b+1}}{b+1}\right] \qquad (21)$$

$$R(t) = 1 - \int_0^t at^b.exp\left[-\frac{at^{b+1}}{b+1}\right]dt$$

$$R(t) = exp\left[-\frac{at^{b+1}}{b+1}\right] \qquad (22)$$

$$MTTF = \frac{\Gamma\left(\frac{1}{1+b}\right)}{(b+1)\left[\frac{a}{b+1}\right]\left(\frac{1}{b+1}\right)} \qquad (23)$$

This general form is known as the Weibull model and generates a wide range of curves for various sets of a and b. When $b = 0$ it represents the constant hazard model and when$b = 1$it represents the linearly increasing hazard model. The parameter a affects the amplitude and b the shape of $h(t)$; therefore, they are called scale and shape parameters, respectively.

Stress dependent hazard models

Basically, the reliability of an item is defined under the stated operating and environmental conditions. This implies that any change in these conditions can affect the failure rate of the item and consequently its reliability. The failure rate of almost all components is stress-dependent. A component can be influenced by more than one kind of stress. For such cases, a power function model of the form $h(t){\sigma_1}^{a_1}{\sigma_2}^{a_2}$ can be postulated for each piece of equipment, where a_1, a_2 are constants, $\sigma_1$and σ_2 are stress ratios for two different kinds of stresses, and $h(t)$ is the failure rate at rated stress conditions.

These models are used for the accelerated testing of components. In addition to the stress factor, there can be other factors, such as the complexity factor, application factor, quality factor, etc., which should be used for the correct estimation of failure rates.

Distributions

Some of the commonly used failure-time distributions are briefly listed below.

Exponential distribution

This is a distribution of the time to an event when the probability of the event occurring in the next small time interval does not vary through time. It is also the distribution of the time between events when the number of events in any time interval has a Poisson distribution.

Exponential distribution is the most commonly used distribution in reliability. This distribution possesses a convenient closure property that applies to the systems made up of exponentially distributed components. If the system fails when the first component fails, and all the components operate independently, then the system life distribution is also exponential.

The probability density function of exponential distribution is

$$f(t) = \lambda exp(-\lambda t)\ ;\ \lambda > 0, t \geq 0$$

Where λ is a parameter and the reliability function is

$$R(t) = exp(-\lambda t), \qquad t \geq 0$$

The hazard function for the exponential density function is

$$h(t) = \frac{f(t)}{R(t)} = \lambda$$

Deemer et al. (1955) studied the estimation of parameters of truncated exponential distribution. Tong (1974) has given a note on the estimation of $P(Y < X)$ for exponential cases. Raghava Char et al. (1983) obtained reliability expressions when the stress and strength of the system were attenuated in cascade reliability. Uma Maheswari et al. (1993) studied the reliability of single strength under n-stresses when stress and strength follow exponential distribution. Uma Maheswari et al (1992) studied the reliability comparison of n-cascade system with addition of n-strengths system.

Normal distribution

Normal distribution is a two parametric distribution of a continuous random variable. "The parameters μ (location parameter) and $\sigma > 0$(scale parameter) are arbitrary and represent the mean and standard deviation of the random variable." The theoretical range for normal variates is from $-\infty$ to ∞. The sensible range for stress–strength is from 0 to ∞. However, in practice the mean in terms of standard deviation will be considerably above the "natural zero" value and hence the non-negative nature of the stress and strength variables will not be practically incompatible with the two-sided infinite range of the theoretical normal variable. If this is not the case one can assume some truncated normal distributions for stress and strength (Kapur and Lamberson 1977).

The probability density function of normal distribution is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}\ , x\epsilon(0,\infty), -\infty < \mu < \infty, 0 < \sigma < \infty$$

Davis (1952) states that for those items under close control of both manufacturing processes and the conditions of tests, a normal theory of failure seems to be constant with the data. The normal distribution has an increasing failure rate.

There are two principal applications of normal distribution in reliability:

(i) The relationship of variable characteristics of equipment to specifications.
(ii) The wear-out life of components and the distribution of component failures due to extended periods of operation (Shooman 1966).

