Zero for Parents and Teachers, or (Almost) All You Need to Know about Mathematics for Young Children

Zero for Parents and Teachers, or (Almost) All You Need to Know about Mathematics for Young Children

By

Anne D. Cockburn and Ioannis Papadopoulos

Illustrations

Anna Papadopoulou

Cambridge Scholars Publishing

Zero for Parents and Teachers, or (Almost) All You Need to Know about Mathematics for Young Children

By Anne D. Cockburn and Ioannis Papadopoulos

Illustrations by Anna Papadopoulou

This book first published 2020

Cambridge Scholars Publishing

Lady Stephenson Library, Newcastle upon Tyne, NE6 2PA, UK

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

ISBN (10): 1-5275-5561-5
ISBN (13): 978-1-5275-5561-7

Contents

PREFACE

Whether you like it or not we all have to do at least some mathematics everyday of our lives. There is no getting away from it despite the fact that the vast majority of the population fear and even dislike the subject often finding it totally confusing.

We – the navigators of this book - are among the fortunate few who loved mathematics from an early age. Yiannis (who, more formally, is called Ioannis) has been particularly lucky being one of the tiny minority who seemed to understand mathematics from the moment he was born. Anne thought she did only to become totally frustrated and confused when she was at secondary school. As will become apparent, it was not until her early twenties that she began to understand some of the most basic aspects of mathematics. Now, in her sixties, it is clear she still has much to learn but she is getting there! Anna – our wonderful illustrator – also found herself becoming increasingly mystified as she made her way through secondary education.

Despite the setbacks Anne and Anna have always enjoyed playing with numbers but we are well aware that many are not so fortunate. It is for you we have written this book.

Having spent more years than we care to recall observing young people struggling with mathematics we are convinced that many fear and loathe the subject because they have never really understood it. One of the reasons for this is that, in the past, the teaching of mathematics was somewhat limited.

Taking 2 apples away from a bowl of 6 was easy ($6 - 2 = \square$) but what on earth was $\square + 3 = 5$, or $7 - (-9) = \square$ all about?

Much of what you know about how numbers, measurements and shapes work may not be wrong but, they may be rather a narrow view and have restricted your thinking in much the same way as Anne's. By way of analogy take a look out of the window: you can see a fair amount but, should you be able to see through the walls enclosing you, what is on offer is likely to be so much greater.

This book takes us back to the beginning of early mathematics. Some of it may be familiar but we suspect, like Anne and Anna, you will make some interesting discoveries along the way. Our intention is to make our work as accessible as possible to as many people as possible. Thus, for example, there are stories, pictures and activities which we hope will appeal to adults and young children alike. Indeed, we hope you will find plenty of opportunities to enjoy them together across the generations.

You may find yourself reading the book from cover to cover or you may prefer to dip in and out of it. If you are of a nervous disposition feel free to skip bits. You never know, in time you might feel ready to tackle them, but, for now, we suggest we begin at the beginning….

CHAPTER 0

ZERO FOR PARENTS AND TEACHERS

For a long time, Anne was under the impression that ‘0’ was an insignificant number meaning ‘nothing’: nothing in my pocket, nothing hiding under the table etc. In many ways this is not surprising given how most of us begin counting, ‘One, two, three…’. But, if you stop to think about it, when launching a rocket, we count backwards and zero (‘0’) becomes a highly significant number!

Imagine if you have zero teeth: it might be fine for a baby but not so good for an adult! Or how might a child feel if he had zero money in his piggybank…

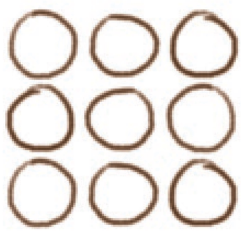
Zero – 0 – in both cases is likely to be highly relevant and considerably more than ‘nothing’ in your thoughts. More accurately, in these examples, it will be the ***absence of something*** that will almost certainly be preying on your mind. Other uses of ‘zero’ in our everyday language indicating the absence of something include, ‘zero tolerance’ and ‘zero effort’. Strangely we can’t think of a sporting example, but the meaning is the same when we groan about the ‘nil’ in a soccer score or a ‘love’ (e.g. forty love) in tennis.

Interpreting '0' as the absence of something rather than nothing may seem relatively trivial – and possibly overly fussy – when learning to count but, as we hope to demonstrate, this book is about helping young children develop a sound foundation in mathematics rather than floundering their way through the subject in a blur of confusion for evermore.

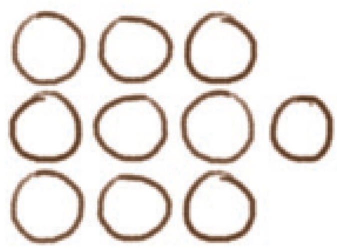
We will return to learning to count in a moment, but before we do, why are we so keen to stress that '0' signifies the absence of something rather than nothing? Two responses spring to mind. Firstly, think about '10'. If '0' is nothing why don't we simply write, '£1' rather than '£10'?

In short, we write '10' as way of representing 9 and 1 more or, put another way, one ten and zero units. Over the years we have seen various ways in which this process is demonstrated to young children. These range from binding sticks together in bundles of ten, to baking biscuits to fit packets of ten and, we suspect, you might recall yet another technique you were encouraged to use.

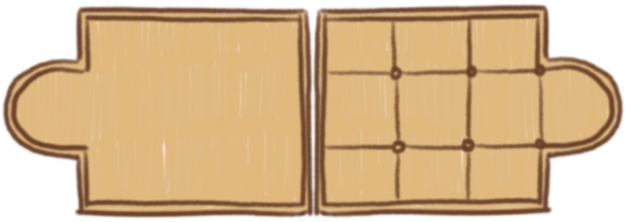
We do not find any of these strategies particularly satisfactory as they largely rely on the children remembering how many sticks, biscuits etc you need to group together to complete the bundle, packet etc. Is it 5? Or 8? Or any number you wish? With this in mind we have devised our own visual representation which we have developed into a story. We started with the idea that if you set out items in the following way you can quickly see that there are 9 circles.



The above image is easy to remember and we are confident that, if they see it frequently, young children will instantly recognise it in next to no time. Once it is familiar, they will also find it easy to notice if another circle has been added to one side as does not fit into the array of 3 x 3.



Wishing to take advantage of such recognisable images we then extended the idea by creating a story involving packing tokens into a box which, in effect, matched that of 3 x 3 array above with an obvious space for a tenth token. We called this a 10-token box.



Once the final – tenth – token has been added to the box then, and only then, may it be closed. At this point it signifies one ten (box).

Taking this a step further '40' is equivalent to four tens (boxes) and zero units as shown by using four closed 10-token boxes and an empty one beside them.

Should you wish to add any units to 40 it is easy to do so as there is plenty of opportunity to add up to 9 without altering 'the tens column'. Thus, for example, 40 tokens and 6 more may be represented by 46 as shown by 4 closed boxes (with 10 tokens in each) and 6 more.

A common problem in the past was that, when asked to write forty-six, some children would record it as 406. As an adult you may think that writing 406 is ridiculous here. If you stop to think about it, it may well seem perfectly logical to a young child who was taught to represent six as '6' and later learnt that the number forty is written as '40': forty (40) and six (6) or 406. Once children become familiar with our idea of 10-token boxes, however, we hope that such errors will become a thing of the past.

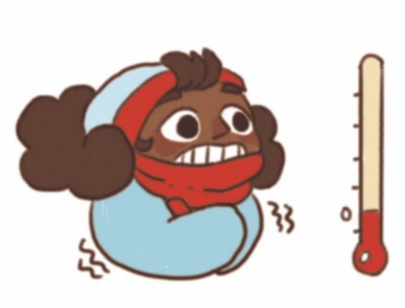
There are several more challenges when we learn to count which we will discuss later but, before we do, let us consider the second reason why it is important that young children develop a suitably broad understanding of zero as soon as possible. Basically, it is very hard to alter a person's fundamental thoughts and ideas if they were acquired in infancy. Perhaps the best way to demonstrate this is for you to think of something you thought to be true but which, in later life, you found to be no longer the case. A classic example might be that you thought your parents knew everything: it may well have come as a shocking realisation when you learnt that this was no longer the case!

