The Life of Cracks

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Theory and Application

By

Srečko Glodež and Boris Aberšek

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TABLE OF CONTENTS

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FOREWORD

The term *fracture and damage mechanics* is somewhat unsettling to many people. This is because, until recently, the major emphasis in mechanics was on the strength and resistance of materials. To speak of fracture is as uncomfortable for some as it is to speak of a fatal illness. However, just as in preventing a fatal disease, one must know its nature, symptoms, and behavior; to ensure the strength of a structure, one must be aware of the causes and nature of its potential failure.

The problem of fracture is vital in the science of strength of materials. However, not only has fracture mechanics, as an independent branch of the mechanics of deformable solids, originated quite recently, but its boundaries are not yet clearly defined. Therefore, it is of paramount interest to combine the efforts of representatives from many different branches of science and engineering for a complete study of the fracture concept. It is also important that differences in terminology (that are usual for different sciences), and the widespread conviction that the solution to everything lies in a particular portion of the general problem, do not lead to a situation in which disputes about the concepts are replaced by arguments about the words.

At present, routine fracture mechanics is the study of conditions under which a crack or a system of cracks undergoes propagation. However, cracks are of different natures, and are considered on different scale levels. The case on one extreme is the fracture of a crystal grain, which initiates with a submicroscopic crack when two atomic layers move apart by such a distance that the forces of interaction between the atoms may be neglected. An example of the other extreme is a crack occurring in a welded turbine rotor in a nuclear reactor, when the crack's length and width may amount to centimeters; this is referred to as a macroscopic fracture.

In the first case, the condition for crack propagation is defined by the configuration of atoms at the crack tip. Considered here is a discrete crystal lattice formed by atoms rather than a continuous medium; therefore, the very concept of the "crack tip" becomes uncertain. The study of this kind of submicroscopic crack and its behavior in interaction with other lattice defects is, essentially, in the province of solid-state physics rather than mechanics; however, the methods of classical theory of elasticity are fully applicable to problems of this nature. The line between modern physics and mechanics is not well defined; nevertheless, it must be drawn to avoid possible terminological confusion.

A macroscopic fracture has dimensions exceeding by several orders the size of the largest structural constituent of the material (the constituent must contain a sufficient number of crystal grains for its properties not to differ from those of any other element of similar size which may be isolated from the material). It is precisely this condition that makes it possible to solve such a crack problem within the framework of mechanics of a solid body. The formulated condition refers to an ideal situation in order to make the theory applicable; in real conditions one may depart from this stringent requirement, but this in no way makes the theory groundless. Assuming the material to be continuous, homogeneous, and elastic, and using the techniques of the classical theory of elasticity, we inevitably arrive at the paradoxical conclusion that the stresses grow infinitely near the crack tip. This paradox is a sort of penalty paid for the simplicity associated with using the linear theory of elasticity in a region where its application is knowing to be invalid.

So-called linear fracture mechanics assume that a physically impossible singularity is a reality. Such an approach is not new and not so unusual for continuum mechanics; recall, for example, the vortex filaments with zero cross section and finite circulation. It appears that the work of crack propagation, which is done either as a result of increase of external forces or reduction of the elastic energy of the body with the crack size increase, is expressed directly through the coefficient of the singular term in the formula for stress. This coefficient is referred to as the stress intensity factor, and is of fundamental importance for the entire theory. The work of crack propagation may be associated with overcoming the forces of surface tension (Griffith's concept), or the plastic deformation in the small region of the immediate neighborhood of the crack tip, or other physical causes. The factor to be emphasized is that the size of the region, where the laws of the linear theory of elasticity are in some way violated, must be very small. The ability of the crack to further propagate is then determined by the sole characteristic: the work per unit length of the propagation path, or the critical stress intensity factor. If the size of the zone, where the relations of the linear theory of elasticity are violated, is large, one should consider the laws of nonlinear fracture mechanics. It appeared at the beginning that formal indifference of linear fracture mechanics to both the object and the scale, mathematical equivalence of problems associated with entirely different physical phenomena, would make it possible to establish nonlinear mechanics in a similar uniform manner. It was later found to be quite different.

The principal problem, on which the efforts of scientists have been focused in recent years, concerns the conditions of either equilibrium or the propagation of a large crack in a sufficiently plastic material. Scientists have been involved in the theory and practical applications of fracture mechanics for evaluating the strength of large-scale structural elements. They have shown that the plastic zone ahead of a crack is sufficiently extensive so that the macroscopic theory of plasticity, which assumes that the medium is continuous and homogeneous, holds good. For the plane state of stress, the Leonov-Panasyuk-Dugdale model, which substitutes the plastic zone by a no-thickness segment extending the crack, appears to be satisfactory. In particular, this book presents an analysis of the corresponding elastic-plastic problem that is solved numerically by using the finite element method (FEM). The presented FEM-solution confirms the validity of the model used.

This book by S. Glodež and B. Aberšek is one of the first Slovenian monographs in international space on the above-discussed subject. It is based mainly on the results obtained by the authors during their original research and concerns the problems of fatigue and fracture mechanics. The greater part of the book is devoted to fatigue problems related to gears and other mechanical elements.

In spite of certain limits imposed by linear fracture mechanics, a wide variety of problems may be reliably solved using its methods. Development of this theory is focused on accumulating data from already solved elasticity problems concerning cracks of various shape in various bodies. The amount of such information continually grows both abroad and in Slovenia. Many results obtained by foreign authors became available by means of numerous books and published articles.

The present book may be considered as a significant contribution to the database of fracture mechanics, especially for gears. Some features of the book deserve special mention. First, it is the new variational principle that makes it possible to approximately solve numerous problems, in particular, to find the trajectory of crack propagation in a nonuniform stress field. Secondly, a straightforward approach for an approximate determination of the stress intensity factor is included; it enables one to obtain a reasonable evaluation for those cases where an exact solution of the elasticity problem is impossible, and the numerical computation is extremely laborious. In addition, a series of newly solved dynamic problems for bodies subjected to cyclic (periodic) loading is provided.

CHAPTER 1

INTRODUCTION

Fatigue of engineering components and structures is a localised damage process produced by cyclic loading. In general, it has been observed that the fatigue process involves the following stages [1.1, 1.2]: (1) crack nucleation; (2) short crack growth; (3) long crack growth; and (4) final fracture. Fatigue cracks usually start on the localised shear plane or near high stress concentrations (persistent slip bands, inclusions, pores, etc.). Crack nucleation is the first phase in the complete fatigue process. Once the initial crack is nucleated and cyclic loading continues, the fatigue crack tends to grow along the plane of maximum shear stress.

Figure 1.1 shows a schematic representation of the fatigue process under cyclic loading where the crack nucleation (stage I) starts along the persistent slip bands. The next stage in the fatigue process is crack growth, which can be divided into short (stage II) and long (stage III) crack growth. Crack nucleation and short crack growth (stages I and II) are generally considered as a crack propagation across a few crystal grains in the plane of maximum shear stress. Here, the crack tip plasticity is greatly affected by slip band characteristics, grain size and their orientation, because the crack length is comparable to the material microstructure. Stage III corresponds to the long crack growth in the direction normal to the principal tensile stress. The long crack growth is less affected by the material microstructure because the crack tip plastic zone is much larger if compared to the microstructural properties (size of crystal grains). The fatigue process is finished when the crack reaches a critical length and final fractures occur (stage IV in Figure 1.1).

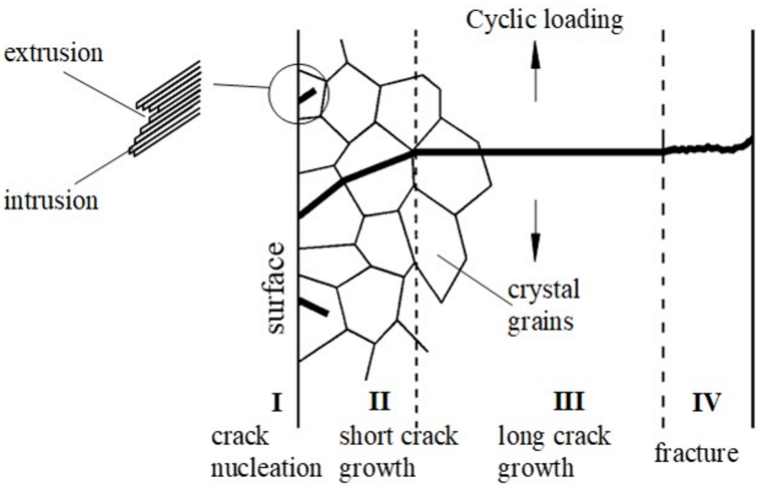

Figure 1.1: The fatigue process under cyclic loading [1.2, 1.3]

In engineering applications, the first two stages (crack nucleation and short crack growth) are usually termed as "*crack initiation period N_i*", while the last two stages (long crack growth and final fracture) are characterised as "*crack propagation period N_p*". The complete fatigue life of an analysed engineering component can then be determined from the number of stress cycles N_i required for fatigue crack initiation and the number of stress cycles N_p required for a crack to propagate from the initial to the critical crack length, when the final failure can be expected to occur:

$$N = N_\mathrm{i} + N_\mathrm{p} \tag{1.1}$$

An exact definition of the transition period from the initiation of an "engineering" crack to its propagation is usually not possible. However, for engineering components made of steels the size of initial crack a_i after stages I and II (see Figure 1.1) is of the order of a few crystal grains of the material. This crack size usually ranges from about 0.1 to 1.0 mm. According to Dowling [1.4], the crack initiation size can also be estimated by the following equations:

$$a_i \approx \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta S_e}\right)^2 \qquad \text{smooth specimen} \tag{1.2}$$

$$a_i \approx (0.1 \ldots 0.2) \cdot R \quad \text{notched specimen} \tag{1.3}$$

where ΔS_e is the stress range at the fatigue limit, ΔK_{th} is the range of the threshold intensity factor, and R is the notch-tip radius.

Once a crack has formed and propagated until the final fracture, the fatigue fracture surface can be inspected. A bending fatigue failure, as presented in Figure 1.2, generally leaves behind clamshell or beach markings. It is evident that the crack is nucleated at one side of the shaft and then propagates away from the nucleation site, usually in a radial manner. A semi-elliptical pattern is left behind. When the crack reaches a critical length, the final fracture occurs. In some cases, inspection of the size and location of the beach marks left behind may indicate where a different period of crack growth began or ended.

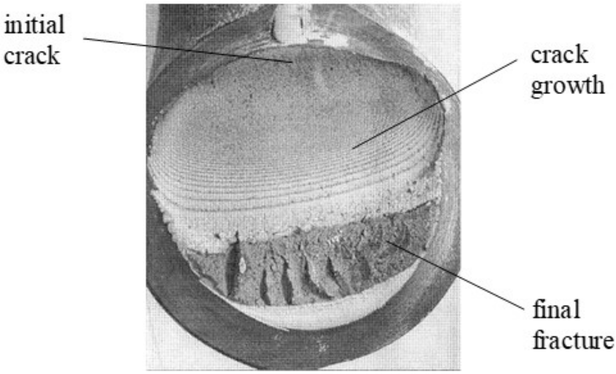

Figure 1.2: Fatigue fracture surface from a shaft in bending [1.5]

Figure 1.3 schematically shows fatigue fracture surfaces of cylinder specimens subjected to the axial or bending loading as a function of load magnitude (nominal stress) and geometry (notch effect) of the specimen. The white coloured regions identify the crack nucleation and the subsequent crack propagation while the grey coloured region represents the surface of final fracture. The direction of crack growth is marked with arrows where the beginnings of the arrows show the location of the crack nucleation. In each case shown in Figure 1.3, the fatigue cracks nucleate at the surface and then propagate in the plane of maximum tensile stress. In the case of uniaxial tension, tension/compression and unidirectional bending initial cracks are nucleated only at one side of the specimen, which corresponds to the locations of tensile stress. For reversed bending, cracks usually nucleate at opposite sides, since both sides are subjected to repeated tensile stress. In rotating bending, the crack may be initiated around the whole circumference of the specimen. The surface of the final fracture is significantly influenced by the magnitude of nominal stress and notch effect of the specimen.

Similar presentations for fracture surfaces could also be drawn for round specimens subjected to torsional loading and also for flat specimens. Detailed information about these problems can be found in [1.2, 1.4].

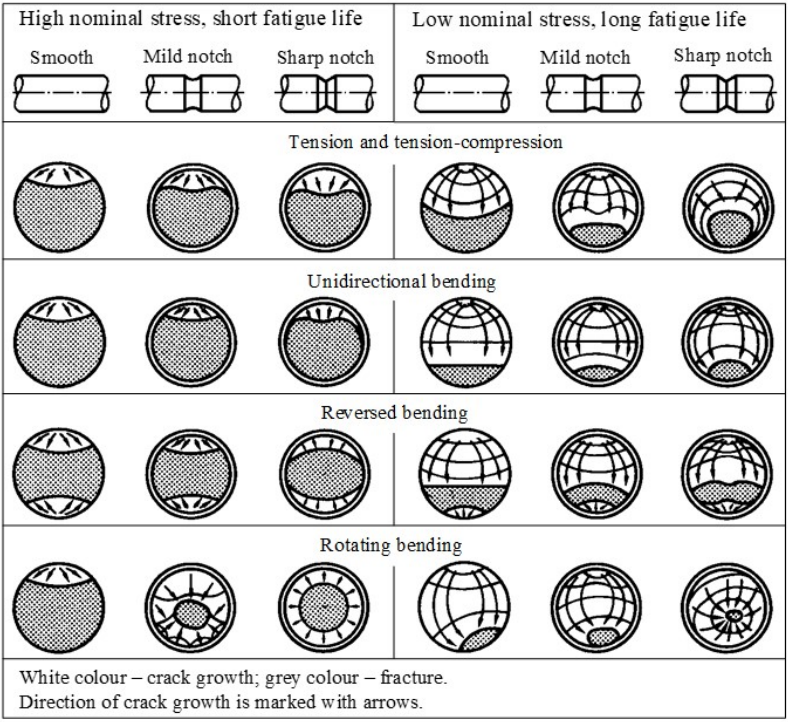

Figure 1.3: Schematic fatigue fractures of round specimens [1.2, 1.6]

1.1 Fatigue design approaches

The available fatigue design approaches have many similarities but also differences: an engineering component may be safety critical or nonsafety critical, the structure may be very simple or very complex, and failures may be a nuisance or catastrophic. Choosing the appropriate fatigue design approach is a crucial decision for engineers when dimensioning dynamically loaded structural components or machine parts. Three main approaches currently exist in fatigue design [1.2]:

- Stress-life approach (*S*–*N*),
- Strain-life approach (ε–*N*),
- Fatigue crack growth approach (d*a*/d*N*–Δ*K*).

The stress-life approach has been in use for more than 150 years, while the other two approaches have been available only since the 1960s. Some authors [1.2, 1.4] have proposed a fourth model, the so-called "two-stage approach", which consists of combining the strain-life approach and the fatigue crack growth approach to incorporate both, macroscopic fatigue crack initiation and the subsequent fatigue crack growth.

1.1.1 Stress-life approach

The stress–life approach is the oldest method for dimensioning dynamically loaded machine parts and structural components. This approach is based on the *S*–*N* curves that are commonly plotted in terms of stress amplitude σ_a versus number of loading cycles to failure *N*. The most basic *S*–*N* curve is considered to be the one for zero mean stress $\sigma_m = 0$, which corresponds to the stress ratio $R = \sigma_{min}/\sigma_{max} = -1$. The relationship between the stress amplitude and the number of cycles to failure may be mathematically expressed as follows:

$$\sigma_a = \sigma_f{}'(2 \cdot N)^b \tag{1.4}$$

where σ_f’ is the fatigue strength coefficient and *b* is the fatigue strength exponent. Material parameters σ_f’ and *b* can be determined experimentally, usually by means of the rotating bending test [1.7, 1.8].

S–*N* curves vary with the material and its prior processing (thermal treatment). They are also affected by mean stress and geometry, especially the presence of notches, as well as by surface finish, residual stresses, loading frequency, environment conditions, etc. If an unnotched structural component is loaded with the mean stress $\sigma_m \neq 0$, the equivalent completely reversed stress amplitude σ_{ac} should be obtained when determining the fatigue life using eq. (1.4):

$$\sigma_{ac} = \frac{\sigma_a}{1-\frac{\sigma_m}{\sigma_f{}'}} \quad \text{Morrow correction} \tag{1.5}$$

$$\sigma_{ac} = \sqrt{\sigma_{max} \cdot \sigma_a} \quad \text{SWT correction} \tag{1.6}$$

The equivalent completely reversed stress amplitude σ_{ac} actually represents the applied combination of stress amplitude σ_a and mean stress σ_m, which results in the same fatigue life as the stress amplitude σ_{ac} applied at zero mean stress. Beside the equations (1.5) and (1.6), other criteria to determine the value of σ_{ac} may be found in specialist literature [1.9, 1.10]. The choice of the appropriate criterion is mainly dependent on the material of the treated structural component.

Equation (1.4) basically represents the relationship between the stress amplitude and the number of cycles when uniaxial fatigue loading is applied. However, this equation may also be generalized and used to determine fatigue life under multiaxial loading. In such cases, the values σ_a and σ_m in eq. (1.4) are replaced by equivalent stresses σ_{aE} and σ_{mE}. Here, the equivalent stress amplitude σ_{aE} is proportional to the amplitude of the octahedral shear stress while the equivalent mean stress σ_{mE} is proportional to the hydrostatic stress due to mean stresses in three directions.

If more than one amplitude or mean stress level occurs (variable amplitude loading), the complete loading spectrum may be divided into the *M*-loading intervals inside which the values σ_{ai} and σ_{mi} are constant. The fatigue life of treated structural component can then be obtained using the Palmgren-Miner rule:

$$\sum_{i=1}^{M} \frac{n_i}{N_i} \leq 1 \tag{1.7}$$

where n_i is the real number of loading cycles inside the i^{th} interval and N_i is the number of loading cycles to failure from the *S–N* curve for σ_{ai} and σ_{mi} in this interval.

Machine parts and structural components often comprise different geometric discontinuities (holes, fillets, grooves, keyways, etc.), which are unavoidable in their design. These stress raisers (notches) reduce fatigue strength and require careful attention in the design process. In the stress-life approach, the notch-effect is taken into account by modifying the unnotched *S–N* curve considering the fatigue notch factor K_f:

$$K_f = 1 + q \cdot (K_t - 1) \tag{1.8}$$

where K_t is the stress concentration factor based on the linear elastic theory and q is the notch sensitivity factor dependent on the material and notch radius. A value $q = 0$ indicates that the notch effect can be neglected ($K_f = 1$), whereas a value $q = 1$ indicates full notch sensitivity ($K_f = K_t$). Two approaches which are often used for determining the notch sensitivity factor q are Neuber's method [1.11], and Peterson's [1.12] method.

In engineering design, the stress-life approach is usually applied by dimensioning machine parts and structures according to the *Infinite-Life Design* criterion, or the *Safe-Life Design* criterion (see section 1.2). Both criteria assume that the stresses in the critical cross-sections are in the linear elastic area. The calculation procedure is focused on the appearance of a final failure (fracture) of the treated component in the critical cross-section, and is not conditioned with crack nucleation and its growth.

1.1.2 Strain-life approach

The strain-life approach is based on the knowledge of stresses and strains that occur at locations where fatigue crack nucleation is likely to start, such as holes, fillets, grooves, etc. In this approach, the behavior of the material is characterized by the use of the stable cyclic stress-strain curve (eq. 1.9) and the strain-life curve (eq. 1.10) from uniaxial loading:

$$\sigma_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{K'}\right)^{\frac{1}{n'}} \tag{1.9}$$

$$\varepsilon_a = \frac{\sigma_f{}'}{E}(2N_i)^b + \varepsilon_f{}' \cdot (2N_i)^c \tag{1.10}$$

where σ_a is the stress amplitude, E is Young's modulus, K' is the cyclic strength coefficient, n' is the cyclic strain hardening exponent, σ_f' is the fatigue strength coefficient, b is the fatigue strength exponent, ε_f' is the fatigue ductility coefficient, c is the fatigue ductility exponent, and N_i is the number of loading cycles required for fatigue crack initiation (according to the eq. 1.1). Material parameters K', n', σ_f', b, ε_f' and c can be determined experimentally, usually by means of uniaxial fatigue tests according to the ASTM E606 standard [1.11]. These parameters vary over a range of different engineering materials and their processing histories (such as thermal treatment).

Equation (1.10) generally applies for zero mean stress $\sigma_m = 0$, which corresponds to completely reversed straining, $R = \varepsilon_{min}/\varepsilon_{max} = -1$. However, a mean strain or stress can be present in many engineering applications. In this case, the effect of mean stress σ_m can be considered using the following equations [1.2, 1.4]:

$$\varepsilon_a = \frac{\sigma_f{}' - \sigma_m}{E}(2N_i)^b + \varepsilon_f'(2N_i)^c \qquad \text{Morr} \tag{1.11}$$

$$\varepsilon_a = \frac{\sigma_f{}' - \sigma_m}{E}(2N_i)^b + \varepsilon_f'\frac{\sigma_f{}' - \sigma_m}{\sigma_f{}'}(2N_i)^c \quad \text{Manson} \tag{1.12}$$

$$\sigma_{max}\varepsilon_a E = \left(\sigma_f{}'\right)^2(2N_i)^{2b} + \sigma_f{}'\varepsilon_f' E(2N_i)^c \quad \text{SWT} \tag{1.13}$$

If multiaxial loading occurs, then the stress-strain and the strain-life relations need to be used in a more general form. Analogous to the equivalent stress approach (see section 1.1.1), the equivalent strain amplitude ε_{aE} can be used when determining fatigue life N_i. The most commonly used theories for determining the equivalent strain amplitude ε_{aE} under proportional loading conditions are the maximum principal strain theory (appropriate for brittle materials), and the maximum shear strain theory (appropriate for ductile materials):

$$\varepsilon_{aE} = \varepsilon_{a1} \qquad \text{maximum principal strain theory} \tag{1.14}$$

$$\varepsilon_{aE} = \frac{\varepsilon_{a1} - \varepsilon_{a3}}{1+\nu} \qquad \text{maximum shear strain theory} \tag{1.15}$$

In equations (1.14) and (1.15), ν is Poisson's ratio, and ε_{a1} and ε_{a3} are principal alternating strains, where $\varepsilon_{a1} > \varepsilon_{a3}$. Once an equivalent strain amplitude, ε_{aE}, has been calculated using the appropriate strain theory, the value ε_a is replaced with the value ε_{aE} to determine fatigue life using equations (1.10) to (1.13).

In the case of variable amplitude loading, the strain-life approach accounts for load sequence effects and is generally advantageous for cumulative damage analyses of notch members where significant plasticity usually occurs due to notch effect. The strain-life approach for fatigue of notched members consists of two stages. First, the stress and strains around the notch are determined (usually numerically by FEM). Afterwards, a life prediction is made using the appropriate strain-life equation with consideration of the notch root stresses and strains.

The strain-life approach is a comprehensive approach, which can be applied for determining fatigue life of structural components in both, low-cycle fatigue (LCF) and high-cycle fatigue (HCF) regimes. In the low-cycle region, the component of plastic strain is dominant, while in the high-cycle region the elastic strain component is dominant. From that perspective, ductile materials have better fatigue resistance at large strain, whereas the material strength is the crucial parameter against fatigue failure at smaller strains.

The strain-life approach can also be used in combination with the fatigue crack growth approach (see section 1.1.3) to obtain total fatigue lives for crack initiation and the subsequent crack growth. From this point of view, the *Safe-Life Design* criterion and the *Fail-Safe Design* criterion (see section 1.2) are usually used when determining the fatigue life of cyclic loaded components.

1.1.3 Fatigue crack growth approach

The fatigue crack growth approach requires the use of fracture mechanics and its integration into the fatigue crack growth theory to obtain the number of loading cycles required for crack propagation form initial to the critical crack length, when final fracture of the treated component can be expected to occur. This approach can be applied to determine total fatigue life when it is used in conjunction with information on the existing initial crack, which has been, for example, detected by previous examination. The "two-stage approach" (see section 1.1) means that the fatigue crack growth approach is combined with the strain-life approach. In this case, total fatigue life can be determined from the number of stress cycles N_i required for fatigue crack initiation (previously determined using the strain-life approach) and the number of stress cycles N_p required for a crack to propagate from the initial to the critical crack length (subsequently determined using the fatigue crack growth approach); see also equation (1.1).

In engineering design, the fatigue crack growth approach is usually used by dimensioning the machine parts and structures considering the *Fail-Safe Design* criterion (see section 1.2). Moreover, this approach is the most often used approach in different case studies presented in this book. In that respect, the fatigue crack growth approach is described in more detail in section 2.

1.2 Fatigue design criteria

As presented in the previous section, three basic approaches can be used for dimensioning structural elements subjected to cyclic loading. However, from a philosophical point of view, four fatigue design criteria can be combined with these three approaches [1.2]:

- Infinite-Life Design,
- Safe-Life Design,
- Fail-Safe Design,
- Damage-Tolerant Design.

Infinite-Life Design

Infinite-Life Design is the oldest fatigue design criterion. This criterion is based on the *S–N* curve with the assumption that the engineering component is going to reach “infinite” life (usually several millions of cycles). According to this criterion, the stresses are in the elastic area and should not exceed the fatigue limit of the material.

This criterion is suitable for dimensioning dynamically loaded machine parts or structures which are in the framework of their fatigue life actually exposed to millions of cycles (engine valve springs, axes and bearings of railway wagons, shafts and gears of high stages in change-speed gear drives, etc.). However, most engineering components undergo significant variable amplitude loading, and the pertinent fatigue limit is difficult to obtain. In addition, this criterion may not be economical in many design situations where the expected fatigue life is shorter (shafts and gears of low stages in change-speed gear drives, certain parts in the aircraft industry, etc.).

The infinite-life design criterion is exclusively combined with the stress-life approach (see section 1.1.1). The main advantage of this method is the fact that fatigue time is known for most engineering materials, and that this information is available in specialist literature. Furthermore, certain design parameters (surface roughness, notch effect, residual stresses, temperature, corrosion, etc.), which may significantly influence the fatigue strength, are also well understood.

Safe-Life Design

Safe-Life is a fatigue design criterion where the engineering component is designed for a finite life, which is often known in advance. The safe-life criterion should include a margin for the scatter of fatigue results and for other unknown factors. The dimensioning process may be based on the stress-life approach, if stresses are in the elastic area, or on the strain-life approach, if plastic deformation occurs in the critical cross section of the treated component.

This criterion is suitable for dimensioning dynamically loaded machine parts or structures with an expected specific finite life (i.e., reverse gears in car drives, pressure vessels design, jet engine design, etc.). Similar to the infinite-life design criterion, some design parameters (such as surface roughness, notch effect, residual stresses, etc.) should also be considered when determining the finite fatigue life.

Fail-Safe Design

Fail-Safe Design is a fatigue design criterion, which assumes that some initial failures (cracks) may appear in individual parts of the engineering structure but these failures are not critical and do not lead to a catastrophic failure of the structure. This fatigue design criterion was developed in the aircraft industry. Namely, aircraft engineers could not tolerate the added weight required by large safety factors, or the danger to life created by small safety factors, or the high cost of the safe-life design. In that respect, the fail-safe design is based on the requirement that the system does not fail if one part fails. This principle recognizes that fatigue cracks may occur, and structures are arranged so that cracks will not lead to failure of the structure before they are detected and repaired. Following this idea, multiple load paths, load transfer between members, crack stoppers built at intervals into the structure, and inspection are some of the means used to achieve a fail-safe design. Although this approach was originally applied mainly to aircraft structures (fuselages, wings), it is now used in many other applications.

Damage-Tolerant Design

The *Damage-Tolerant Design* criterion is actually a refinement of the Fail-Safe Design criterion. It is based on the assumption that cracks or initial defects exist in engineering structures which were caused either by mechanical and thermal treatment of components during the manufacturing process, or by fatigue. A fracture mechanics analysis can then be performed, in order to determine whether such cracks will grow large enough to produce failures before they are detected by periodic inspection. When dimensioning machine parts and structures using the damage-tolerant design criterion, at least three issues should be taken into account:

- crack detection involving nondestructive inspection,
- residual strength of treated component, and
- fatigue crack growth behavior.

In recent decades, several nondestructive inspection methods have been developed to detect possible defects (cracks) in a treated engineering component. If a crack is detected, the residual strength of the treated component should be obtained using fracture mechanics theory. As a crack propagates under cyclic loading, the residual strength decreases up to the critical crack length, when final failure (fracture) occurs. If there is no crack, the residual strength is equal to the ultimate tensile strength or yield stress of the material. Apart from those described above, some other influencing parameters, such as environmental conditions, load history, statistical evaluation, etc., should also be incorporated into this methodology.

The damage-tolerant design criterion is often used when evaluating the residual strength of complex and expensive engineering components, which should be retired from service because they have reached their designed safe-life service life, based upon analytical and experimental results. However, it has often been established that such components could have significant additional service life. To allow for possible extended service life, damage tolerant methodology based upon both, analytical (or numerical) analyses and additional experimental testing, is required.

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CHAPTER 2

FATIGUE AND FRACTURE MECHANICS – THEORETICAL BACKGROUND

The presence of a crack in an engineering component can significantly reduce its fatigue life, as already discussed in Chapter 1. This chapter introduces the concept and use of *fracture mechanics* in the fatigue crack growth approach (see section 1.1.3) when dimensioning machine parts and structural components according to the fail-safe design criterion or the damage-tolerant design criterion (see section 1.2). When a crack is detected in a treated engineering component, the following questions are usually topical for designers:

- What is the residual strength of the treated engineering component?
- What is the critical crack length that still assures the safety operation of the component?
- What is the operation time (number of loading cycles) needed for crack extension from the initial to the critical length?
- What is the fatigue life of a component with a detected micro-crack (i.e., as a consequence of mechanical or thermal treatment)?
- How frequent are periodic inspections of components with detected cracks?

As implied above, the effective use of fracture mechanics requires periodic inspections of cracked components in order to determine what sizes and geometries of cracks are present or might be present. Such periodic inspections are often performed on aircraft, bridges, nuclear constructions, pressure vessels and many other engineering applications. According to this philosophy, a crack cannot grow to a dangerous length and the critical part of the structure will be repaired before a critical situation could happen. Methods of inspection to detect the presence of possible cracks can be simple visual examinations, or more comprehensive examinations such as X-ray photography or ultrasonics. Repairs necessitated by cracks may consider replacing a part or repairing it, for example, by machining away a small crack to leave a smooth surface [2.1–2.3].

This chapter provides the theoretical background for fracture mechanics and its use when solving engineering problems related to the fatigue of cyclically loaded machine parts and structural components. The topic is focused only on that part of the fracture mechanics theory, which is then actually used in the subsequent case studies presented in Chapters 3 to 6 of this book. In specialist literature, different concepts may be found that explane the evaluation of a component with a crack [2.4–2.6]:

- *K*-concept (the stress intensity factor concept)
- *G*-concept (the energy release rate concept)
- *COD*-concept (the crack opening displacement concept)
- *J*-concept (the J-integral concept)

This chapter focuses predominantly on the *K*-concept and its application to fatigue problems.

2.1 Linear elastic fracture mechanics

Linear Elastic Fracture Mechanics (*LEFM*) is used to determine crack growth in components under the basic assumption that material conditions are predominantly linear elastic during the fatigue process. This is usually the case in the High Cycle Fatigue (HCF) regime, or when using hard and brittle materials (i.e., high strength steels), where the actual stresses are lower if compared to the yield stress of the material.

Basic investigations related to LEFM have been made by Griffith [2.7], who defined the energy release rate, *G*, which represents the elastic energy per unit crack surface area required for crack extension. However, Griffith's researches were limited on very brittle materials (glass). Irwin [2.8] and Orowan [2.9] made significant advances by applying Griffith's theory to metals while taking into account small plastic deformations at the crack tip. The next stage was the energy approach to crack extension, which was developed by Irwin [2.10] and further improved through the implementation of the stress intensity factor *K* [2.11, 2.12.].

2.1.1 Loading modes

The theoretical background of fracture mechanics is based on three loading modes of cracked components as shown in Figure 2.1:

- *Mode I* – opening mode

- *Mode II* – shearing mode
- *Mode III* – tearing mode

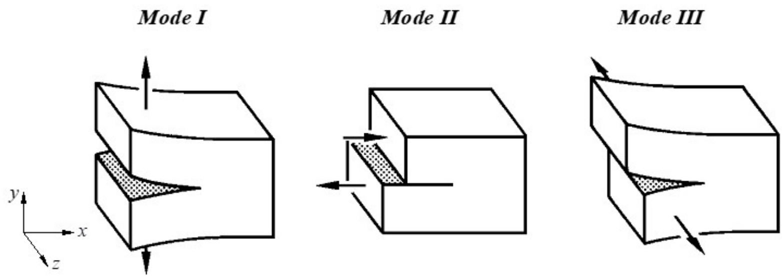

Figure 2.1: The basic loading modes of cracked components

Mode I (*opening mode*) is the most common when solving uniaxial fatigue problems and has received the greatest amount of investigation in the past. In this mode, tensile loading acts perpendicular to the crack front and causes a crack opening in the plane of the maximum tensile stress. In Mode II (*shearing mode*), shear loading acts on the plane x-y along the crack front and causes the sliding of crack faces relative to one another along the plane of the maximum shear stress. Mode III (*tearing mode*) also involves relative sliding of the crack faces, but now in plane y-z (perpendicular to the crack front).

Machine parts and structural elements are often subjected to external loading where more than one mode is present. In such cases, the crack propagates according to the *mixed-mode crack extension*. An example of mixed-mode crack extension I and II is shown in Figure 2.2, where appropriate stress intensity factors K_I and K_{II} are also designated. This involves the axial loading of a crack inclined for an angle β in respect to the x-axis. When β = 0°, the pure mode I is present (K_{II} =0), when β = 90°, the pure mode II is present (K_I =0). For all other values of β, a mixed-mode of crack extension appears.

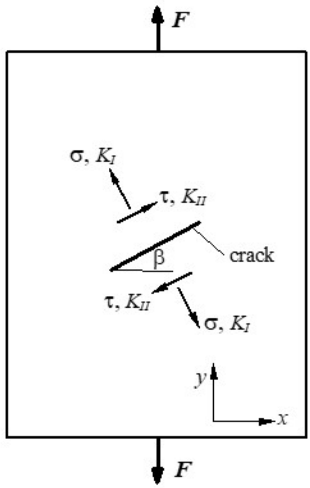

Figure 2.2: Mixed-mode I and II

2.1.2 Stress intensity factor

In general, the *stress intensity factor K* characterizes the intensity of the stresses in the vicinity of an ideally sharp crack tip in a linear-elastic and isotropic material [2.1]. Figure 2.3 shows the elastic stress σ_y near the crack tip (r/a << 1) in an elastic isotropic body subjected to the pure mode I loading. It is evident that the magnitude of this stress at a given point is dependent entirely on K_I.

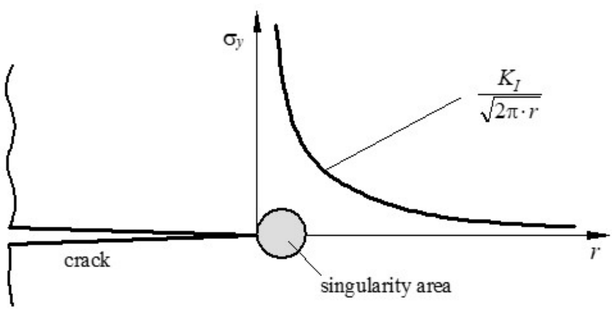

Figure 2.3: Elastic stresses near the crack tip for pure mode I loading [2.5]

It can be seen from Figure 2.3, that the elastic stress distribution σ_y approaches infinity ($\sigma_y \to \infty$) as r approaches zero ($r \to 0$). Following this finding, the stress intensity factor for mode I loading can be defined mathematically as:

$$K_I = \lim_{r \to 0} \sigma_y \cdot \sqrt{2\pi r} \tag{2.1}$$

Similar equations can also be expressed for mode II and mode III loading. Values of K are generally dependent on the external loading, crack length and geometry of the cracked member. Explicit equations for the determination of K can be found in specialist literature [2.13]. On the other hand, K can also be determined numerically using the available numerical approach (i.e., FEM).

When the crack is small compared to other dimensions of the component, the crack is viewed as being contained within an infinite body (Figure 2.4). If nominal stress σ_{nom} acts at a large distance from the crack, the stress intensity factor results in:

$$K_I = \sigma_{nom} \sqrt{\pi \cdot a} \tag{2.2}$$

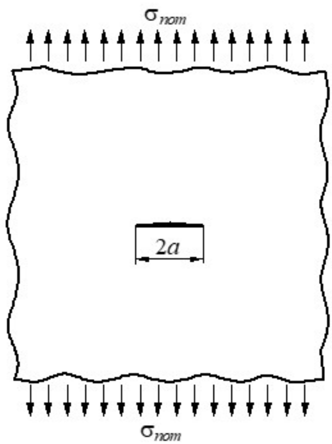

Figure 2.4: Middle crack of length $2a$ in an infinity body

For other crack configurations and external loadings, the stress intensity factor can be determined with the modification of Eq. (2.2) as follows:

$$K_I = \sigma_{nom} \sqrt{\pi \cdot a} \cdot f\left(\frac{a}{W}\right) \tag{2.3}$$

where $f(a/W)$ is the dimensionless function dependent on the crack length, geometry of the specimen and loading. For many standardized crack configurations and specimen geometries, the appropriate equations for the function $f(a/W)$ may be found in different standard procedures [2.14, 2.15], or in specialist literature related to fracture mechanics [2.5, 2.13].

2.1.3 Critical stress intensity factor

It can be seen from eqs. (2.2) and (2.3) that the stress intensity factor increases with the increase of crack length and loading. The critical stress intensity factor K_c refers to the condition when a crack propagates in an unstable manner. According to the eq. (2.3), K_c can be expressed mathematically as follows:

$$K_c = \sigma_c \sqrt{\pi \cdot a_c} \cdot f\left(\frac{a_c}{W}\right) \tag{2.4}$$

where σ_c is the applied nominal stress at crack instability and a_c is the crack length at instability. The critical stress intensity factor K_c is the main designing parameter when dimensioning cyclically loaded components using the damage-tolerant design criterion. Namely, it represents the critical value of the stress intensity factor K for a given load, as well as the crack length and geometry required to cause the fracture.

In general, K_c mainly depends on the material and geometry (thickness) of the cracked member (see Figure 2.5). It is evident that the highest K_c corresponds to the thinnest specimen where the plane stress appears in the critical cross section. As the thickness of the specimen increases the value of K_c, it decreases up to the thickness B_I when plane strain conditions appear. The minimum value of K_c is called *fracture toughness* K_{Ic} and strictly refers to the plane strain conditions. The type of fracture (brittle or ductile) is also dependent on the loading conditions (plane stress or plane strain) as it can be seen in Figure 2.5.

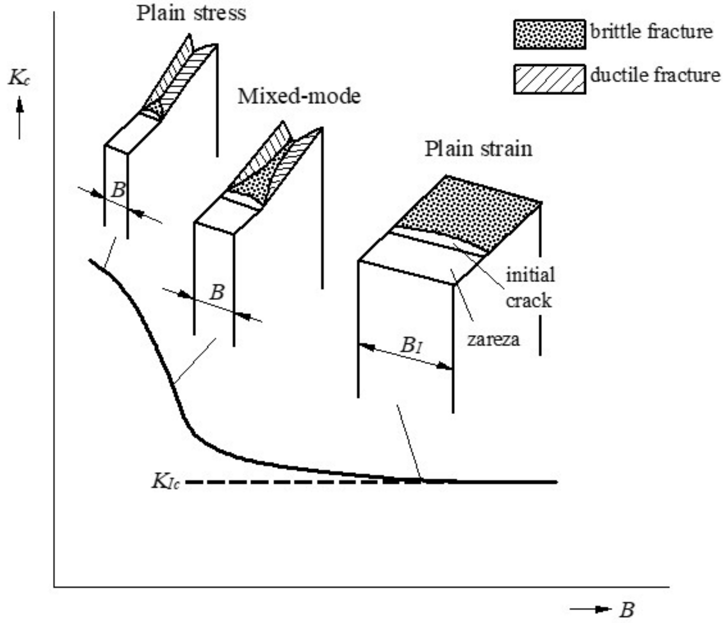

Figure 2.5: Critical stress intensity factor K_c for given material [2.2]

Fracture toughness K_{Ic} is a material parameter and is independent of the geometry of the cracked member. The values of K_{Ic} for different engineering materials can be found in specialist literature [2.16, 2.17]; the fields of K_{Ic}-values for typical engineering materials are also shown in Figure 2.6. From this diagram, it can be seen that the values of K_{Ic} decrease with the increase of the yield stress of the material. Therefore, high strength materials (i.e., high strength steels) are more sensitive to the occurrence of cracks if compared to low strength materials. Furthermore, fracture toughness is also dependent on temperature (see Figure 2.7). As the temperature decreases, the value of K_{Ic} decreases.