Two-sided confidence limits for reliability based on normal samples have been studied by Govindarajulu (1967). The reliability factor for normally distributed stress and strength in a cascade system has been obtained by Raghava Char et al. (1987). Uma Maheswari (1993) also worked on normal distribution.

Log-normal distribution

Log-normal distribution is applicable to random variables that are constrained by zero but have a few very large values.

Log-normal distribution is a very useful and flexible model for reliability. It is closely related to normal distribution, since the logarithm of a log-normal random variable has a normal distribution. The log-normal distribution can be used for the successful modelling of failures due to chemical reactions or molecular diffusion or migration. Thus this distribution has an increasing failure rate.

The probability density function of log-normal distribution is

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} exp\left(\frac{-(\log x - \mu)^2}{2\sigma^2}\right), \quad x \geq 0$$

where μ and σ are parameters such that $-\infty < \mu < \infty$ and $\sigma > 0$.

Gupta (1962) considered the lognormal distribution as yet another model that merits consideration in view of its undoubted links with the normal distribution and the fact that the variable in question is connected to the positive half of the real axis. The log-normal distribution gives a good fit to repair time distributions.

Weibull distribution

The Weibull variate is commonly used as a lifetime distribution in reliability applications. The two-parameter Weibull distribution can represent decreasing, constant, or increasing failure rates. These correspond to the three sections of the “bathtub curve” of reliability, referred to also as “burn-in,” “random,” and “wear-out” phases of life.

The probability density function for two-parameter Weibull distribution is:

$$f(x) = \left[\frac{\beta x^{\beta-1}}{\eta^{\beta}}\right] exp\left(-\left(\frac{x}{\eta}\right)^{\beta}\right), \quad 0 \leq x < \infty$$

where$\eta > 0$ is the scale parameter and $\beta > 0$ is the shape parameter.

Further flexibility can be introduced into the Weibull distribution by adding a third parameter, which is a location parameter and is usually denoted by the symbol gamma (γ). The probability density is zero for $x < \gamma$ and then follows a Weibull distribution with its origin at γ. In reliability applications, gamma is often referred to as the minimum life, but this does not guarantee that failures will not occur below this value in the future.

The probability density function is

$$f(x) = \left[\frac{\beta(x-\gamma)^{\beta-1}}{\eta^{\beta}}\right] exp\left\{-\left[\frac{(x-\gamma)}{\eta}\right]^{\beta}\right\}, x \geq \gamma, \gamma \leq x \leq +\infty$$

where$\eta > 0$ is a scale parameter,$\beta > 0$ is a shape parameter, and $\gamma > 0$ is the location parameter.

The hazard function decreases for $\beta < 1$, increases for $\beta > 1$, and is constant when β is exactly one. When $\beta < 1$, the Weibull distribution takes a hyper exponential shape and reduces to the exponential when $\beta = 1$ and Rayleigh for $\beta = 2$.

Zelem and Dannemillen (1961) discussed the Weibull distribution as a failure-time distribution. Different types of situations could also be accommodated in this model on account of the extra degree of randomness. The problems of Weibull distribution are worked out by Mann (1972) and Bennet (1973).

Gamma distribution

The gamma distribution includes the chi-squared, Erlang, and exponential distributions as special cases, but the shape parameter of the gamma is not confined to integer values. The gamma distribution starts at the origin and has a flexible shape.

The failure density function for a gamma distribution is

$$f(x) = \frac{\lambda^{\eta}}{\Gamma(\eta)} x^{\eta-1} exp(-\lambda x), x \geq 0, \eta > 0, \lambda > 0$$

whereη is the shape parameter and λ is the scale parameter.

The gamma failure-density function has shapes that are very similar to Weibull distribution. Harris and Singapurwala (1968) have adopted this approach by considering the parameter to be governed by either rectangular or gamma distribution. Gupta (1960), Wilk (1962), and Wani and Kabe (1971) have considered gamma distribution as a lifetime distribution. Uma Maheswari (1994) obtained the reliability of single stresses under $n-$ strengths of life distribution when stress and strength follow gamma distribution.

Rayleigh distribution

The probability density function of Rayleigh distribution is

$$f(x) = \lambda x.exp\left(-\frac{\lambda x^2}{2}\right), \qquad 0 \leq x < \infty, \lambda > 0$$

Rayleigh distribution is a particular case of Weibull distribution when $\beta = 2$.Rayleigh distribution finds application in reliability when system components are characterised by linearly increasing failure rates. In a bathtub curve, the wearing-out period follows Rayleigh distribution.

