Ball Lightning as a Messenger from a New World of Circulating Light

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By

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PREFACE

This book describes a theory of the finally unraveled nature of ball lightning and the consequences of this event for physics in the field of the interaction of light with matter. The description is based on more than eight dozen articles published since 2003 in leading international journals in physics and optics. The Introduction presents convincing arguments that ball lightning is completely different from what was assumed by the numerous researchers who have put forward more than 500 different hypotheses and theories about the nature of ball lightning so far. It is shown that ball lightning is an object whose existence in nature no one suspected, and after its description, few can agree that such an object can exist.

This object is so simple that its essence can be described in one short sentence. This is a bubble, the spherical shell of which is a thin film of highly compressed air, in which ordinary white light circulates in all possible directions. Since 2003, in numerous publications, we have shown that the behavior of such a bubble in the Earth's atmosphere, in accordance with the generally accepted laws of physics and optics, completely coincides with the mysterious, intriguing and inexplicable behavior of natural ball lightning.

We consider a unique object that can detect a flying airliner, catch up and accompany it, even though the object should be blown away by airflow, the speed of which is several times higher than in the strongest hurricane. It can enter rooms through window panes. It can move upwind. It can find holes and cracks and penetrate them while changing its shape if necessary. If a science fiction writer described such an object, then the reader's reaction would be you can fantasize, but you need to know when to stop.

Taking into account that the behavior of the bubble is exactly the same as that of ball lightning, and applying the well-known duck test to the bubble - "If it looks like a duck, swims like a duck and quacks like a duck, then it is probably a duck" - we can say that this bubble is probably a ball lightning.

It remains to explain the inexplicable - how can such a light bubble exist in nature? This is the subject of this book.

At the same time, it turned out that the existing knowledge about the interaction of light with matter is incomplete. This concerns the magnitude of the momentum of light in an optical medium, the reasons for its increase compared to the momentum of light in free space, the magnitude of the forces acting on the optical medium in which light propagates, and the magnitude of the pressure created by light in various media, including gases. Therefore, before proceeding to the justification of the stability of the bubble, it is necessary to have a clear understanding of the laws on the basis of which such a justification is carried out.

Initially, the laws that are used to determine the necessary conditions for the existence of the bubble are presented. As for the experimental confirmation of the existence of such bubbles, nature has created experimental samples, and numerous eyewitness accounts can be considered as documentation of the observed properties of such bubbles. So the experiments have already been carried out and their results coincide with theoretical predictions.

After the laws of physics and optics necessary for stability analysis are presented, which are consistent with other generally accepted laws, it is shown that the stability of the bubble arises with an extremely high compression of air in the bubble shell and an extremely high light energy density. Other parameters of the bubble were also determined.

No modern knowledge about quantum mechanics, the theory of relativity, nuclear physics, thermodynamics, etc. is required. Knowledge at the level of the 18-19th century is sufficient for understanding. The book can be used as a textbook on optics and physics in the field of the interaction of light with matter.

This is the only book in the world in which for the first time the correct physical theory of a ball lightning is presented and the physical phenomena responsible for its appearance and anomalous behavior are considered. The owners of the book will witness that they lived at a time when the centuries-old mystery of ball lightning was solved and a new stage in the development of mankind began.

INTRODUCTION

No great discovery was ever made without a bold guess.
I. Newton

Piotr Kapitsa, a Nobel Prize winner, studied the phenomenon of ball lightning, and at the end of his life was forced to admit that this nut was too strong for his teeth. At the same time, he assumed that ball lightning was a phenomenon from another world unknown to him. Indeed, he was right. There is a whole world in which circulating light plays a major role. Ball lightning is one of the most famous and mysterious representatives of this world, which we call the world of circulating light.

This statement implies that the nature of the light in ball lightning is well understood. It really is. Since 2003, when the first publication about the completely unexpected nature of ball lightning appeared (Torchigin 2003, 108-116), more than eight dozen articles have been published in leading international journals in physics and optics, which not only explain the physical nature of ball lightning, but also show that the mysterious, intriguing and paradoxical behavior of ball lightning in the earth's atmosphere has a very simple explanation.

Typically, the Introduction describes the previous state of the problem discussed in the book. The phenomenon called ball lightning has been known for several centuries. Currently, more than 500 different theories and hypotheses about the nature of this phenomenon are known, and several thousand articles have been published. But the mysterious and intriguing behavior of ball lightning, which is described in many books and databases, remains unexplained. As the author of one of the scientific books about ball lightning said out of desperation: "All theories are united by one thing, they do not work" (Sagan 2004). The same author complained that: how can one describe the physical nature of ball lightning if it violates the basic laws of physics?

Since we are spared the need to describe the existing theories about the nature of ball lightning, we can describe the physical nature of ball lightning immediately in this Introduction, since the structure of ball lightning is very simple, and its description takes only a few lines. Ball lightning is a completely unusual physical object. Previously, no one suspected the existence of such objects. Therefore, it could not be studied either theoretically or experimentally. Of course, this object does not have a name.

