A Semi-Analytical Approach to Nonlinear Mechanical Engineering Issues

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CHAPTER 1

INTRODUCTION

1.1. Preface

Most users of differential equations in the mathematical and engineering sciences face the difficulty of being nonlinear. There has not been a particularly effective way to solve different kinds of nonlinear differential equations in diverse fields of study up until now.

It is important to note that although academics have recently developed a few semi-analytical techniques for solving a small number of groups of nonlinear differential equations, none can be applied to various nonlinear equations. To understand better, it is necessary to mention that some techniques now used for solving nonlinear equations include the homotopy analysis method (HAM)[1], homotopy perturbation method (HPM)[2], VIM[3], DTM[4], ADM[5], etc. However, each technique works well for a subset of nonlinear differential equations.

Additionally, the use of every approach has occasionally resulted in calculations that contain serious errors. As a result, only some of the nonlinear issues can be solved using the abovementioned methods, which do not apply to all nonlinear situations. To further elaborate, the second drawback of these procedures is the drawn-out and challenging process.

In this book, a practical method is presented for all the various types of nonlinear differential equations and various sets of nonlinear equations, allowing for straightforward analytical solutions to all the differential equations and the final solution of each differential equation as an algebraic function. By selecting an answer function for a differential equation with

[1] Homotopy Analysis Method
[2] Homotopy Perturbation Method
[3] Vibrational Iteration Method
[4] Differential Transformation Method
[5] Adomian Decomposition Method

constant coefficients, which may be obtained by applying initial or boundary conditions in a certain way, AGM seeks to solve all nonlinear differential equations algebraically.

Also, semi-analytical methods can be divided into two groups based on how they solve problems; for the sake of simplicity, we will refer to them as the Iterate-Base Method and the Trial Function-Base Method. The number of iterations is a crucial aspect that influences the solving procedures in iterate-based methods like (HPM), VIM, ADM, and others. Although we can use trial functions in this procedure that are based on our independent functions, to solve each step, we must first solve the stages that came before it. The reasons above make it clear that we will encounter issues that impede our problem-solving processes when iteration results in higher phases inaccessible to the relevant software.

Additionally, these approaches typically require more time to find solutions.

The primary element that affects the solving procedures in trial function-based methods like Collocation Method (CM)[6], least square method (LSM)[7], AGM, and others are trial function. This approach assumes a practical trial function with various constant coefficients based on the problem's boundary and initial conditions.

After that, we must solve the constant coefficients because of each approach's fundamental premise. Most of the time, a collection of polynomials can be solved to obtain the constant coefficients readily. Although the number of terms in our trial function in these approaches might be referred to as the number of needed iterations, it is essential to note that used constants will be obtained concurrently in solution procedures. Therefore, these techniques do not have iteration issues.

AGM has high efficiency and accuracy for solving nonlinear problems with high nonlinearity, according to recent achievements from this approach and the Trial Function-Base properties of this method. It is important to note that the following can be used as a summary of this method's superiority versus other approaches: In the process of solving differential equations, boundary conditions are required in the order of the differential equations; however, if the number of boundary conditions is less than the order of the differential equation, this method may result in the creation of additional

[6] Colocation Method
[7] Least Square Method

new boundary conditions for the internal differential equation and its derivatives.

1.2. The basic concept of AGM

Every engineering science branch uses linear or nonlinear differential equations as its governing equations to solve physics-based problems. It is necessary to apply sufficient boundary or beginning conditions to the physics of these problems and the mathematical formulation they have been given to solve the difficulties under consideration. We can understand the significance of these boundaries and initial conditions in determining the accuracy and efficiency of analytical methods achieving acceptable solutions due to the physics of problems since procedures of applying analytical methods for obtaining the solution of linear and nonlinear differential equations are not an exception from mentioned fact. The entire process has been defined in plain terms to make it easier to understand the supplied method in this book.

The general approach to a differential equation depends on the boundary conditions and is as follows:

$$p_k : f\,(u, u', u'', \ldots, u^{(m)}) = 0; \quad \mathrm{u} = u(x) \tag{1.1}$$

The nonlinear differential equation of p, which is a function of u, the parameter u, which is a function of x, and their derivatives are assumed as follows:

Boundary conditions:

$$\begin{cases} u(x) = u_0, u'(x) = u_1, \ldots, u^{(m-1)}(x) = u_{m-1} & \text{at} \quad \mathrm{x} = 0 \\ u(x) = u_{L_0}, u'(x) = u_{L_1}, \ldots, u^{(m-1)}(x) = u_{L_{m-1}} & \text{at} \quad \mathrm{x} = 0 \end{cases} \tag{1.2}$$

The sequence of letters in the nth order with constant coefficients that we assume as the solution of the first differential equation is taken into consideration to solve the first differential equation concerning the boundary conditions in x = L in Eq. (1.2) as follows:

$$u(x) = \sum_{i=0}^{n} a_i x^i = a_0 + a_1 x^1 + a_2 x^2 + \ldots + a_n x^n \tag{1.3}$$

The additional series statements in Eq. (1.3) lead to a more accurate solution for Eq (1.1). There are (n + 1) unknown coefficients that require (n + 1) equations to be stated to achieve the solution of the differential equation (1.1) about the series from degree (n). A group of (n+1) equations is solved using the boundary conditions of equation (1.2).

1.2.1. Applying the boundary conditions

(a) The boundary conditions are used in the following way to solve differential equation (1.3):

When x = 0:

$$\begin{Bmatrix} u(0) = a_0 = u_0 \\ u'(0) = a_1 = u_1 \\ u''(0) = a_2 = u_2 \\ \vdots \quad \vdots \quad \vdots \end{Bmatrix} \tag{1.4}$$

And when x= L:

$$\begin{aligned} &u(L) = a_0 + a_1 L + a_2 L^2 + \ldots + a_n L^n = u_{L_0} \\ &u'(L) = a_1 + 2a_2 L + 3a_3 L^2 + \ldots + n a_n L^{n-1} = u_{L_1} \\ &u''(L) = 2a_2 + 6a_3 L + 12a_4 L^2 + \ldots + n(n-1) a_n L^{n-2} = u_{L_{m-1}} \\ &\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \end{aligned} \tag{1.5}$$

(b) The boundary conditions are applied to the differential Eq. (1.1) in the following manner after substituting Eq. (1.5) into Eq. (1.1):

$$\begin{aligned} &p_0 : f\,(u(0), u'(0), u''(0), \ldots, u^{(m)}(0)) \\ &p_1 : f\,(u(L), u'(L), u''(L), \ldots, u^{(m)}(L)) \\ &\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \end{aligned} \tag{1.6}$$

To create a system of equations with (n+1) equations and (n+1); unknowns, based on the selection of n; ($n \prec m$) phrases from Eq. (1.3), we must deal with several extra unknowns that are the identical coefficients of Eq (1.3). We must first apply the boundary conditions to the differential equations indicated above to solve this issue before deriving m times from Eq. (1.1) by the new unknowns.

$$
\begin{aligned}
&p_k' : f\,(u',u'',u''',\ldots,u^{(m+1)}) \\
&p_k'' : f\,(u'',u''',u^{(IV)}\ldots,u^{(m+2)}) \\
&\vdots \qquad \vdots \qquad \vdots \qquad \vdots
\end{aligned}
\tag{1.7}
$$

(c) The boundary conditions in Eq. (1.7) are applied to the derivatives of the differential equation P_k as follows:

$$
p_k' : \begin{Bmatrix} f\,(u'(0),u''(0),u'''(0),\ldots,u^{(m+1)}(0)) \\ f\,(u'(L),u''(L),u'''(L),\ldots,u^{(m+1)}(L)) \end{Bmatrix}
\tag{1.8}
$$

$$
p_k'' : \begin{Bmatrix} f\,(u''(0),u'''(0),\ldots,u^{(m+2)}(0)) \\ f\,(u''(L),u'''(L),\ldots,u^{(m+2)}(L)) \end{Bmatrix}
\tag{1.9}
$$

It is possible to construct (n+ 1) equations from Eq. (1.4) to Eq. (1.9) to determine the (n +1) unknown coefficients of Eq. (1.3), such as a_0, a_1, a_2,..., and a_n. Finding the coefficients of Eq. (1.1) will lead to solving the nonlinear differential equation (1.3).

CHAPTER 2

THE APPLICATION OF AGM IN SOLID MECHANICS

2.1. Swinging Oscillation of a Solid Circular Sector Object

In this problem, a new and innovative semi-analytical method called AGM has been applied to solve nonlinear equations of the semicircular oscillator.

Consider a homogeneous solid circular sector object with angle α and radius R, as shown in Fig. 2.1.1, that rolls in an oscillatory motion back and forth on a flat, stationary support with no sliding effect. When α becomes radian, no oscillatory swinging motion will occur. It may be easily verified that the governing equation of the oscillation is as follows:

$$(\frac{3}{2}R^2-\frac{4\sin(\alpha)}{3\alpha}R\cos(\theta))\ddot{\theta}+$$

$$\mathrm{R}(\frac{2R\sin(\alpha)}{3\alpha}\sin(\theta))\dot{\theta}^2+(\frac{2\sin(\alpha)}{3\alpha}g)\sin(\theta)=0 \qquad (2.1.1)$$

$$\theta(0)=\mathrm{A}\,,\ \dot{\theta}(0)=0$$

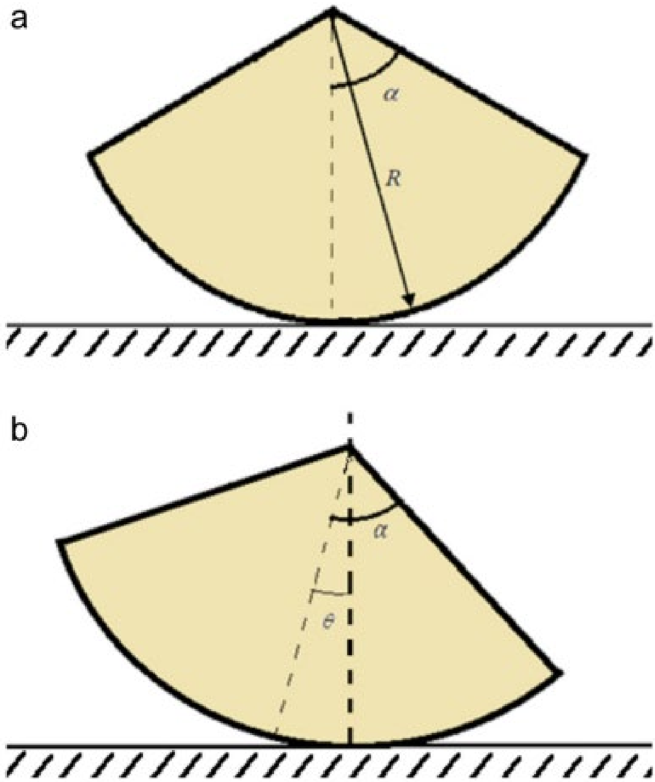


Fig. 2.1.1. Geometric parameters of the homogeneous solid circular sector body

By substitution of the relatively accurate approximations $\sin(\theta) \approx \theta - \frac{\theta^3}{3!}$ and $\cos(\theta) \approx 1 - \frac{\theta^2}{2!}$ Into Eq. (2.1.1), the governing equation would be in the following order:

$$(\frac{3}{2}R^{2}-\frac{4\sin(\alpha)}{3\alpha}(1-\frac{\theta^{2}}{2!}))\ddot{\theta}+$$
$$\text{R}(\frac{2R\sin(\alpha)}{3\alpha}(\theta-\frac{\theta^{3}}{3!})\dot{\theta}^{2}+(\frac{2\sin(\alpha)}{3\alpha}g)(\theta-\frac{\theta^{3}}{3!})=0 \qquad (2.1.2)$$

$$\theta(0)=\text{A}\ ,\ \dot{\theta}(0)=0$$

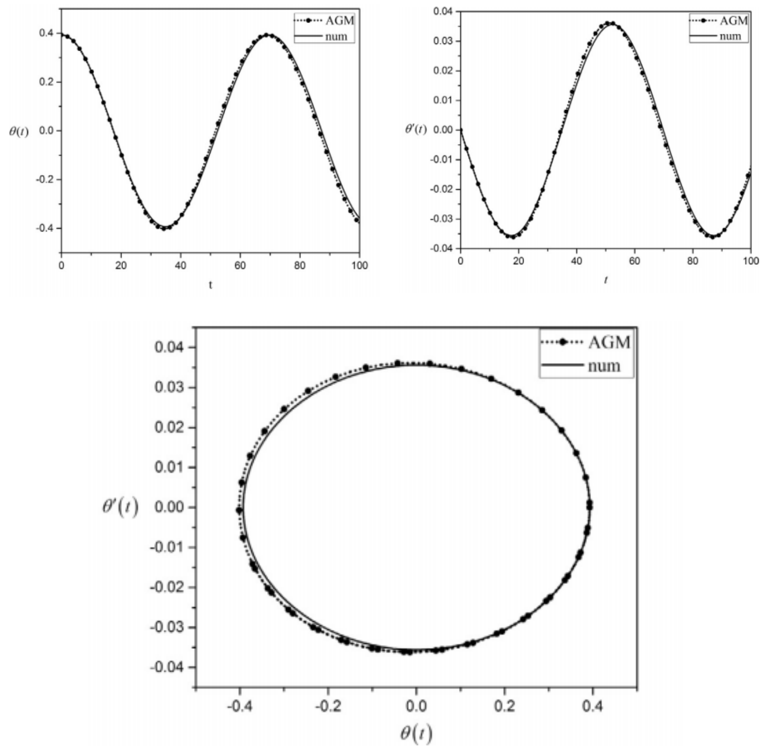


Fig. 2.1.2. (a, b, c) A comparison between the solutions obtained by AGM and the numerical method for R = 15, α = 2π/3.

According to the basic idea of the proposed method, first, we rewrite Eq. (2.1.1) in the following order:

$$\text{G(t)}:\left(\frac{3}{2}R^{2}-\frac{4\sin(\alpha)}{3\alpha}\left(1-\frac{\theta^{2}}{2!}\right)\right)\ddot{\theta}+ \\ \text{R}\left(\frac{2R\sin(\alpha)}{3\alpha}\left(\theta-\frac{\theta^{3}}{3!}\right)\dot{\theta}^{2}+\left(\frac{2\sin(\alpha)}{3\alpha}g\right)\left(\theta-\frac{\theta^{3}}{3!}\right)=0\right. \tag{2.1.3}$$

For solving the nonlinear differential equation by AGM, it is necessary to consider a function as the solution to the presented problem as follows:

$$\theta(t)=e^{-at}\left(b_{0}\cos(\omega t+\phi_{0})+b_{1}\cos(2\omega t+\phi_{1})\right) \tag{2.1.4}$$

The term (e^{-at}) in Eq. (2.1.4) indicates a damping component in the oscillating system. Since no damping components exist in the mentioned example, the term (a) in Eq. (2.1.4) will automatically be zero after applying the initial conditions in AGM. Moreover, the constant coefficient (b), the initial vibrational phase (φ), and the angular frequency (ω) can be computed by applying the initial conditions. According to the above theory, the solution to the problem is assumed as follows:

$$\theta(t)=b_{0}\cos(\omega t+\phi_{0})+b_{1}\cos(2\omega t+\phi_{1}) \tag{2.1.5}$$

Regarding the proposed physical model, there are no boundary conditions, so the constant coefficients of Eq. (2.1.5) are acquired only concerning the given initial conditions presented in Eq. (2.1.3). Notably, initial or boundary conditions are applied in two manners in the following form:

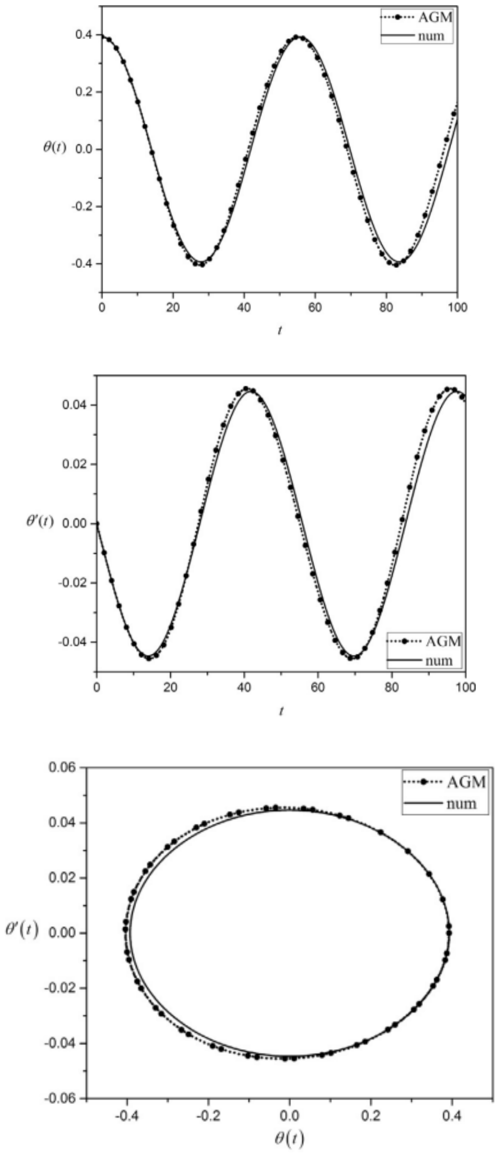


Fig. 2.1.3. (a, b, c) A comparison between the solutions obtained by AGM and the numerical method for $R = 15$, $\alpha = \pi/2$.

1) The initial conditions are applied to Eq. (2.1.5) in the form of

$$\theta = \theta(IC) \tag{2.1.6}$$

the initial condition (IC) is the abbreviation of the initial conditions. As a result, applying the initial conditions to Eq. (2.1.5) is performed as follows:

$$\theta(0) = A \rightarrow a_0 \cos(\varphi_0) + a_1 \cos(\varphi_1) = A \tag{2.1.7}$$

$$\dot{\theta}(0) = 0 \rightarrow -a_0\omega\sin(\varphi_0) - 2a_1\omega\sin(\varphi_1) = 0 \tag{2.1.8}$$

2) The initial conditions are applied to the main differential equation, which in the problem is Eq. (2.1.3), and on its derivatives in the following general forms:

$$G(\theta(t)) \rightarrow G(\theta(IC)) = 0,\ G'(\theta(IC)) = 0, \ldots \tag{2.1.9}$$

Therefore, after substituting Eq. (2.1.5), which has been considered as the solution to the main differential equation, into Eq. (2.1.3), the initial conditions are applied to the equation obtained and its derivative based on Eq. (2.1.9) as follows:

$$\begin{aligned} G(\dot{\theta}(0)) &: (\frac{3}{2}R^2 - \frac{4}{3}\frac{R\sin(\alpha)\cos(\psi)}{\alpha})(\Gamma_2) \\ &+ \frac{2}{3}\frac{R^2\sin(\alpha)\sin(\psi)(\Gamma_1)^2}{\alpha} \\ &+ \frac{2}{3}\frac{g\sin(\alpha)\sin(\psi)}{\alpha} = 0 \end{aligned} \tag{2.1.10}$$

$$\begin{aligned}
G'(\dot{\theta}(0)) &= \frac{4}{3}\frac{R\sin(\alpha)\sin(\psi)(\Gamma_1)(\Gamma_2)}{\alpha} \\
&+(\frac{3}{2}R^2 - \frac{4}{3}\frac{R\sin(\alpha)\cos(\psi)}{\alpha})(\Gamma_3) \\
&+\frac{2}{3}\frac{R^2\sin(\alpha)\cos(\psi)(\Gamma_1)^3}{\alpha} \\
&+\frac{4}{3}\frac{R^2\sin(\alpha)\sin(\psi)(\Gamma_1)(\Gamma_2)}{\alpha} \\
&+\frac{2}{3}\frac{g\sin(\alpha)\cos(\psi)(\Gamma_1)}{\alpha} = 0
\end{aligned} \tag{2.1.11}$$

$$\begin{aligned}
G'(\dot{\theta}(0)) &: \frac{4}{3}\frac{R\sin(\alpha)\cos(\psi)(\Gamma_1)^2(\Gamma_2)}{\alpha} \\
&+\frac{4}{3}\frac{R\sin(\alpha)\sin(\psi)(\Gamma_2)^2}{\alpha} \\
&+\frac{8}{3}\frac{R\sin(\alpha)\sin(\psi)(\Gamma_1)(\Gamma_3)}{\alpha} \\
&+(\frac{3}{2}R^2 - \frac{4}{3}\frac{R\sin(\alpha)\cos(\psi)}{\alpha})(\Gamma_4) \\
&-\frac{2}{3}\frac{R^2\sin(\alpha)\sin(\psi)(\Gamma_1)^4}{\alpha} \\
&+\frac{10}{3}\frac{R^2\sin(\alpha)\cos(\psi)(\Gamma_1)^2(\Gamma_2)}{\alpha} \\
&+\frac{4}{3}\frac{R^2\sin(\alpha)\sin(\psi)(\Gamma_2)^2}{\alpha} \\
&+\frac{4}{3}\frac{R^2\sin(\alpha)\sin(\psi)(\Gamma_1)(\Gamma_3)}{\alpha} \\
&-\frac{2}{3}\frac{g\sin(\alpha)\sin(\psi)(\Gamma_1)^2}{\alpha} \\
&+\frac{2}{3}\frac{g\sin(\alpha)\cos(\psi)(\Gamma_2)}{\alpha}
\end{aligned} \tag{2.1.12}$$

To simplify, we use the following statements:

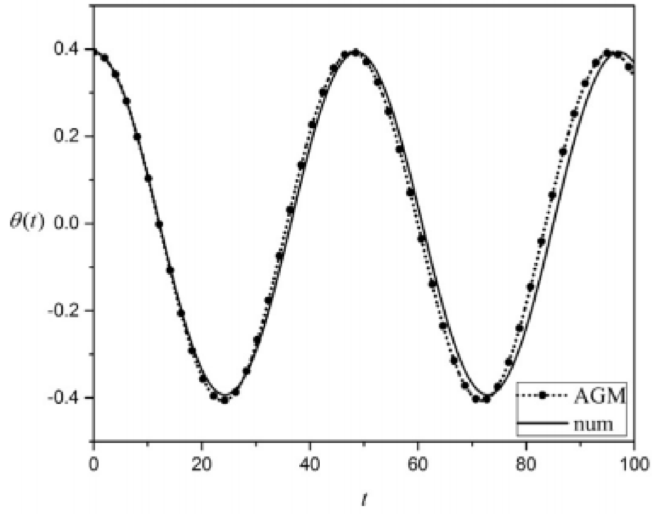
$$
\begin{aligned}
&\psi = b_0 \cos(\phi_0) + b_1 \cos(\phi_1),\\
&\Gamma_1 = -b_0\omega \sin(\phi_0) - 2b_1\omega \sin(\phi_1),\\
&\Gamma_2 = -b_0\omega^2 \cos(\phi_0) - 4b_1\omega^2 \cos(\phi_1),\\
&\Gamma_3 = b_0\omega^3 \sin(\phi_0) + 8b_1\omega^3 \sin(\phi_1),\\
&\Gamma_4 = b_0\omega^4 \cos(\phi_0) + 16b_1\omega^4 \cos(\phi_1).
\end{aligned}
\tag{2.1.13}
$$

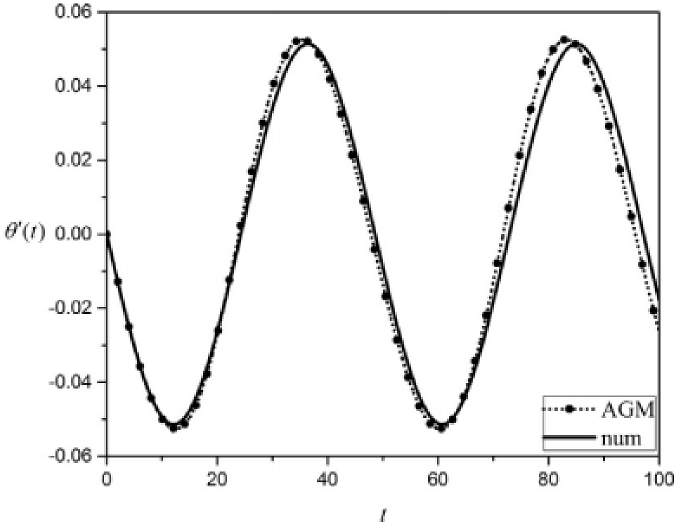
Solving a set of five algebraic equations with five unknowns from Eqs. (2.1.7) and (2.1.8) and Eqs. (2.1.10)-(2.1.12), with the assumption of $A = \dfrac{\pi}{8}\ R = 15, \alpha = \dfrac{2\pi}{3}, g = 9.81,$ the constant coefficients b_0, b_1, ϕ_0, ϕ_1 and ω Eq. (2.1.5), can easily be yielded as follows:

$$
\begin{aligned}
&b_0 = -0.3972120591,\ \mathrm{b}_1 = 0.004512977299,\\
&\phi_0 = -9.424777961,\ \phi_1 = -9.424777961,\\
&\omega = -0.09096384741
\end{aligned}
\tag{2.1.14}
$$

After substituting the obtained values from Eq. (2.1.14) and into Eq. (2.1.5), the solution to the problem with the assumed physical constants will be obtained as follows:

$$
\begin{aligned}
&\theta(t) = -0.3972120591\ \times\\
&cos\,(0.09096384741t + 9.424777961)\\
&+0.004512977299\times\\
&cos\left(0.1819276948t + 9.424777961\right)
\end{aligned}
\tag{2.1.15}
$$


0.4
0.2
$\theta(t)$ 0.0
-0.2
-0.4
0
20
40
60
80
100
t
AGM
num


0.06
0.04
0.02
$\theta'(t)$ 0.00
-0.02
-0.04
-0.06
0
20
40
60
80
100
t
AGM
num

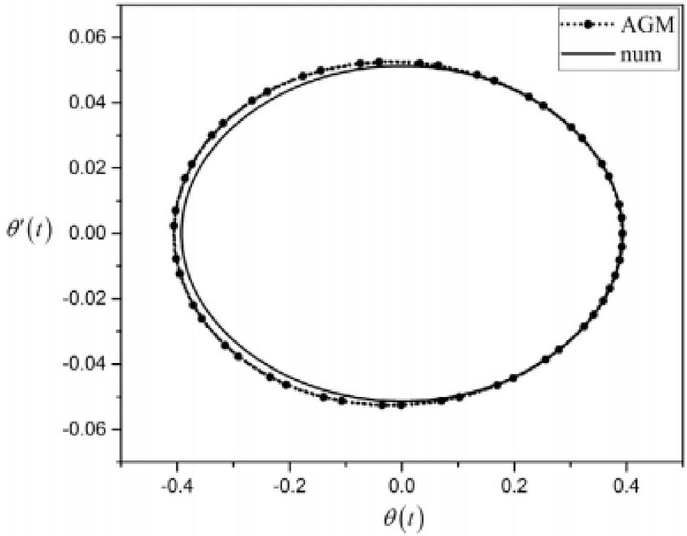


Fig. 2.1.4. (a, b, c) A comparison between the solutions obtained by AGM and the numerical method for R = 15, α = π/3.

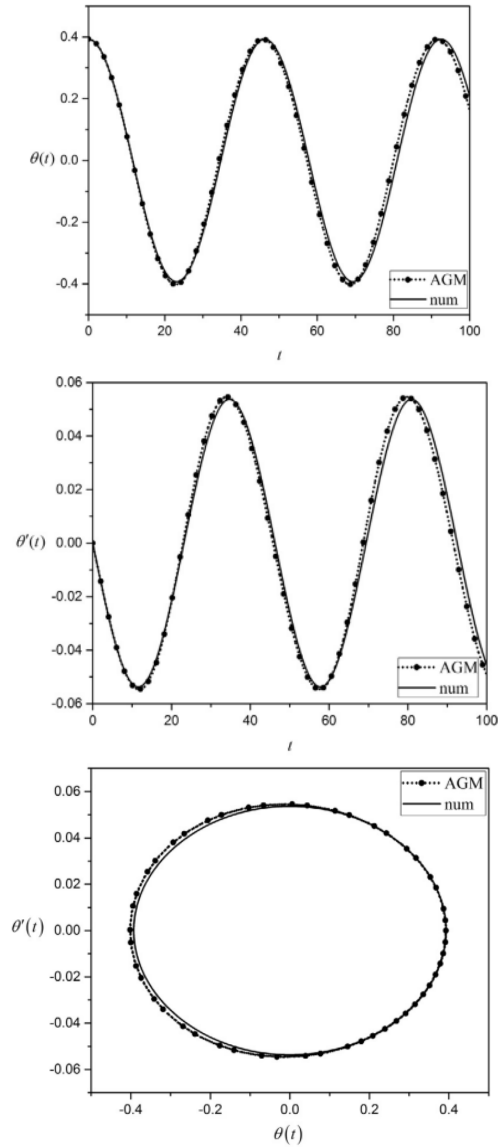


Fig. 2.1.5. (a, b, c) A comparison between the solutions obtained by AGM and the numerical method for R = 10, α = 2π/3.

In this problem, AGM has been utilized to solve the nonlinear differential equation of the swinging oscillation of a solid circular sector, and the results have been compared with the numerical solution. Although the processes of obtaining an analytical solution were accomplished in the previous section to make comprehensive comparisons and illustrate the accuracy and efficiency of the proposed method, we have investigated the effects of semicircular radius (R) and semicircular angle (α). Therefore, we have assumed the following appropriate amounts for our purposes:

$$1) R = 15 \rightarrow \alpha = \frac{2\pi}{3}, \alpha = \frac{\pi}{2}, \alpha = \frac{\pi}{3}$$

$$2) R = 10 \rightarrow \alpha = \frac{2\pi}{3}, \alpha = \frac{\pi}{2}, \alpha = \frac{\pi}{3}$$

$$3) R = 5 \rightarrow \alpha = \frac{2\pi}{3}, \alpha = \frac{\pi}{2}, \alpha = \frac{\pi}{3}$$

Figs. 2.1.2–2.1.4 demonstrate that the oscillation frequency will increase considerably by reducing the semicircular angle (α) with a constant semicircular radius (R). Notably, the frequency of the oscillation determined with AGM is generally higher than that determined with the numerical solution. Because the oscillation frequency is negatively correlated with the oscillation frequency, the oscillation period will increase. Apart from the mentioned results, these figures depict that the oscillation trends are somehow harmonic and stable. This is mainly because the vibration amplitude does not change at all. The same results are achieved in Figs. 2.1.5–2.1.10. A comparison of Fig. 2.1.2, Fig. 2.1.5, and Fig. 2.1.8 illustrates that the oscillation frequency will increase by reducing the semicircular radius (R). In addition, the velocities reach their peaks when the semicircular radius (R) or the semicircular angle (α) decreases (Tables 2.1.1–2.1.3).

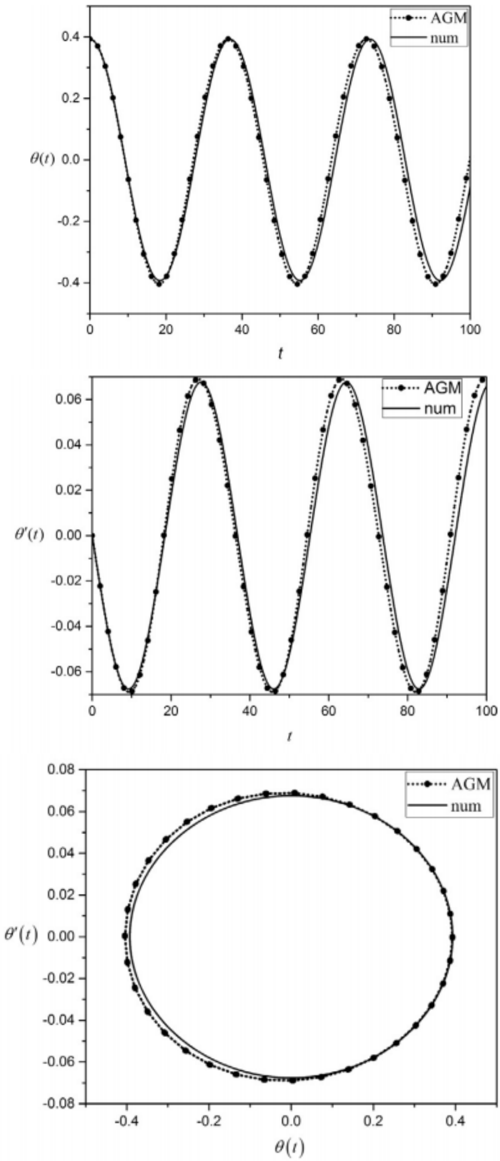


Fig. 2.1.6. (a, b, c) A comparison between the solutions obtained by AGM and the numerical method for R = 10, α = π/2.

Table 2.1.1. The results obtained by the Numerical Solution and AGM for R = 15 and α=2π/3

t	*Num*	*AGM*	*Error* = |*Num* − *AGM*|
0	0.39269908	0.39269908	0.00000000
10	0.24512253	0.24501045	0.00011208
20	−0.08993096	−0.09371990	0.00378893
30	−0.35555274	−0.36692738	0.01137464
40	−0.35254525	−0.35162485	0.00092039
50	−0.08307460	−0.06065639	0.02241821
60	0.25054818	0.26978783	0.01923964
70	0.39263866	0.39135302	0.00128564
80	0.23961867	0.21847002	0.02114865
90	−0.09675804	−0.12620233	0.02944428
100	−0.35844921	−0.37955984	0.02111063

Table 2.1.2. The results obtained by the numerical solution and AGM for R=15, α=π/2

t	*Num*	*AGM*	*Error* = |*Num* − *AGM*|
0	0.39269908	0.39269908	0.00000000
10	0.17012305	0.16964486	0.00047818
20	−0.24887845	−0.25944763	0.01056917
30	−0.38187454	−0.38749464	0.00562010
40	−0.08143699	−0.05286683	0.02857015
50	0.31332380	0.33172291	0.01839910
60	0.34991634	0.33408668	0.01582966
70	−0.01205181	−0.04827707	0.03622526
80	−0.36004875	−0.38611530	0.02606655
90	−0.29838583	−0.26304319	0.03534264
100	0.10482796	0.16555209	0.06072413

In this problem, AGM has been used to solve a nonlinear equation of circular sector oscillation systems. The plots and tables represent that AGM has acceptable accuracy compared with the numerical method. We can claim that AGM is a strong analytical method for solving linear and nonlinear equations, especially vibrational problems.

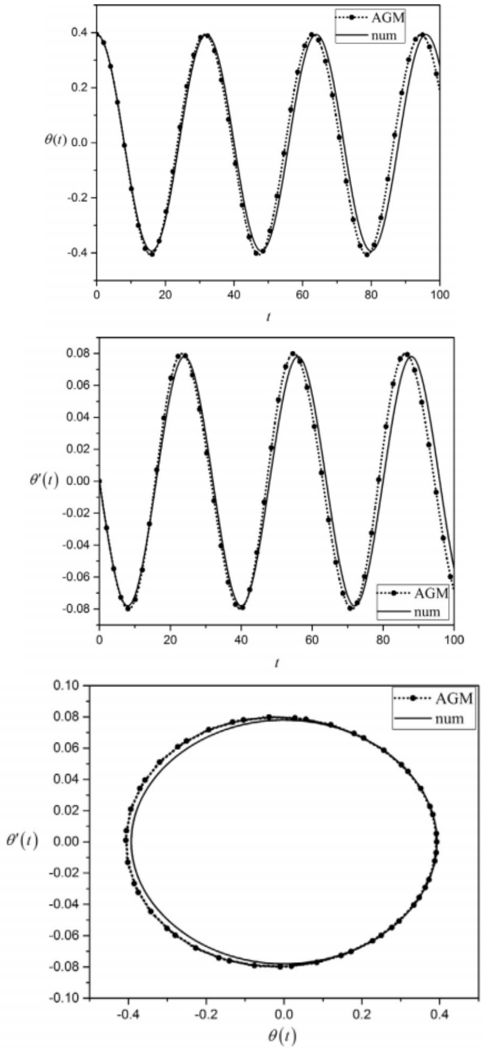


Fig. 2.1.7. (a, b, c) A comparison between the solutions obtained by AGM and the numerical method for R = 10, α = π/3.

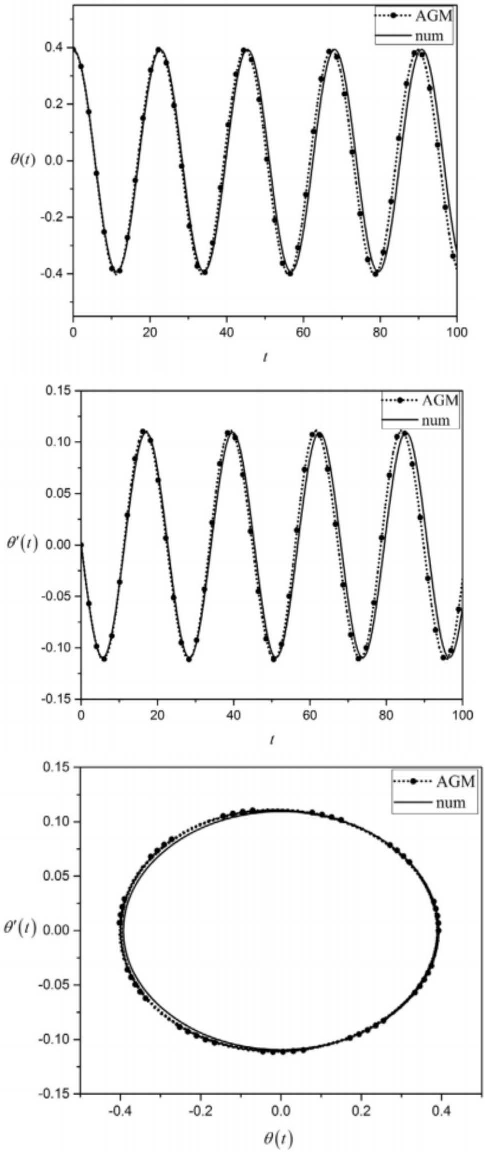


Fig. 2.1.8. (a, b, c) A comparison between the solutions obtained by AGM and the numerical method for R = 5, $\alpha = 2\pi/3$.

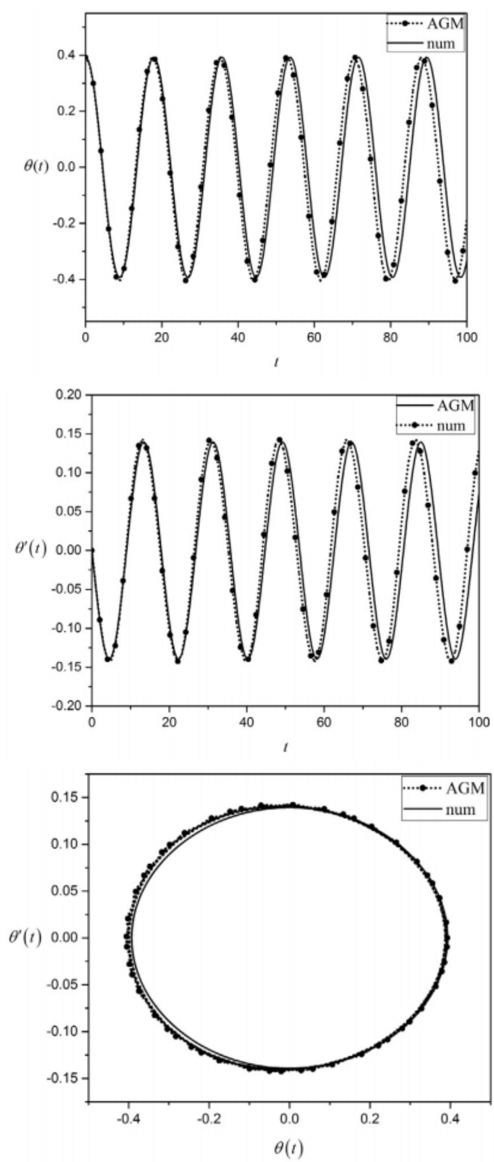


Fig. 2.1.9. (a, b, c) A comparison between the solutions obtained by AGM and the numerical method for R = 5, α = π/2.

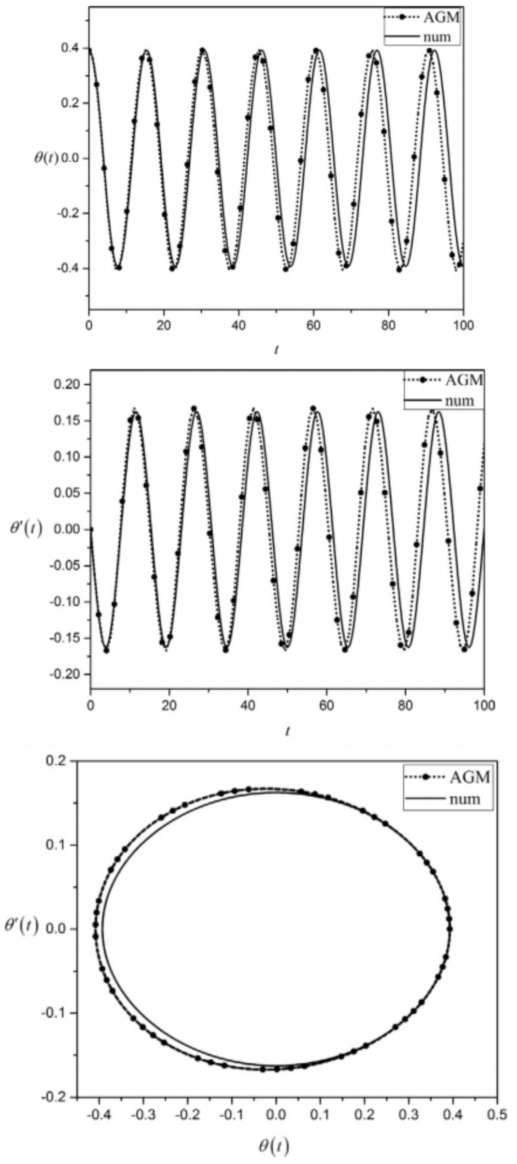


Fig. 2.1.10. (a, b, c) A comparison between the solutions obtained by AGM and the numerical method for R = 5, α = π/3.

Table 2.1.3. The results obtained by the numerical solution and AGM for R=15, $\alpha=\pi/3$

t	*Num*	*AGM*	*Error* = |*Num* − *AGM*|
0	0.39269908	0.39269908	0.00000000
10	0.10875275	0.10762438	0.00112836
20	−0.33471506	−0.35104698	0.01633191
30	−0.29201803	−0.27870234	0.01331569
40	0.17597552	0.20864948	0.03267395
50	0.38631310	0.37789019	0.00842291
60	0.03769972	−0.00330218	0.04100190
70	−0.36626688	−0.39470691	0.02844003
80	−0.23945080	−0.18427538	0.05517541
90	0.23706674	0.29236833	0.05530158
100	0.36732664	0.33439609	0.03293055

In AGM, it is very convenient and easy to find the solution to the differential equation and the angular frequency (ω) simultaneously by merely selecting a function as the solution to the differential equation regarding the kind of operating system. In addition to the explanations above, after applying initial conditions to the considered answer, we exit from the field of differential equations into a set of algebraic equations. By solving a set of algebraic equations, a simple method, constant response coefficients are considered, and the angular frequency can be easily obtained.

2.2. Vibration in Arched Beam

Analysing and modelling the vibrational behaviour of arched bridges during an earthquake to decrease the damage to the structure is a very hard task. This has been performed analytically for the first time in the present study.

In general, vibrational equations and their initial conditions are defined for different systems as follows:

$$f\,(\ddot{u}, \dot{u}, u, F_0 \sin(\omega_0 t)) = 0 \tag{2.2.1}$$

Parameter (ω_0) is the angular frequency of the harmonic force exerted on the system, and (F_0) is its maximum amplitude. The initial conditions are as follows: