Trussed Frames and Arches

Trussed Frames and Arches:

Schemes and Formulas

By

Mikhail Kirsanov

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Introduction

Nowadays, when numerical systems based on the finite element method have captured all the dominant heights in the field of structural analysis, accurate calculation formulas do not lose their significance. On the other hand, there are few such formulas and therefore the engineer has to turn to numerical methods. With this approach, one necessarily encounters standard shortcomings that are common for approximate calculations. First of all, there is a problem with calculations accuracy. In a simple case, when the design is simple and the number of its elements, for example, rods, is limited, this inaccuracy is not that significant. However, dealing with very large-scale structures containing thousands of elements is a real issue. Take as an example the overlapping of stadiums or large-span bridges (Fig. 1 – 3). Doing calculations in such cases is laborious, and no one can guarantee the accuracy of them. The well-known problem, so called the «curse of dimension», is a guaranteed loss of accuracy when solving a system of linear equations with the number of equations increased.

Fig. 1. Royal Albert Bridge, river Tamar, United Kingdom

For large order of the system, the solution for individual unknowns may have an error of 100% or more. In such cases simple and reliable formulas, containing as parameters not only the dimensions of the structure, but also the number of elements could be very useful. The number of elements here does not affect accuracy. Such formulas can be derived by induction for regular systems having periodicity elements in their structure. Experience shows that such formulas can be derived by induction for regular systems having periodicity elements in their structure. For trusses, the periodicity element may be a panel containing a number of rods. The periodicity element for trusses could be a panel containing a number of rods.

Fig. 2. Story Bridge, Brisbane, Queensland, Australia

This handbook provides a very practical and simple approach for overcoming these issues. There are 72 schemes of planar statically determinate trusses with formulas for the dependence of deflection and forces in some (usually critical) rods on the number of panels.

Fig. 3. Andreyevsky Bridge, Moscow, Russia

For each schema formulas are given for two cases: distributed or concentrated load. For the structures that contain a movable support, there are additional formulas for the magnitude of its horizontal displacement. This handbook consists of six chapters. Trusses schemes are conventionally divided into certain types in the first five chapters. The division by trusses types is not straightforward. For example, some trusses may belong to both arches and frames. Chapter 5 shows console calculations. All the schemes in the handbook are original, except the simplest ones. The sixth chapter describes the program written in Maple [76] language. In addition to the problem of allocating time for work, by breaking away from teaching, the author had two difficulties while he was writing this handbook.

The first and main problem was to come up with an original scheme of a statically determinate regular truss in a way that is kinematically unchanged, and the formula for its calculation would not be very cumbersome.

Publishing voluminous solutions containing half page trigonometric functions, is not a good idea due to small practical value. It is not a practical approach to use a formula the perfection of which is hard to check due to its size and complication. However, it is a good point here to talk about the second challenge, which is related to the reliability of the obtained solutions.

How do I guarantee the reliability of the derived formulas? The following technique was found, previously used by the author in his first handbook [28]. Firstly, the conclusion is duplicated and in order to avoid elementary typos, it is checked by typing from a sheet of a finished manuscript in numerical form. Secondly, all formulas for each truss are placed in separate files in text format. These files are available for free download (http://vuz.exponenta.ru/Trusstxt2020.zip). Thus, one has an opportunity to avoid typing large formulas from the book sheet. Just copy the formula from the file and place it into the program that performs calculations. I think, the best approach is to use Pascal and Maple based languages for such programs.

The handbook is a continuation of the first handbook [28]. The author hopes that the formulas could be useful to engineers in their practice, to students — for training purposes. The initial idea of generating formulas for calculating regular systems (not necessarily trusses) could be helpful for researchers in their scientific activities.

Please do not hesitate to contact the author via following email: c216@ya.ru.

Chapter 1

Arches

Truss 1.1

A truss with a height of $(m+1)h$, $h = a\sqrt{3}/2$ consists of $8n + 8m + 3$ rods of length a (Fig. 4).

1.1.1. Concentrated force in the mid-span

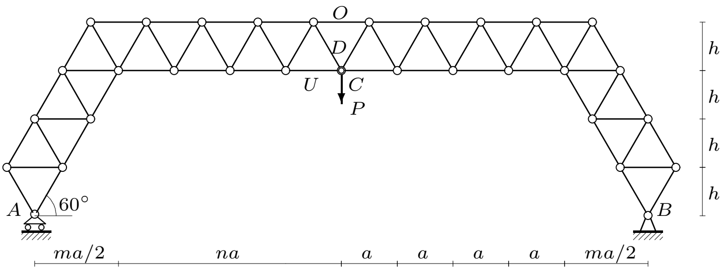

Fig. 4. Truss, $m = 3$, $n = 4$

The deflection (vertical displacement of the middle node C in the lower zone) has the form

$$\Delta = Pa(4m^3 + 3(1+4n)m^2 + 2(13+12n^2)m+$$
$$+2n(25+8n^2))/(36EF).$$

Support A offset:

$$\delta_A = Pa\sqrt{3}\,(4m^3+3(3+4n)m^2+2(6n^2+3n+4)m+6n^2)/(18EF).$$

Forces:

$$O = -P\sqrt{3}\,(2n+m)/6,\ D = P\sqrt{3}/3,\ U = P\sqrt{3}\,(2n-1+m)/6.$$

Supports reactions: $Y_A = Y_B = P/2$, $X_B = 0$.

1.1.2. Load on the upper belt

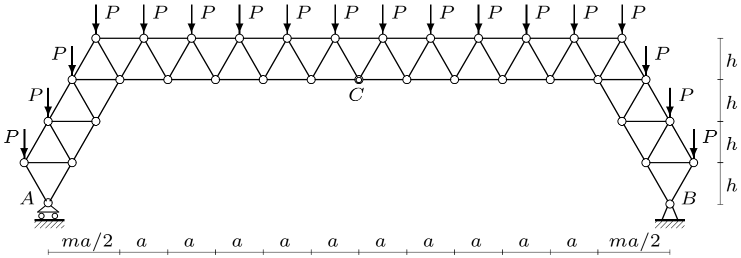

Fig. 5. Truss, $m = 3$, $n = 5$

The deflection (vertical displacement of the middle node C in the lower zone) has the form

$$\Delta = Pa(5m^4 + (20n - 1)m^3 + (36n^2 - 6n + 10)m^2 + \\ +(40n^3 - 12n^2 + 30n + 16)m + 4n^2(7 + 5n^2))/(36EF).$$

Support A offset:

$$\delta_A = Pa\sqrt{3}\,(5m^4 + (7 + 20n)m^3 + 2(12n^2 + 6n - 7)m^2 + \\ +4(n - 1)(4n^2 + 7n + 4)m + 8n(n^2 - 1))/18.$$

Forces:

$$O = -P\sqrt{3}\,(m^2 + (2n - 1)m + 2(n^2 - 1))/6,\ D = 0,\ U = -O.$$

Supports reactions: $Y_A = Y_B = (n + m)P$, $X_B = 0$.

Truss 1.2

A truss with a height of $h + 2b$ (Fig. 6) containing $2n + 4$ panels, consists of $8n + 17$ rods. The total length of the rods in the truss is

$$L_{sum} = 2(2a + c)(n + 1) + 2d + g + 2h(n + 4),$$

where $c = \sqrt{a^2 + h^2}$, $d = \sqrt{4a^2 + h^2}$, $g = \sqrt{4a^2 + 9h^2}$, $b = h/2$.

The deflection (vertical displacement of the middle node C in the lower zone) has the form

$$\Delta = P(C_1a^3 + C_2c^3 + C_3h^3 + C_4(d^3 + g^3))/(6h^2EF). \qquad (1.1)$$

Support A offset:

$$\delta_A = P(A_1 a^3 + A_2 c^3 + A_3 h^3 + A_4 d^3 + A_5 g^3)/(9ahEF). \qquad (1.2)$$

1.2.1. Concentrated force in the mid-span

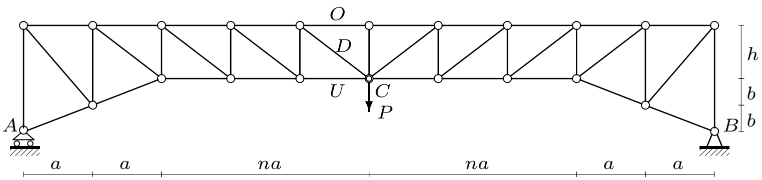

Fig. 6. Truss, $n = 3$

Deflection. The coefficients in (1.1):

$C_1 = (6n^3 + 36n^2 + 75n + 40)/3$, $C_2 = (9n + 16)/3$,

$C_3 = 14 + 3n$, $C_4 = 1/6$.

Support offset. The coefficients in (1.2):

$A_1 = (27n^2 + 99n + 40)/2$, $A_2 = 8$, $A_3 = 21$, $A_4 = 1$, $A_5 = 1/4$.

Forces:

$O = -Pa(n + 2)/(2h)$, $D = Pc/(2h)$, $U = Pa(n + 1)/(2h)$.

Supports reactions: $Y_A = Y_B = P/2$, $X_B = 0$.

1.2.2. Load on the upper belt

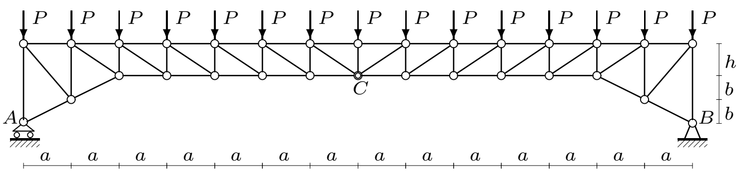

Fig. 7. Truss, $n = 5$

Deflection. The coefficients in (1.1):

$C_1 = (15n^4 + 120n^3 + 363n^2 + 436n + 168)/6$,

$C_2 = (9n^2 + 32n + 24)/3,\ C_3 = 3n^2 + 34n + 54,$

$C_4 = (2n + 3)/6.$

Support offset. The coefficients in (1.2):

$A_1 = (36n^3 + 207n^2 + 287n + 84)/2,$

$A_2 = 16n + 12,\ A_3 = 42n + 81,$

$A_4 = 2n + 3,\ A_5 = (2n + 3)/4.$

Forces:

$O = -Pa(n + 2)^2/(2h),\ D = Pc/(2h),$

$U = Pa(n + 1)(n + 3)/(2h).$

Supports reactions:

$Y_A = Y_B = (2n + 5)P/2,\ X_B = 0.$

Truss 1.3

A truss with a height of $(2n + 1)h$ (Fig. 8) containing $2n$ panels, consists of $8n - 1$ rods [6].

The deflection (vertical displacement of the middle node C in the lower zone) has the form

$$\Delta = P(C_1a^3 + C_2c^3 + C_3h^3)/(h^2EF), \qquad (1.3)$$

where $c = \sqrt{a^2 + h^2}$.

The total length of the rods in the truss is

$$L_{sum} = 4(3c + h)n + 2a(1 + 2n) - 2c.$$

Support A offset:

$$\delta_A = P(A_1a^3 + A_2c^3 + A_3h^3)/(ahEF). \qquad (1.4)$$

1.3.1. Concentrated force in the mid-span

Deflection. The coefficients in (1.3):

$C_1 = n(2n + 1),\ C_2 = n(2n^2 + 1)/3,\ C_3 = n.$

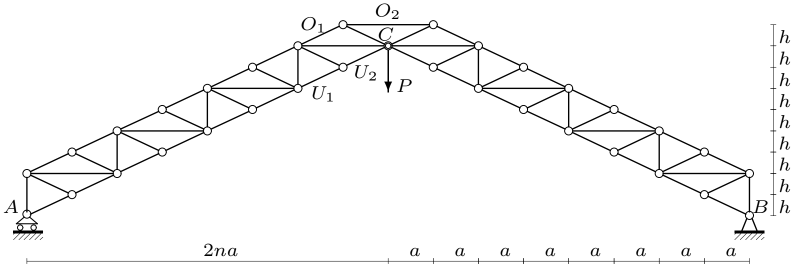

Fig. 8. Truss, $n = 4$

Support offset. The coefficients in (1.4):

$A_1 = 2n(2n + 1)$, $A_2 = n(4n^2 + 3n - 1)/3$, $A_3 = 2n$.

Forces:

$O_1 = -Pcn/(2h)$, $O_2 = -Pan/h$,

$U_1 = U_2 = Pc(n - 1)/(2h)$, $V = -P/2$.

Supports reactions: $Y_A = Y_B = P/2$, $X_B = 0$.

1.3.2. Load on the upper belt

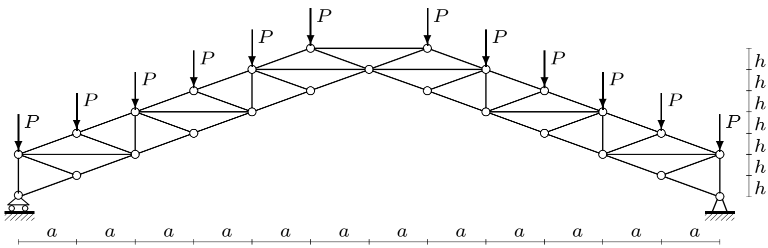

Fig. 9. Truss, $n = 3$

Deflection. The coefficients in (1.3):

$C_1 = 4n^3$, $C_2 = n(20n^3 - 8n^2 + 7n - 7)/12$, $C_3 = 2n(n + 1)$.

Support offset. The coefficients in (1.4):

$A_1 = 8n^3$, $A_2 = n(20n^3 + 8n^2 - 11n - 5)/6$, $A_3 = 4n(1 + n)$.

Forces:

$O_1 = -Pc(2n^2 - n + 1)/(2h)$, $O_2 = -Pan(2n - 1)/h$,

$U_1 = U_2 = Pc(2n^2 - n - 1)/(2h)$, $V = -2P$.

Supports reactions: $Y_A = Y_B = 2nP$, $X_B = 0$.

Truss 1.4

A truss with a height of $2(n + 1)h$ (Fig. 8) containing $2n$ panels, consists of $8n + 1$ rods. The total length of the rods in the truss is

$$L_{sum} = 4(a + 3c + h)n + 2h,$$

where $c = \sqrt{a^2 + h^2}$.

The deflection (vertical displacement of the middle node C in the lower zone) has the form

$$\Delta = P(C_1a^3 + C_2c^3 + C_3h^3)/(h^2EF). \quad (1.5)$$

Support A offset:

$$\delta_A = P(A_1a^3 + A_2c^3 + A_3h^3)/(ahEF). \quad (1.6)$$

1.4.1. Concentrated force in the mid-span

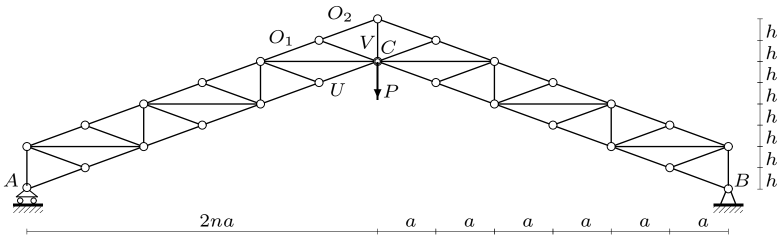

Fig. 10. Truss, $n = 3$

Deflection. The coefficients in (1.5):

$C_1 = n$, $C_2 = n(1 + 2n^2)/3$, $C_3 = n(1 + 2n)$.

Support offset. The coefficients in (1.6):

$A_1 = 2n,\ A_2 = n(4n^2 + 3n - 1)/3,\ A_3 = 2n(1 + 2n).$

Forces:

$O_1 = O_2 = -Pcn/(2h),\ U = Pc(n - 1)/(2h),\ V = Pn.$

Supports reactions: $Y_A = Y_B = P/2,\ X_B = 0.$

1.4.2. Load on the upper belt

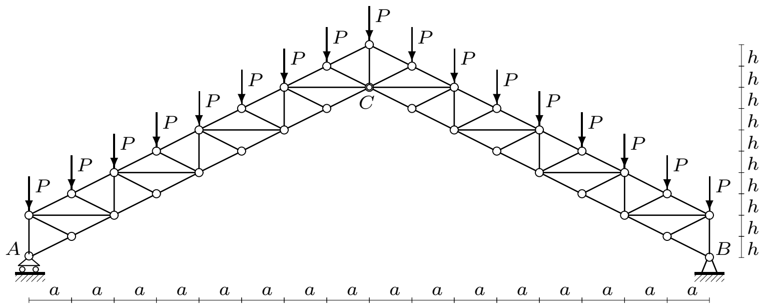

Fig. 11. Truss, $n = 4$

Deflection. The coefficients in (1.5):

$C_1 = n(2n + 1),\ C_2 = n(20n^3 + 7n + 3)/12,$

$C_3 = n(4n^2 + 2n + 1).$

Support offset. The coefficients in (1.6):

$A_1 = 2n(1 + 2n),\ A_2 = n(20n^3 + 16n^2 - 5n - 1)/6,$

$A_3 = 2n(4n^2 + 2n + 1).$

Forces:

$O_1 = -Pc(1 + 2n^2)/(2h),\ O_2 = -Pcn^2/h,$

$U = Pc(n^2 - 1)/h,\ V = (2n^2 - 1)P.$

Supports reactions: $Y_A = Y_B = (4n + 1)P/2,\ X_B = 0.$

Truss 1.5

A truss with a height of $nf+(m+1)h$ (Fig. 12) containing $2(n+m)$ panels, consists of $8(n+m)+1$ rods. The total length of the rods in the truss is

$$L_{sum} = 2ma + 4mc + 3dn + (2m + 2n + 1)h,$$

where $c = \sqrt{a^2 + h^2}$, $d = \sqrt{4a^2 + h^2}$, $f = h/2$.

The deflection (vertical displacement of the middle node C in the lower zone) has the form

$$\Delta = P(C_1a^3 + C_2c^3 + C_3d^3 + C_4h^3)/(16h^2EF). \tag{1.7}$$

Support A offset has the form

$$\delta_A = P(A_1a^3 + A_2c^3 + A_3d^3 + A_4h^3)/(8ahEF). \tag{1.8}$$

1.5.1. Concentrated force in the mid-span

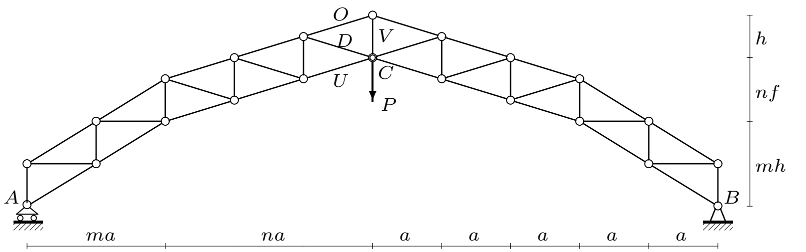

Fig. 12. Truss, $n = 3$, $m = 2$

Deflection. The coefficients in (1.7):

$C_1 = 8m$, $C_2 = 8m(1 + 2m^2)/3$,

$C_3 = 2(n^3 + 3n^2m + (2 + 3m^2)n)/3$,

$C_4 = 2(2n^2 + 4(m + 1)n + 3m^2)$.

Support offset. The coefficients in (1.8):

$A_1 = 8m$, $A_2 = 4m(4m^2 + 3m - 1)/3$,

$A_3 = (2n^3 + 3(3m + 1)n^2 + (12m^2 + 6m + 1)n)/6,$

$A_4 = 2(n^2 + (3m + 2)n + m + 3m^2).$

Forces:

$O = -Pd(n + m)/(4h),\ D = Pd/(4h),$

$U = Pd(n + m - 1)/(4h),\ V = P(n + m)/2.$

Supports reactions: $Y_A = Y_B = P/2,\ X_B = 0.$

1.5.2. Load on the upper belt

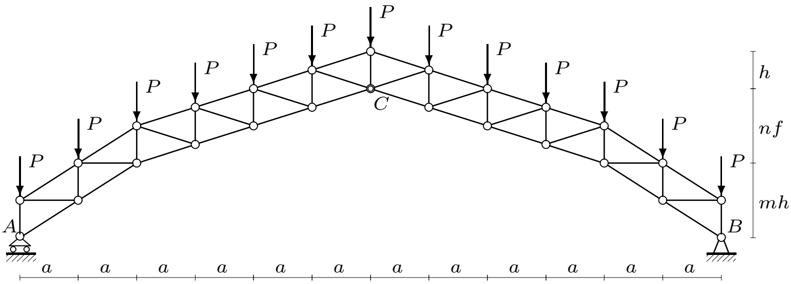

Fig. 13. Truss, $n = 4$, $m = 2$

Deflection. The coefficients in (1.7):

$C_1 = 8m(2n + m),\ C_2 = 4m(4(1 + 2m^2)n + m(1 + 5m^2))/3,$

$C_3 = (5n^4 + 20n^3m + (7 + 30m^2)n^2 + 2m(6m^2 - 1)n)/6,$

$C_4 = 2(2n^3 + 2(2 + 3m)n^2 + 4(2m^2 + 1)n + 3m^3 + 2m^2 + 2m).$

Support offset. The coefficients in (1.8):

$A_1 = 8m(2n + m),$

$A_2 = 4m(m + 1)(2(4m - 1)n + 5m^2 - m - 1)/3,$

$$A_3 = (5n^4 + 4(2 + 7m)n^3 + (1 + 24m + 54m^2)n^2 + \\ +2(2m + 1)(6m^2 - 1)n)/12,$$

$A_4 = 2(n^3 + (4m + 2)n^2 + (7m^2 + 2m + 2)n + m(3m^2 + 3m + 2)).$

Forces:

$O = -Pd(n+m)^2/(4h)$, $D = Pd/(4h)$,

$U = Pd((n+m)^2 - 1)/(4h)$, $V = P((n+m)^2 - 2)/2$.

Supports reactions: $Y_A = Y_B = (2n+2m+1)P/2$, $X_B = 0$.

Truss 1.6

A truss with a height of $(m+1)h$ (Fig. 14) containing $2(n+m+2)$ panels, consists of $8(m+n)+7$ rods. The total length of the rods in the truss is

$$L_{sum} = 4(n+1)a + 2(2m+1)c + 2(m+n+1)d + 2(m+n)h,$$

where $c = \sqrt{a^2+h^2}$, $d = \sqrt{4a^2+h^2}$.

The deflection (vertical displacement of the middle node C in the upper zone) has the form

$$\Delta = P(C_1a^3 + C_2c^3 + C_3d^3 + C_4h^3)/(2h^2EF). \qquad (1.9)$$

Support A offset has the form

$$\delta_A = P(A_1a^3 + A_2c^3 + A_3d^3 + A_4h^3)/(ahEF). \qquad (1.10)$$

1.6.1. Concentrated force in the mid-span

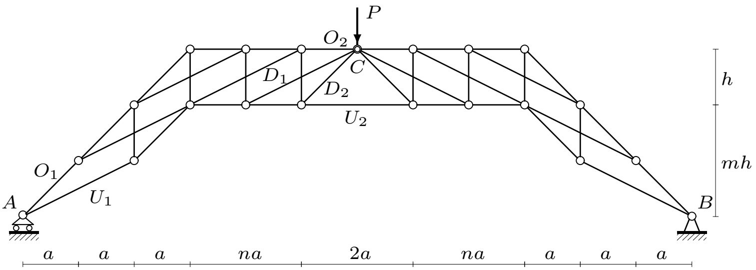

Fig. 14. Truss, $n = 2$, $m = 2$

Deflection. The coefficients in (1.9):

$$C_1 = (4n^3 + 12(m+2)n^2 + 4(3(-1)^m + 17 + 12m + 3m^2)n + \\ +12(-1)^m - 3(-1)^{n+m} + 12m^2 + 42m + \\ +45 + 6(-1)^m m - 6(-1)^n)/6,$$

$$C_2 = (12m^2 + 26m + 27 + 4m^3 + 12(-1)^m + 3(-1)^{n+m} +$$
$$+6(-1)^m m + 6(-1)^n)/6,$$
$$C_3 = (4(2(-1)^m + 3)n + 4m + 10 - 2(-1)^{n+m} -$$
$$-4(-1)^n + 4(-1)^m)/8,$$
$$C_4 = (2(2(-1)^m + 3)n + 4(-1)^m m + 6(-1)^m +$$
$$+4m^2 + 14m + 7 + (-1)^{n+m} + 2(-1)^n)/4,$$

Support offset. The coefficients in (1.10):

$$A_1 = (2(2m+1)n^2 + 2(4m^2 + 10m + 3(-1)^m + 9)n + 20m + 15 +$$
$$+6(-1)^m + 8m^2 - 3(-1)^n + 4(-1)^m m - 2(-1)^{n+m})/4,$$
$$A_2 = (8m^3 + 18m^2 + 4(3(-1)^m + 7)m +$$
$$+3(-1)^n + 24 + 18(-1)^m + 3(-1)^{n+m})/12,$$
$$A_3 = (6(1 + (-1)^m)n + 4m + 7 - (-1)^n - (-1)^{n+m} + 3(-1)^m)/8,$$
$$A_4 = (8m^2 + 4(2(-1)^m + 5)m + 5 + 9(-1)^m +$$
$$+(-1)^{n+m} + 6(-1)^m n + (-1)^n + 6n)/8.$$

Forces:

$O_1 = -Pc/h$, $O_2 = -Pa(2n + (-1)^n(2 + (-1)^m) + 2m + 1)/(4h)$,

$D_1 = Pd((-1)^n(2 + (-1)^m) - 1)/(4h)$,

$D_2 = Pc((-1)^n(2 + (-1)^m) + 1)/(4h)$,

$U_1 = Pd/(2h)$, $U_2 = Pa(n + 2 + m)/(2h)$.

Supports reactions:

$Y_A = Y_B = P/2$, $X_B = 0$.

1.6.2. Load on the upper belt

Deflection. The coefficients in (1.9):

$$C_1 = (10n^4 + 40(2+m)n^3 + 4(15m^2 + 60m + 77 + 12(-1)^m)n^2 +$$
$$+2(12m^3 + 96m^2 + 8(29 + 3(-1)^m)m + 221 - 6(-1)^n -$$
$$-3(-1)^{n+m} + 60(-1)^m)n + 24m^3 + 6(23 + 3(-1)^m)m^2 +$$

$+6(45-(-1)^{n+m}+12(-1)^m)m+198-18(-1)^n-15(-1)^{n+m}+$
$+75(-1)^m)/12,$

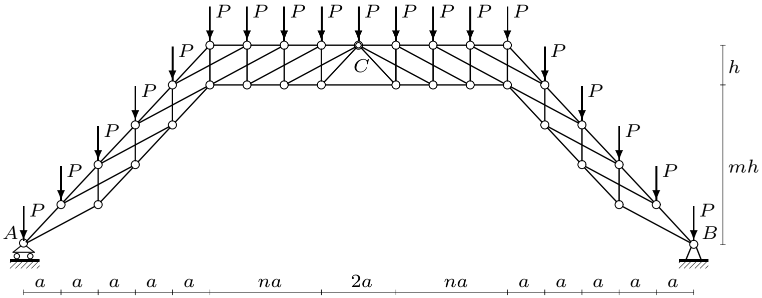

Fig. 15. Truss, $n = 3$, $m = 4$

$$C_2 = (10m^4 + 8(2n + 7)m^3 + 2(67 + 24n + 9(-1)^m)m^2+$$
$$+2(4(3(-1)^m + 13)n + 101 + 3(-1)^{n+m} + 36(-1)^m)m+$$
$$+6(17 + 2(-1)^n + (-1)^{m+n} + 8(-1)^m)n + 141 + 75(-1)^m+$$
$$+12(-1)^{n+m} + 12(-1)^n)/12,$$
$$C_3 = (2(11 + 8(-1)^m)n^2 + 2(2(2(-1)^m + 3)m + 23 - 2(-1)^n+$$
$$+16(-1)^m-(-1)^{n+m})n+6m^2+2(11-(-1)^{n+m}+2(-1)^m)m+$$
$$+10(-1)^m + 25 - 5(-1)^{n+m} - 6(-1)^n)/8,$$
$$C_4 = (8m^3 + 2(8n + 25 + 6(-1)^m)m^2 + 2(41 + 12(-1)^m n+$$
$$+22(-1)^m + (-1)^{n+m} + 30n)m + 2(11 + 8(-1)^m)n^2+$$
$$+2(25 + (-1)^{n+m} + 2(-1)^n + 24(-1)^m)n+$$
$$+21 + 2(-1)^n + 38(-1)^m + 3(-1)^{n+m})/8.$$

Case $m = n$:

$$C_1 = (134n^4 + 536n^3 + 2(455 + 57(-1)^n)n^2 + 20(35 + 9(-1)^n)n+$$
$$+183 + 57(-1)^n)/12,$$
$$C_2 = (26n^4 + 104n^3 + 14(17 + 3(-1)^n)n^2 + 4(79 + 33(-1)^n)n+$$
$$+153 + 87(-1)^n)/12,$$

$C_3 = (2(5+3(-1)^n)n^2 + 8(2+(-1)^n)n + (-1)^n + 5)/2,$

$C_4 = (6n^3+(33+13(-1)^n)n^2+(24(-1)^n+34)n+6+10(-1)^n)/2.$

Support offset. The coefficients in (1.10):

$$A_1 = (8(1+2m)n^3 + 12(4m^2+10m+8+3(-1)^m)n^2+$$
$$+(36(-1)^m m + 190 - 12(-1)^n + 272m - 6(-1)^{n+m}+$$
$$+84(-1)^m + 156m^2 + 24m^3)n + 99 + 174m + 18(-1)^m m^2+$$
$$+60(-1)^m m - 15(-1)^n + 114m^2 - 15(-1)^{n+m} + 24m^3+$$
$$+51(-1)^m - 6m(-1)^{n+m})/12,$$

$$A_2 = (4(12+9(-1)^m + 4m^3 + 6(-1)^m m + 14m + 9m^2)n + 69+$$
$$+108m + 18(-1)^m m^2 + 60(-1)^m m + 10m^4 + 48m^3 + 86m^2+$$
$$+51(-1)^m)/12,$$

$$A_3 = (12((-1)^m+1)n^2 + 2(6m + 2(-1)^m m + 11(-1)^m + 15)n+$$
$$+22m+20+2(-1)^m m-(-1)^{n+m}+6m^2+6(-1)^m-(-1)^n)/8,$$

$$A_4 = (12((-1)^m+1)n^2 + 2(8m^2 + 2(5(-1)^m + 11)m + 17(-1)^m+$$
$$+13)n + 8 + 50m + 8m^3 - (-1)^n + 38(-1)^m m-$$
$$-(-1)^{n+m} + 26(-1)^m + 6(2(-1)^m + 7)m^2)/8.$$

Case $m = n$:

$$A_1 = (44n^4 + 154n^3 + (45(-1)^n + 241)n^2+$$
$$+2(88+33(-1)^n)n + 18(-1)^n + 42)/6,$$

$$A_2 = (26n^4 + 84n^3 + 2(21(-1)^n + 71)n^2 + 12(13+8(-1)^n)n+$$
$$+69 + 51(-1)^n)/12,$$

$$A_3 = (2(15+8(-1)^n)n^2 + 4(6(-1)^n + 13)n + 19 + 5(-1)^n)/8,$$

$$A_4 = (24n^3 + (98+44(-1)^n)n^2 + 4(18(-1)^n + 19)n+$$
$$+7 + 25(-1)^n)/8.$$

Forces:

$O_1 = -Pc(2m+2n+3)/h,$

$$O_2 = -Pa(m^2 + 4m + 2 + 2nm + 4n + (-1)^{n+m}(m + n + 2)+$$
$$+2(-1)^n(1 + n) + n^2)/(2h),$$
$$D_1 = Pd((-1)^{n+m}(n + m + 2) + 2(-1)^n(1 + n) - 1)/(2h),$$
$$D_2 = -Pc((-1)^{n+m}(n + m + 2) + 2(-1)^n(1 + n))/(2h),$$
$$U_1 = Pd(2m + 2n + 3)/(2h),$$
$$U_2 = Pa(m^2 + 4m + 4 + 2nm + 4n + n^2)/(2h).$$

Supports reactions:

$$Y_A = Y_B = P(2n + 2m + 5)/2,\ X_B = 0.$$

Truss 1.7

A truss with a height of $nf + h$ (Fig. 16) containing $2n$ panels, consists of $8n + 1$ rods. The total length of the rods in the truss is

$$L_{sum} = 2(2c + h + d)n + h,$$

where $c = \sqrt{a^2 + f^2}$, $d = \sqrt{a^2 + (h - f)^2}$.

1.7.1. Concentrated force in the mid-span

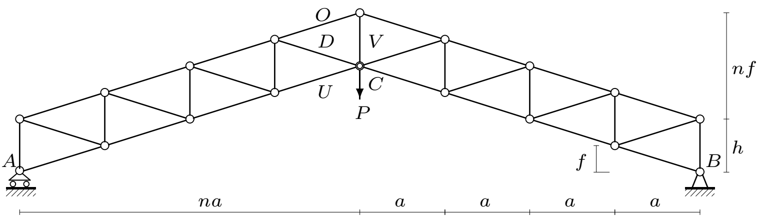

Fig. 16. Truss, $n = 4$

The deflection (vertical displacement of the middle node C in the lower zone) has the form

$$\Delta = Pn(2c^3n^2 + 6f^2hn + c^3 + 3d^3 + 3h^3)/(6h^2EF).$$

Support A offset has the form

$$\delta_A = Pn(4c^3fn^2 + 3h(4f^3 + c^3)n + 2c^3f - 3c^3h+$$

$+6d^3f + 6fh^3)/(6h^2aEF).$

Forces:

$O = -Pcn/(2h)$, $D = Pd/(2h)$,

$U = Pc(n-1)/(2h)$, $V = Pfn/h$.

Supports reactions:

$Y_A = Y_B = P/2$, $X_B = 0$.

1.7.2. Load on the upper belt

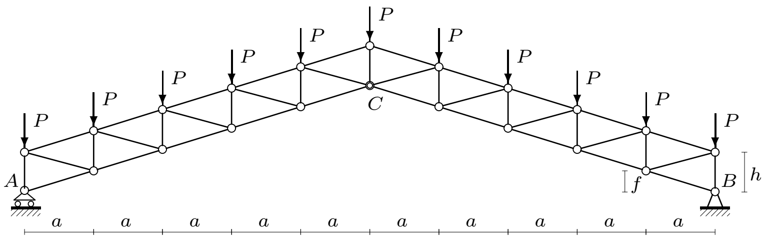

Fig. 17. Truss, $n = 5$

The deflection (vertical displacement of the middle node C in the lower zone) has the form

$$\Delta = Pn(5c^3n^3 + 12f^2hn^2 + (c^3 + 6h^3 + 6d^3)n + 12h^2(h-f))/(12h^2EF).$$

Support A offset has the form

$$\delta_A = Pn(5c^3fn^3 + 4h(c^3 + 3f^3)n^2 + (6d^3f + c^3f + 6fh^3 - 3c^3h)n -$$
$$-h(c^3 - 12hf(h-f)))/(6ah^2EF).$$

Forces:

$O = -Pcn^2/(2h)$, $D = Pd/(2h)$,

$U = Pc(n^2-1)/(2h)$, $V = P(fn^2/h - 1)$.

Supports reactions:

$Y_A = Y_B = P(2n+1)/2$,

$X_B = 0$.

Truss 1.8

A truss with a height of $(n+1)h$ (Fig. 18) containing $2n+2$ panels, consists of $8n+5$ rods. The total length of the rods in the truss is

$$L_{sum} = 2(n+1)a + 2(2n+1)c + (2n+1)h,$$

where $c = \sqrt{a^2+h^2}$.

The deflection (vertical displacement of the middle node C in the lower zone) has the form

$$\Delta = P(C_1a^3 + C_2c^3 + C_3h^3)/(h^2EF). \quad (1.11)$$

Support A offset:

$$\delta_A = P(A_1a^3 + A_2c^3 + A_3h^3)/(ahEF). \quad (1.12)$$

1.8.1. Concentrated force in the mid-span

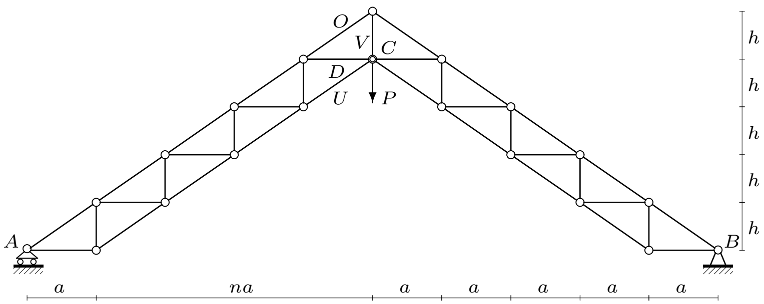

Fig. 18. Truss, $n=4$

Deflection. The coefficients in (1.11):

$C_1 = (n+1)/2,$

$C_2 = (2n^3+6n^2+7n+3)/6,$

$C_3 = (2n^2+5n+2)/2.$

Support offset. The coefficients in (1.12):

$A_1 = n+1,\ A_2 = n(4n^2+9n+5)/6,\ A_3 = n(2n+3).$

Forces:

$O = -Pc(n+1)/(2h)$, $D = Pa/(2h)$,

$U = Pcn/(2h)$, $V = P(n+1)$.

Supports reactions:

$Y_A = Y_B = P/2$, $X_B = 0$.

1.8.2. Load on the upper belt

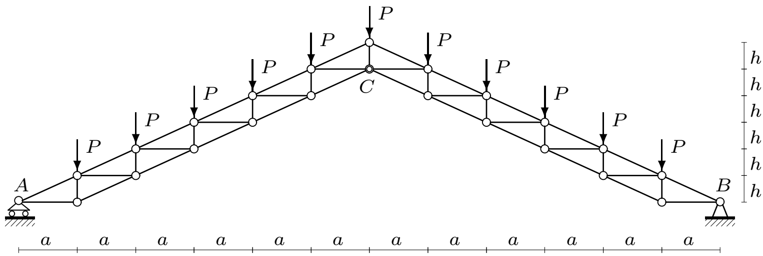

Fig. 19. Truss, $n = 5$

Deflection. The coefficients in (1.11):

$C_1 = (n+1)^2/2$,

$C_2 = (5n^4 + 20n^3 + 31n^2 + 22n + 6)/12$,

$C_3 = n(2n^2 + 7n + 6)/2$.

Support offset. The coefficients in (1.12):

$A_1 = (n+1)^2$, $A_2 = n(5n^3 + 16n^2 + 16n + 5)/6$,

$A_3 = n(2n^2 + 5n + 2)$.

Forces:

$O = -Pc(n+1)^2/(2h)$, $D = Pa/(2h)$,

$U = Pcn(n+2)/(2h)$, $V = Pn(n+2)$.

Supports reactions:

$Y_A = Y_B = P(2n+1)/2$, $X_B = 0$.

Truss 1.9

A truss with a height of $(n+1)h$ (Fig. 20) containing $2n$ panels, consists of $8n+1$ rods. The total length of the rods in the truss is

$$L_{sum} = 2(a+2c+h)n+h,$$

where $c = \sqrt{a^2+h^2}$.

The deflection (vertical displacement of the middle node C in the upper zone) has the form

$$\Delta = P(C_1a^3 + C_2c^3 + C_3h^3)/(h^2EF). \quad (1.13)$$

Support A offset has the form

$$\delta_A = P(A_1a^3 + A_2c^3 + A_3h^3)/(ahEF). \quad (1.14)$$

1.9.1. Concentrated force in the mid-span

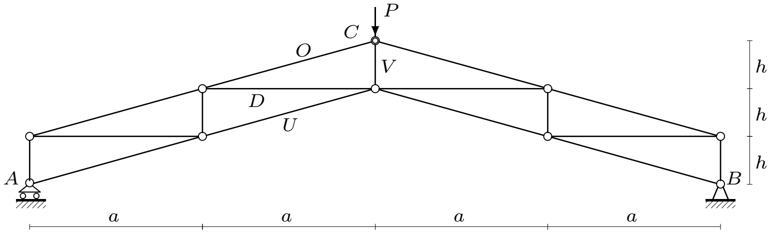

Fig. 20. Truss, $n = 2$

Deflection. The coefficients in (1.13):

$C_1 = n/2$, $C_2 = n(2n^2+1)/6$, $C_3 = (2n^2-3n+2)/2$.

Support offset. The coefficients in (1.14):

$A_1 = n$, $A_2 = n(4n^2+3n-1)/6$, $A_3 = n(2n-1)$.

Forces:

$O = -Pcn/(2h)$, $D = Pa/(2h)$,

$U = Pc(n-1)/(2h)$, $V = P(n-1)$.

Supports reactions: $Y_A = Y_B = P/2$, $X_B = 0$.

1.9.2. Load on the upper belt

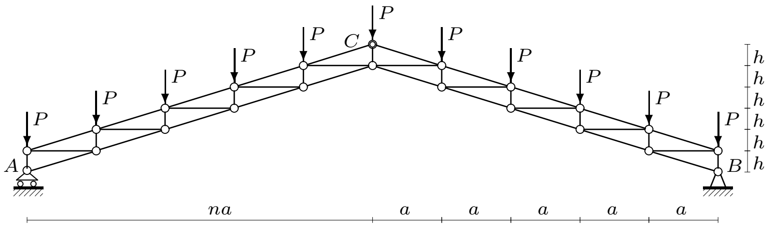

Fig. 21. Truss, $n = 5$

Deflection. The coefficients in (1.13):

$C_1 = n^2/2$, $C_2 = n^2(1 + 5n^2)/12$, $C_3 = (2n^3 - n^2 + 2)/2$.

Support offset. The coefficients in (1.14):

$A_1 = n^2$, $A_2 = n(5n^3 + 4n^2 - 2n - 1)/6$, $A_3 = n^2(2n + 1)$.

Forces:

$O = -Pcn^2/(2h)$, $D = Pa/(2h)$,

$U = Pc(n^2 - 1)/(2h)$, $V = P(n^2 - 1)$.

Supports reactions: $Y_A = Y_B = P(2n + 1)/2$, $X_B = 0$.

Truss 1.10

A truss with a height of $(n + m)h$ (Fig. 22) containing $2n + 2m$ panels, consists of $8n + 8m + 5$ rods. The total length of the rods in the truss is

$$L_{sum} = (2m + 2n + 3)a + 3mc + 4gn + (1 + 2n)h,$$

where $c = \sqrt{a^2 + 4h^2}$, $g = \sqrt{a^2 + h^2}$.

The case $d = a$, $b = a/2$ is considered.

The deflection (vertical displacement of the middle node C in the upper zone) has the form

$$\Delta = P(C_1a^3 + C_2c^3 + C_3g^3 + C_4h^3)/(h^2EF). \qquad (1.15)$$