# Syntacticism and Functional Completeness

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Cambridge
Scholars
Publishing

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This book first published 2024

Cambridge Scholars Publishing

Lady Stephenson Library, Newcastle upon Tyne, NE6 2PA, UK

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library



ISBN (10): 1-5275-5636-0
ISBN (13): 978-1-5275-5636-2

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# FUNCTIONAL COMPLETENESS AND SYNTACTICISM: LOGICAL-PHILOSOPHICAL SIGNIFICANCE OF A METALOGICAL CONCEPT

Syntacticism is a term used often to refer to a philosophical school of thought about the nature of logic. (For example, Anderson, 2019, ch. 13, albeit in a more restricted sense relative to how we define it here). Copious effort is devoted in this study to characterizing this approach, which parallels what is known as formalism in the philosophy of mathematics, in the sense promoted by David Hilbert and understood as a philosophic view of the characterization of mathematics (rather than in the more common senses of effecting a formalization or constructing a formal system or using symbolic means). Notwithstanding the appeal of such thinking about logic to the influential analytical school, its deep connections to nominalistic and empiricist thinking, and the fact that the Tractatarian Wittgenstein propounded its tenets, syntacticism has to be modeled in some detail, as we attempt to do in this project: the support for the position tends to be programmatic, even literally emerging in practice – as in the construction of Rudolf Carnap's early logical systems – and incidental to discussions of more concrete themes in the philosophy of logic rather than having dedicated treatises, supporting or polemical or both, lavished specifically on it. Moreover, the claims of syntacticism can often strain credulity – as Frege thought about formalism in arithmetic in a famous invective he penned against it: building and expanding on an idealized version of the theory can be intellectually challenging.

The connection to functional completeness, which we pursue, emerges by virtue of the crucial significance that syntacticism must attach to certain metalogical expressive qualities, as we will show. A criticism of syntacticism can be eked out from examining what the syntacticist view

must regard as anomalies and pathologies surrounding expressivity and specifically functional completeness.

At first blush, it may seem that the obscure metalogical subject of functional completeness will not repay extended philosophical attention. As a metalogical property, functional completeness is known as the characteristic of a set of logical functions, possibly a singleton, and also as a characteristic of a formal system of logic that is constructible as containing a functionally complete set of logical functions. Refinements, generalizations, and relevant distinctions are in order, and will be interspersed throughout the present study in technically colorful detail and to appropriate effect to bolster certain logical-philosophical observations. What is at stake in having functionally complete sets of logical functions is availability of formal and systematic reductions of expressive resources. The logical functions in a functionally complete set can be deployed according to the systematically specified formal methods for finitary composition of functions to define, only by themselves, all the mathematically definable logical functions of the logic to which the presented system belongs. As the above statement clearly shows, this subject belongs more broadly to the metalogical investigations into the expressivity of symbolic resources and issues related to existence and definability of the logical elements of a formal system for logic. All this makes the subject a staple within the ambit of metalogical investigations, but the present project delves into the logical-philosophical significance attaching to the study of functional completeness, and other topics in symbolic expressivity, motivated by the objective of understanding and critiquing a school of thought known as syntacticism.

- The significance that is attached by syntacticism to the availability of functional completeness results is known and easily documented in the historical record. We track the historical vicissitudes of analytic philosophy's endorsements of syntacticism, reflecting the early endorsement of this way of thinking about logic (partially) by the tractatarian Wittgenstein. Syntacticism's appealing nominalistic and logical-empiricist credentials bring this philosophy of logic to the forefront for analytic philosophers. In that context, we explore in depth the system known as protothetic (10.1-10.2.2), as a landmark in the development of formalisms that are motivated by a deeply entrenched syntacticist view of logic, as we also highlight challenges and reverses building up to plausible criticisms. Another case that we examine (7.2.2 – 7.2.2.2) is that of combinatory logic, given its initial motivations and

availability as a versatile instrument for probing deep questions about the nature of logic.
- As we seek to grasp the characterizing attributes of syntacticism and to document challenges that arise for the syntacticist program, we build an idealized version of this influential school of thought.
- We show how analysis of metalogical results pertaining to symbolic expressivity (including functional completeness) can support a critique of this philosophical view of logic.
- In the course of developing the case, we are compelled to navigate through extensive technical presentations of subjects related to the metalogical property known as functional completeness and to the logical-philosophic themes surrounding it.

Although functional completeness receives immediate attention – as there are also historical reasons for this – there are other metalogical vicissitudes affecting formal expressivity by means of symbolic resources, which we will also examine, and which also cast a pall over the characterizations of logic promoted by the syntacticist school.

Given the labyrinthine meandering needed to navigate such topics – from what logic is to how specifically formal systems are constructed and how metalogical properties are investigated – this project is ineluctably tedious at times, cannot fail to omit significant swaths of material, may be lapsing into simplifications, and inevitably takes sides now and then when it comes to notoriously controversial issues. The objective is to present and evaluate a school of thought about what logic, known as syntacticism (also symbolicism, foreshadowed by Wittgenstein in the *Tractatus*, anticipated by members of the Warsaw-Lvov school of logic, and later waxing influential among members of the analytic school of philosophy).

My purpose is not to unfold historical accounts (except briefly, see chapters 7, 11, 12) but to sketch a configuration of fundamental principles, trying to do justice to this approach as an ideal type rather than dwelling on historical details. We aim to show how results arising from the study of functional completeness, and other notions of symbolic expressivity, raise doubts about the claims made by syntacticism with respect to the nature of logic. This is a rather neglected, track of approaching the claims of, and trying to inflict criticism on, syntacticist. Arguably, this is not a direct confrontation with the philosophic view as much as it is a massive innuendo about how expressivist anomalies can be recruited to undermine syntacticism's credibility. It may be objected, on the other hand, that as the proverb has it

– who lives by the sword dies by the sword – reflections on the metalogics of expressivist issues arising around formal systems catapult us right at the central case that syntacticism makes with respect to what logic is. Moreover, the portrait of syntacticism that is cultivated, far from being set up straw-person style, opts for sensitivity that is supportive and keen on grasping the nuances of so radical a position. If anything, the idealization that is attempted smooths edges in favor of syntacticism, and in light of debates in the philosophy of logic which have already transpired. It is hoped that whatever intrinsic incongruities arise in this idealized version are reflective of deeper deficits that haunt the syntacticist philosophy itself but, of course, we take full responsibility for this modeling of syntacticism and plead only that we do exercise logical charity throughout.

The overarching case is as follows: If syntacticism represents correctly what logic is, then only those formal systems ought to be considered as properly conducting the business of logic which are functionally complete or can be extended to formal languages that are functionally complete in the strict sense of the meaning of this concept (and as it is to be differentiated from related notions); nevertheless, not only functional completeness fails to obtain as a metalogical property for formal systems that can be independently and plausibly argued to be proper systems for logic, but there are also other characteristic pathologies surrounding expressivity by means of symbolic resources: such lapses cannot plausibly be adduced to deny logical credentials to the formal systems that are so affected. Ergo, the view of logic underwritten by syntacticism stands impugned. Moreover, a plethora of expressive anomalies arise, as we examine, to which the syntacticist view, as we model it ideally, seems to be at a loss to respond.

We allow for extravagance in covering material in formal logic and the philosophy of logic in this project, continuously belaboring the relevant points while we also provide background information that may be needed. The technical details are hard to resist, and, so, there is some justification to claiming functional completeness as such to be standing alongside syntacticism as protagonists.

## 1.1 Functional Completeness, Expressive Completeness, and Pathologies of Expressivity

The subject of functional completeness, once one wanders into it, exudes at first an aura of triviality about it – in the technical sense in which a trivial claim is extracted uninformatively from the mere meanings of words

(including logic words). In this case, the investigation as to the functional-completeness credentials of a set of functions or of a formal system seems trivial in the sense that whether such a property obtains or not reflects just characteristics of the resources and formal methods by means of which the system is constructed. The paucity, or, indeed, the absence of logical-philosophical works devoted specifically to functional completeness seems understandable. Nonetheless, there is a case that can be made for embarking on a project dedicated to discussion of functional completeness in pursuit of logical-philosophical aspirations: not only are the metalogical explorations promising of elegant results in themselves but there is a logical-philosophical dimension to the subject.

Preliminarily, let us think of what functional completeness reveals about a formal system of logic and, a fortiori, about the nature of logic itself insofar as we agree to think of logic as whatever is shown through a formal system that has the proper credentials. I am not trying to give a definition of logic, in which case I would be running into circularity, but to reflect on plausible ways for assessing not only formal systems but also logic itself. Insofar as we can agree that there are independent initial requirements as to the formal characteristics of a system that can properly serve as a logical system, we have one half of our assessment task, so to speak; if we also agree on certain characteristics of logic as an enterprise – a tough task provoking ample controversy, but it can be provisionally carried through – then we have what we need: characteristics of appropriate formal systems as well as characteristics of logic as an enterprise.

With respect to an attempt to characterize logic, inevitably suppressing the multitudinous issues around the subject toward a preliminary blueprint, we may agree on the following.

- ❖ It is evident that our subject lies within what is approached under the designation “deductive logic,” excluding altogether inductive and arguably – if it is accepted – so-called abductive logic. We take logic, in the relevant sense, to have a characteristic feature of formality. No property of logic is uncontroversial but, without navigating such depths, we understand by logic not the empirical investigation of principles by which reasoners operate but non-reified collections of formal rules that exercise a normative effect on relevantly applied instances of reasoning.
- ❖ Unlike other disciplines, logic as a field of analysis applies to itself in ways that implicate how logic as such is characterized. While, for instance, in the study of biological processes, the operation of such

biological processes in the course of the study is irrelevant, this is not the case for logic. Even if reasons are accepted for a rigidified separation of levels of formal languages (possibly in accord with implementing some theory or types to prevent semantic paradoxes from arising), the examination of the logical system continues to call into focus the fundamental features of logic, whatever those may be. Even in the case of studying language itself, the study is conducted, inevitably, by use of language but does not itself count as an active ground for proving how the grammar of language *should* be used (which is rather presupposed). In the case of logic, however, binding decisions about the characterizing features of logic remain active also within the study of logic itself. Another example is that of the study of history: while history is the study of the historical events, the study of such events is not itself a historical event to be counted or excluded for purposes of carrying out the study; thus not only is "history" disambiguated but the study that we understand as history cannot be implicated in the kinds of self-referential paradoxes that are familiar in the case of philosophical investigations – including foundational theories *about* mathematical theories.

- In addition to formality and normativity, logic, as we understand it for our purposes, is what is sometimes called "pure" as distinguished from applied logic. More strongly, we take the proper subject of a philosophic view about logic to exclude matters of application (although not necessarily of formalizations pertaining to applicability), while issues as to application are to be thought as belonging to the realm of empirical inquiry. This also means that motivating interpretations are seen as empirical, possibly pragmatically instigated or psychologically adapted to presumed intuitions, and, as such, are not counted among the proper subjects of logic. Of course, all this is controversial, but we simply state our operational assumptions.
- A related trope in this respect relates to attempted comparisons of logic to geometry. A classical locus for drawing and restricting this parallelism is (Rescher, 1969). Because the vicissitudes of Euclidean geometry, ranging from the heavily metaphysical defense of its postulates all the way to its stunning removal from the toolkit of the modern, relativistic, theories of physics, have been painstakingly documented, there is an understandable pull toward seeking to draw parallels. (Rescher, 1969) presents reasons for rejecting the analogy. Specifically in the case of logic, it is not to be accepted that logic is motivated by pre-systematic views, relegating this whole enterprise of fit between logical systems and applied interpretations to an extra-logical empirical realm. Nor is it accepted that there is a distinction

between logic and geometry in that there is one correct physical geometry: pragmatic reasons may compel uses of different geometries indeed, not to mention that the commitment to speaking of correctness in this sense both incurs a burdensome metaphysical debt and runs afoul of widespread experiences of overdetermination. Moreover, once the view of dependence of logic on pre-formal standards is rejected, it is not the case that only geometries, but not logics, are all formally correct in themselves. (Rescher, 1969, 228-231) presents that the criteria for what counts as logic properly and for the characterizing logicality properties of the subjects of logic as study are furnished by pre-formal, pre-systemic means but this encapsulates a view that should be rejected toward drawing an ideal version of the philosophical theory of logic, which is called syntacticism. Arguably, Rescher's position has some intuitive force behind it but only as the path of least resistance, so to speak: quandaries as to ultimate grounds of justification often beckon in the direction of metaphysical solutions or, down another facile path, toward mentalistic and psychological assumptions. The pre-systematic criteria Rescher alludes to have been traditionally sought and presumed found in nature, in abstract mind-independent structures, in mental operations.

- ❖ It may appear that we are stacking the deck since we also intend to highlight the (syntacticist) view, according to which logic is what can be in principle formalized by means of syntactically regulated symbolic resources – which may be compared to a view that language is only what is in principle expressible by means of systematic manipulations of symbols (even taking the biochemistry of brain processes to constitute physically executable operations that are so expressible). It is admitted, however, that the purpose is to showcase an idealized variant of this theory and then subject it to criticism. Logical charity requires such concessions. As to how a principle of logical charity is itself to be defended, we adduce a point already raised: justification can be viewed as broader than the relevant notion of justification of claims specifically pertaining to logic. We have multiple occasions to observe that this view evokes a move made by Wittgenstein in his "Lecture on Ethics" by which he allowed for what seems to be a mode of justification of value-judgments even though he expounded a non-cognitivist approach with respect to value-claims.
- ❖ It may appear that inductive and abductive logics are question-beggingly robbed of their credentials. (We may bypass here interesting technical speculations as to whether combined inductive and deductive justificatory frames are constructible or are indeed at play in the course

of reasoning outside of what is standardly characterized as deductive logic). The opacity of inductive logic to formalization – having admitted formalizability as an overarching criterion in the case of logic – may itself be a matter of empirical exigency insofar as this claim is not itself the result of a formal proof – and no such proof can be produced formally without begging the question as to whether the criterion of formalism is provable by formal means without incurring self-referential paradoxes. Indeed, inductive logic is often forgotten, time and again victimized by oversight in the development of "commonsense" reasoning programs, although it may even claim the lion's share of everyday reasoning tasks.

- As indicated above, we may plead that justification may be thought of as broader than logical justification. Such a device is available to press a case for restricting attention to so-called deductive logic. But it is not rhetorical obfuscation as to definitions that we have in mind. Disambiguations as to the meaning of "justification" may be needed but what is discussed is, basically, a question as to ultimate grounds of the characteristic normativity that attaches to logic. Carving out normative authorities for other, non-logical, processes is a substantive, not a trivial option.
- Adopting this way of approaching the issue, one may wonder if, for example, moral justification or justification of moral principles can be counted as possible in some extra-logical fashion even if a non-cognitivist stance is adopted. This appears to be the stance taken by Ludwig Wittgenstein in his "Lecture on Ethics," which is a proto-manifesto for non-cognitivism in ethics and, at the same, albeit exercising markedly less influence, also hints that the realm of ethics, and aesthetics too, is not to be "denigrated" insofar as, presumably, they are somehow accessible. It is scandalizing to take this as extra-linguistic, possibly as exemplarily mystical unmediated apprehension. If, on the other hand, justification is assumed to be broader than what we take as justification as we define logic, then a less shocking position can be eked out. This presupposes, of course, that non-logical justification is not nonsensical as a phrase. As a testament to how difficult this subject is, we seem to have come back full circle: we might demand to know if the criteria for extra-logical justification themselves are to be extra-logical – and if the grounds for this are further queried, we appear to have triggered an infinite regress.
- We may appeal to the brute fact of the separate operation of inductive inference from deductive logical consequence. It is interesting to keep in mind, without entering into details, that, surprisingly perhaps, deeper

grounds of justifying either deductive or inductive inference are hard to come by. (See Haack, 1976)

- Denying justificatory valence to anything but in-principle formalizable inference may encourage philosophical flight – in one direction to something like a, perhaps caricatured, "Humean" skepticism about the status of observed law-like regularities, and, at another far-flung direction, toward the phantasmagoric construct of Leibniz who, as it is known, took all truths to be analytic and attributed impediments to reasoning to inability to continue the analysis as far as it needs to be taken.
- Rational thinking that proceeds by means of inductive inferences is admittedly susceptible to analysis; moreover, as it is known, characteristic types of informal fallacies are identifiable, stemming from: imposition of irrelevant or unsupported (presumed) premises, suppression of relevant premises, an imprecisely assessed low degree of likelihood that the inferential connection obtains, ambiguity and vagueness affecting the meanings of words and phrases in the argument, and psychological or rhetorical foisting of an impression that a proof has been presented when this is not the case.
- The relevant question for us is: does the fact that inductive logic is in-principle non-formalizable also undermine the syntacticist delineation of what logic is? This would be automatic if inductive logic were to be taken as logic in the relevant sense; by not granting inclusion of inductive logic, we further extend logical charity for the position we will be examining.
- Moreover, restricting logic to what may be thought of narrowly as deductive logic accords with possession, by whatever logic is taken to be, of certain inexorable features – like the ones presented below. The defense of these features can be carried through without appealing to a distinction between deductive and inductive logic – so, there is no circularity in making this case.
- It does not follow, of course, that non-deductive inference is exclusively and narrowly a psychological activity insofar as – and this should be controversial, but we venture it anyway – the selection of the justificatory standards for the analysis of non-deductive arguments is in principle defensible by deductive arguments that can be accepted as valid. Whether such arguments are also sound reflects exactly the issue in discussion – of how the foundations of non-deductive logic are to be fathomed, which, further, and by definition of what we mean by non-deductive, entails that no foundational issue can be at stake in this subject.

- ❖ On the other hand, we are not oblivious to the challenges facing the justification of deductive inference – which may sound surprising – parallel in a way to the more readily intuited problems in the justification of inductive inference. (See Haack, 1976) This is another subject we need to bypass.
- ❖ As another pertinent case as to how we may think about such issues, let us consider the formalized family of logics known as relevantist. Here is an example in which considerations of relevance are taken as integrally logical rather than relegated to what, in our suggested schematization of justification, would be issues of soundness and inductive strength. This, however, does not pose a threat. As argued above, the defense of criteria for inductive strength may be justified deductively; accordingly, claiming such criteria as inherently logical is not contradictory to the case that extra-logical justification can be demarcated separately. In fact, the case of relevantist logic can be enlisted in support of the syntacticist view that if formalization is feasible, then we are dealing with logic properly understood. The task is this: if one claims that some feature properly falls under the providence of logic, then the test is to show that formalization is feasible under the overarching constraints that may be accepted for formal systems. Such formal programs, like the one famously undertaken by Belnap and Anderson with respect to relevantist logic, are not to be thought of as "discoveries" but as operational constructions of logical systems.
- ❖ It may be questioned, though, if the motivations are extra-formal in a case like this, and, indeed, historically across the varied vicissitudes of formal logic. A claim that the standards of normativity that apply are tendered by extra-logical motivating considerations – trying to "get it right" by means of the formalism, appears to advocate against a syntacticist position and will be addressed shortly.
- ❖ Without belaboring such issues – given our current purposes in this text – we simply admit that the delineational restriction of the view of what logic is to deductive logic is presupposed.

We can now lay out some basic characteristics of what we mean by logic.

- Logic has a characteristic feature of meaning-normativity: logic dictates correctness standards for the assessment of meaning.
- Contentiously, the exercise of normative authority may be regarded as local rather than universal; as an example, accepting a quantum logic as the proper normative system may be dictated with respect to the standard theory that is in effect in quantum physics. This is readily seen as a

pragmatic motivation but a logical-pluralistic case can be made that it is about logic in a sense: the choice of a formalization of quantum theory that has "packed" in it the logic called "quantum logic" can be justified by appealing to criteria, perhaps themselves without further justification; if we are to say, additionally, that this *is* the proper quantum logic, as far as the characterization of a logic is concerned, this does not necessarily imply that some reified prior logical language is to be assumed; thinking of logic itself, any logic, as generated by formal means, we never have to account for some provenance of the logic outside of the manifold of motivating circumstances within which the logic is constructed and applied. We may think of the quantum theory, for instance, as coming in variants that are co-generated with the embedded logics and, then, the choice of one such variant, be it pragmatic, applies not to accounting for the logic but for the choice of the whole variant.

- Putting it along the above lines seems to adopt a local-pluralistic view, defined as a view that maintains a notion of "correctness" in the application of logical formalism but regards motivated choices as (possibly pragmatically) adjusted to capture principles that have been accepted extra-formally. Nevertheless, there is an instrumentalist view as to the provenance of logic, which rejects sense for any standards of correctness as such. (Haack, 1978, 227ff). The syntacticist philosophy ought to comport with the instrumentalist view unless "correctness" is to be conceptualized differently and without appeals to some extra-systematic notion of valid logical consequence. This last desideratum also seems to present itself decisively in the case of syntacticism. Insofar as the key challenge is, "how does one choose among competing logics" in the absence of extra-formal and extra-systemic criteria, this is the target that the syntacticist has to address.
- But this question hints, even if indirectly, at some grandiose vista of deep metaphysical grounds (and this subject ought to be properly handled under "logic and metaphysics" in a philosophy of logic text). This is precisely what the radical syntacticist rejects. A commitment to some ongoing search for the correct logic, or logics if locally instigated in accordance with theoretical needs, smacks of the metaphysical attachments that are rejected wholesale by syntacticism.
- There is, however, a way to speak of correctness, and indeed of local correctness, in the motivation of different systems. This view is certainly instrumentalist but it can still afford to deploy a properly defined notion of "correctness," a syntacticist notion of correctness so to put it. This is crucial because assessments as to correctness of applied logical formalisms are central to the overarching view of what logic is and,

further, a way for syntacticism to speak of correctness immunizes syntacticism from certain criticisms.

- A way to do this is by distinguishing between the standards for the correctness of the logical system itself from standards of "correctness" of the motivated application. The philosophical discussions around this issue, understandably, take the latter sense of "correct" as the relevant notion, notwithstanding the overused analogies drawn to applications of geometrical systems toward constructing a theory of physics. The systems themselves cannot be incorrect on account of deficiencies arising in the application. Formal systems can indeed be "incorrect" insofar as they lack with respect to any of the identifiable features that logical systems must show that they have. (This may be an open subject, and one in which it is critical to try to fend off circularity when defending such features of logical systems).
- The analytic school of philosophy is notoriously dedicated to bowing deferentially to the needs of science, but it seems that a deeply syntacticist approach to thinking about what logic is as such does not necessitate justifying the logical system itself on the basis of such deference. The logical system ought not to be justified on the basis of extra-formal considerations. In a sense, we may say, logical systems, as systematic syntactical operational programs, lack justification of the kind that one may be thinking of when considering, for instance, pragmatic motivations for choice. It can be said even that it is nonsensical to speak of justification of a logical system insofar as such talk alludes to some other system – within the formal structure of which the other system is justified: but the second system itself would then be in need of such justification – and, once again as it often happens in philosophic speculation, this triggers an infinite regress. The view that "justification of a logical system" is nonsensical may then be adopted and put to rest. We can still speak of motivation – as above, in the narrower sense of practically motivating application. What is being justified in such a case is not the logical system as such.
- Logic, whether more specifically defined through characteristic logical consequence relations or through collections of designated formulas, is formal, in the sense that even though pronouncements as to the correctness of specific logical targets are concrete, the pronouncements themselves are justified on the basis of general law-like patterns or shape-like so-called schemata or what Aristotelian syllogistic called "figures"; this can be accepted broadly and regardless of how one may handle a problem as to how to distinguish types from tokens.

- Thus, the symbolic character of logic is in evidence even in a reasonable minimal demarcation. This is consistent with certain old historical claims to the effect of, for instance, enthroning as founder someone like Aristotle, who grasped that logic has a systematically formal character after other interested speculative thinkers had not discerned this. It never ceases to surprise the novice who is prone to intuitive expectations that logic must be bound in decisive ways by the content of proclamations we may make about experienced events and depictions of natural objects and their objective arrangements.
- It should be clear that in this attempted minimalist definition of logic, what is arguably smuggled is that logic is as logic does by means of formal expressibility (in principle, not necessarily in the sense that a formalization is actually available or obtainable by known means). Not only are we to forswear use of such expressions as "the logical constitution of reality" on pain of nonsensical speech but along the lines of declaring as nonsensical a phrase like "I know it but cannot possibly express it," we have it that it is nonsense, based on definition of what logic is, to think that there are regulative restraints on assessment of meaning, which are inexpressible by symbolic means. Another way of saying this is: logic is such if and only if it is (in principle) symbolic logic.
- Why adopt a view like this? Should this inherently symbolic character of logic be laid down definitionally, or is it a theoretically open and inherently contestable point?
- We restrict the definition this way in deference to the syntacticist view that we examine closely; we set up a deferential view methodically, given the critical stance toward syntacticism: this is along the lines of the familiar strategy of giving the opponent as much as possible before proceeding to identify lacunae and anomalies. It is to be noted that, even if the syntacticist view is criticized, it does not mean, of course, that this view is wrong in every single respect. The characterization of logic as inherently symbolic is broader than the specific theory of syntacticism, which we will be exploring.
- If the restrictedly symbolic character of logic were to be relaxed as a definitional feature, the question would arise as to the status of the "logical forms" on which plausible consensual agreement can be obtained as to the nature of logic. It appears that remaining alternatives are Platonistic – taking the forms to be verities in the sense of mind-independent abstract entities – or mentalistic. To define logic without committing to either view is possible, however, and this is accomplished by incorporating an emphatically symbolicist characterization of logic.

- One important view is that the deeper origin of the correctness of the logical patterns or forms accrues due to the definitions of the ascribed meanings of the characterizing (logical) elements, or connectives if we use the semantic term. In this sense, logical truths are analytic, although not all analytically true statements are logical truths as the logical necessity of the truth assigned may be due to the defined meaning of non-logical words and expressions composed of non-logical words. The distinction between logical and non-logical formal (symbolic) components and their syntactical rules is part of the systematic study of logic but facts about the formal features of the non-logical components should not enter into deep characterizations of logic itself. Indeed, to do otherwise seems to straightforwardly lead down to paths into metaphysical views about the constitution of reality itself, possibly including teleological views, credulity-rousing logical fatalistic views, and so on.
- There is a burden to show that logic should be thought of as a rule-based operational enterprise rather than as some reified array of abstract forms or schemata of rules or theses. Wittgenstein elaborates this point in his *Tractatus*, as we trace further in 11.1.1.
- Logic is not a science, without thus forfeiting its authoritative status (as Wittgenstein argues in the *Tractatus*). An argument to this effect is that logic, as such, cannot be itself subjected to higher regulative or normative authority (we are not speaking here of motivations as to which logic is in effect but about the logic that, for whatever motivating reason, is considered as regulative). Therefore, if all sciences as such are to be considered as subject to the normative authority of some logic, logic itself cannot be a science. (There are other arguments to make this case). In this sense, logic is pre-scientific (to be disambiguated from "primitive," "mythological," or "historically antedating the development of modern science.") But this does not mean that logic imposes on science an authority that it inherits from metaphysically or transcendentally derived sources.
- To serve as system of logic, a formal system must be supplied with specified expressive resources and syntactical arrangements for the management of such resources in a systematic manner.
- Syntax is assigned the twain task of specifying rules for well-formedness (what counts as a grammatically correct as "readable" concatenation of specified symbolic expressions), and of specifying regulative procedures for the formal construction of proofs in the system.
- Some disambiguation is sorely needed with respect to "syntax." (Church, 1956, 50, fn. 116; 58, para. 08; 59-60) disambiguates between

"syntax" as the assemblage of stipulated formation rules (and sanctioning decision tests for well-formedness of symbolic expressions, on the one hand, and the broader term "logical syntax," which, however, may itself be referred to as simply "syntax" if no risk of ambiguity arises. The broader notion of "logical syntax" contains the interpretation, broadly understood as the task of assigning denotata inclusive of objects and truth values, while the narrower notion is the formalist or strictly grammatical edifice for construction of symbols and formation rules. A dedicated metalanguage may be constructed, an appropriately minimal fragment of some natural language (for instance, English), enhanced with symbolic resources for naming symbols in the object language (possibly autologously, or by using the same symbols but placed within some metalinguistic auxiliary symbols to indicated that they are mentioned rather than used).

- It is crucial that semantics be reducible to syntax from a syntacticist point of view. As we survey the historical background (ch. 7), a saga emerges that, notwithstanding the syntacticist principled opposition to semantics, and even under the pall cast by stern warnings about reopening the floodgates of metaphysical profligacy by admitting semantics, Rudolf Carnap changed his mind on this subject, even producing a major opus on semantics. Nevertheless, we can see, controversially perhaps, inclusion of semantics as a challenge of a different sort. In spite of misleading impressions that can arise from taking the term "syntacticist" literally, the philosophic view – as we model it here – considers logic – not other enterprises of knowledge but specifically logic – to consist in syntactical manipulation of symbolic resources according to specified rules. Insofar as the semantics can be carried out by syntactical means – in other words, seeing the relevant rules about manipulation of symbols to extend beyond well-formedness and rule application and to reach even stipulations about alleged objects – the inclusion of semantics itself does not topple the syntacticist program – except, perhaps, we may say that this dethrones a certain radical version of the view. It is arguable, moreover, that this threatened version is not radical – because its empiricist and anti-metaphysical temperament is retained anyway; it is possibly confused as to how conducting semantics may still be regarded as a syntactical affair under satisfaction of certain appropriate conditions.
- The use of substitutional interpretation in first-order logic, instead of the so-called object or objectual interpretation comes to mind in this regard. (See also 1.2)

- To this effect, the significance of the work of Alfred Tarski rightly claims preeminence. (See, e.g., Tarski, 1936; contrast Carnap, 1959, for some reservations. See below, 2.0.2) A strictly syntactical property of well-formedness is definable, as Tarski showed, which is coextensive with the semantical property of being-true in a language that is rich enough symbolically to express its own syntax. Relegation of the so-constructed semantic predicates to ever higher-level languages is enforced, in the light of the insidious semantic paradoxes and with a view to precluding such paradoxes from arising. What is needed for this syntactic reduction of the semantic aspect is to lay down the rules for what it is to take value, as object or truth value, in the same way that well-formedness rules are instigated within a formal system. The apparent independence or irreducibility of the semantics originates, as it turns out, from taking the meaning of "assigning value" as prior and then enforcing regulations for matching the syntactically constructed language with its assigned interpretation.
- Not taking semantic concepts as given in advance, but instead legislating them by syntactical means, does the trick. Tarski showed how a consistent formal language with its characteristic truth-for-language predicate can be constructed. Indeed, a multiplicity of interpretations may be so assigned to a formal system. (Church, 1956, 66, fn. 143) analogizes this maneuver to the device by which assigning different meanings to a particular straight line and a particular elliptic involution of such a line generates differentiation between Euclidean metric geometry and plane projective geometry.

It all gets much more complicated beyond such first steps, but the minimalist flavor of such requirements is sufficient to show that a formal system must present perspicacity of display through its symbolic resources and systematicity of any permissible reductions of its symbolic resources. The point is not that what is displayed through the system is external or metaphysically supported, so that the formalism has to match some transcendental target: the formal system must be able to stand on its own; the display that we may speak of in the case of a formal system is delineated completely and ought to be delineated precisely by means made available through the constructive architecture of the system. Problems as to the correspondence of the system to a motivating interpretation are external to logic as conducted by formal means and seeking to address such problems requires empirically ascertaining whether some application of a formal system is successful according to appropriate criteria. More narrowly, a formal system for logic must have characteristic capacities of perspicuous

display and systematicity of reductions with a view to succinct expression, which arise out of its presumed character as, specifically, a logical system. As (Frege, 1893, ¶ 93) puts it, in a diatribe against the formalist school of philosophy about arithmetic (which parallels in significant ways the syntacticist view of logic), the symbols are to be regarded as "empty" of (external or prior) referent and the syntactically constructed expressions are ciphers tending toward more succinct representations of the formal theory's theses.

For the syntacticist, logic accrues from the concrete regulated operations for the manipulation of symbolic resources that are made systematically available within a specified system. Logic in the broad sense - a vexingly complicated definitional challenge - may be thought of as what can be correctly expressed through operational transactions of properly constructed formal systems. We will erect an idealized scaffold for our purposes (starting at 1.2), which is not necessarily one all concerned would agree about, but it is proffered as some tenable variant. We may think roughly of the syntacticist view along the following lines: echoing (some of the views presented by) Wittgenstein in the *Tractatus* (6.126, 6.127, 6.1265), and thinking for now of logic as characterizable through the corresponding logical consequence relation, we can say that syntacticism views the correct operational derivation of new sentences as a matter of manipulating signs in accordance with specified systematic rules. It is all "in the signs alone," to put it rather crudely at first, and, enigmatically, it is averred that every logical thesis "ultimately depends only on itself." There are both structural rules and functional or connective rules in a system of the kind initiated by Gentzen, and it appears that the characteristically logical rules are those governing the manipulation of the connective symbol (which are introduction and elimination rules).

The Tractatarian Wittgenstein caused confusion on this score, vacillating between seeking for ways to accommodate claims to relevance of the "ordinary" or natural language, on the one hand, with the obviously obfuscating influence of its historically evolving linguistic grammar, and the formal syntax of an ideal language for logical meaning: although the *Tractatus* labors tenuously under this tension (see the contradictions underlined in, for instance, Copi, 1957), Wittgenstein himself states clearly that "ordinary language is obscure and misleading" and "[natural] language disguises the thought" and "[conceals] the … propositional sign." (4.002) To read this as a syntacticist proto-manifesto – since we are interested in an idealized prototype of the theory rather than in historically ascertaining

exactly what Wittgenstein thought – we need to understand it along the following lines (see further chapter 11).

- It is not that there is a pre-symbolic and pre-linguistic depository of logical meaning, which is amenable to be more or less perspicuously expressed by different languages by dint of the ways they are constructed – with the formal idioms of logic claiming preeminence in this respect. Wittgenstein's attraction to a logical-atomistic view and his interest in a pictorial-representational theory of truth, which also claim eminence in the project of the *Tractatus*, pull in such a direction. The view may also be intuitive but that is another issue. The syntacticist view can be culled by thinking of the "sign" as being the relevant "fact" pertaining to logical meaning generation, and of the syntactical construction being integrally contributory to the constitution. Thinking along these lines, we can make sense of the other critical point raised by Wittgenstein, that natural language "disguises and misleads" and we can account for this without having to invoke any metaphysical or empirical props in grasping what is fundamental to logic.
- Copi (1958, 145), in an attempt to negotiate certain detected inconsistencies in the *Tractatus* poignantly corrects an earlier commentator (Daitz, 1953, 188-189) who had read Wittgenstein's project as seeking to "assimilate sentences to facts:" it is like taking to task a book about Latin grammar for not making true assertions about Chinese grammar, Copi points out. Wittgenstein is interested in what came to be called, under his own broader influence, the logical grammar or, as we may put it, the correct grammar that matches the logical consequence relation – or the grammar of the logical form, if we may speak imprecisely for now. This is still noncommittal as to the provenance of logic – the source of the standards of correctness, relative to which the formal grammar "gets it right." Even unintentionally, though, Copi's remark, pithily, brings up a comparison of grammars – English and Chinese, in the example – adumbrating that what may be at issue is syntactical construction.
- Presumably, unless empirical investigation shows otherwise, let us accept that logical consequence does not shift as between English and Chinese: the grammars of the two languages do not come into play but it is a different matter when it comes to the definitional arrangements about the logical components of the languages. At this point, we may think of such a possible shift, in the definitions of the logical components, as being itself dependent on the linguistic grammar but this

is the error that one ought to become aware of once awakened to the distinction between linguistic and logical grammar.

- The perspicacity of systematic display is not itself a secondary feature; it is fundamental: hypothetically, we could think of two natural languages, the grammar of one of which is more effectively suited, expressively, to "showing" the logical grammar than the other. Nevertheless, such happenstance would not in any way affect the basic claims about a distinction between the linguistic and the logical grammar. It is merely incidental that one of the two languages, in our hypothetical, is relatively less "obscure" or "misleading;" and, indeed, to attempt a precise approach to making such a comparison presupposes that we can make appeals to standards by which the formal syntax is expressively effective in showing forth – or, rather, constructing – the logic.
- All this is with respect to the logic, though, and the subject becomes muddy if attempts are made to bring logic in contact with the realm of empirical facts, as the pictorial theory purports to do. That side of Wittgenstein's attempted project in the *Tractatus* creates corresponding problems, which Rudolf Carnap tried to remove by recruiting simply syntactical means in the construction of formal systems for logic. (See 7.3)

Wittgenstein was attuned to the standard first-order logic, attempted to define a functionally complete logical function, unsuccessfully as it turns out, and was further committed to a project, later abandoned, of in-principle development of a prelogical systematic language built of atomic sentences that ate taken to correspond to atomic facts (the so-called logical atomism program). Notwithstanding this historical background, and well-known emphatic preferences in favor of first-order logic, we take the view, perhaps contentiously, that syntacticism, as a view about the demarcation of logic, is rather compatible with a pluralistic view of logic provided that the standards of correctness for assessing application of the system are not themselves regarded as logical (i.e. there are no extra-systemic or extra-formal logical considerations but there are, of course, other, possibly pragmatic considerations).

The initially puzzling contention about the self-dependence, by way of deducibility, of the logical theses draws attention at what we will take as a fundamental syntacticist thesis. Formally, there a couple of different ways of envisioning this principle, which is presented as stemming from the deep understanding of what logic is, as this is understood through the lens of

syntacticism. We may think of a single axiom, like Nicod's, constructed by means of a single functionally complete function, so that the entire logical system is derivable from this and by use of the systemic rules. Alternatively, if we insist that logic should be conceptualized by examining logical consequence relations, we may think of a constructible formal system of derivations, such that finitary operations of deriving theses are effectively sufficient when carried out by application of the system's rules for logical functions so that every derivation step is fully justified (or better, "described", Hacking, 1979) by reference to the system's rules for manipulation of symbols.

In light of the above, syntacticism gives rise to a requirement to the effect that anything that can be counted as logic has to be in principle accessible through a formal system that regulates the syntactical manipulation of the symbolic resources made available in the system. Syntacticism reaches beyond and into the relation of meaning and symbolic agglomerates across the range of meaning-rendering enterprises, but our present focus is specifically on the syntacticist view of logic. Accepting the syntacticist view, that perspicuous display should be available is not just a nod to convenience; it is not the result of a preference for aesthetic elegance; not an ancillary to propaedeutic, assistant to discovery, or arising as a reflection of some transcending theoretically motivating purpose: perspicuous display is a necessary systemic attribute for formal systems of logic on pain of disqualifying those that fail in this regard.

Considering that "perspicuous display" is vague as an initially presented notion, we may right away point to the options of expressivity made available for formal systems – specifically narrowing in on functional completeness – as diagnostics, so to speak, of the system's proper credentialization as a logical system. Wittgenstein's celebration of "Sheffer's discovery" of a single functionally complete logical function is a case in point. Syntacticist stress is also placed on a requirement of systematicity of reductions of symbolic resources in accordance with the formal arrangements specified in the system. We may be accused of straw-person fallacy since we set up such a case of syntacticist stringency about these requirements: it begs incredulity to demand that formal systems that fail in characteristic ways with respect to expressivity features – like functional completeness – ought to be rejected as properly logical systems; it appears, however, that there are facts about symbolic expressivity, which we will be canvassing throughout this study, that show precisely how far-fetched the demands of syntacticism are – at least as portrayed, and,

hopefully, not caricatured but extracted from a plausible template of syntacticism. By the same token, syntacticism becomes open to criticism in a way that has not attracted attention to itself directly – although essentially these issues and reservations run throughout the literature we survey – possibly because of the afore-mentioned impression that the study of expressivity is a metalogical prerogative but devoid of logical-philosophical significance when it comes to grandiose subjects like the characterization of logic as such. Nevertheless, it may be said, it is syntacticism that precisely elevates – and can even be summarily defined as the view that elevates – systematic expressivity of logical symbolic resources to constituting the "essence" of logic (without attaching any essentialist imputations to this use of "essentially" of course).

The semantic side, the logical truths of the language, ought to be expressible in strictly syntactical terms, and the consistent syntacticist must think of semantic definitions of the connectives and fundamental notions like consistency and completeness of the formal language as carried out by purely syntactic arrangements and for the concept of truth that is specifically in the formal language (not excluding whatever adjustments to the relevant notions, like "truth" are needed depending on the kind of logic that is at play). This can be done even within the strictures imposed by textbook-like austerity of execution (as in Church, 1956). The definitions of connectives are to be considered, more appropriately, as internal formal definitions, specifications or characterizations and as being exhaustively based on stipulations for the manipulation of symbolic resources (as the case is, paradigmatically, with the proof-theoretic view of connectives as depending on rules for introduction and/or elimination of their symbols). This places the syntacticist view in alignment with the traditional view of logical truths as analytic, as based on the definitions of their logical components – albeit with the added caveat, it may be contended, that no analytic truths can be deployed in justifying selection of one logical system over another. The traditional view in the analytic, logical empiricist and nominalistic traditions (within whose overlapping realm syntacticism thrives) has not been necessarily pluralistic but any burden on this matter ought to be placed on a universalistic side: if there are supervenient, extra-systemic principles governing assessments of how symbolic resources are constructed and managed syntactically, this can only be defended on pain of infinite regress if the extra-logical principles themselves are to be regarded (as the syntacticist must regard them) as formally justifiable.

For now, we may return to the subject of expressivity and functional completeness, having preliminarily highlighted the integral connection of this school of thought about logic to what we define as syntacticism. (See further 1.2)

- Functionally complete sets of functions, if available, answer to both requirements: they make, in principle, display of logical operations perspicuous through systematically available reductions of symbolic resources. Moreover, since the syntactic transformations of the symbolic resources of the formal systems are systematic, the reductions themselves, at the service of perspicacity of display, must be tending in-principle toward available functionally complete sets of logical functions, which are of relatively lower cardinality (smaller rather than larger number of function symbols in the set).
- Hence, the case of a singleton set of functionally complete functions is not a mere curiosity but the ultimately "well-behaved" template for how a formal system is fit to serve as a system for logic while, at the other extreme, recourse to the expedient of parading all the mathematically definable functions and claiming trivially functional completeness for that set makes manifest a fundamental defect relative to the (perspicuous) display and reduction requirements for a formal system.
- A vital notion for the methodology of obtaining metalogical results about functional completeness, beyond the case of two-valued logic, is that of functionally precomplete sets: a functionally precomplete set S can be defined as a maximally closed set of functions (in the sense that all functions that are definable from the given functions are in the set), other than the empty set or the universal set of functions, and which is a proper subset of the set of all definable functions and such that this latter set – the set of all definable functions – is the only (degenerately) functionally complete set that has S as a subset. (See 1.3, 2.1).
- The familiar semantic approach of the standard propositional logic, which defines the connectives of the given set by means of the truthtabular device, sets up a target relative to which functional completeness results can be presented, and even the notion of functional completeness can be defined. The set may have pairwise mutually independent functions or not, and it may be minimally functionally complete (as in the case of the available singleton functionally complete sets) or simply functionally complete.
- Relative to initially given functions, which is of course a member of the powerset of all definable functions, functions may be added or withdrawn with a view to obtaining intended results about expressivity: