Advances in Applied Nonlinear Optimal Control

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By

Gerasimos Rigatos and Electra Karapanou

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In memory of my mother Diamantina Rigatou
1939-2018

Contents

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Preface

The monograph presents advances in applied nonlinear optimal control, comprising both theoretical analysis of the developed control methods and case studies about their use in robotics, mechatronics, electric power generation, power electronics, micro-electronics, biological systems, biomedical systems, financial systems and industrial production processes. The monograph is developed around new theoretical results allowing for solution of the nonlinear optimal control problem through approximate linearization of the controlled system's dynamics and through application of H-infinity control methods. Because of the nonlinearity of the state-space model of the considered dynamical systems under control, other approaches to solve the associated optimal control problem are of questionable performance. Therefore, the monograph's results go beyond other control approaches, such as the typical model predictive control (MPC) and the nonlinear model predictive control (NMPC). For instance, it is widely acknowledged that MPC is a linear control method, which in the case of the nonlinear dynamics of the considered complex dynamical systems cannot assure the stability of the control loop. Besides, it is known that the NMPC's iterative search for an optimum is dependent on initial parametrization and is not always of assured convergence. On the other side, the use of global linearization-based methods for the control of the considered complex dynamical systems requires the definition of the linearizing outputs in a case-based manner, and the application of complicated change of state-space variables. Moreover, such methods may come against singularity problems due to including also additional transformations being-based on matrices inversions. Alternatively, the application of backstepping control to the considered complex systems requires to express previously their state-space description into the triangular form, and this is not always a straightforward procedure. Finally, sliding-mode control cannot be directly applied to complicated multivariable dynamical systems because these are not found in a canonical linear form and consequently there is no systematic manner in defining the sliding surface. For the reasons explained above, the monograph's findings on nonlinear optimal control, can be a substantial contribution to the areas of nonlinear control and complex dynamical systems and can find use in several research disciplines and practical applications. The monograph is concerned with applied nonlinear optimal control, and one of its primary objectives is to demonstrate potential applications for its theoretical developments. Prospective application areas are outlined as follows:

1) industrial robotics: robotic manipulators and networked robotic systems. Applications to fully actuated robotic manipulators, redundant manipulators, underactuated manipulators, cranes and load handling systems, time-delayed robotic systems. closed kinematic chain manipulators, flexible-link manipulators, micromanipulators.

2) transportation systems: autonomous vehicles and mobile robots. Applications to two-wheel and unicycle type vehicles, four-wheel drive vehicles, four-wheel steering vehicles, articulated vehicles, truck and trailer systems, unmanned aerial vehicles, unmanned surface vessels, autonomous underwater vessels, underactuated vessels.

3) motion generation and transmission systems: actuators and motors. Applications to mechatronic systems and actuators, switched reluctance motors, permanent magnet synchronous motors, permanent magnet linear motors, synchronous reluctance motors, induction motors, induction linear motors doubly-fed reluctance machines and multi-phase machines.

4) electric power systems: power generators and power electronics. Applications to photovoltaic units, fuel cells units, synchronous generators, permanent magnet synchronous generators, doubly-fed induction generators, doubly-

fed reluctance generators, gas-turbine electric power units, hybrid-excited synchronous generators, steam-turbine electric power units, wind-turbine electric power units, hydropower generators, multi-phase generators, distributed electric power transmission and distribution systems. Applications to power electronics, such as DC-DC converters, DC-AC inverters, AC-DC converters, DC-AC inverters and active power filters, power transformers, batteries and capacitors, VSC-HVDC transmission systems, components of the smart grid.

5) biosystems: bioprocesses in the pharmaceutical industry, controlled drugs infusion. Applications to the pharmaceutical industry, processes of controlled protein and hormone synthesis, systems of haemodialysis, controlled anaesthesia, controlled drug infusion for diabetes, regulation of heart's functioning, control of biological oscillators, and to several other types of biosystems.

6) financial systems: risk prevention and assets management. Applications to optimal control of macroeconomic systems, optimal control of models of markets dynamics and business cycles, management for the elimination of loans and investments risk, decision making for the mitigation of companies' default risk, optimized planning of transactions and investments in the commodities market, optimal management of capitals and assets.

The prospective audience of the monograph comes from both the academic field and from engineers working on practical optimal control and optimization problems. There is need for generic and systematic methods of nonlinear optimal control, in robotics, mechatronics, electric power generation, power electronics, micro-electronics, biological systems, biomedical systems, financial systems and industrial production processes The monograph's methods are of proven stability and convergence and exhibit also robustness to model uncertainty and external perturbations. The stages of the developed nonlinear optimal control methods are clear and easy to follow and implement. Taking into account the above, it is expected that the monograph will have a good acceptance by a wide audience, in both the academic and the engineering communities. It is anticipated that the interest of the academic, research and engineering community in the topics presented by this monograph will grow in the forthcoming years. This is because optimization in functioning of nonlinear dynamical systems is becoming a prerequisite for a wide spectrum of applications including engineering systems, biomedical systems and financial systems. Besides, as the monograph's methods for nonlinear optimal control are characterized by global stability and robustness features they are not going to get outdated or scientifically depreciated. Furthermore, starting from the applications examples presented in the monograph one can find more areas for using the provided results on nonlinear optimal control. Consequently, the monograph's findings are expected to be well disseminated among researchers and engineers and the book is anticipated to keep on being of interest in the following years, for both research institutes or universities and for engineers.

The monograph presents advances in applied nonlinear optimal control, comprising both theoretical analysis of the developed control methods and case studies about their use in robotics, mechatronics, electric power generation, power electronics, micro-electronics, biological systems, biomedical systems, financial systems and industrial production processes. The advantages of the nonlinear optimal control approaches which are developed in the monograph are outlined as follows: (i) by applying approximate linearization of the controlled systems' state-space description, one can avoid the elaborated state variables transformations (diffeomorphisms) which are required by global linearization-based control methods, (ii) the control input is applied directly on the controlled systems and not on an equivalent linearized description of theirs. Thus one can avoid the inverse transformations met in global linearization-based control methods and the appearance of singularity problems, (iii) the monograph's control methods retain the advantages of linear optimal control, that is best trade-off between accurate tracking of reference setpoints and moderate variations of the control inputs. The monograph's findings on nonlinear optimal control, can be a substantial contribution to the areas of nonlinear control and complex dynamical systems and can find use in several research disciplines and practical applications.

In particular, with respect to approaches attempting to treat optimal control problems for complex dynamical systems it can be pointed out that the present monograph is developed around new theoretical results allowing for solution of the nonlinear optimal control problem through approximate linearization of the controlled system's dynamics and through application of H-infinity control methods. Because of the nonlinearity of the state-space model of the considered dynamical systems under control, other approaches to solve the associated optimal control problems are of questionable performance. Therefore, the monograph's results go beyond other control approaches such as Model Predictive Control (MPC) and Nonlinear Model Predictive Control, (NMPC). For instance, it is widely acknowledged that MPC is a linear control method which in the case of the nonlinear dynamics of the considered complex dynamical systems cannot assure the stability of the control loop. Besides, it is known that the NMPC's iterative search for an optimum is dependent on initial parametrization and is not always of assured convergence.

Primarily, the book is addressed to the research and academic community. The monograph can be a reference for researchers working on nonlinear control problems. Moreover, the content of the book can be used for teaching undergraduate or postgraduate courses in nonlinear control. Therefore, it can be considered by both academic tutors and students as a reference book for such courses. A significant part of the book's readership is also expected to come from the engineering community. Engineers working in the design and development of robotic and mechatronic systems, electric power systems, biomedical systems or cyberphysical systems may come against nonlinear control problems which can be solved using the guidelines of the monograph. The monograph is anticipated to attract the interest of a significant part of the academic and engineering community. The timeliness of the monograph's topics is not expected to decline in the following years because the developed control methods are of proven stability and robustness while potential applications cover a wide spectrum that ranges from engineering systems to biomedical and financial systems. Since the monograph's methods and approaches offer complete and reliable solutions to nontrivial problems of robotics, mechatronics, electric power generation, power electronics, micro-electronics, biological systems, biomedical systems, financial systems and industrial production processes, their scientific value is difficult to be depreciated in the course of time. On the other hand, since the monograph's methods have excellent performance and contribute to the reliable functioning of several types of nonlinear dynamical systems, it is expected this book to become a useful reference for the academic and engineering community.

Dr. Gerasimos Rigatos
Athens. Greece
October 2020

Glossary

AGV: Automatic Ground Vehicle
ARE: Algebraic Riccati Equation
AUV: Autonomous Underwater Vessel
CT; Computed Torgue Method
DARE: Differential Algebraic Riccati Equation
DOF: Degrees of Freedom
DFIG: Doubly Fed Induction Generator
DFRM: Doubly-fed Reluctance Machine
EGR: Exhaust Gas Recirculation
FC: Fuel Cells
GT: Gas Turbine
H_∞ control: H-infinity Control
H_∞ Kalman Filter: H-infinity Kalman Filter
HESG: Hybrid Excited Synchronous Generator
HT: Hydro Turbine
HJB: Hamilton-Jacobi-Bellman equation
HVDC: High Voltage Direct Current line
IM: Induction Motor
LIM: Linear Induction Motor
LQR: Linear Quadratic Requlator
LQG: Linear Quadratic Gaussian
MAGLEV: Magnetic Levitation Train
MPC: Model Predictive Control
NMPC: Nonlinear Model Predictive Control
PID: Propotrional Integral Derivative
PMSG: Permanent Magnet Synchronous Generator
PMLSM: Permanent Magnet Linear Synchronous Motors
PMSM: Permanent Magnet Synchronous Motor
PWM: Pulse Width Modulation
RMSE: Root Mean Square Error
SG: Synchronous Generator
ST: Steam Turbine
SwRM: Switched Reluctance Machine
SRM: Synchronous Reluctance Machine
STATCOM: Static Synchronous Compensator
UAS: Unmanned Aerial Systems
UGV: Unmanned Ground Vehicle
USV: Unmanned Surface Vessel
UAV: Unmanned Aerial Vehicle
VSI: Voltage Source Inverter

VSC: Voltage Source Converter
WT: Wind Turbine
4WD: Four-wheel drive vehicle
4WS: Four-wheel steering vehicle

Chapter 1

Nonlinear optimal control for dynamical systems

1.1 Overview of the optimal control concept

1.1.1 Optimal control for continuous-time systems with Bellman's principle

Optimal control and its application exhibit a rapid development in the last years [1147], [677], [679], [133]. However, nonlinear optimal control problems remain difficult to handle and a conclusive solution about them has not been achieved so far [402], [63], [103], [972]. In the area of optimal control, Bellman's optimality principle stands for one of the most efficient approaches [267]. The following continuous-time dynamical system is considered

$$\dot{x} = f(x,u,t), \quad x(t_0) = x_0 \tag{1.1}$$

The objective is to select the control input $u(t), \quad t{\in}[t_0, t_f]$ so as to achieve minimization of the following cost function

$$J(u) = \int_{t_0}^{t_f} L(x,u,t)dt \tag{1.2}$$

To this end, one defines

$$J^0(x,t) = \min_{u\in[t,t_f]} \int_t^{t_f} L(x,u,t)dt \tag{1.3}$$

where $J^0(x,t_0) = J^0(x_0)$. According to the principle of optimality, optimization can take place in two stages: (i) from t to $t+\Delta t$, (ii) from $t+\Delta t$ to t_f. Consequently, the previous cost function is decomposed as follows:

$$\begin{aligned} J^0(x,t) &= \min_{u\in[t,t_f]} \{\int_t^{t+\Delta t} L(x,u,t)dt + \int_{t+\Delta t}^{t_f} L(x,u,t)dt\} \\ &= \min_{u\in[t,t_f]} \{\int_t^{t+\Delta t} L(x,u,t)dt + J^0(x+\Delta x, t+\Delta t)\} \end{aligned} \tag{1.4}$$

After taking the Taylor series expansion of $J^O(x+\Delta x, t+\Delta t)$ one obtains

$$J^0(x+\Delta x, t+\Delta t) = J^0(x,t) + \tfrac{\partial J^0}{\partial x}\Delta x + \tfrac{\partial J^0}{\partial t}\Delta t + O(\Delta) \tag{1.5}$$

where $O(\Delta)$ stands for higher-order terms to be omitted from the Taylor series expansion. By substituting Eq. (1.5) in Eq. (1.4) one obtains

$$0 = \min_{u \in [t,t_f]} \{L\Delta t + \frac{\partial J^0}{\partial x}\Delta x + \frac{\partial J^0}{\partial t}\Delta t + O(\Delta)\} \quad (1.6)$$

and after dividing by Δt and taking the limit $\Delta t \to 0$ one arrives at the relation

$$-\frac{\partial J^0}{\partial t} = \min_{u \in [t,t_f]} \text{H} \quad (1.7)$$

$$H = L(x,u,t) + \frac{\partial J^0}{\partial x}\frac{dx}{dt} = L(x,u,t) + \frac{\partial J^0}{\partial x} f(x,u,t) \quad (1.8)$$

In the above relations, Eq. (1.7) is known as the Hamilton-Jacobi-Bellman equation , whereas Eq. (1.8) is the Hamiltionian function of the optimal control problem. By solving the Hamilton-Jacobi-Bellman equation one obtains both the control input $u(t)$ $t \in [t,t_f]$ that minimizes the objective function $J^0(x,t)$, as well as the minimized value of $J^0(x,t)$.

The solution of the Hamilton-Jacobi-Bellman equation is a non-trivial problem and in the case of nonlinear dynamical systems it may become a complicated or even non-tractable problem. If the system is a linear one, and the cost function comprises quadratic terms, then the solution of the Hamilton-Jacobi-Bellman equation is substituted by the solution of a Riccati differential equation.

1.1.2 Solution of the optimal control problem in closed form

The solution of the optimal control problem in closed form can be achieved in the case of linear continuous-time systems with a quadratic cost function . For a linear system of the form:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \quad (1.9)$$

and for a cost function that comprises quadratic terms in the form [267]:

$$J = \int_{t_0}^{t_f} L dt \quad (1.10)$$

with

$$L = \frac{1}{2}x^T Q(t)x + \frac{1}{2}u^T Ru \quad (1.11)$$

then the optimal value of the cost function is

$$J^0(x,t) = \frac{1}{2}x^T(t)P(t)x(t) \quad (1.12)$$

with $P(t)$ being a symmetric positive definite matrix . For such a system the Hamiltonian function is [267]

$$H = \frac{1}{2}x^T Qx + \frac{1}{2}u^T Ru + \frac{1}{2}x^T P(Ax + Bu) + \frac{1}{2}(Ax + Bu)^T Px \quad (1.13)$$

By deriving the Hamiltonian function with respect to the control input u and by taking its extremum condition one gets

$$\frac{\partial H}{\partial u} = Ru + B^T Px = 0 \quad (1.14)$$

By solving the above equation with respect to $u(t)$ one finds the optimal control input that minimizes the quadratic cost function of the system. This is given by

$$u^0(t) = -R^1(t)B^T(t)P(t)x(t) \quad (1.15)$$

By substituting the optimal control input to the Hamiltonian of Eq. (1.13) one obtains

$$\begin{aligned} H^0 = \min_{u\in[t,t_f]} \ & H = \tfrac{1}{2}x^TQx + \tfrac{1}{2}x^TPBR^{-1}B^TPx + \tfrac{1}{2}x^TA^TPx \\ & +\tfrac{1}{2}X^TPAx - x^TPBR^{-1}B^TPx \\ & = \tfrac{1}{2}x^T(Q + A^TP + PA - PBR^{-1}B^TP)x \end{aligned} \tag{1.16}$$

Next, by differentiating the cost function of Eq. (1.12) one obtains

$$\frac{\partial J^0(x,t)}{\partial t} = \frac{1}{2}x^T(t)\frac{dP(t)}{dt}x(t) \tag{1.17}$$

Next, by equating Eq. (1.16) to Eq. (1.17) one has the following formulation of the Hamilton-Jacobi-Bellman equation

$$\begin{aligned} & -\tfrac{1}{2}x^T(t)\tfrac{dP(t)}{dt}x(t) = \\ \min_{u\in[t,t_f]} \ & H = \tfrac{1}{2}x^TQx + \tfrac{1}{2}x^TPBR^{-1}B^TPx + \tfrac{1}{2}x^TA^TPx \\ & +\tfrac{1}{2}X^TPAx - x^TPBR^{-1}B^TPx \\ & = \tfrac{1}{2}x^T(Q + A^TP + PA - PBR^{-1}B^TP)x \end{aligned} \tag{1.18}$$

Through intermediate computations one arrives at the following differential Riccati equation [267]

$$-\frac{dP(t)}{dt} = A^TP + PA + Q - PBR^{-1}B^TP \tag{1.19}$$

The boundary condition for the solution of this differential equation comes from the relation

$$J^0(x,t_f) = \frac{1}{2}x(t_f)P(t_f)x(t_f) = 0 \tag{1.20}$$

having also $P(t_f) = 0$. By integrating Eq. (1.19) in inverse time from t_f to t_0 one can find matrix $P(t) : \ t\in[t_0, t_f]$. Besides, using Eq. (1.15) one can find the optimal control input to be applied to the system. Moreover, using Eq. (1.12) one can find the minimum value of the system's cost function $J^0(x,t)$.