Extreme value distribution

Material or equipment failure is related to the weakest point or the weakest component; the extreme value distribution for the smallest value is the one usually encountered in reliability work.

Extreme value distributions are applicable where the phenomena causing failure depend on the smallest or the largest value from a sequence of random variables. Probabilistic extreme value theory deals with the stochastic behaviour of the maximum and minimum independent and identically distributed random variables. The distributional properties of extremes (maximum and minimum), extreme and intermediate order statistics, and exceedances over (below) high (low) thresholds are determined by the upper and lower tails of the underlying distribution.

There are three asymptotic distributions for either the smallest order statistics or the largest order statistics. Distributions of the smallest value are

$$Type-I \qquad F(x) = 1 - exp\left\{-exp\left(\frac{x-\delta}{\theta}\right)\right\} - \infty < x < \infty, \theta > 0$$

$$Type-II \qquad F(x) = 1 - exp\left\{-\left(-\frac{x-\delta}{\theta}\right)^{-\beta}\right\} - \infty < x \leq \delta, \theta > 0, \beta > 0$$

$$Type-III \qquad F(x) = 1 - exp\left\{-\left(\frac{x-\delta}{\theta}\right)^{\beta}\right\} \delta \leq x < \infty, \theta > 0, \beta > 0$$

Distributions of the largest extreme values are

$$Type - I \qquad F(x) = exp\left[-exp\left\{-\left(\frac{x-\delta}{\theta}\right)\right\}\right] - \infty < x < \infty, \theta > 0$$

$$Type - II \qquad F(x) = exp\left[-\left(\frac{x-\delta}{\theta}\right)^{-\beta}\right] x \geq \delta, \theta > 0, \beta > 0$$

$$Type - III \qquad F(x) = exp\left[-\left(-\frac{x-\delta}{\theta}\right)^{\beta}\right] x \leq \delta, \theta > 0, \beta > 0$$

The theory of extreme values has been applied to analyse gust velocities, gust loads, and landing loads for aircrafts. In general, stress distributions are described by the largest extreme value distributions. If the underlying distribution is exponential or normal, type I largest extreme value distribution is applicable. Type II largest extreme value distribution has been used for the analysis of maximum wind speeds.

The theoretical developments of the 1920s and mid 1930s were followed in the late 1930s and 1940s by a number of papers dealing with practical applications of extreme value statistics in distributions of human lifetimes, radioactive emissions (Gumbel 1937a–b), strength of materials (Weibull 1939), flood analysis (Gumbel1941, 1944, 1945, 1949a), Rantz and Riggs 1949), seismic analysis (Nordquist 1945), and rainfall analysis (Potter 1949).

Gumbel was the first to call the attention of engineers and statisticians to possible applications of formal "extreme-value" theory to certain distributions which had previously been treated empirically. David (1981) and Arnold, Balakrishnan, and Nagaraja (1992) provide a compact account of the asymptotic theory of extremes. Reiss (1989) discussed various convergence concepts and rates of convergence associated with extremes (and also with order statistics). Castillo (1988) has successfully updated Gumbel (1958) and presented many statistical applications of extreme value theory with emphasis on engineering. Beirlant, Teugels, and Vynekier (1996) provide a lucid practical analysis of extreme values with emphasis on actuarial applications. Tirumala (2011) has done a paper on the reliability of an n-cascade system when stress and strength follows extreme value distribution. In the context of reliability modelling, extreme value distributions for the minimum are frequently encountered.

Pareto distribution

Pareto distribution was first proposed as a model for the distribution of incomes. It is also used as a model for the distribution of city populations within a given area. Pareto distributions arise as tractable "lifetime" models in many areas, including actuarial science, economics, finance, life-testing, survival analysis, and telecommunications. The probability density function of Pareto distribution is

$$f(x) = \frac{\alpha k^{\alpha}}{x^{\alpha+1}}, \qquad k \leq x < \infty, \qquad \alpha . k > 0$$

whereα is the shape parameter and k is the location parameter.