Or, taking a more everyday example, as a young child you may have cleaned your teeth in a certain way only to be told by a dentist as you grew older that you should use a different technique. If you have not experienced such a phenomenon you can take Anne's word for it that it is remarkably challenging to change one's tooth cleaning routine after years of using a different approach! It is important to stress here that Anne's parents were highly conscientious and had her very best interests at heart when teaching her how to clean her teeth. At the time she had fewer teeth than she has now, less was known about gum disease, toothbrushes were less sophisticated and no doubt there were other contributing factors not least that they – and probably Anne – wanted to make the process as easy as possible.

Anne's realisation that '0' meant much more than nothing was even more of a shock to her than being told by the dentist that she was not cleaning her teeth properly.

Even though she knew the reason behind such necessary changes in her previous thinking (that zero always means 'nothing') or approach (how to clean her teeth and gums using manual rather than an electric toothbrush) it was not easy to incorporate it into her everyday way of using numbers or cleaning her teeth! Far easier is to have a sound knowledge and understanding – be it of numbers, teeth cleaning or whatever – right from the start.

In the ideal world we would chat to each and every one of you and endeavour to discover how you think and feel about mathematics. We would then tailor our approach specifically for you in much the same way as if we were a highly empathetic understanding mathematical genius who knows all there is to know and can explain it in a clear and kindly manner. Sadly – as we suspect is the case with the vast majority of our readers – we cannot claim to know everything but we are fairly knowledgeable and believe our intentions are good!



So, to return to zero, we would be doing you a disservice if we simply led you to believe that '0' signifies the absence of something. Consider, for example, 0^{o} C. We know that this means that it is pretty chilly outside but it certainly does not mean that there is no temperature!

Another example might be one you have come across in a lift:

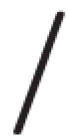
Floor '0' definitely does exist! In many countries it is at street level with floor -1 being underground and one floor up being shown as '1'.

Perhaps of particular relevance to parents and a young child with a new-born sibling is that there is plenty action, noise and joy before the milestone of the first birthday! Another reminder that counting does not begin at 1. It is beyond the scope of this book to discuss the full range of meanings of zero but, suffice it to say that, if you can introduce a young child to a range of mathematical possibilities such as discussed above, then you are well on the way to success.

Before we move on to the next section take a few moments to reflect on all the different places you see '0' in your everyday life. In which situations does it mean the absence of something? Can you think of other occasions where it acts as a significant marker such as on a thermometer? Take a minute or two to reflect on the different ways in which it is displayed. For example, is it always presented horizontally?

Where might you see it as part of a circular dial?

Are there any occasions where it might be seen diagonally such as shown here?



We suggest you encourage young children in your quest to seek out '0's' in their everyday environments and together you could discuss what they might mean. The more settings in which you encounter '0' the better as each one helps develop the idea that it serves a variety of functions and can appear in a range of different guises and orientations.

Fun activities

One of the most important messages we wish to convey is that mathematics can be stress-free and fun. Below are some ideas we have developed to try with young children. They focus on the idea of the absence of someone or something. There is no need to use the term 'zero' unless you feel comfortable doing so. We do, however, recommend having a chat about noticing that something significant is missing. The idea behind this will become clearer at the end of Chapter 1.

Before we present the activities we would like you to…..

Meet the 6 friends

I am Luka and I am 6 years old.
This is my sister Emily.
She is 5 years old!

We have lots of friends. Here are four of them:

Sami Kim Sky Carlo

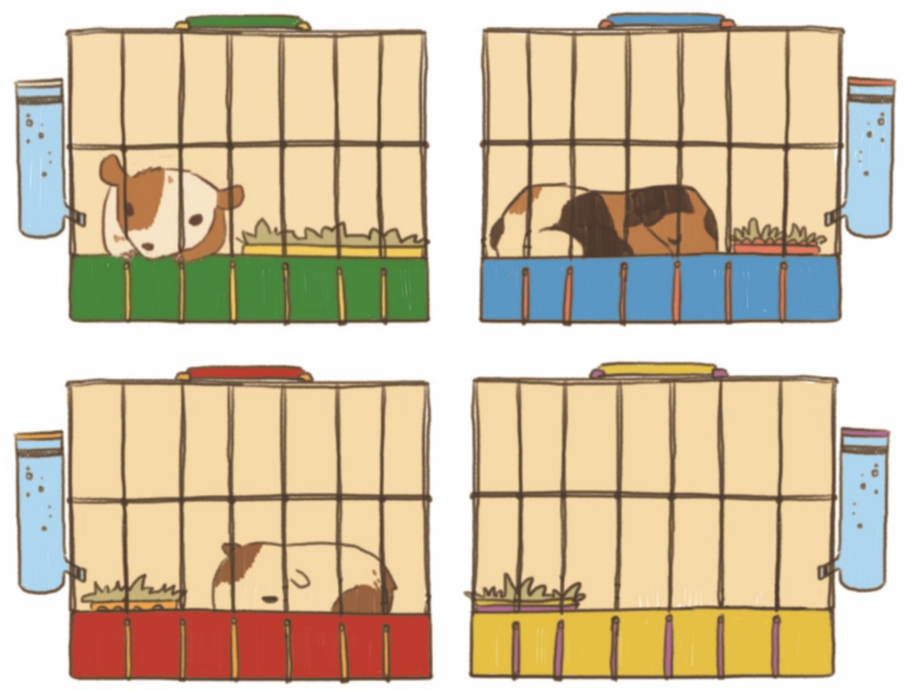
You will be seeing a lot of us in this book as we have been busy doing all sorts of interesting things. If you look further on in this chapter you will meet some of our pets. I hope you like them! They have been very busy too. As you will discover, we need some help looking after them.

Later you will find us going on several adventures.
We hope you will join us!

1: Missing!

Puzzle 1

Carlo has some guinea pigs. One of them has escaped! Can you find her on one of the following pages? We hope so!



Which cage do you think is her cage? Why?

Puzzle 2

Luka, Sky and Sami went out for a ride. Sky fell off! I wonder where her horse went. Can you find it?

How many horses are available in the picture above for Sky?

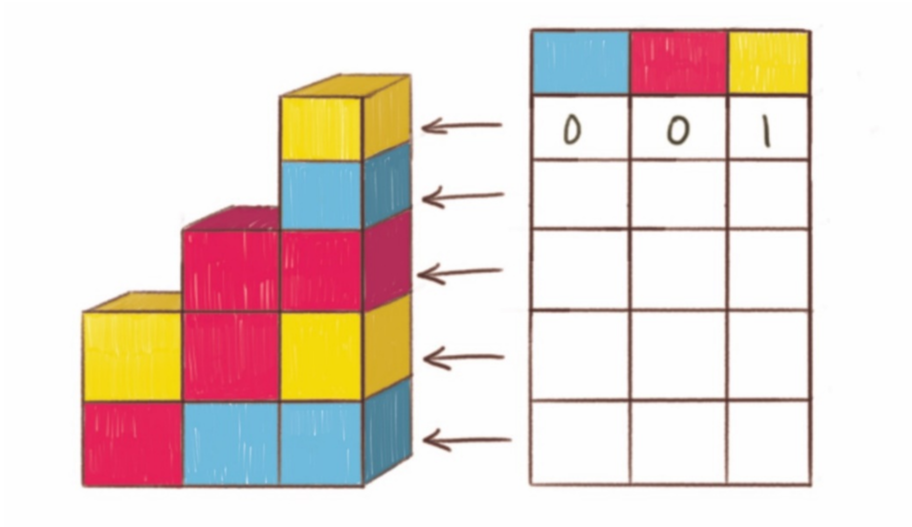
Puzzle 3

Emily is feeding the puppies, but she can't find one of them. Can you?

How many puppies in this picture are eating from the red bowl?

2: Colourful building

Sami has been building a tower with different coloured blocks. On some floors she has included a blue block but on others she hasn't. We wonder why? There is no correct answer to this question, but we see it as an opportunity to start a discussion. We cannot predict the direction in which such a conversation will go but we imagine that it might include phrases such as, 'There are no …', 'There are zero... '



Can you construct your own building and talk about it with your friends?

3: The bananas

Kim took 3 bananas with him on the train. During the trip he started eating them one after the other. After eating the last banana how many were left?

And finally…

See how many examples you can find where zero matters. Which of them can you share with young children?
We have thought up several other activities relating to zero, but we have included these at the end of the next chapter as they are more appropriate to try once a child has started counting.

Pondering on zero

A - The birth of zero

When people first began counting, the idea of zero did not exist. Its absence, however, created considerable difficulties for many including the Ancient Babylonians. Nowadays we use the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 but, unlike us, the Babylonians had only one symbol, the . They used this to represent 1, 60, or 3600. We guess it must have been fairly easy to tell which of these three numbers people were talking about by the content of the conversation. For example, if they were discussing the number of stars in the sky the symbol would mean 3600 but if they were considering how many peas they might have for lunch it is much more likely that it was worth 60.

But life became more complicated when they wished to be more specific. For example,  could mean 61 (60+1) or 3660 (3600+60) or 3601 (3600+1). They realised that another symbol was required and that the invention of zero was the solution to the problem. So, around 300 BC they started using two slanted wedges  as a placeholder to represent an empty space which made easier to interpret numbers correctly and distinguish between 61 ( : 60+1) and 3601 ( : 3600+0+1). So, zero took its birth from the need to give a unique meaning to a sequence of digits.

B – It seems paradox

Zero is a number and has a specific value but sometimes we use it in a way that shows quite the opposite. Its placement on the number line is before ‘1’.

If you look at your computer’s keyboard however, you will realize that 0 comes after 9! Is this also the case with your smartphone, calculator, remote control? Do you think this might confuse children? How is it possible for 0 - which is less than 1 - to be put in a position implying that it is bigger than 9? From the mathematical point of view there is only one appropriate place for zero on a number line, just before 1. No other place makes sense. Interestingly, there is an explanation for its placement next to 9 on a keyboard and it's not a mathematical one. In typewriters 0 was put after 9 as usually it follows other numbers - for example, 40, 90 etc. - making it easier for people to type quickly. By the time computer keyboards and smartphones appeared this was a recognised convention and still helps us type large numbers as rapidly as possible.

C – Zero: What a naughty number!

Yiannis (the everyday version of the official name of Ioannis, one of the book's authors) sometimes describes zero to his student teachers as 'the naughty number'. He explains:

> *Have you ever thought that zero is a 'naughty' number and why its weird behaviour is the reason that makes zero so different from all the other numbers?*
>
> *Let me explain. Archimedes (born in Sicily in 287 BC) proposed that if you have two numbers such as '3' and '5' then it is always possible to add the smaller - in this case 3 - to itself enough times so as to exceed the larger number which, in this case, is 5. In other words, the smaller number gradually becomes bigger as you add it to itself (e.g. 3 + 3 + 3...). However, when you try that with zero it refuses to become bigger. Moreover, if you add zero to any other number it never makes the other number any bigger. Add eight and zero and you get eight. If you think about it the same is true for subtraction. Try it.*

This 'problematic' behaviour becomes even more serious in multiplication and division. Zero times any number makes zero. Have you come across the phrase, 'Division by zero is undefined?' Have you ever wondered why? As we will discuss in Chapter 3 you can use division to undo multiplication. For example, since 2 x 3 = 6 then 6 ÷ 3 = 2. In the same spirit you might think that dividing by zero should be the opposite of multiplying by zero. You might imagine that since 6 x 0 = 0 it should be 0 ÷ 0 = 6. Similarly, since 5 x 0 = 0, it should be 0 ÷ 0 = 5 and so on but this would mean that 0 ÷ 0 equals 5, 6, 7, …. any number which does not make sense!

Some suggested reading

Although many authors discuss zero, they usually do so in passing and as part of a bigger concept such as counting. If you would like to learn more about it, however, you might be interested in a chapter written by Anne and her colleague - Paul Parslow-Williams - entitled 'Zero: understanding an apparently paradoxical number'. It can be found in:

Cockburn, A.D. and Littler, G. (2008) *Mathematical Misconceptions*. London: Sage.

The complexities of zero and how they should be tackled gradually by children are described in the paper of Glenda Anthony and Margaret Walshaw.

Anthony, G. and Walshaw, M. (2004) Zero: a 'none' number? *Teaching Children Mathematics, 11*(1), 38-42.

If you are interested in studying the history of zero more deeply then you might well be interested in Charles Seife's book:

Seife, C. (2000) *Zero. The biography of a dangerous idea*. New York: Viking Penguin.

CHAPTER 1

I'M COUNTING ON YOU

Most children love learning to count! It is amazing to see the thrill they experience when they realise that they can recite the numbers up to 100. They behave as if they are positively grown up. There may, however, be one or two hurdles for both you and them along the way for learning to count is, in fact, quite a complicated business.

As you might expect one of the earliest steps is learning to recite the numbers in the correct order. At this age when we talk about numbers, we refer to natural numbers (or counting numbers, the numbers we use for counting, i.e., 1, 2, 3, …, going on forever). If children hear you frequently counting aloud, they will soon pick up some of the number names. Initially they may appear to say them at random but, in due course, they may well start using sequences such as '1, 2, 4, 5, 8'. You may find it a bit frustrating but, rest assured, there is no need to worry if a child recites the number sequence incorrectly. Listen carefully. For example, if Sami consistently says, '1, 2, 4, 5, 8', it means that she recognises that the number names always follow the same order which is a crucial building block in learning to count. If you stop to think about it, it would be totally hopeless if sometimes we counted a collection of items '1, 2, 3, 7, 9' and another time '2, 4, 6, 7, 8'!

To help children learn the names of the numbers in the correct order we recommend that you make a habit of counting objects as you point to them and reciting the number names as you do so. If this is simply part of everyday life – for example when you count out plates for lunch or chairs to put round a table – then children will start remembering the numbers in the correct order when they are ready to do so and without feeling any pressure. Although it may seem odd at first, we also suggest that you start the sequence with ‘0’ as discussed in the previous chapter. For example, you might say, ‘There are zero plates on the table, now there are one, two….’

Whenever you – or older children – can, make a point of pointing to or touching objects as you count them out in front of a child. Anna has drawn some examples of how this might be done.

On hearing a child recite the correct numbers in the correct order you might reasonably assume that they can count but, if you observe them at play, you may discover that they are not quite as competent as you had imagined. More specifically, in order to count effectively, you need to associate each number you say with a different object. It would appear that Sky, in the picture below, has grasped the idea of 1-1 matching of number to object.

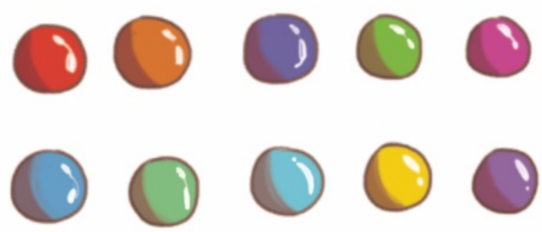
Sky has also realised that the last number she counted is the total number of eggs she can see.

In contrast Kim counted the beach balls below reciting the numbers correctly as he did so and then announced….

Although he recited correctly, the total number of beach balls are fewer than the last number he counted to.

As children become more experienced at counting it is interesting to see how they cope with different arrangements of items. For example, it is relatively easy to count these teddies as they are in a neat line.