The above procedure for finding a solution to the optimal control problem of a linear dynamical system arrives at the design of a Linear Quadratic Regulator. If matrices A and B which appear in the state-space model of the system of Eq. (1.9) are time invariant and the weight matrices Q and R which appear in the quadratic cost function of Eq. (1.10) and Eq. (1.11) are time-invariant too, and the finite-time $t_f = \infty$ (infinite time horizon) then the solution of the differential Riccati equation $P(t)$ arrives at a steady-state value, that is $\frac{dP(t)}{dt} = 0$ for $t\to\infty$. In such a case, the differential Riccati equation of Eq. (1.19) becomes an algebraic Riccati equation of the form:

$$A^TP + PA + Q - PBR^{-1}B^TP = 0 \tag{1.21}$$

In such a case the optimal feedback control becomes

$$u = R^{-1}B^TP \tag{1.22}$$

where P now is the solution of the algebraic Riccati equation of Eq. (1.21). Thus, the optimized cost function becomes $J^0 = \frac{1}{2}x^T(0)Px(0)$.

1.1.3 Stability conditions for optimal control

The stability of a linear optimal control loop is related with the stabilizability properties of such a system . The linear system $\dot{x}(t) = Ax(t) + Bu(t) \ \ x\in R^n \ \ u\in R^m$ is stabilizable if the control input $u(t)$ can be chosen such that all closed-loop poles are stable. The stabilizability of a system is a more generic property than controllability . It actually means that some of the poles of the system can be placed at the left complex semi-plane through state feedback, while the rest of the system poles which cannot be relocated with feedback are inherently found in the left complex semi-plane. The conditions that assure global stability for the closed-loop of an optimal linear quadratic control scheme are given in the following theorem:

Theorem: Assume C to be a $n{\times}n$ square root of matrix Q where Q is the weight matrix appearing in the quadratic cost function of Eq. (1.10) and Eq. (1.11). This signifies that $C^TC = Q$. Assume next, that the matrices pair (A, B) is stabilizable , and the matrices pair (A, C) is observable . Then the following hold:

(i) there exists a unique symmetric positive-definite steady-state solution P of the differential Riccati equation given in Eq. (1.19, which is not dependent on the final condition $P(t_f)$).(ii) the aforementioned matrix P is also the unique solution of the algebraic Riccati equation given in Eq. (1.21) (iii) the closed-loop system is globally asymptotically stable

According to this theorem, the stability properties of an optimal linear quadratic control loop depend on the structure of the open loop system, as defined by matrices A and B, and on the structure of the weight matrix Q which appear as design parameters in the quadratic cost function. The stabilizability of the pair (A, B) assures that through state-feedback all poles of the closed-loop system can be finally found in the left complex semi-plane. The observability of the pair $(A, \sqrt{Q})$ signifies that all state variables of the system appear finally in the cost function J which is subject to minimization. The poles of the closed-loop system, and consequently the transient characteristics of the optimal control loop are determined by the selection of matrices $Q{\geq}0$ and $R > 0$.

1.2 Design of an H-infinity nonlinear optimal controller

1.2.1 Equivalent linearized dynamics of the system

The proposed nonlinear optimal control method is addressed to nonlinear dynamical systems in the generic form:

$$\dot{x} = f(x) + g(x)u \tag{1.23}$$

where $x{\in}R^n$, $f(x){\in}R^n$, $g(x){\in}R^n$ and $u{\in}R^m$. After linearization around its current operating point, the system's dynamic model is also written as

$$\dot{x} = Ax + Bu + d_1 \tag{1.24}$$

Parameter d_1 stands for the linearization error in the system's dynamic model appearing in Eq. (1.24). The reference setpoints for the system's state vector are denoted by $\mathbf{x_d} = [x_1^d, \cdots, x_n^d]$. Tracking of this trajectory is achieved after applying the control input u^*. At every time instant the control input u^* is assumed to differ from the control input u appearing in Eq. (1.24) by an amount equal to Δu, that is $u^* = u + \Delta u$

$$\dot{x}_d = Ax_d + Bu^* + d_2 \tag{1.25}$$

The dynamics of the controlled system described in Eq. (1.24) can be also written as

$$\dot{x} = Ax + Bu + Bu^* - Bu^* + d_1 \tag{1.26}$$

and by denoting $d_3 = -Bu^* + d_1$ as an aggregate disturbance term one obtains

$$\dot{x} = Ax + Bu + Bu^* + d_3 \tag{1.27}$$

By subtracting Eq. (1.25) from Eq. (1.27) one has

$$\dot{x} - \dot{x}_d = A(x - x_d) + Bu + d_3 - d_2 \tag{1.28}$$

By denoting the tracking error as $e = x - x_d$ and the aggregate disturbance term as $\tilde{d} = d_3 - d_2$, the tracking error dynamics becomes

$$\dot{e} = Ae + Bu + \tilde{d} \tag{1.29}$$

The above linearized form of the system's model can be efficiently controlled after applying an H-infinity feedback control scheme .

1.2.2 The nonlinear H-infinity control

The initial nonlinear model of the system is in the form

$$\dot{x} = \tilde{f}(x,u) \quad x{\in}R^n, \ u{\in}R^m \tag{1.30}$$

Linearization of the system is performed at each iteration of the control algorithm around its present operating point $(x^*, u^*) = (x(t), u(t-T_s))$, where T_s is the sampling period. The linearized equivalent model of the system is described by

$$\dot{x} = Ax + Bu + L\tilde{d} \quad x{\in}R^n, \ u{\in}R^m, \ \tilde{d}{\in}R^q \tag{1.31}$$

where matrices A and B are obtained from the computation of the Jacobians

$$A = \begin{pmatrix} \frac{\partial \tilde{f}_1}{\partial x_1} & \frac{\partial \tilde{f}_1}{\partial x_2} & \cdots & \frac{\partial \tilde{f}_1}{\partial x_n} \\ \frac{\partial \tilde{f}_2}{\partial x_1} & \frac{\partial \tilde{f}_2}{\partial x_2} & \cdots & \frac{\partial \tilde{f}_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial \tilde{f}_n}{\partial x_1} & \frac{\partial \tilde{f}_n}{\partial x_2} & \cdots & \frac{\partial \tilde{f}_n}{\partial x_n} \end{pmatrix} |_{(x^*,u^*)} \quad B = \begin{pmatrix} \frac{\partial \tilde{f}_1}{\partial u_1} & \frac{\partial \tilde{f}_1}{\partial u_2} & \cdots & \frac{\partial \tilde{f}_1}{\partial u_m} \\ \frac{\partial \tilde{f}_2}{\partial u_1} & \frac{\partial \tilde{f}_2}{\partial u_2} & \cdots & \frac{\partial \tilde{f}_2}{\partial u_m} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial \tilde{f}_n}{\partial u_1} & \frac{\partial \tilde{f}_n}{\partial u_2} & \cdots & \frac{\partial \tilde{f}_n}{\partial u_m} \end{pmatrix} |_{(x^*,u^*)} \tag{1.32}$$

and vector $\tilde{d}$ denotes disturbance terms due to linearization errors . The problem of disturbance rejection for the linearized model that is described by

$$\begin{matrix} \dot{x} = Ax + Bu + L\tilde{d} \\ y = Cx \end{matrix} \tag{1.33}$$

where $x{\in}R^n$, $u{\in}R^m$, $\tilde{d}{\in}R^q$ and $y{\in}R^p$, cannot be treated efficiently if the classical LQR control scheme is applied. This is because of the existence of the perturbation term $\tilde{d}$. The disturbance term $\tilde{d}$ apart from modeling (parametric) uncertainty and external perturbation terms, can also represent noise terms of any distribution.

In the H_∞ control approach, a feedback control scheme is designed for trajectory tracking by the system's state vector and simultaneous disturbance rejection, considering that the disturbance affects the system in the worst possible manner. The disturbances' effects are incorporated in the following quadratic cost function:

$$J(t) = \frac{1}{2}\int_0^T [y^T(t)y(t) + ru^T(t)u(t) - \rho^2\tilde{d}^T(t)\tilde{d}(t)]dt, \quad r,\rho > 0 \tag{1.34}$$

The significance of the negative sign in the cost function's term that is associated with the perturbation variable $\tilde{d}(t)$ is that the disturbance tries to maximize the cost function $J(t)$ while the control signal $u(t)$ tries to minimize it. The physical meaning of the relation given above is that the control signal and the disturbances compete to each other within a min-max differential game . This problem of mini-max optimization can be written as

$$min_u max_{\tilde{d}} J(u, \tilde{d}) \tag{1.35}$$

The objective of the optimization procedure is to compute a control signal $u(t)$ which can compensate for the worst possible disturbance, that is externally imposed to the system. However, the solution to the min-max optimization problem is directly related to the value of parameter ρ. This means that there is an upper bound in the disturbances magnitude that can be annihilated by the control signal.

1.2.3 Computation of the feedback control gains

For the linearized system given by Eq. (1.33) the cost function of Eq. (1.34) is defined, where coefficient r determines the penalization of the control input and weight coefficient ρ determines the reward of the disturbances' effects.

It is assumed that (i) The energy that is transferred from the disturbances signal $\tilde{d}(t)$ is bounded, that is $\int_0^{\infty} \tilde{d}^T(t)\tilde{d}(t)dt < \infty$, (ii) matrices $[A, B]$ and $[A, L]$ are stabilizable , (iii) matrix $[A, C]$ is detectable . Then, the optimal feedback control law is given by

$$u(t) = -Kx(t) \tag{1.36}$$

with

$$K = \frac{1}{r} B^T P \tag{1.37}$$

where P is a positive definite symmetric matrix which is obtained from the solution of the Riccati equation

$$A^T P + PA + Q - P(\frac{2}{r} BB^T - \frac{1}{\rho^2} LL^T)P = 0 \tag{1.38}$$

where Q is also a positive semi-definite symmetric matrix. The worst case disturbance is given by

$$\tilde{d}(t) = \frac{1}{\rho^2} L^T P x(t) \tag{1.39}$$

The diagram of the considered control loop is depicted in Fig. 1.1.

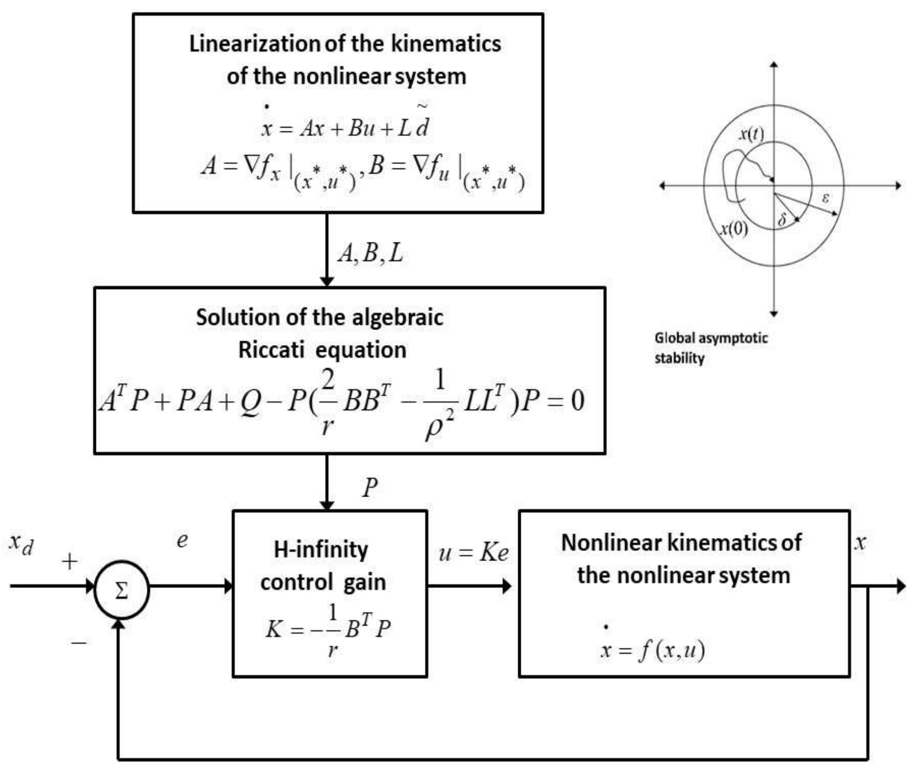

Figure 1.1: Diagram of the control scheme for the nonlinear system

1.2.4 Riccati equation coefficients in H-infinity control robustness

Parameter ρ in Eq. (1.34), is an indication of the closed-loop system robustness. If the values of $\rho > 0$ are excessively decreased with respect to r, then the solution of the Riccati equation is no longer a positive definite matrix. Consequently there is a lower bound ρ_{min} of ρ for which the H_∞ control problem has a solution. The acceptable values of ρ lie in the interval $[\rho_{min}, \infty)$. If ρ_{min} is found and used in the design of the H_∞ controller, then the closed-loop system will have elevated robustness. Unlike this, if a value $\rho > \rho_{min}$ is used, then an admissible stabilizing H_∞ controller will be derived but it will be a suboptimal one. The Hamiltonian matrix

$$H = \begin{pmatrix} A & -(\frac{1}{r}BB^T - \frac{1}{\rho^2}LL^T) \\ -Q & -A^T \end{pmatrix} \tag{1.40}$$

provides a criterion for the existence of a solution of the Riccati equation Eq. (1.38). A necessary condition for the solution of the algebraic Riccati equation to be a positive definite symmetric matrix, is that H has no imaginary eigenvalues [894], [689].

1.2.5 Lyapunov stability analysis

Through Lyapunov stability analysis it will be shown that the proposed nonlinear control scheme assures H_∞ tracking performance for the nonlinear system, and that in case of bounded disturbance terms asymptotic convergence to the reference setpoints is achieved.

The tracking error dynamics for the nonlinear system is written in the form

$$\dot{e} = Ae + Bu + L\tilde{d} \tag{1.41}$$

where in the case of a nonlinear system with a state vector of dimension equal to n, one can define $L = I \in R^n$ with I being the identity matrix. Variable $\tilde{d}$ denotes model uncertainties and external disturbances of the system's model. The following Lyapunov equation is considered

$$V = \frac{1}{2}e^T Pe \tag{1.42}$$

where $e = x - x_d$ is the tracking error. By differentiating with respect to time one obtains

$$\begin{array}{c} \dot{V} = \frac{1}{2}\dot{e}^T Pe + \frac{1}{2}e^T P\dot{e} \Rightarrow \\ \dot{V} = \frac{1}{2}[Ae + Bu + L\tilde{d}]^T Pe + \frac{1}{2}e^T P[Ae + Bu + L\tilde{d}] \Rightarrow \end{array} \tag{1.43}$$

$$\begin{array}{c} \dot{V} = \frac{1}{2}[e^T A^T + u^T B^T + \tilde{d}^T L^T]Pe + \\ +\frac{1}{2}e^T P[Ae + Bu + L\tilde{d}] \Rightarrow \end{array} \tag{1.44}$$

$$\begin{array}{c} \dot{V} = \frac{1}{2}e^T A^T Pe + \frac{1}{2}u^T B^T Pe + \frac{1}{2}\tilde{d}^T L^T Pe + \\ \frac{1}{2}e^T PAe + \frac{1}{2}e^T PBu + \frac{1}{2}e^T PL\tilde{d} \end{array} \tag{1.45}$$

The previous equation is rewritten as

$$\begin{array}{c} \dot{V} = \frac{1}{2}e^T(A^T P + PA)e + (\frac{1}{2}u^T B^T Pe + \frac{1}{2}e^T PBu) + \\ +(\frac{1}{2}\tilde{d}^T L^T Pe + \frac{1}{2}e^T PL\tilde{d}) \end{array} \tag{1.46}$$

Assumption: For given positive definite matrix Q and coefficients r and ρ there exists a positive definite matrix P, which is the solution of the following matrix equation

$$A^T P + PA = -Q + P(\frac{2}{r}BB^T - \frac{1}{\rho^2}LL^T)P \tag{1.47}$$

Moreover, the following feedback control law is applied to the system

$$u = -\frac{1}{r}B^T P e \tag{1.48}$$

By substituting Eq. (1.47) and Eq. (1.48) one obtains

$$\begin{array}{c} \dot{V} = \frac{1}{2}e^T[-Q + P(\frac{2}{r}BB^T - \frac{1}{\rho^2}LL^T)P]e+ \\ +e^T PB(-\frac{1}{r}B^T Pe) + e^T PL\tilde{d} \Rightarrow \end{array} \tag{1.49}$$

$$\begin{array}{c} \dot{V} = -\frac{1}{2}e^T Qe + \frac{1}{r}e^T PBB^T Pe - \frac{1}{2\rho^2}e^T PLL^T Pe \\ -\frac{1}{r}e^T PBB^T Pe + e^T PL\tilde{d} \end{array} \tag{1.50}$$

which after intermediate operations gives

$$\dot{V} = -\frac{1}{2}e^T Qe - \frac{1}{2\rho^2}e^T PLL^T Pe + e^T PL\tilde{d} \tag{1.51}$$

or, equivalently

$$\begin{array}{c} \dot{V} = -\frac{1}{2}e^T Qe - \frac{1}{2\rho^2}e^T PLL^T Pe+ \\ +\frac{1}{2}e^T PL\tilde{d} + \frac{1}{2}\tilde{d}^T L^T Pe \end{array} \tag{1.52}$$

Lemma: The following inequality holds

$$\frac{1}{2}e^T PL\tilde{d} + \frac{1}{2}\tilde{d}L^T Pe - \frac{1}{2\rho^2}e^T PLL^T Pe \leq \frac{1}{2}\rho^2\tilde{d}^T\tilde{d} \tag{1.53}$$

Proof: The binomial $(\rho\alpha - \frac{1}{\rho}b)^2$ is considered. Expanding the left part of the above inequality one gets

$$\begin{array}{c} \rho^2 a^2 + \frac{1}{\rho^2}b^2 - 2ab \geq 0 \Rightarrow \frac{1}{2}\rho^2 a^2 + \frac{1}{2\rho^2}b^2 - ab \geq 0 \Rightarrow \\ ab - \frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2 \Rightarrow \frac{1}{2}ab + \frac{1}{2}ab - \frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2 \end{array} \tag{1.54}$$

The following substitutions are carried out: $a = \tilde{d}$ and $b = e^T PL$ and the previous relation becomes

$$\frac{1}{2}\tilde{d}^T L^T Pe + \frac{1}{2}e^T PL\tilde{d} - \frac{1}{2\rho^2}e^T PLL^T Pe \leq \frac{1}{2}\rho^2\tilde{d}^T\tilde{d} \tag{1.55}$$

Eq. (1.55) is substituted in Eq. (1.52) and the inequality is enforced, thus giving

$$\dot{V} \leq -\frac{1}{2}e^T Qe + \frac{1}{2}\rho^2\tilde{d}^T\tilde{d} \tag{1.56}$$

Eq. (1.56) shows that the H_∞ tracking performance criterion is satisfied. The integration of $\dot{V}$ from 0 to T gives

$$\begin{array}{c} \int_0^T \dot{V}(t)dt \leq -\frac{1}{2}\int_0^T ||e||_Q^2 dt + \frac{1}{2}\rho^2\int_0^T ||\tilde{d}||^2 dt \Rightarrow \\ 2V(T) + \int_0^T ||e||_Q^2 dt \leq 2V(0) + \rho^2\int_0^T ||\tilde{d}||^2 dt \end{array} \tag{1.57}$$

Moreover, if there exists a positive constant $M_d > 0$ such that

$$\int_0^\infty ||\tilde{d}||^2 dt \leq M_d \tag{1.58}$$

then one gets

$$\int_0^\infty ||e||_Q^2 dt \leq 2V(0) + \rho^2 M_d \tag{1.59}$$

Thus, the integral $\int_0^\infty ||e||_Q^2 dt$ is bounded. Moreover, $V(T)$ is bounded and from the definition of the Lyapunov function V in Eq. (1.42) it becomes clear that $e(t)$ will be also bounded since $e(t) \in \Omega_e = \{e|e^T Pe \leq 2V(0) + \rho^2 M_d\}$. According to the above and with the use of Barbalat's Lemma one obtains $lim_{t\to\infty}e(t) = 0$.

Elaborating on the above, it can be noted that the proof of global asymptotic stability for the control loop of the nonlinear system is based on Eq. (1.56) and on the application of Barbalat's Lemma. It uses the condition of Eq.

(1.58) about the boundedness of the square of the aggregate disturbance and modelling error term $\tilde{d}$ that affects the model. However, as explained above the proof of global asymptotic stability is not restricted by this condition. By selecting the attenuation coefficient ρ to be sufficiently small and in particular to satisfy $\rho^2 < ||e||_Q^2/||\tilde{d}||^2$ one has that the first derivative of the Lyapunov function is upper bounded by 0. Therefore, for the i-th time interval, it is proven that the Lyapunov function defined in Eq (1.42) is a decreasing one. This also assures that the Lyapunov function of the system defined in Eq. (1.42) will always have a negative first-order derivative.

1.2.6 Robust state estimation with the use of the H_∞ Kalman Filter

The control loop has to be implemented with the use of information provided by a small number of sensors and by processing only a small number of state variables. To reconstruct the missing information about the state vector of the nonlinear system, it is proposed to use a filtering scheme and based on it to apply state estimation-based control [896], [382], [1013]. The recursion of the H_∞ Kalman Filter , for the model of the nonlinear system, can be formulated in terms of a *Measurement update* and a *Time update* part

Measurement update:

$$\begin{array}{l} D(k) = [I - \theta W(k)P^-(k) + C^T(k)R(k)^{-1}C(k)P^-(k)]^{-1} \\ K(k) = P^-(k)D(k)C^T(k)R(k)^{-1} \\ \hat{x}(k) = \hat{x}^-(k) + K(k)[y(k) - C\hat{x}^-(k)] \end{array} \tag{1.60}$$

Time update:

$$\begin{array}{l} \hat{x}^-(k+1) = A(k)x(k) + B(k)u(k) \\ P^-(k+1) = A(k)P^-(k)D(k)A^T(k) + Q(k) \end{array} \tag{1.61}$$

where it is assumed that parameter θ is sufficiently small to assure that the covariance matrix ${P^-(k)}^{-1} - \theta W(k) + C^T(k)R(k)^{-1}C(k)$ will be positive definite. When $\theta = 0$ the H_∞ Kalman Filter becomes equivalent to the standard Kalman Filter. One can measure only a part of the state vector of the nonlinear system and can estimate through filtering the rest of the state vector elements.

1.3 Optimal state estimation with the H-infinity Kalman Filter

1.3.1 Outline of filter

Throughout this monograph the H-infinity Kalman Filter is often used as a robust optimal state estimator [382],[1013]. This is a variant of the typical Kalman Filter. This filter is the result of an optimization procedure which maximizes the robustness of estimation against noise and against errors about the initial value of the system's state vector. The computational stages of the H-infinity Kalman Filter are given in Fig. 1.2 and Fig. 1.3. The functioning and the performance of the filter is assessed as follows:

(i) Due to the nonlinearity of the systems under control, state estimation should be normally performed with a nonlinear filter or state-observer. This problem cannot be treated with the use of the linear Kalman Filter or with the use of linear state observers. However, the monograph's approach allows the use of the H-infinity Kalman Filter, although this is a linear filter too. This is achieved by applying approximate linearization around a time-varying operating point (equilibrium). The linearization relies on Taylor series expansion and on the computation of the associated Jacobian matrices.

(ii) Usually the problem of state estimation for nonlinear systems is treated with nonlinear filtering techniques. However, the related Extended Kalman Filtering methods are not particularly robust against linearization modelling errors and against measurement noise. Furthermore, Sigma Point Kalman Filtering methods are not of proven convergence and stability. Additionally, Particle Filtering techniques demand high computation power and their slow convergence shows that they are not the recommended approach for real-time state estimation purposes [897],

[896]. Comparing to the aforementioned nonlinear filtering schemes, the H-infinity Kalman Filtering approach exhibits elevated robustness against noise and is of proven convergence and stability.

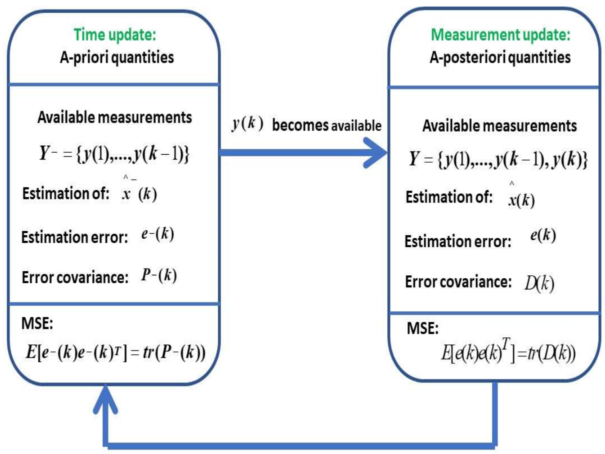

Figure 1.2: Diagram of the H-infinity Kalman Filter, comprising a Time update and a Measurement update stage

(iii) To elaborate on the matrices which appear in the *Measurement update* part and in the *Time update part* of the H-infinity Kalman Filter, the following can be noted: Matrix $R(k)$ is the measurement noise covariance matrix, that is the covariance matrix of the measurement error vector of the system. Matrix $P^-(k)$ is the a-priori state vector estimation error covariance matrix of the system, that is the covariance matrix of the state vector estimation error prior to receiving the updated measurement of the system's outputs. Matrix $W(k)$ is a weight matrix which defines the significance to be attributed by the H-infinity Kalman Filter to minimizing the state vector's estimation error, relatively to the effects that the noise affecting the system may have. Finally, matrix $D(k)$ stands for a modified a-posteriori state vector estimation error covariance matrix, that is the covariance matrix of the state vector estimation error after receiving the updated measurement of the system's outputs. Conclusively, the H-infinity Kalman Filter retains the structure of the typical Kalman Filter, that is a recursion in discrete time comprising a Time update part (computation of variables prior to receiving measurements) and a Measurement update part (computation of variables after measurements have been received). There is a modified a-posteriori state vector estimation error covariance matrix, which in turn takes into account a weight matrix that defines the accuracy of the state estimation under the effects of elevated noise.

(iv) The stability of the H-infinity Kalman Filter and the convergence of the state estimation procedure that is performed by it relies on the detectability / observability properties of the nonlinear sytems treated in the monograph and not on initialization of the matrices that appear in the filter's recursion. The monograph's concept on performing nonlinear state estimation with the use of the H-infinity Kalman filter is clear and widely applicable [897], [896. The nonlinear state-space model of the monitored system undergoes approximate linearization around a temporary operating point, which is recomputed at each time-step of the estimation algorithm. For the approximately linearized model, state estimation is performed with the use of the H-infinity Kalman Filter. The effects of measurement noise, as well as the modelling error which is due to truncation of higher-order terms in the Taylor series expansion is considered to be a perturbation which is asymptotically compensated by the robustness of the H-infinity Kalman Filter.