Saralees Nadarajah and Samuel Kotz (2003) have considered several Pareto distributions and derived the corresponding forms for the reliability. Mette Rytgaard (1990) estimated the Pareto distribution. Joseph Lee Petersen (2000) estimated the parameters of Pareto distribution by introducing a quantile regression method. Naser Odat (2010) studied the estimation of reliability based on Pareto distribution.

Binomial distribution

Binomial distribution is one of the most widely used discrete random variable distribution methods in reliability. It has applications in reliability engineering, when one is dealing with a situation in which an event is either a success or a failure. The probability mass function of the distribution is given as

$$P(X = x) = \binom{n}{x} p^{x}(1-p)^{n-x}, \qquad x = 0,1,2, \dots . n$$

Where *n*= number of trials, *x*= number of success, and p = single trial probability of success.

Poisson distribution

Poisson distribution can be used in a manner similar to binomial distribution, it is used to deal with events in which the sample size is unknown. This is also a discrete random variable distribution whose probability mass function is given by

$$P(X = x) = \frac{e^{-\alpha t}(\alpha t)^{x}}{x!} \qquad x = 0,1,2, \dots .$$

Whereα=constant failure rate and$x$ = number of events.

Geometric distribution

Geometric distribution is the number of failures before the first success. The variable of interest is the number of trials required to obtain the first success.

The probability mass function of the distribution is given by

$$P(X = x) = q^{x} p, \quad x = 0,1,2, \dots$$

p = single trial probability of success, x = number of events.

Reliability improvement

To improve reliability, we can use superior components and parts with low failure rates. The components of high reliability will require more time and money for development. They may also be larger in size and weight. Generally, the objective is not merely to produce a system with the highest reliability but to evolve a system that reflects an optimum total cost. The major items contributing to the total cost are research and development, production, spares, and maintenance. In order to design and develop a highly reliable component or unit, we would undoubtedly require a correspondingly high investment in research and development activities, which will be reflected in considerable measure in the total cost. Similarly, the production facilities must be sufficiently sophisticated to enable manufacture of precision components, with the result that the production cost also would increase with the requirement of greater reliability. On the other hand, the cost of maintenance and spares would reduce with an increase in the reliability factor.
The objective in the majority of designs will be attain this optimum cost.

If the state of the art is such that either it is not possible to produce highly reliable components or the cost of producing such components is very high, we can improve the system reliability through the technique of introducing redundancies.

Redundancy

Redundancy is the provision of alternative means or parallel paths in a system for accomplishing a given task such that all means must fail before causing a system failure. Redundancy is an effective way of increasing the system reliability without changing the reliability of the components. The problem of constructing reliable system by appropriate redundant use of relatively unreliable components was first studied by John Von Newman (1950). The various approaches for introducing redundancy in the system are:

1. Unit redundancy:

The simplest and most straight-forward approach is to provide a duplicate path for the entire system itself. This is known as unit redundancy.

2. Component redundancy:

Component redundancy is another approach to system reliability. It provides redundant paths for each component individually. Component redundancy provides higher reliability than unit redundancy.

3. Weakest-link technique:

In this method the weakest component should be identified and strengthened for reliability. This approach is useful when we consider reliability and cost-optimisation problems.

4. Partial redundancy:

Another important redundancy technique is the partial redundancy popularly known as the$k$- out of – m system. We know that the$k$- out of – m system becomes a series structure when $k = m$ and a parallel structure when $k = 1$.

5. Mixed redundancy:

This approach is an appropriate mix of the above techniques, depending upon the system configuration and reliability requirements.

6. Standby redundancy:

Not all components and equipment are suitable for 'active' redundancy. In such cases, another kind of redundancy popularly known as standby redundancy is useful. In standby redundancy, the failed equipment or unit is replaced manually or automatically by its "equivalent."

For standby redundancy, it is convenient to speak about "a socket" and a set of units that are put into it one by one after each failure. All units for the socket are considered to be identical. In reality, a standby unit of some type must be able to replace the main unit only with a unit of the same type.

It can be statedthat a system consists initially of n workable units of which only one unit will be actually working, the other (n-1) units remaining as standbys. The system fails only when all the n units fail. There are three types of standby redundancy:

(i) Hot-standby:

If the standby unit can fail even when it is not active, and if it has the same law of failure as the active unit, then it is called a hot (or active) standby. The reliability of hot-standby units is independent of the instant at which they take the place of the failed units. Hence this is the same as in the case of parallel redundancy. Thus

$$t_s = max[t_j]$$

If $R_i(t) = P(t_i) > t$ are the reliabilities at time t of the individual units, system reliability in this case is

$$R_s(t) = 1 - \big(1 - R_1(t)\big)\big(1 - R_2(t)\big) \ldots \ldots \big(1 - R_2(t)\big)$$

If $R_i(t) = R(t),\ i = 1, 2, \ldots, n$ we get

$$R_s(t) = 1 - [1 - R(t)]^n$$

Examples of hot standbys are components such as A/V switches, computers, network printers, and hard disks.

(ii) Cold-standby:

In the case of a cold standby, the primary component operates and one or more secondary components are placed as standbys. It is assumed that the secondary components in the standby mode will not fail. In such a case, the system time to failure is "do not fail." Thus, the system time to failure is

$$t_s = \sum_{i=1}^{n} t_i$$

Hence, $t_s$has a distribution that is the convolution of the individual distributions $R_i(t)$. If the failure times are independent and identically distributed exponentially, the distribution of t_s as gamma leads to

$$R_n(t) = \sum_{k=0}^{n-1} \frac{e^{-\lambda t}(\lambda t)^k}{k!}$$

(iii) Warm or tepid standby:

The redundant units are in a partially energised state up to the instant they take the place of active units. During the inactive period, they can fail, though usually with lesser probability than in the case of active units. For example, components having rubber parts deteriorate over time and ultimately affect the reliability of standby components.

Stress–strength reliability models

In some cases, a system does not depend on time and may work for an infinite period if all the factors are within the prescribed tolerances. The failure of a system may occur due to certain types of stresses acting on it. If these stresses do not exceed a certain threshold value, they may work for a very long period; on the other hand if the stresses exceed the threshold, they may fail in no time. Hence the stress applied to the system plays a major role in its working or its failure, working within prescribed tolerances.

Stress in an operating system of this type may originate from diverse causes or situations. For example, pressure, load, velocity, resistance, temperature, humidity, vibrations, and voltage are all possible factors that may contribute to the operating of the system. Thus a system composed of components of random strengths will have its strength as a random variable and the stress applied to it will also be a random variable.

A system fails whenever an applied stress exceeds the strength of the system. These types of systems are called stress–strength reliability models. Such models play an important role in the study of structural reliability—a natural criterion for the safety of a structure is that it is reliable. If X and Y denote the stress and strength respectively applied to a component, the reliability R of the component is defined as the probability of not failing, i.e.,

$$R = P(X < Y)$$

The germ of this idea was introduced by Birnbaum (1956) and was developed by Birnbaum and McCarty (1958). The formal term "stress–strength" appears in the title of Church and Harris (1970).

The first attempt to study $P(X < Y)$under certain parametric assumptions on X and Y was undertaken by Owen et al. (1964), who constructed confidence limits for $P(X < Y)$ when X and Y are dependent or independent normally distributed random variables. In the sixties very little was done to investigate a parametric version of the stress–strength model; however, in the seventies investigation of the topic gathered some steam. By the end of the seventies, the estimation of $P(X < Y)$was carried out for the major distributions, such as exponential (Kelly et al 1976; Tong 1974), normal (Church and Harris 1970; Downton 1973; Woodward and Kelley 1977), Pareto (Beg and Singh 1979), and exponential families (Tong 1977). Also, significant advances in the Bayes estimation of $P(X < Y)$ for exponentially or normally distributed X and Y were made by Enis and Geisser (1971). The other milestones of the seventies include the introduction of a non-parametric empirical Bayes estimation of $P(X < Y)$ (Ferguson 1973), Hollander and Korwar (1976), and the study of system reliability (Bhattacharya and Johnson 1974).

By the late eighties, estimators of $P(X < Y)$ were obtained for the majority of common distribution families for the situations when X and Y are independent, by Award and Gharraf (1986), Beg (1980a, b, c), Constantine et al. (1986), Ismail et al. (1986), Iwase (1987), Reiser and Guttman (1986), and Voinov (1984). At the same time, efforts were also