The essence of this object is easiest to explain when comparing it with the well-known soap bubble. Unlike a soap bubble, the shell of this object consists of a thin spherical layer of strongly compressed air where conventional white light is circulating in all possible directions. The light compresses the air to minimize the total energy of the light and compressed air. The reflective index of the compressed air becomes greater than that of the surrounding air. In this case, the shell is a spherical lightguide that prevents radiation of light in free space. Thus, our object is a symbiosis of two elements. These are the air and light. Compressed air ensures the existence of the circulating light, and the circulating light ensures the existence of compressed air. Our object has nothing to do with electricity. Therefore, in order to distinguish references to our imaginary object from references to natural ball lightning, we cut off the tail in the word "lightning" and will call our object ball light or bubble light or circulating light.

This short explanation can be clarified a bit. The air pressures inside and outside the spherical layer of compressed air are the same and equal to atmospheric pressure. Unlike a soap bubble, whose spherical shape is provided by the excess air pressure inside the soap film, the spherical shape of our object is provided by the centrifugal force produced on the layer by the light circulating in it. The glow of ball lightning is explained by the fact that the circulating light is gradually scattered due to the molecular light scattering. Similarly, the sunlight is scattered in the earth's atmosphere. The blue color of the sky and orange color of the Sun near horizon are explained by the light scattering in the earth’s atmosphere.

In these two paragraphs, we have fully described in detail the physical nature of ball lightning. In the next paragraphs, we explain the reasons for its abnormal behaviour. Since the 17th century, it has been known that the light propagating in an inhomogeneous optical medium is deflected in the direction where the reflective index increases. Thus, the circulating light also moves in the direction where the reflective index increases. Using more understandable physical parameters, we can say that the circulating light moves in the direction of increased terrestrial air density. If we take into account that the air pressure is the same in the region where the ball lightning is usually observed, then the air density can be various if the air temperatures in this region are various. The lower the air temperature, the higher the air density.

Thus, the behaviour of ball light obeys the simplest rule. Ball light tends to move in the direction of decreasing air temperature. However, in some cases, when moving over long distances, the ball light moves in the direction of increasing air density. This situation occurs when a ball lightning falls from the sky to the earth or catches up with a flying airliner that disturbs the uniform distribution of air density in such a way that the maximum air density is at the leading edge of the fuselage or wings of the airliner.

Observational properties of ball lightning are given in books by authors such as Singer (1971), Stakhanov (1979), Barry (1980), and Steinhoff (1998), where more than a thousand of the most revealing events are described in detail. Physicists cannot imagine an object obeying accepted laws of physics, the properties of which would coincide with the observed properties of ball lightning.

There is a legitimate question. What are the grounds for asserting that the presented nature of ball lightning in the form of circulating light corresponds to reality, while numerous other theories are erroneous? Nature made sure that the answer to this question was convincing. Nature provided ball lightning, which has a set of unique and anomalous properties that no other object known to date has had. The exception is our object, which has the same paradoxical properties as natural ball lightning. Here are some of the most mysterious.

First, the anomalous property of ball lightning to penetrate into rooms through window panes ought to be taken into account. This wonderful property is confirmed by 46 similar cases described in Grigor’ev, Grigor’eva, and Shiryaeva (1992, 279). It is known that no particles, not even electrons, can penetrate glass. What then does ball lightning consist of? It is difficult to imagine a theory that states that ball lightning consists of nothing.

Second, it is the anomalous property of ball lightning to move in a direction that does not coincide with the direction of the wind. A section entitled "Flies against wind" in Sagan’s 2004 book, based on the declassified ball lightning information from the Oak Ridge National laboratories in the USA, is devoted to a description of this property. Sagan rightly notes that it is difficult to create a theory out of an object that violates the basic laws of physics.

Third, ball lightning has the ability to catch up and accompany a flying airliner. An entire chapter entitled "Airplane Crashes" is devoted to a description of this phenomenon in Sagan (2004). In this case, ball lightning is not blown away by the wind, whose speed exceeds the wind speed in the strongest hurricane.

Bearing in mind these features, we can conclude that none of the known hypotheses satisfies these requirements, since it is difficult to imagine a hypothesis in which it is stated that ball lightning consists of nothing and violates the basic laws of physics.

Such unusual, mysterious properties of ball lightning have a great advantage. They allow one to easily reject various theories and hypotheses about the nature of ball lightning because the objects proposed as ball lightning do not have the same abnormal properties that natural ball lightning has. On the other hand, if our ball light has the same set of anomalous properties as natural ball lightning, we have good reason to assert that ball light and ball lightning are one and the same.

The question may arise - what kind of object can penetrate a window pane without damaging it, because air cannot penetrate through the glass? The answer is simple. Air does not penetrate glass, only light does. On the other side of the glass is just such air. The penetrating light compresses this air. For light, glass is just another optical medium with a different reflective index.

In our first paper, published in 2003, we showed that the behavior of such circulating light in the terrestrial atmosphere completely coincided with the behavior of natural ball lightnings, based on numerous eyewitness accounts. Our explanation was based on the simplest well-known laws of physics and optics. Later, we explained in more detail all the anomalies of natural ball lightnings in numerous publications.

Another argument in favor of the fact that ball light is ball lightning can be the so-called duck test, by which it is possible to identify an unknown object by observing its properties and characteristic behavior. The test sounds like this: "If it looks like a duck, swims like a duck, and quacks like a duck, then it's probably a duck."

We will apply this test to the Ball Light (the unknown subject which properties were considered in the articles) and compare its properties and behavior with that of the natural Ball Lightning (duck), the behavior and characteristics of which are well known from numerous evidences of eyewitnesses. The following similarities ought to be noted.

Both Ball Lightning and Ball Light have a spherical form.

Both Ball Lightning and Ball Light radiate white light, but their temperature is essentially smaller than that of a usual body-radiated white light.

Both Ball Lightning and Ball Light have lifetimes ranging from a few seconds to a few minutes (Torchigin 2019, 2961).

Both Ball Lightning and Ball Light disappear unexpectedly (Torchigin 2019, 65-69).

Both Ball Lightning and Ball Light disappear instantly (Torchigin 2019, 65-69).

Both Ball Lightning and Ball Light disappear without traces (Torchigin 2019, 65-69).

Both Ball lightning and Ball Light bypass obstacles (Torchigin 2018, 167).

Both Ball Lightning and Ball Light fall from sky but do not strike the earth's surface (Torchigin 2018, 149).

Both Ball Lightning and Ball Light can find out holes and slots to penetrate within buildings (Torchigin 2019, 163126.).

Both Ball Lightning and Ball Light change their form to penetrate through small holes and narrow slots (Torchigin and Torchigin 2016, 6155).

Both Ball Lightning and Ball Light can penetrate into rooms through window panes (Torchigin and Torchigin 2016, 5876).

Both Ball Lightning and Ball Light can move against wind (Torchigin 2019, 533-537).

Both Ball Lightning and Ball Light can catch up to a flying airliner and penetrate inside its saloon (Torchigin and Torchigin 2017, 196).

Both Ball Lightning and Ball Light can burn out metal objects (Torchigin 2021, 167528).

In accordance with the duck test, since both Ball Lightning and Ball Light have identical behavior and characteristics, Ball Light is probably Ball Lightning. Thus, our ball light passes the duck test.

Furthermore, it has been shown that circulating light exists not only in ball lightning. There is a whole world of luminous objects with anomalous properties in which circulating light is the cause of their anomaly. We can say that there is a hitherto unknown world of circulating light, inhabited by objects with anomalous properties. The most famous of this world of circulating light is ball lightning. Unlike ball lightning, which is a rare natural phenomenon, other objects from the world of circulating light are obtained in laboratories, usually when trying to get artificial ball lightning. The properties of these objects are well studied, which allows a deeper understanding of the nature of ball lightning. The presence of circulating light in such objects makes it possible to explain their anomalous properties.

Having made sure that the properties and behavior of a bubble of light and ball lightning in the earth's atmosphere completely coincide, we faced a more difficult problem - to show, on the basis of existing knowledge, that the bubble of light could be stable. This was a task where the positive answer known in advance, but for two decades no theoretical physicist could indicate the physical laws and phenomena responsible for the existence of bubbles of light and their stability.

Essentially, compressed air and circulating light both had a tendency to expand, and it was not clear why their interaction should prevent these trends.

We eventually managed to find a solution to this problem (Torchigin 2019, 704; Torchigin 2020, 165098; Torchigin 2021, 166635). The study of the necessary conditions under which stability can exist has led to results according to which the energy densities of compressed air and circulating light should be record highs. Based on the stability conditions, other parameters of the circulating light were determined (Torchigin 2021, 167390). It turned out that the lifetime of circulating light was approximately three orders of magnitude longer than the lifetime of sunlight in the Earth's atmosphere. This conclusion is also supported by observations.

We present the reasons for the arising of stability of the circulating light and the conditions for its occurrence. Skeptics may argue that the paper tolerates everything, but where is the evidence for the correctness of the presented theory? Any theory must be experimentally confirmed. Indeed, we present only a model of ball lightning. Any model only approximately describes the real phenomenon and requires experimental confirmation. Fortunately, nature presents numerous experimental samples in the form of natural ball lightnings and a complete set of information about their properties in the form of numerous eyewitness accounts. From this information, it follows that experimental samples exist. Moreover, these samples have exactly the same set of anomalous properties (inexplicable on the basis of modern knowledge). It is shown that the properties of ball lightning are not anomalous, but are in full accordance with the generally accepted and long-known basic laws of optics and physics.

The great scientist Faraday, after whom the unit of electrical capacity is named, said: "Nothing is too wonderful to be true if it is consistent with the laws of nature." The famous American writer Mark Twain also respected reality. He believed that "Fiction is obliged to stick to possibilities. Truth isn't. "

To substantiate the stability of circulating light, it is necessary to have reliable knowledge about the interaction of light with matter. Unfortunately, there is no such generally accepted knowledge. This applies to almost all issues necessary to determine the stability, such as the magnitude of the momentum of light in the optical medium, the magnitude of the forces acting on the light from the optical medium in which it propagates, and the pressure exerted by light on the air in which it circulates. Without reliable answers to these questions, the study of the stability of circulating light loses its meaning. Therefore, before considering the stability of circulating light, a theory of the interaction of light with matter is presented, which is of independent value and can be used in other applications. It should be immediately noted that the indisputable generally accepted laws of physics and optics without any additional assumptions in all the considerations given below were used. In other words, it can be argued that the results below are a direct consequence of generally accepted laws. In this regard, the book can be seen as a textbook that outlines the current understanding of the above issues.

First of all, this refers to the change in the magnitude of the momentum of light in the optical medium relative to the momentum of the same light in free space. There are irrefutable arguments that the momentum of light in an optical medium increases and equally irrefutable arguments that the momentum of light in an optical medium decreases. Without a reasonable idea of the magnitude of the momentum of light in compressed air, further reasoning loses its meaning.

Since the forces arising at an interaction between light and matter following Newton's laws are determined by the derivative of the momentum over time, any conclusion regarding these forces can be doubted under the pretext that there is no generally accepted idea of the magnitude of the momentum. The first chapter deals with the question of the magnitude of the momentum of light in an optical medium. Regarding the magnitude of the momentum in free space, there is a generally accepted opinion. However, up to now there has been no generally accepted opinion on the magnitude of momentum of light in an optical medium. There is not even a qualitative understanding. The question of increasing or decreasing the momentum of light in an optical medium in comparison with the momentum of light in free space is still being debated.

Without going into this discussion, we have shown that the momentum of light in an optical medium is increased by a factor of n, where n is the reflection index of the optical medium. This conclusion is confirmed by a dozen thought experiments published in leading international journals. Such a result existed before the dispute arose and followed directly from the laws of mechanics. Particularly convincing arguments have been obtained in the analysis of the momentum of other types of waves, such as elastic waves and sound waves. In this case, the momentum of an electromagnetic wave is a special case of a more general law. This makes the results more reliable.

However, another thought experiment, by Balazs, also based on the generally accepted laws of physics and optics, remained inexplicable, from which it follows that the momentum of light in an optical medium decreases compared to the momentum of light in free space. The second chapter is devoted to the reconciliation of these contradictions. The modified thought experiment of Balazs is considered. In contrast to the original experiment, where a light wave was considered, a wave of an arbitrary type was considered. It turned out that the conclusions drawn from the results of this experiment should be interpreted differently. It is shown that the above reconciliation can be achieved under the assumption that additional forces must act on the homogeneous medium in which the wave propagates. For the particular case of an electromagnetic wave, these forces coincide in magnitude with the Abraham forces, the existence of which has not yet ceased disputes. An experimental confirmation of the existence of these forces for an ultrasonic wave propagating in the air is given.

The nature of the momentum of an arbitrary type wave is considered. An explanation is given as to why the momentum of a body with mass is proportional to its velocity, while the momentum of a wave is inversely proportional to its velocity. It is shown that the wave entails the material medium in which it propagates. It is the movement of the material medium in which the wave propagates that ensures the existence of a momentum for the wave. On the assumption that an electromagnetic wave propagating in free space is not an exceptional case, then it should be recognized that such a wave entrains free space (or tries to entrain, creating pressure on it).

The third chapter is devoted to the analysis of the physical nature of the forces produced by waves of various types propagating in a homogeneous medium. Since the existence of these forces is still being debated, it can be assumed that these unknown forces are new forces that can complement knowledge about the nature of waves. It is shown that these forces have a different nature depending on the type of energy carried by a particular wave. Formulas are given for calculating these forces for waves of various physical natures, such as an elastic wave, a sound wave, a torsion wave, a wave in a stretched string, and a sea wave in shallow water. Particular attention was paid to the study of such forces produced by an electromagnetic wave. Formulas are given for such forces produced by an electromagnetic wave in a dielectric. These formulas are compared with similar formulas obtained on the basis of a perfectly different approach using the Lorentz forces, which is currently popular due to the lack of a generally accepted opinion on the magnitude of the momentum of light in matter. It is shown that the formulas have significant differences. A special case is considered to show that the formulas based on Lorentz forces give an incorrect result. Corrections are given to which these formulas must be subjected.

The fourth chapter is devoted to the investigation of the phenomenon that provides compression of the air in the shell of circulating light. This phenomenon was discovered by Helmholtz in 1888 for the dielectric placed in an electrical field. The same phenomenon takes place for an optical medium where the light wave is propagating. In this case, the phenomenon is called optical electrostriction pressure and is considered as one of the possible mechanisms of optical nonlinearity. It is shown that, contrary to the generally accepted concept, this phenomenon manifests itself completely differently in gas than in liquid and solids. Furthermore, it was shown that an increase in the air reflective index could occur not only with an increase in the air density but also with the absorption into the shells of other gases, the reflective indexes of which exceed the air reflective index.

The fifth chapter is devoted to the justification of the stability of circulating light. First of all, the necessary conditions are derived under which the circulating light in a spherical shell of compressed air is stable. Since the stability of such objects has not been previously studied, special attention was paid to substantiating the research methodology. For this purpose, the chosen methodology was initially tested on objects whose stability is not in doubt. These were a soap bubble and a children's balloon. The necessary conditions for the stability of these objects were determined.

After that, exactly the same methodology was used to determine the necessary conditions for the stability of the circulating light. At the same time, immediately before the start of calculations, it turned out that stability can occur at a sufficiently high value of the reflective index of compressed air, which can only be achieved with its extremely high compression. It is shown that the steady state and its stability appear only at the very great air pressure produced by the very great energy intensity of the circulating light.

From these necessary conditions, the parameters of the circulating light were derived, such as stored energy, compressed air pressure, the energy density of circulating light, the lifetime of circulating light, and features of its occurrence and disappearance. The last chapter is devoted to an analysis of the processes connected with the arising, existence and disappearance of the circulating light.

It can be concluded that the presented optical theory of ball lightning makes it possible not only to solve the riddle about the nature of ball lightning, the solution of which no-one suspected, but also to gain new knowledge about the interaction of light with matter.

Almost 20 years have passed since convincing evidence was presented that the behaviours of a bubble of light and a natural ball lightning were the same, and therefore a bubble of light existed in nature. It would seem that the discovery of a new natural phenomenon should have aroused interest in it and in the study of the presence of circulating light in other natural phenomena. However, this did not happen.

Having an advantage over other researchers who only doubted and shook their heads about the existence of circulating light in nature, we, based on the belief that circulating light exists, not only showed its stability, but also began to deduce new knowledge from this fact. We analyzed the interaction of waves of arbitrary types with the mediums in which they propagate, and obtained new results that are valid not only in the context of circulating light, but also in a general case. The book expands our knowledge of nature.

The information presented in the book could be accessible to scientists of the 19th century, when concepts about the reflective index of an optical medium *n*, momentum, force, mass, and energy appeared. True, it is necessary to know the foundations of the mathematical analysis developed by Leibniz and Newton at the same time. We tried to present the book in such a way that the presentation of the new theory of ball lightning was combined with the presentation of modern ideas about the foundations of modern optics, in part concerned the relationship of optical and mechanical phenomena. Therefore, the book can be considered a textbook on the fundamentals of modern optics, which were partially corrected and supplemented during the development of the theory of ball lightning.

This book is the first book where the theoretical grounds that explain an existence of natural ball lightnings and the laws responsible for their existence are presented.

CHAPTER ONE

MOMENTUMS OF ARBITRARY WAVES

Nothing is too wonderful to be true, if it be
consistent with the laws of nature.
M. Faraday

The purpose of this section is to prove that the momentum of light in an optical medium is increased by a factor of n as compared with the momentum of the same light in free space where n is the reflective index of the medium.

The momentum of light in free space is well known, but there is still no generally accepted idea of the magnitude of the momentum of light in an optical medium. There is still no generally accepted opinion about whether the momentum of light in an optical medium increases or decreases compared to the momentum of the same light in free space. With the Minkowski approach, the momentum in the optical medium increases by a factor of n. In the Abraham approach, the momentum in the optical medium decreases by the same factor.

Currently, there is an opinion that both approaches are applicable in different situations. For example, we can read Griffith (2012, 7): "Beginning in the late 1960's something approaching a consensus emerged: Both the Minkowski momentum and the Abraham momentum are "correct," but they speak to different issues, and it is largely a matter of taste which of the two one identifies as the "true" electromagnetic momentum. Correct both forms".

Another compromise was proposed by Mansuripur (2010, 1997-2005), according to whom the momentum of a light wave in an optical medium was the arithmetic average of the values proposed by Minkowski and Abraham, that is, it was equal to $(n + 1/n)/2$.

Most recently, Barnett and Loudon reanalyzed the controversy and argued that both momenta are "correct" because both can be measured, but in different situations (Barnett and Loudon 2010, 927). There is also the Balazs thought experiment (Balazs 1953, 411), the interpretation of which shows that the Abraham approach is correct. Following Balazs' analysis and repeating arguments of his thought experiment, they concluded that "it is difficult to see how any component of our derivation could seriously be open to question … If argument advanced in favour of the Abraham momentum were to be incorrect, then that would bring into question uniform motion of an isolated body as expressed in Newton's first law of motion".

Lengthy attempts to derive the value of optically induced force (OIF) from the Lorentz law for forces acting on a moving charge in electric and magnetic fields are known. These attempts began with the work of Einstein and Laub (1908), resumed in the work of Gordon (1973, 14), and continued more recently in the works of Laudon and Mansuripur (Mansuripur, Zakharian, and Wright 2013, 023826; Loudon and Barnett 2006, 11855). Various formulas for a magnitude of optically induced forces (OIF) have been obtained, but not one of them gives the correct result in the simplest particular case. We do not give here arguments related to the criticism of various incorrect expressions for calculating OIF based on Lorentz forces. The correct expression, in our opinion, is presented in Brevik (2009, 219301).

Such a situation with the momentum of light in the optical medium cannot be considered satisfactory. We cannot develop a theory of circulating light using concepts for which there is no generally accepted recognition. In this case, the theory may cause reasonable doubts about its correctness. Below, we present evidence that the momentum of light in an optical medium increases by *n* times.

First, we calculate the momentum of light in the optical medium for cases where the mention of electricity is not required. Moreover, the magnitude of the momentum is derived for an abstract wave, where a mention of the reflective index is absent, as in the case of the Doppler effect. Then we will show, as an explanation and illustration, that the same result can be obtained if we take into account the expressions known since Faraday's era for forces acting on a dielectric in a constant electric field.

We present arguments based on various physical phenomena that the momentum of a light pulse in the optical medium corresponds to the Minkowski approach and is greater by *n* times than that in free space, where *n* is the reflective index of the optical medium. We then refute a single opposite argument in the form of the Balazs thought experiment, according to which the Abraham approach is correct and the momentum of light in the optical medium decreases by *n* times. Thus, after a dozen arguments presented in favour of the Minkowski approach, we conclude that the ageold debate about the magnitude of the momentum of light in an optical medium should be completed in favour of the Minkowski approach.

Thus, we consider the debate about the magnitude of the momentum of light in the optical medium complete (at least in the context of where the forces act on the light due to a change in its momentum). By the way, a correct magnitude of the momentum of light in the optical medium could be derived from results of Thomson who determined in 1904 the pressure produced by the light at a normal incidence on a flat boundary between the optical medium and free space (Thomson 1904). The knowledge of the magnitude of the momentum of light in an optical medium allows one to calculate the optically induced forces applied to the circulating lights and to calculate their motion without the knowledge that light is an electromagnetic wave.

The momentum of a wave of arbitrary physical nature

The concept of momentum refers not only to bodies and electromagnetic waves, but also to other waves for which such physical concepts as the speed of propagation, frequency, and energy density exist. We believe that the calculation of the momentum in the general case for waves of an arbitrary physical nature allows us to calculate the momentum for electromagnetic waves. In this case, long disputes about the momentum of the electromagnetic wave remain set aside, as disputes about particular cases, when the result in a general case is known.

Notation and basic definitions

First of all, it is worth noting that our notion of the momentum of light was taken from mechanics, where Newton introduced this concept long before a discussion arose about the magnitude of the momentum of light in an optical medium. For a body of mass m moving at speed v, the momentum of the body is defined as follows

$$\boldsymbol{p} = m\,\boldsymbol{v} \tag{1.1}$$

Following Newton, the momentum characterizes the "quantity of motion". A reason for a change of the momentum p is force F that in accordance with the second Newton law is given by

$$\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt} \tag{1.2}$$

In a closed system (one that does not exchange any matter with the outside and is not acted on by outside forces) the total momentum is constant. This fact, known as the law of conservation of momentum, is implied by Newton's laws of motion.

A magnitude of the momentum of a light pulse propagating in a vacuum is generally accepted and is given by

$$M_0 = \mathscr{E} / c \tag{1.3}$$

where $\mathscr{E}$ is the energy of the light pulse.

If a continuous plane light wave is considered, any notion of the energy is senseless, and the energy density W of the wave should be considered. Analogously, the notion of the momentum for a continuous wave is senseless also and the momentum flux density (MFD) that is equal to the momentum of the wave propagating through a cross-section of the unit area per unit time is considered. In other words, MFD is equal to the momentum of the fragment of the continuous wave, the cross-section of which is equal to the unit area, and the length of which is equal to the distance that the wave travels per unit time. The MFD of a continuous plane light wave propagating in a vacuum at speed c is given by $(W_0c)/c=W_0$ [J/m^3], where W_0 is the energy density of the light wave in free space.

For continuous light wave, the density force F in Eq. (1.2) is substituted by the pressure P, and dp/dt is substituted by a difference between input and output MFDs. As a result, the pressure P produced by the wave propagating in an optical medium is determined as follows

$$P = MFD_{IN} - MFD_{OUT} \tag{1.4}$$

For example, a mechanical pressure P applied to a reflector by the light wave at a normal reflection from the reflector in accordance with Eq. (1.4) is equal

$$P = MFD_{IN} - MFD_{OUT} = MFD_{IN} - (-MFD_{IN}) = 2MFD_{IN} \tag{1.5}$$

Thus the pressure produced on the reflector at the normal reflection of a continuous wave, for which MFD is equal to MFD_{IN}, is equal to twice MFD_{IN}.

When we talk about waves in an optical medium and free space, we mean the MFD of this wave. Our immediate goal is to convince the reader of the correctness of the Minkowski approach with the help of numerous thought arguments.

Optically induced forces (OIF) are produced by the light propagating in an inhomogeneous optical medium. As a result, the light interacts with matter (an exchange of the momentums between the light and matter takes place). The law of the conservation of momentum and the third Newton law are valid at this interaction. As a result, each OIF changes the mechanical MFD of matter. In turn, a counterpart of the OIF that arises in accordance with the third Newton law changes the MFD of light. Thus, each interaction is accompanied by redistribution between the mechanical momentum of the optical medium and the momentum of light. The sum of these momentums is not changed.

We ought to note that the dimensions of the following three physical values: energy density *W* [J/m^3], pressure *P* [$N/m^2=Nm/m^3=J/m^3$], and momentum density flux *MFD* [$\frac{J}{m/s}\frac{m/s}{m^3}=\frac{J}{m^3}$] are identical. The difference is that *MFD* is a vector and *W* is a scalar. On the assumption that the *MFD* in an optical medium that increases by *n* times, the magnitudes of the *MFD* and *W* in an optical medium are identical also because *W* increases in an optical medium by *n* times.

It should be noted that the above concepts (force, energy density, momentum flux density, pressure, and wave propagation velocity) are also applicable to waves of a different physical nature (in particular, elastic waves). There is no dispute about the magnitude of the momentum for other types of waves. For waves of any physical nature for which the above concepts are applicable, the MFD of this wave can be determined. After that, the momentum for an electromagnetic wave can be considered as a special case, and the debate will end on this. We begin our consideration with examples valid for waves of various physical natures.

The momentum of arbitrary waves derived from the Doppler law

In accordance with the Doppler law (Eden 1992), an increase of the frequency of the wave reflected from the reflector that moves towards the wave is given by

$$\Delta\omega = \omega\frac{2v_R}{v_W} \tag{1.6}$$

where ω is the frequency of the incident wave, and v_W and v_R are the velocities of the incident wave and reflector, respectively. Let us consider a fragment of the wave of unit area and length v_W and energy $\mathcal{E}$. The momentum of this fragment is equal to the MFD of the wave. The volume V of the fragment is equal to v_w.

For definiteness, we restrict ourselves to considering light and sound waves. Taking into account that the energy of a photon or phonon is proportional to its frequency, we can conclude from Eq. (1.6) that

$$\Delta\mathcal{E} = \mathcal{E}\frac{2v_R}{v_W} = 2Wv_R \tag{1.7}$$

where

$$W = \mathcal{E} / v_W \tag{1.8}$$

is the energy density of the wave. This fragment is reflected from the reflector at a time interval 1s. At this time the reflector moves at distance v_R. The mechanical effort that is spent at this displacement is given by

$$\Delta\mathcal{E} = Pv_R \tag{1.9}$$

where P is the pressure produced by the wave at the reflection. Comparing Eq. (1.7) and Eq. (1.9), we have

$$P = 2W \tag{1.10}$$

The *MFD* of the fragment changes its sign at the reflection. The pressure due to the change of the momentum of the wave is given by

$$P = 2MFD \tag{1.11}$$

Comparing Eq. (1.10) and Eq. (1.11) we can see that

$$MFD = W \tag{1.12}$$

Thus, the MFD of an arbitrary wave is equal to the energy density of the wave. As is seen from Eq. (1.8), the energy density is in inverse proportion with the wave speed.

Concepts and terms inherent for waves of an arbitrary physical nature were used. The light waves can be considered as a special case. We believe that the overall result is more convincing than the specific one.

In the particular case of the light wave, the energy density of the fragment in an optical medium is greater by n times than that in free space. In this case, the MFD in an optical medium in accordance with Eq. (1.12) and Eq. (1.8) is greater by n times than that in free space.

The momentum of an arbitrary wave derived from its adiabatic invariant

Let us compare two plane resonators consisting of two ideal reflectors, as is shown in Fig.1-1. The first resonator is located in the first medium, and the distance between reflectors is equal *to* $\lambda/2$, where λ is the wavelength in the medium. The second resonator is located in the second medium, where the distance between reflectors is equal to $\lambda/(2\gamma)$ and where $\gamma>1$ is the ratio of speeds of waves in the resonators. In this case, the eigen frequencies of the resonators are identical.

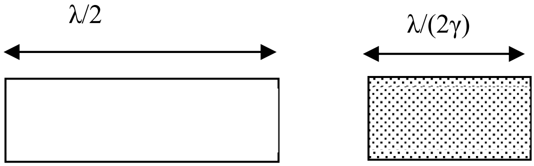

Fig.1-1. Comparison of two plane resonators with identical eigen frequencies.

Let the energy of oscillations also be identical. Let us decrease the lengths of the resonators by one percent. The eigen frequency increases by one percent also. Following the theory of oscillations of lossless conservative isolated oscillations systems, there is the adiabatic invariant under which a change of the energy stored in the resonator is proportional to a change in its eigen frequency at a slow change of any resonator parameters. We can then conclude that the energies stored in the resonators increase by one percent in both resonators.

The mechanical works that are then required to decrease the distance between the reflectors are identical for both resonators. But a change in the distance between reflectors for the second resonator is γ times smaller than for the first one. To have identical mechanical works, the force applied to the reflector should be greater by γ times for the second resonator than for the first one. Since the force overcomes the pressure produced by the wave at the reflection from the reflector, the pressure in the second case is greater by γ times than in the first one. Since pressures are produced by a change of the MFD of the wave at the reflection, the MFD of the wave in the second resonator is greater by γ times than that in the first one. Thus, the MFDs of waves differ by γ times, provided that their energies are identical. We can also conclude that the MFD and energy density of wave in the second resonator is greater by γ times than in the first one.

Since the presented arguments are true for waves of an arbitrary physical nature, they are also true for electromagnetic waves that can be considered as a special case. Assuming that the first medium is free space, and the second medium is an optical medium with a reflective index $n = \gamma$, we can conclude that the MFD of light in an optical medium increases by γ=n times. We believe that the overall result is more convincing than the specific one.

The momentum of an arbitrary wave derived from the balance of its momentums at refraction

Let us consider the refraction of any wave at a boundary between two mediums where the same wave propagates at speeds v_1 and v_2. The incidence angle is α_1 and the refracted angle is α_2, as shown in Fig. 1-2..Let any part of the wave penetrate from medium 1 to medium 2. The horizontal component of the momentum is preserved on the condition given by

$$p_1 sin\alpha_1 = p_2 \sin\alpha_2 \tag{1.13}$$

where p_1 and p_2 are MFDs of waves in mediums 1 and 2, respectively.

The periodicities of these waves at the boundary should be identical. This condition can be written as follows

$$k_1 \sin\alpha_1 = k_2 \sin\alpha_2 \tag{1.14}$$

where k_1 and k_2 are propagation constants in the first and second mediums and

$$k_1 = \omega / v_1,\ k_2 = \omega / v_2 \tag{1.15}$$

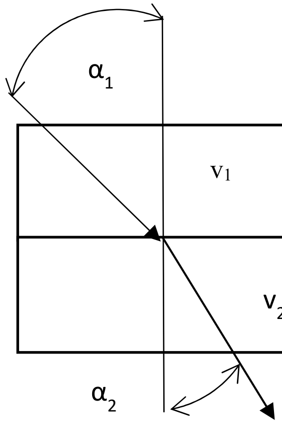

Fig.1-2. Propagation of a wave of an arbitrary physical nature through a boundary of two mediums.

From Eq. (1.14) and Eq. (1.15), we have

$$\frac{\sin\alpha_1}{v_1} = \frac{\sin\alpha_2}{v_2} \tag{1.16}$$

Taking into account Eq. (1.13) and Eq. (1.16), we have

$$\frac{p_1}{p_2} = \frac{v_2}{v_1} \tag{1.17}$$

Setting the light wave at $v_1 = c$, $v_2 = c/n$, we see that the MFD of light in an optical medium increases by n times as compared with that in free space from simple geometric consideration.

The momentum of a wave derived from the analysis of a circulating wave

Let a wave of a whispering gallery type circulate around the equator of the ball. The circulating wave contains an integer number of wavelengths, i.e. the phase shift along the equator is given by

$$2\pi Rk = 2\pi N \tag{1.18}$$

where the propagation constant $k=\omega/v$, ω is the angular frequency, v is the velocity of the wave within the ball, R is the radius of the ball, and N is any integer. Eq. (1.18) preserves at a gradual change the parameters R and k. In this case, we have from Eq. (1.18)

$$\omega R \,/\, v = \omega_0 R_0 \,/\, v_0 \tag{1.19}$$

where ω_0, v_0, R_0 are parameters in any particular case. Furthermore, it is known that the energy $\mathcal{E}$ stored in an isolated conservative oscillating system is proportional to the eigen frequency ω at a slow change of the parameters of the system. In this case, we have $\mathcal{E}R = const$ or

$\ln\mathcal{E} + \ln R = const$ or $\dfrac{d\mathcal{E}}{\mathcal{E}} + \dfrac{dR}{R} = 0$ or

$$\frac{d\mathcal{E}}{dR} = -\frac{\mathcal{E}}{R} \tag{1.20}$$

The term $\dfrac{d\mathcal{E}}{dR}$ in Eq. (1.20) designates the total force F (a sum of modules of all surface forces) applied to the surface of the ball. The total force tends to increase the radius of the ball. Eq. (1.20) can then be rewritten as follows

$$F = \frac{\mathcal{E}}{R} \tag{1.21}$$

This force is equal to the centrifugal force applied to the entire surface of the ball. The force is produced by circulating light, the total momentum