A Statistical Theory of Gravitating Body Formation in Extrasolar Systems

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Dedicated to my dear parents

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Part II: The Statistical Theory of Gravity in Solar and Extrasolar System Applications

INTRODUCTION

Despite the significant successes and achievements in astrophysics and geophysics over recent decades, the problems of the origin of the Solar system and the formation of planets remain important and relevant, if only because there is no general or consistent scenario for the formation of the proto-Sun and the protoplanetary system from a protosolar nebula (molecular cloud).

In particular, in astrophysics, there is the problem of the gravitational condensation of an *infinitely spread* gas-dust cosmic matter which is closely related to the problem of gravitational instability and the well-known Jeans criterion [1]. The main difficulty of the theory of Jeans is associated with the *gravitational paradox*, that is, for an infinite homogeneous substance, there exists no potential of the force of gravity [2]. In other words, due to the absence of the gravitational field inside a spread molecular cloud, gravitational tightening could not arise.

Recently, the general problems of the formation of protoplanetary systems, the study of their dynamical behavior, and the formation and evolution of the planets have begun attracting additional attention among the scientific community in connection with the discovery of *extrasolar planets*, considered one of the greatest achievements of modern astronomy.

Our understanding of our place in the Universe changed measurably in 1995 when Michel Mayor and Didier Queloz of Geneva Observatory in Switzerland announced the discovery of an extrasolar planet around a star, 51 Pegasi, similar to our Sun [3]. Geoff Marcy and Paul Butler in the United States soon confirmed their discovery, and the science of observational extrasolar planetology was born. The field has exploded in recent years, resulting in publications showing numerous planetary systems in 2019 (see http://exoplanet.eu/ and http://exoplanets.org/). Most of these systems contain one or more gas-giant planet close, or very close, to their parent star. In that, they do not resemble our Solar system. In this connection, the recent paper [4] tallies:

> The discovery of the gas-giant planet – named 51 Pegasi b after its parent star, 51 Pegasi – came as a surprise. Gas-giant planets, such as Jupiter, are located in the outer parts of the Solar System. The prevailing theory was, and still is, that the formation of these planets requires icy building blocks that are available only in cold regions far away from stars. Yet Mayor and Queloz found 51 Pegasi b to be orbiting about ten times closer to its host star than Mercury is to the Sun... One possible explanation is that the planet formed farther out and then migrated to its current location.

Nevertheless, earlier detection of planets with masses approximately equal to the mass of our Earth M_{Earth} is evidence that there exist extrasolar planets with low masses. In addition to obtaining important knowledge about the formation and structure of new planetary systems, that is, *exoplanetary systems*, these discoveries provoke genuine interest among the scientific community regarding the prospects for finding life in the Universe.

However, the questions considered in this monograph deal mainly with the problems of cosmogony and only partially touch upon cosmology. *Cosmological* bodies include large-scale space objects (for example, galaxies and their clusters) based on the fact that cosmology is a science that studies the properties and evolution of the Universe as a whole. In this context, *cosmogonical* bodies unite stars, protostars, interstellar molecular clouds, planetary systems, protoplanetary gas-dust disks, planets, protoplanets, and natural satellites of planets.

Generally speaking, the cosmogony, according to O. Yu. Schmidt [5], includes both *planetary cosmogony* and *stellar cosmogony*, that is, such directions have been developing within the framework of various cosmogonical theories.

Several cosmogonical theories are known to explain the formation of the Solar system, the formation of planets and the estimation of planetary orbits [1–16]:

- *electromagnetic theories* based on the works of O. K. Birkeland [17], H. P. Berlage [18], H. Alfvén [9, 19, 20], and others;
- *gravitational theories* based on the works of O. Yu. Schmidt [5, 6, 21], L. E. Gurevich and A. I. Lebedinsky [22, 23], M. M. Woolfson [14, 24], V. S. Safronov [2], S. H. Dole [25], A. V. Vityazev [12], and others;
- *nebular theories* based on the works of C. F. von Weizsäcker [26, 27], G. P. Kuiper [28, 29], F. Hoyle [30, 31], D. Ter Haar [7, 32], T. Nakano [33], A. G. W. Cameron [7, 10] and others;
- *quantum mechanical theories* based on the works of E. Nelson [34, 35], L. Nottale [36, 37], G. Ord [37, 38], M. De Oliveira Neto [15, 39], A. G. Agnese and R. Festa [40], M. S. El Naschie [41, 42], E. G. Sidharth [43] and others.

The state and achievements of cosmogonical theories are described by Stephen G. Brush in his review “Theories of the origin of the Solar system 1956–1985” [11]:

> Attempts to find a plausible naturalistic explanation of the origin of the Solar system began about 350 years ago but have not yet been *quantitatively*[1] successful. The period 1956–1985 includes the first phase of intensive space research; new results from lunar and planetary exploration might be expected to have played a major

[1] Emphasis added.

> role in the development of ideas about lunar and planetary formation. While this is indeed the case for theories of the origin of the moon (selenogony), it was not true for the Solar system in general, where ground-based observations (including meteorite studies) were frequently more decisive. During this period most theorists accepted a monistic scenario: the collapse of a gas-dust cloud to form the sun with a surrounding disk, and condensation of that disk to form planets, were seen as part of a single process. Theorists differed on how to explain the distribution of angular momentum between sun and planets, on whether planets formed directly by condensation of gaseous protoplanets or by accretion of solid planetesimals, on whether the Solar nebula "was ever hot and turbulent enough to vaporize and completely mix its components, and on whether an external cause such as a supernova explosion triggered" the initial collapse of the cloud. Only in selenogony was a tentative consensus reached on a single working hypothesis with quantitative results.

Despite a large amount of research and a huge number of works aimed at studying the formation of the Solar system, because of a lack of *quantitative results* these theories cannot fully explain all the phenomena observed – in particular, the four groups of facts following Ter Haar [6, 44 p. 277]. (The last concerns the distribution of angular momentum: although the Sun has more than 99% of total mass of the Solar system, only 2% of the total angular momentum belongs to Sun, while the remaining 98% belongs to the planets (see also Introduction in Chapter 6).)

In this context, beginning in 1996 the *statistical theory* for a cosmogonical body formation has been developed [45–70] by the author of this monograph, based on the so-called model of a spheroidal body forming through numerous gravitational interactions of its parts and particles (see also recent articles, book, and chapters [16, 71–79]).

The notion of a *spheroidal body* means a sphere-like body whose iso-surfaces (the surfaces of equal mass density) are spheroids (in the case of rotation of this body) or spheres (in the absence of visible motion). The area of investigations within the framework of the statistical theory mainly relates to Newtonian gravity and partly affects Newtonian quantum gravity (this area is highlighted by the arc in Figure I.1 which was proposed on the website of the Bremen University in 2003 (http://www.zarm.uni-bremen.de/2forschung/gravi/gravity_main.htm). In this sense, the present book continues the analytical tradition of Cambridge Scholars, starting with the works of I. Newton, G. Stokes, J. Maxwell, Lord Rayleigh (J. Strutt), J. Jeans, A. Eddington, R. Lyttleton, R. Fowler, and other scientists.

This book has two parts. Part I (Chapters 1–5) seeks to acquaint the readers with the developing statistical theory of gravitating cosmogonical body formation. Within the framework of this theory, the models, as well as the evolutionary equations of the statistical mechanics, are proposed. The well-known problem of gravitational condensation of infinite distributed cosmic substance (in particular, the Jeans gravitational instability) is solved by the proposed statistical model of spheroidal bodies. For the first time, the statistical model of slow-flowing gravitational condensation based on the anti-diffusion process allows a solution to the gravitational paradox for an infinite homogeneous spatial spread substance. With the use of this statistical model, a new nonlinear time-dependent Schrödinger-like undulatory equation describing the processes of cosmogonical body formation is derived.

In detail, in Chapter 1, the problem of the gravitational condensation of the spread cosmic matter is considered for the formation of protoplanets in the gravitational field. In particular, Sections 1.1–1.6 describe the main problems of the theory of gravitational condensation and the theory of gravitational instability applied to the molecular (gas-dust) cloud. In Section 1.7, the general evolutionary equation for the distribution function is obtained [16, 65, 73] which generalizes the well-known Jeans equation characterizing the dynamical behavior of the gas-dust cloud.

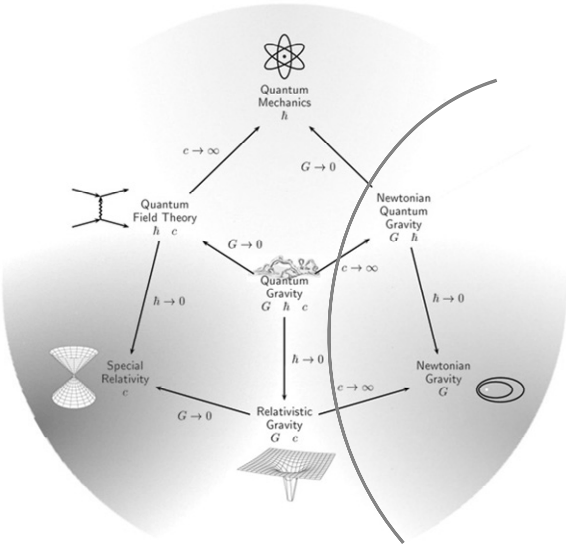

Figure I.1.

As shown in Chapter 2, the particle distribution functions obtained in Section 2.1, as well as the mass density of a *sphere-like gaseous body* (an immovable spheroidal body) [45–61], characterize *the first stage of evolution*: from a molecular cloud (nebula) to a forming core (proto-Sun) together with the outer shell (protosolar nebula). In Sections 2.4 and 2.5 the gravitational potential and the potential energy of a gravitating sphere-like gaseous body, are calculated. The probabilistic interpretation of physical values describing the gravitational interaction of particles in a sphere-like gaseous body is considered in Section 2.6. Under the condition of mechanical equilibrium, the pressure inside a sphere-like gaseous body is calculated in Section 2.8, as well as its internal energy in Section 2.9.

Chapter 3 is devoted to the study of statistical models of a *rotating* and gravitating *spheroidal body* to describe the evolution of a protoplanetary gaseous (gas-dust) cloud around a forming star. In particular, Section 3.1 considers the statistical interpretation of Poincaré's well-known general theorem and the Roche model for a *slowly* rotating and gravitating spheroidal body, that is, for a sphere-like gaseous body. In Section 3.3, the *equilibrium distribution function* of liquid particles relative to the spatial coordinates is derived, and the mass density function is also obtained for a uniformly rotating and gravitating spheroidal body with a small angular velocity. In Section 3.4 the distribution function of the specific angular momentum and the angular momentum density for a uniformly rotating spheroidal body are derived. The average value of specific angular momentum and the total angular momentum of a rotating spheroidal body being in relative mechanical equilibrium are calculated [16, 73]. The determination of the gravitational potential in the case of a uniformly rotating spheroidal body is discussed in Section 3.6. Section 3.7 estimates the potential energy of a uniformly rotating and gravitating spheroidal body. Moreover, as shown here, the disk-shaped spheroidal body does not possess its own gravitational energy. As noted in Section 3.8, the derived mass density function characterizes the flattening process: from initial spherical shapes (for an immovable spheroidal body) through flattened ellipsoidal shapes (for a rotating spheroidal body) to spheroidal disks. So, in Chapter 3 *the second stage of evolution* is described: from the protosolar nebula to the forming protoplanetary gas-dust disk.

Chapter 4 considers equations of a forming spheroidal body (both the centrally symmetric and the axially symmetric spheroidal body) in the process of its initial gravitational condensation. In Section 4.1, the basic *anti-diffusion equation* of the initial gravitational condensation of a non-rotating (or slowly rotating) spheroidal body from an infinitely spread matter is derived, and in Section 4.2, the general differential equations for physical values describing the anti-diffusion process of initial gravitational condensation of a spheroidal body in the vicinity of mechanical equilibrium are obtained. In other words, these equations show that the gravitational field tightening of the molecular cloud (nebula) is preceded by its initial anti-diffusion condensation [16, 65, 68]. Namely, two particular cases of the basic equation of slow-flowing initial gravitational condensation are considered in Sections 4.3 and 4.5. In Section 4.6, the possible dynamical states of the forming of a centrally symmetric spheroidal body from an infinitely spread gas-dust matter are systematized. The *general anti-diffusion equation* for a slowly evolving process of initial gravitational condensation of an axially symmetric spheroidal body (which is formed as a result of its rotation) is derived in Section 4.7.

Chapter 5 is devoted to the derivation of a new time-dependent nonlinear Schrödinger-like equation of a cosmogonical body formation [68, 71, 73, 77, 78] and the development of a scenario for the gravitational field origin based on an avalanche anti-diffusion mass transfer in a forming spheroidal body (see Sections 5.1–5.4), when in addition to the anti-diffusion velocity of particles, the usual (hydrodynamic) velocity arises. Nevertheless, the main result of the research in this chapter is the derivation of the generalized nonlinear Schrödinger-like equation in Section 5.6, describing not only the state of virial mechanical equilibrium and quasi-equilibrium gravitational compression state close to the mechanical equilibrium (with a slowly varying anti-diffusion coefficient) but also gravitational instability states leading to the formation of a cosmogonical body (see Section 5.7). In particular cases, the nonlinear time-dependent generalized Schrödinger-like equation becomes the well-known time-dependent Schrödinger equation or the generalized Schrödinger equation in Nottale's form (see Section 5.5). The *cubic* time-dependent Schrödinger-like equation describing cosmogonical body formation in the state of soliton disturbances is derived in Section 5.7. Since this equation has a soliton solution, the cubic Schrödinger-like equation can describe an evolution of the envelope of a wave packet of Jeans' substantial waves that propagate in a nonlinear and dispersive medium of a forming cosmogonical body (following the gravitational instability theory of Jeans [1]).

Part II of this monograph (Chapters 6–9) explores theoretical and practical approaches to investigating our Solar system and other exoplanetary systems. In particular, a new universal stellar law (USL) for extrasolar planetary systems connecting the temperature, size, and mass of each star is justified. Within the framework of the developed statistical theory, a new law (generalizing the famous law of O. Yu. Schmidt, for example) for the distribution of the planetary distances in our Solar system is proposed.

In detail, *the third stage of evolution* is considered in Chapter 6: from a protoplanetary flattened gas-dust disk to originating protoplanets [16, 65, 73]. The proposed statistical theory is applied primarily to develop *two models* of protoplanet formation (see Sections 6.1 and 6.2) and explaining the distribution law of planetary distances in the Solar system (see subsection 6.1.3), although the results presented in subsection 6.2.2 are also suitable for the construction of models of formation of exoplanetary systems. In more detail, the obtained distribution function of a specific angular momentum for a rotating uniformly spheroidal body (as a gas-dust flattened protoplanetary cloud) is used in

Section 6.1. Since the specific angular momentums (for particles or planetesimals) are averaged during a conglomeration process (under a planetary embryo formation) the specific angular momentum for a protoplanet of the Solar system is found in subsection 6.1.1. As a result, a new law for planetary distances (which generalizes Schmidt's law) is derived theoretically in subsection 6.1.2. Moreover, unlike the well-known planetary distance laws, the proposed law is established by a physical dependence of planetary distances from the value of the specific angular momentum. Within the framework of the second model, Section 6.2 develops an alternative heat emission model of protoplanet formation. As shown in subsection 6.2.1, in the state of relative mechanical equilibrium of particles moving in elliptical orbits in the gravitational field, an equation for the heat distribution function of the specific angular momentum is derived. Within the framework of this model, only 0.8% of the total number of particles of the Solar system composing the protoplanetary cloud has the angular momentum 15.6 times higher than the angular momentum of the remaining 99% of particles in the Solar system. This conclusion is in full agreement with Ter Haar's above-mentioned four facts of a nonuniform distribution of the angular momentum in the Solar system [7, 32].

Chapter 7 investigates the orbits of moving planets and bodies in the centrally symmetric gravitational field of a gravitating and rotating spheroidal body simulating the protostar with the flattened gas-dust disk during the *planetary stage* of its evolution. Though orbits of moving bodies and particles into a flattened rotating spheroidal body are circular initially, they could however be distorted by collisions with planetesimals and gravitational interactions with neighboring originating protoplanets during the evolutionary process of protoplanetary formation. This chapter shows that the orbits of moving particles are formed by the action of the centrally symmetric gravitational field mainly in the final stage of decay of a gravitating and rotating spheroidal body when the particle orbits become Keplerian. In Section 7.1, the estimation of the gravitational potential in the remote zone based on the general solution of the Poisson equation and the general expression for the gravitational potential of an axially symmetric spheroidal body is obtained. Section 7.2 investigates the orbits of moving planets and bodies in the centrally symmetric gravitational field of a gravitating and rotating spheroidal body during the planetary stage of its evolution. In Section 7.3, calculation of the orbit of the planet Mercury as well as estimation of angular displacement of Mercury's perihelion based on the statistical theory of gravitating spheroidal bodies is carried out. As a result, this section shows that according to the proposed statistical theory of gravitating spheroidal bodies the turn of perihelion of Mercury's orbit is equal to 43.93" per century, which is consistent with the conclusions of Einstein's general theory of relativity (his analogous estimation is equal to 43.03") and astronomical observation data ($43.11'' \pm 0.45''$).

In Chapter 8, the statistical theory of gravitating spheroidal bodies is applied to derive and develop a USL for the investigation of extrasolar systems [75, 76]. A preliminary estimate of an average gravitational potential energy of interaction of a particle with the gravitational field of a spheroidal body is given in Section 8.1. Section 8.2 then considers the derivation of the equation of the state of an ideal stellar substance taking into account an extended substance called the *stellar corona*. In other words, the stellar corona together with the star is described through the model of a rotating and gravitating spheroidal body in Section 8.2. Using the virial theorem as well as the theorem of uniform distribution of energy on freedom degrees for each particle inside a rotating and gravitating spheroidal body, the USL for a star including its stellar corona is justified. In the case of the Sun, the verification of USL shows its validity with the relative error equal to 3.37% (see Section 8.3). Section 8.3 then considers the modification of the USL, taking into account the ratio of the temperature of the Solar corona to an effective temperature of the Sun's surface. The verification of the modified USL for other stars belonging to the different spectral classes and types is carried out in Section 8.3. Theoretical estimations of temperatures of stellar coronas for stars belonging to the different spectral classes as well as orbital and thermodynamical characteristics of multi-planet extrasolar systems are investigated in Sections 8.4 and 8.5. So, the knowledge of some characteristics for multi-planet extrasolar systems permits us to refine a star's own parameters. In this context, comparison with estimations of temperatures using the regression dependences for multi-planet extrasolar systems attests the results obtained. In Section 8.6, the known Hertzsprung–Russell dependence is derived from USL directly.

In the final chapter (9) of this monograph, the stability of planetary orbits based on the statistical theory of gravitating spheroidal bodies is investigated [16, 45–79]. Using the obtained USL and its modification connecting temperature, size, and mass of a star the combination of Kepler's 3rd law with the universal stellar law (3KL-USL) is derived in Section 9.1 [79]. As shown in Section 9.1, the combined 3KL-USL law connects among themselves both the mechanical values (the Keplerian angular velocity Ω_K and the major semi-axis a of a planetary orbit) and the statistical (thermodynamic) values (the parameter of gravitational condensation α and the temperature T). The proposed 3KL–USL thus explains the stability of planetary orbits in extrasolar systems. In this context, Section 9.2 investigates the additional periodic force causing the radial and axial *orbital oscillations* (which modify initial circular orbits of bodies) based on the approach of Alfvén and Arrhenius [9, 19, 20]. A prediction of the Alfvén–Arrhenius specific additional periodic force within the framework of the Newtonian theory of gravity is considered in Section 9.3. As shown in Section 9.4, from the point of view of the theory of retarded potentials the *wave gravitational potential* and the Alfvén–Arrhenius specific additional periodic force arise in the remote zone II of the gravitational field under the orbital motion of a body around the central gravitating body. The obtained spectral representations correspond entirely to the analogous spectral expansion derived in the statistical theory of gravitating spheroidal bodies (see Chapter 5). Thus, the proposed statistical *theory of the formation* of planetary systems pointing to the regular and wave gravitational potentials origin is confirmed by the *theories of existence* (Newtonian and retarded potentials). Section 9.5 finds that additional periodic force is similar to Hooke's force which affects free oscillations of a body in orbit. Due to dissipation, these oscillations are damped gradually, so that they need support through the periodic impact of the additional periodic force by analogy with the principle of an anchoring mechanism in a clock. Section 9.6 justifies that the spatial deviation of the gravitational potential of an ellipsoid-like rotating cosmogonical body from the centrally symmetric field $1/r$ – gravitational potential (of a sphere-like body) implies different values of the radial and the axial orbital oscillations.

The investigations presented in this monograph in the field of theoretical statements on the processes of self-organization in a spread gas-dust cosmic media and the development of statistical models for the formation of planetary systems and the origin of planets (including planets in our Solar system and other extrasolar systems) have been widely discussed and reported at several international conferences, in particular, under the aegis of the General Assembly of the European Geosciences Union (EGU) and the European Planetary Science Congress (EPSC). Indeed, the author of this monograph was the organizer (as Convener, co-Convener, Chairman) of sessions: PS15 "Models of Solar system forming" (April 2–7, 2006, Vienna, Austria), PS7.1 "Extrasolar planets and planet formation" (April 16–20, 2007, Vienna, Austria), PS9 "Extrasolar planets and planet formation" (April 13–18, 2008, Vienna, Austria) and PS8 "Extrasolar planets and planet formation, exoplanetary magnetospheres and radio emissions" (April 19–24 2009, Vienna, Austria) of the EGU General Assembly; ON1 "Planetary formation and the origin of the Solar system" (September 18–22, 2006, Berlin, Germany) and OG1 "Origin and evolution" (September 14–18, 2009, Potsdam, Germany) of EPSC; a member of the organizing committee and chairman of international scientific conferences: 2nd International Conference and Exhibition "Satellite & Space Missions" (July 21–23, 2016, Berlin, Germany), the 3rd International Conference and Exhibition "Satellite & Space Missions" (May 11–13, 2017, Barcelona, Spain), the 4th International Conference and Exhibition "Satellite & Space Missions" (June 18–20, 2018, Rome, Italy) and the 12th International Conference "Chaotic Modeling and Simulation (CHAOS-2019)" (June 18–22, 2019, Chania, Crete, Greece).

The topic of this research corresponds to the priority direction of fundamental and applied scientific research of the Republic of Belarus on the mathematical and physical modeling of systems, structures, and processes in nature. This work has been carried out by the author in the Laboratory of Self-Organization System Modeling at the United Institute of Informatics Problems of the National Academy of Sciences of Belarus within the framework of the projects: "Methods of

Mathematical Modeling of Self-organization Processes in Active Media" (2006–2010), "Scientific Foundations and Tools of Space Technologies" (2011–2013), "Investigation of Gas-Dust and Plasma Media and their Impact on the Motion and State of Spacecraft" (2013–2015). This work has also been supported partially by the grant of the President of the Republic of Belarus in science (2019).

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