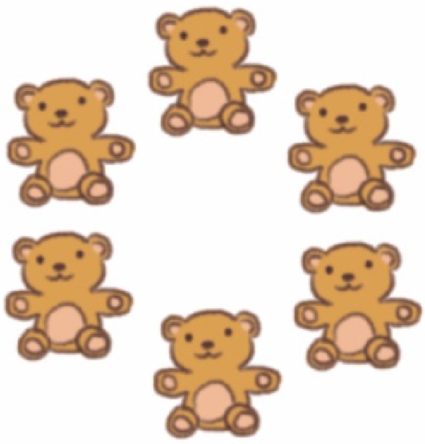
Once a young child can do that with relative ease you might like to try the following experiment to see how they respond. Place some items – for example 5 marbles – in a row. Place the same number of marbles immediately below the first line as shown here:



Ask the child which row has more marbles in it. They are likely to reply that there are the same number of marbles in each row or something similar. Now space out the second row and repeat the question.

You may find that the child replies that there are more marbles in the second row and, if asked, that there are fewer marbles in the top row. It is not entirely clear why some children give such answers but, as will be discussed further in Chapter 9, it may be because they think they are required to give a different response to the one they gave initially. Or, it may be that they conclude that, because the marbles in the second row take up more space than those in the first, there are more of them.

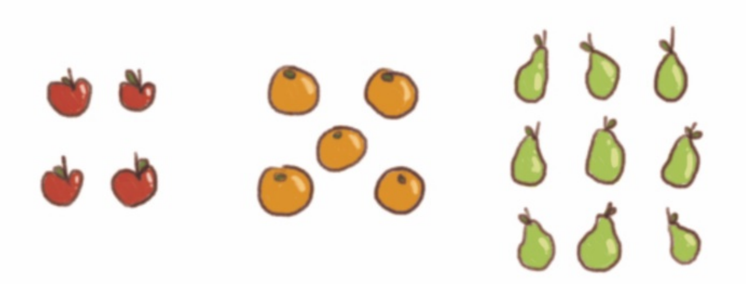
It becomes even more complicated to count objects if they are arranged in a circle as it can be tricky to remember which one you counted first.



Even as adults it can be a challenge to count a lot of items if they are arranged in certain formations …especially if you lose concentration!

It becomes considerably easier if they are laid out in a more organised manner.

As adults we can help young children with counting a group of objects by placing them into patterns that we find easy to recognise such as those below.



It may be that seeing the above you had no need to count them as you instantly recognised the patterns and knew there to be 4, 5 and 9 objects. Such instant recognition without the need for counting is called ***subitizing***. It can prove very useful especially when playing board games involving dice.

Once a young child is familiar with the different pattern shapes for different numbers of items, they can use the knowledge to check their own counting. If, for example, Sky is asked to count out 4 coins. She can lay them out on the table as shown below and immediately know that she has put out the requested number of pennies.

Before we move on to discussing higher numbers, we need to explain one more important aspect of learning to count which may cause confusion for a young child. If you were asked how many animals there are 'in the field' below you might well count:

1 2 3 4

If, however, the pig started chasing the rabbit and everybody changed position you would still have 4 animals in the field, but you might count them in a different order such as:

1 2 3 4

Now the pig is no longer '1' nor the lamb '3'. Appreciating that a number is not permanently attached to a specific object is termed the ***order irrelevant principle***. In other words, you can count any group of objects in any order and it does not matter as you will always end up with the same total number of objects.

Thus, however Luka places and then counts the muffins below, he will always have 6 of them until, of course, he eats some!

Summing up thus far, in order to be able to count effectively a young child needs to:

1) Know how to recite numbers consistently in the same order (***The stable order principle***)
2) Recognise that when counting a group of objects one number needs to be **temporarily** (see point 4) assigned to one and only one object (***The one-to-one principle***)
3) Appreciate that the final number they have counted is the same as the total number of objects they have counted (***The cardinal principle***)
4) Realise that you can count any group of objects in any order and you will still have the same total number of objects (***The order irrelevance principle***).

So far with one exception, we have only mentioned numbers below ten in this chapter. Life becomes rather more confusing for young children when they first encounter the numbers 11 to 20. This is the case in many languages with Welsh being a notable exception: